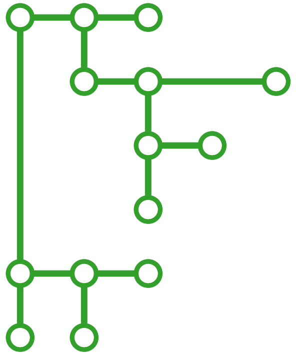
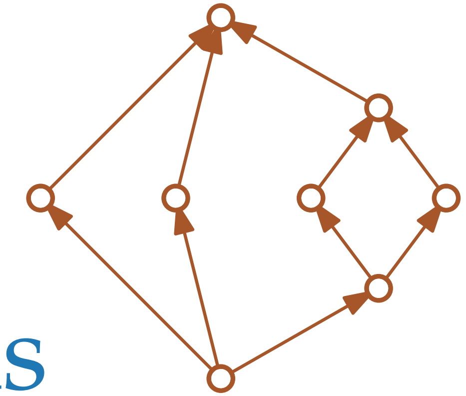
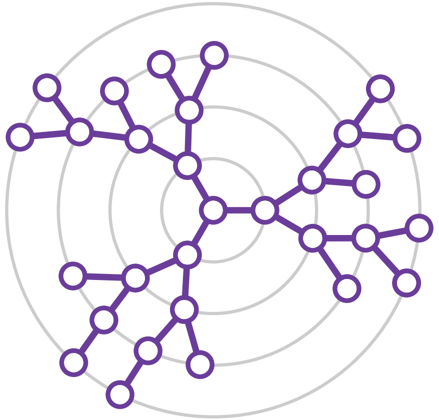
Universität Trier



Visualization of Graphs

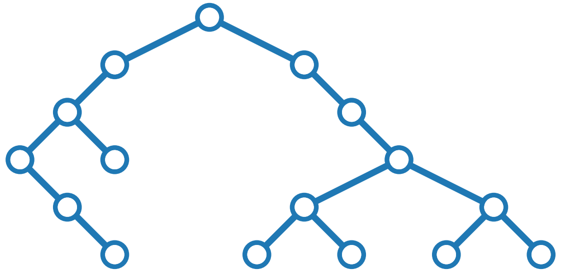


Lecture 2:
Drawing Trees and Series-Parallel Graphs



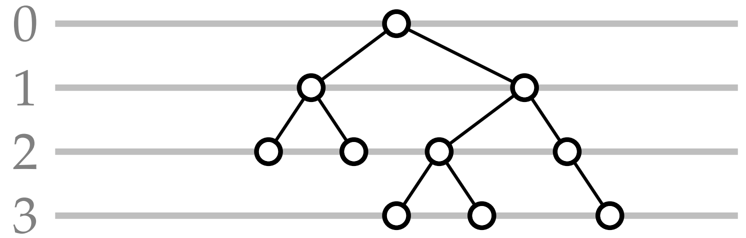
Part I:
Layered Drawings

Philipp Kindermann



First Grid Layout of Binary Trees

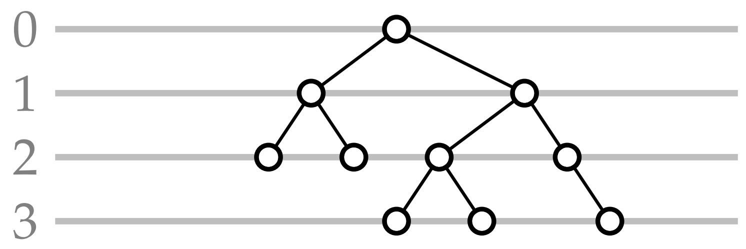
1. Choose y-coordinates: $y(u) = \text{depth}(u)$



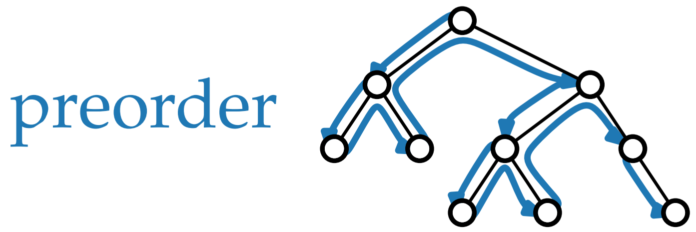
2. Choose x-coordinates:

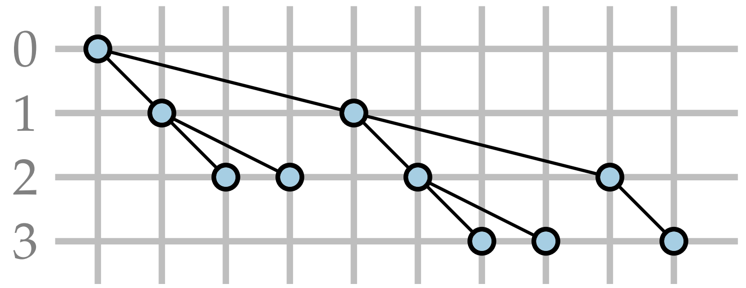
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) = \text{depth}(u)$



