Algorithmik
Fachbereich IV – Informatik
Universität Trier

Trier, June 26, 2023

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Exercise sheet 5
Visualization of Graphs

Exercise 1 – Higher degree vertices in orthogonal layouts

Let $G = (V, E)$ be an arbitrary graph with faces F and outer face f_0. Our goal is to draw G orthogonally, while preserving the embedding, such that all vertices of degree greater than 4 are represented by rectangles instead of points. To achieve this, we replace every vertex v having $\deg(v) > 4$ by a ring of vertices $v_1, \ldots, v_{\deg(v)}$, such that the edges incident to v are distributed among the vertices $v_1, \ldots, v_{\deg v}$ (see figure below). The embedding $\mathcal{E}$ is modified accordingly during this step. Let G' with faces F' and outer face f'_0 be the result of this replacement step.

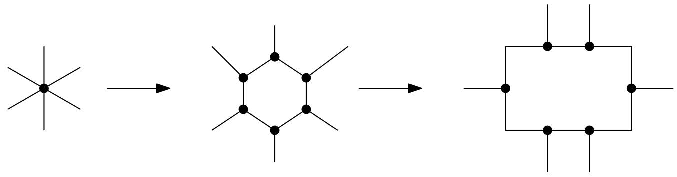

Figure 1: Replacement of a vertex of higher degree by a ring, which shall be represented by a rectangle in the orthogonal drawing.

Modify the flow network by Tamassia such that it provides a bend-minimal orthogonal description of (G', F', f'_0) in which every ring representing a vertex of higher degree is a rectangle such that no vertices are placed in any of its four corners and such that every side of the rectangle contains at least one vertex. **7 Points**

Hint: Consider the set V' of the new vertices and the set E' of the new edges of a ring that are added to the graph after the modification. Think of the additional constraints to the flow model to enforce the structure of the ring and its vertices.

Exercise 2 – From flow to orthogonal representation

Let $G = (V, E)$ be a plane graph with faces F and outer face f_0. Let X be a flow of cost k in the corresponding flow network $N(G)$. We consider the orthogonal description $H(G)$ belonging to X (as in the lecture).

Show that $H(G)$ fulfills property (H3) on the angle sum of the faces for orthogonal descriptions, that is, argue that

$$\sum_{r \in H(f)} C(r) = \begin{cases} -4 & \text{if } f = f_0 \\ +4 & \text{otherwise.} \end{cases}$$

holds for every face f.

4 Points

Exercise 3 – Edge bending left and right in orthogonal representation

Let $G = (V, E)$ be a plane graph with faces F and outer face f_0 and let $H(G)$ be a bend-minimal orthogonal representation of (G, F, f_0). Is it possible that there exists an edge such that, in $H(G)$, it bends to the right as well as to the left?

Prove this claim (by giving an example) or disprove it (by showing that such an edge cannot exist). **3 Points**

Exercise 4 – Algorithm for a refinement step

The flow networks for compaction in the lecture only work for orthogonal representations where every face is rectangular. In the lecture, we learned how to refine the faces such that every face becomes rectangular.

Formulate the algorithm to do the refinement of an inner face. Write a method (in pseudocode) that takes as input the face representation of a face f and returns an orthogonal representation of the refinement of the face. **6 Points**

This assignment is due at the beginning of the next lecture, that is, on May 31 at 16:00. Please hand in your solutions online via Moodle. The exercises on this assignment will be discussed in the tutorial session on June 03 at 09:00.