2. Choose x-coordinates:





First Grid Layout of Binary Trees

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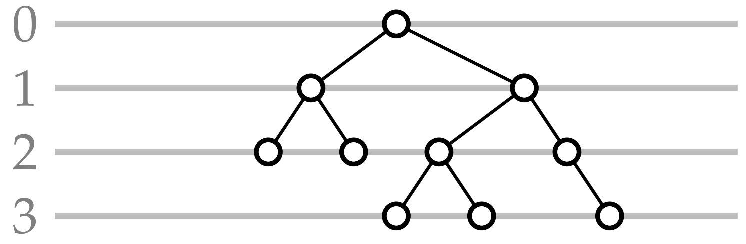
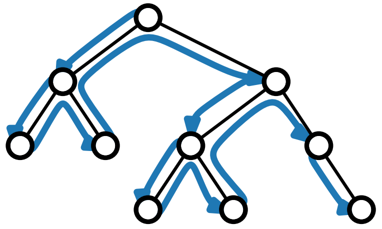
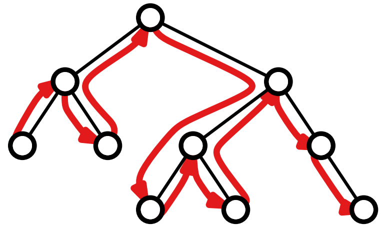


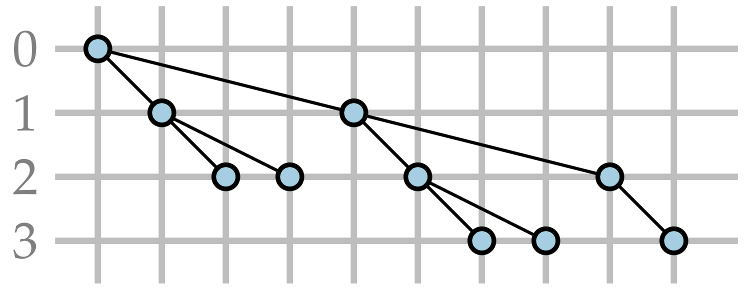
2. Choose x-coordinates:

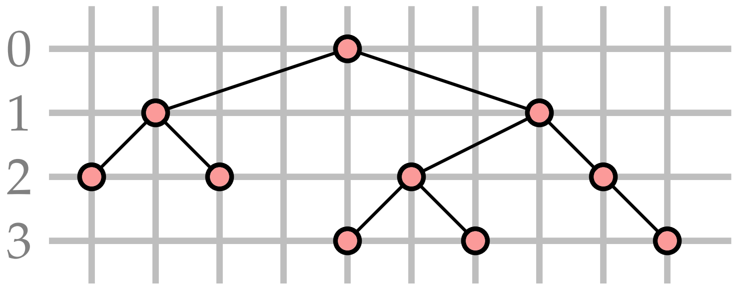
preorder



inorder







First Grid Layout of Binary Trees

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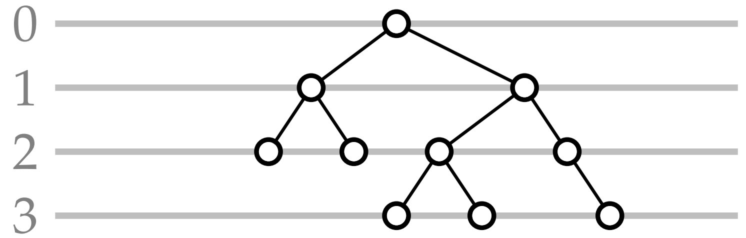

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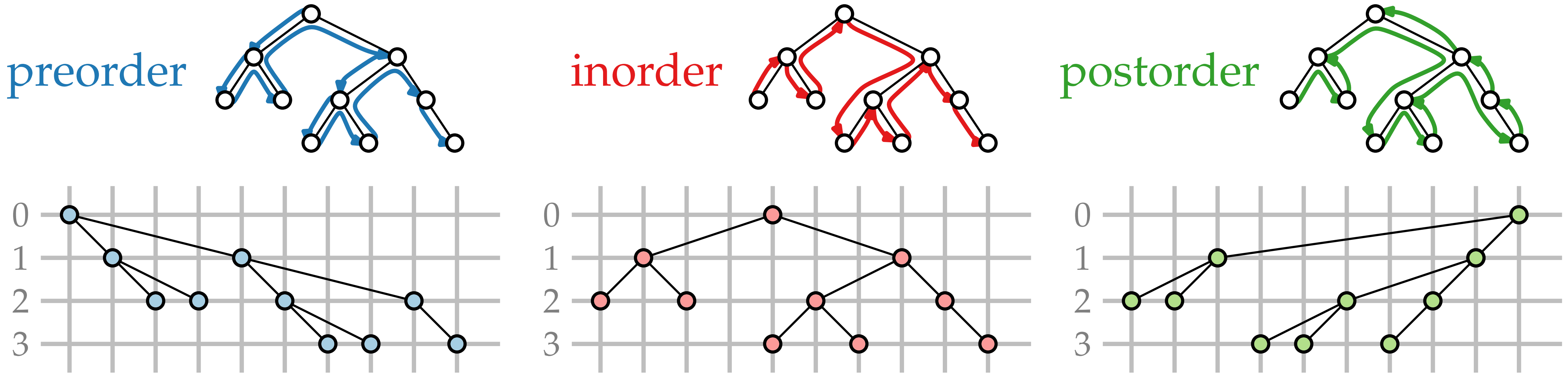
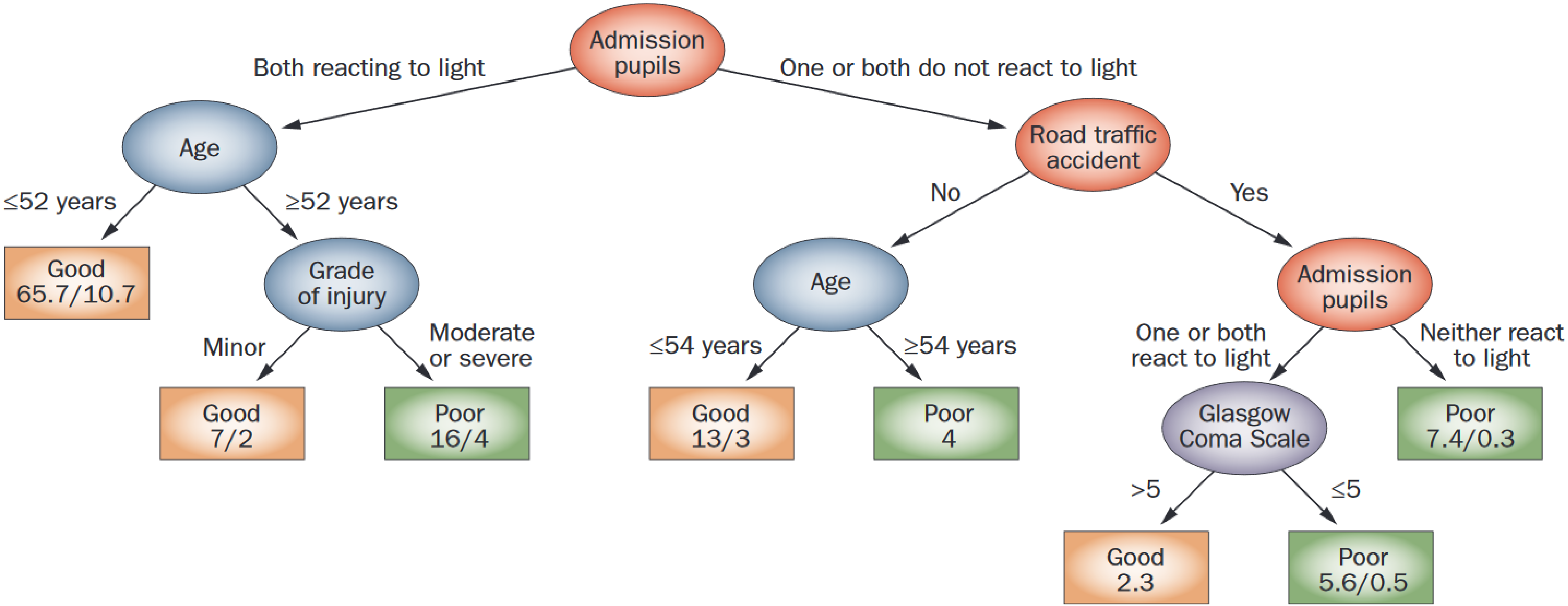


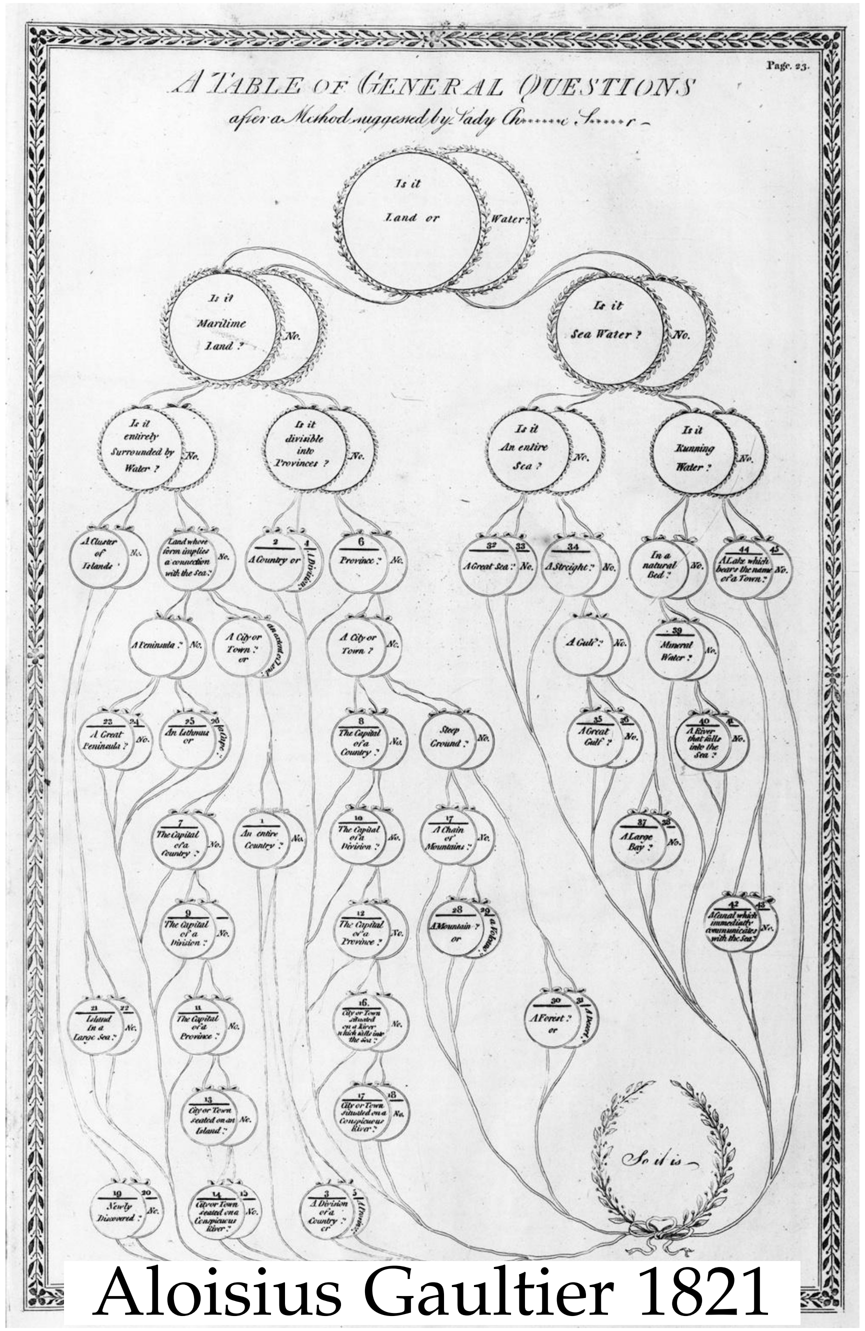
Layered Drawings – Applications



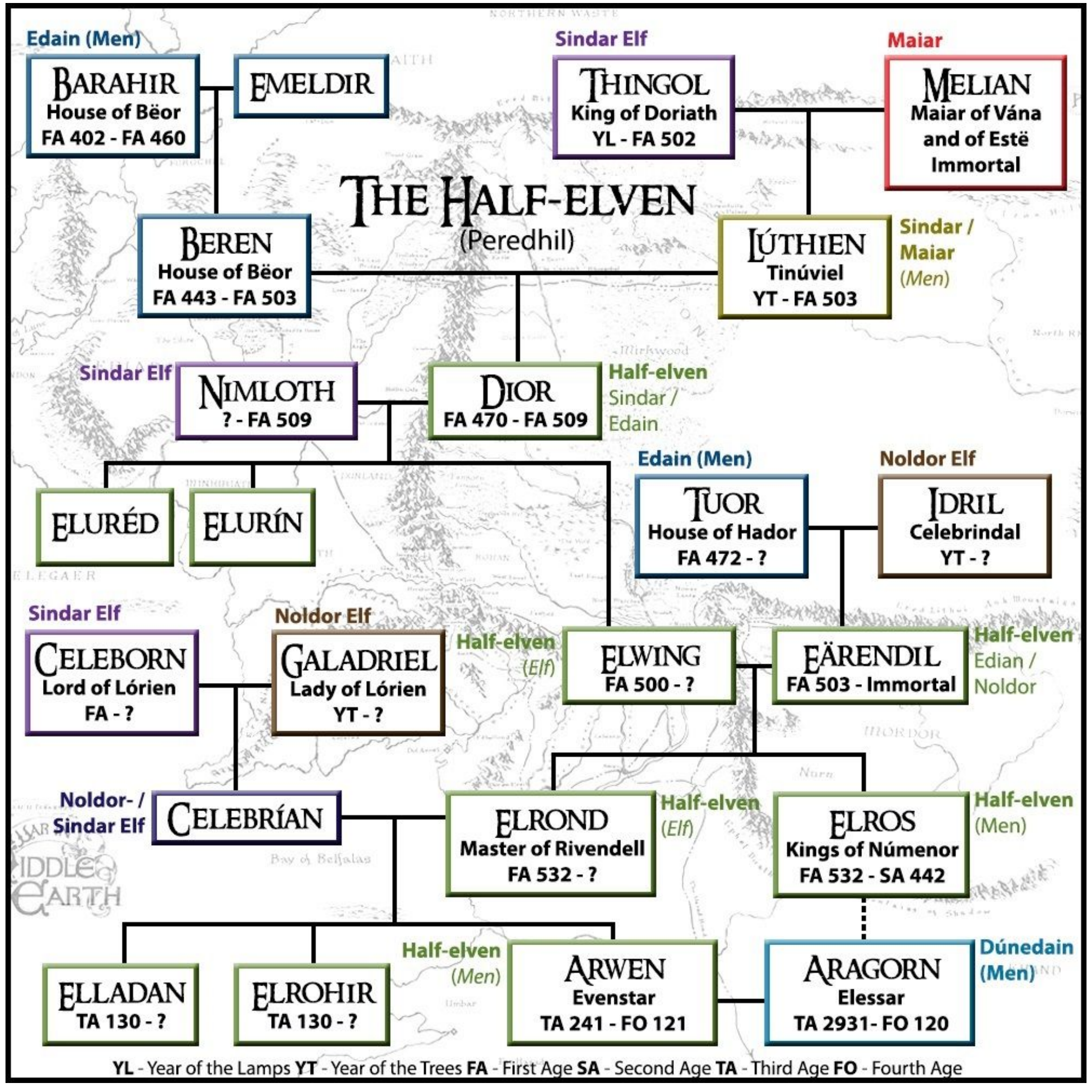
Decision tree for outcome prediction after traumatic brain injury

Source: Nature Reviews Neurology

Layered Drawings – Applications

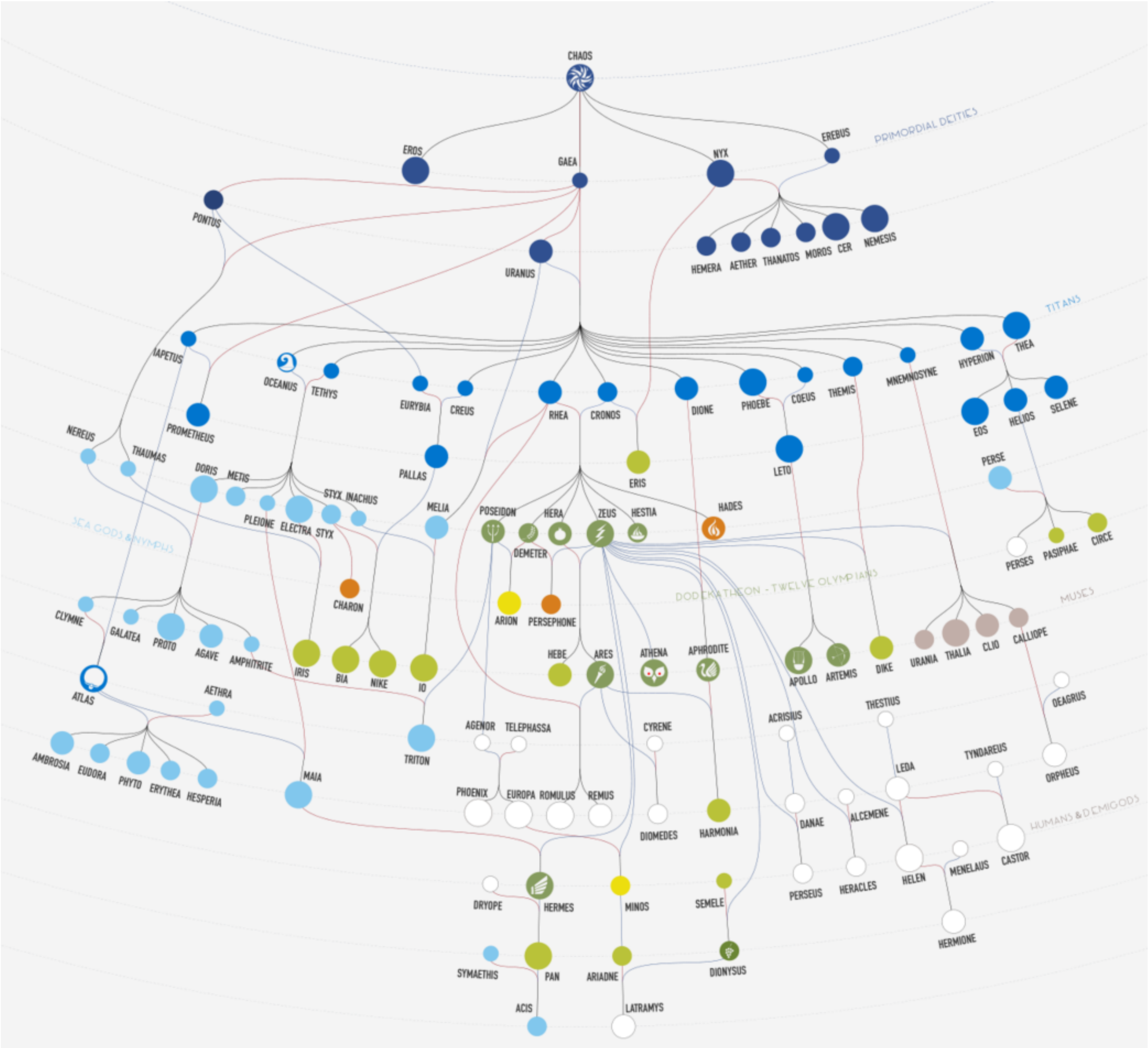


Aloisius Gaultier 1821



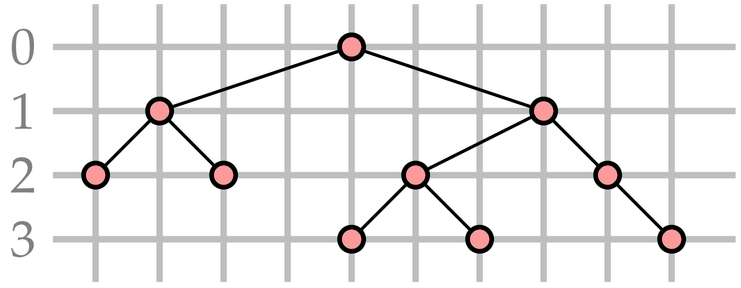
Family tree of LOTR elves and half-elves

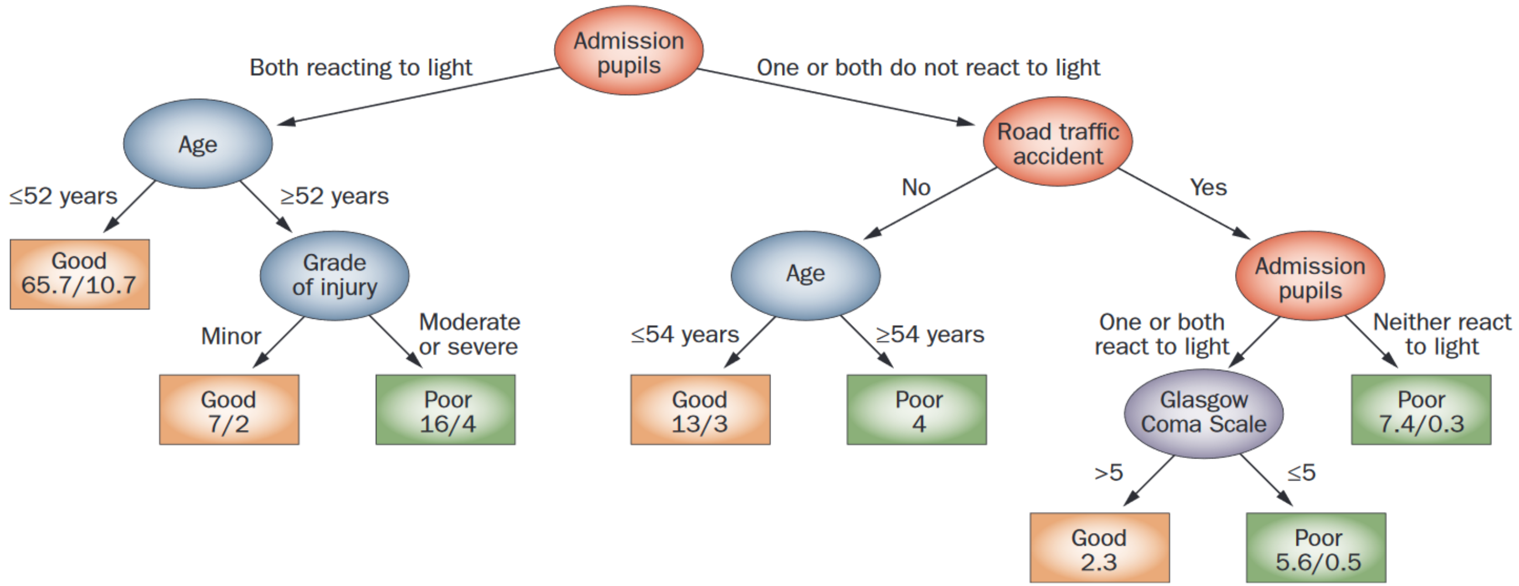
Layered Drawings – Applications



J. Klawitter, T. Mchedlidze, *Link:* go.uniwue.de/myth-poster

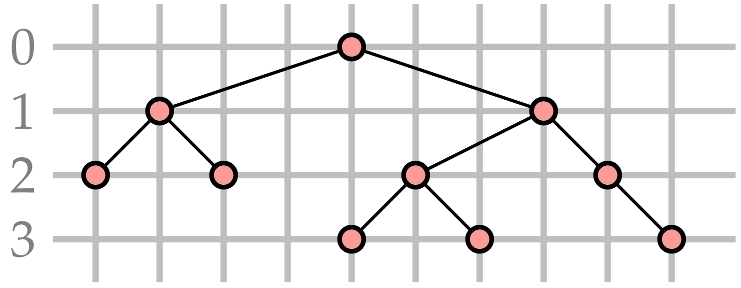
Layered Drawings – Drawing Style

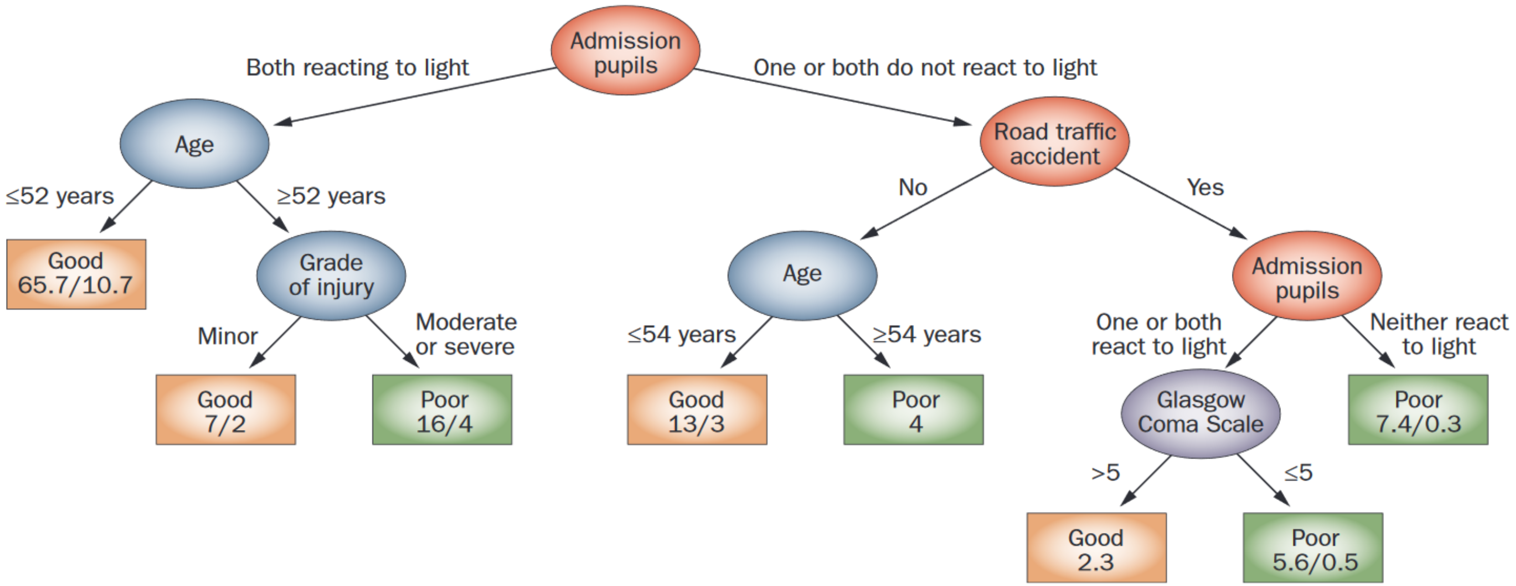




- What are properties of the layout?

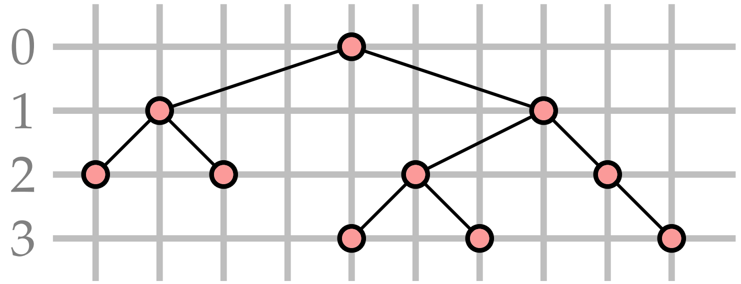
Layered Drawings – Drawing Style

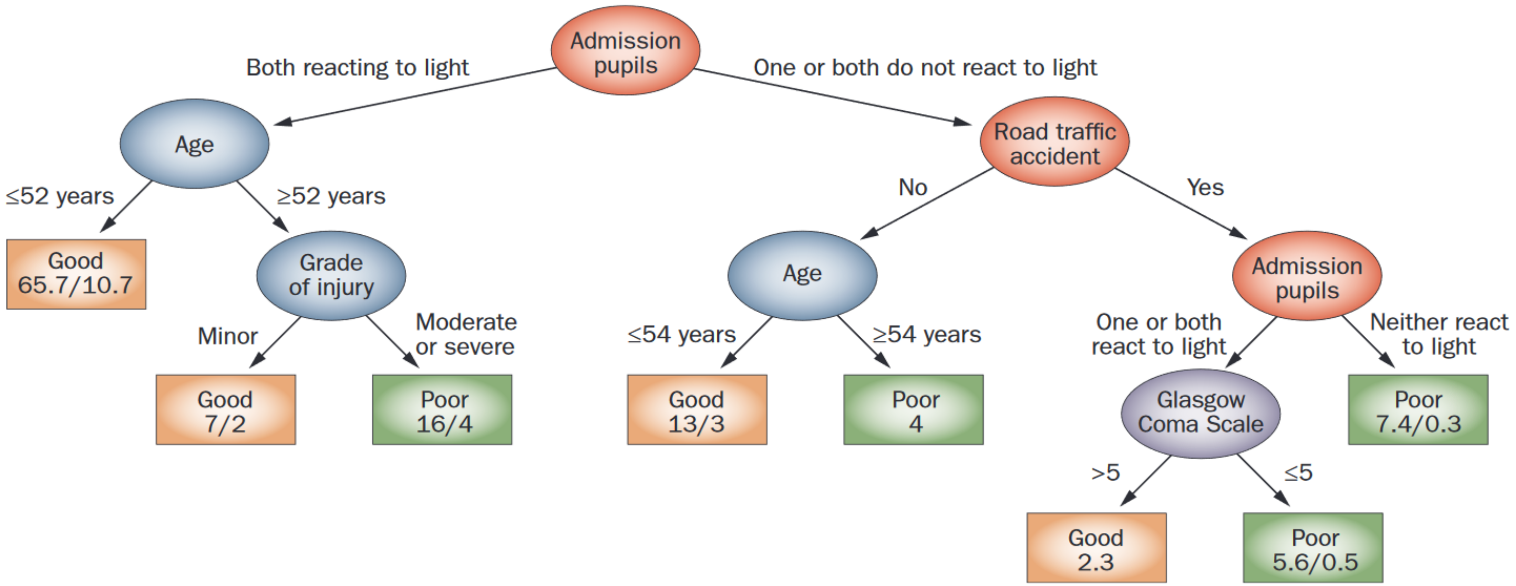




- What are properties of the layout?
- What are the drawing conventions?

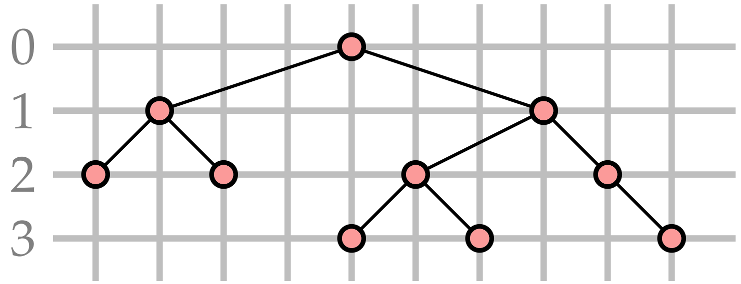
Layered Drawings – Drawing Style

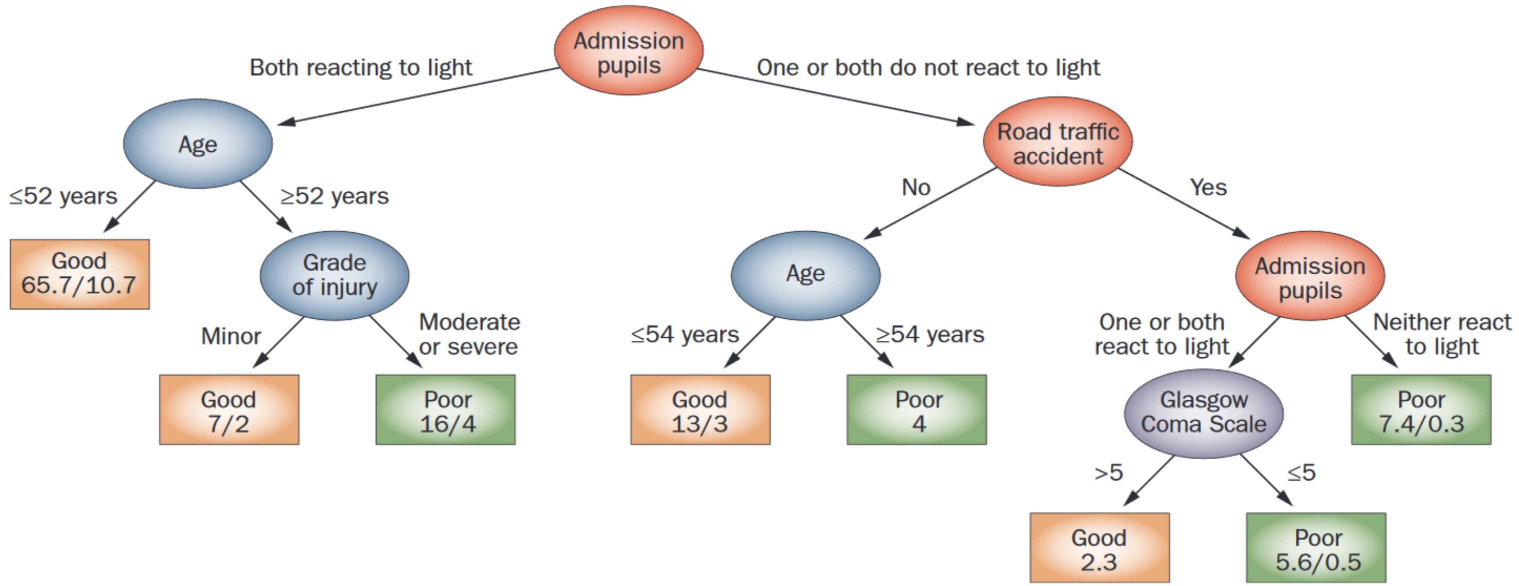




- What are properties of the layout?
- What are the drawing conventions?
- What are aesthetics to optimize?

Layered Drawings – Drawing Style





Drawing conventions

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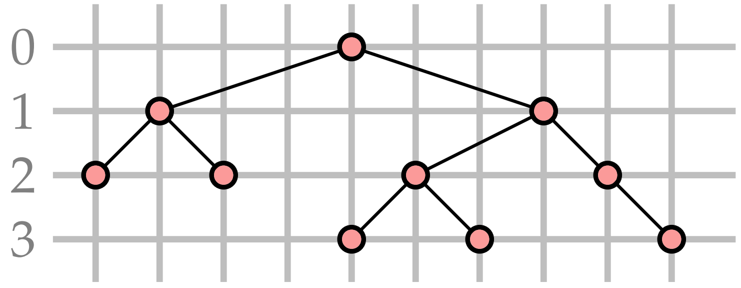
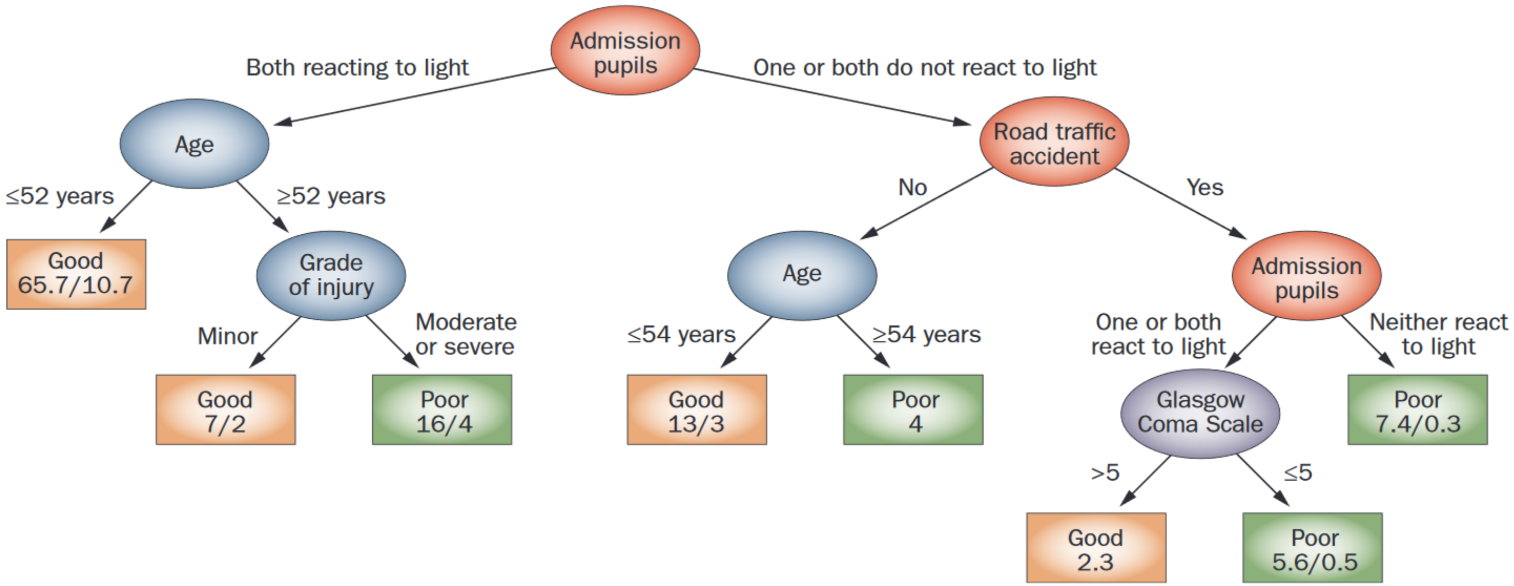
Layered Drawings – Drawing Style





Drawing conventions

- Vertices lie on layers and have integer coordinates

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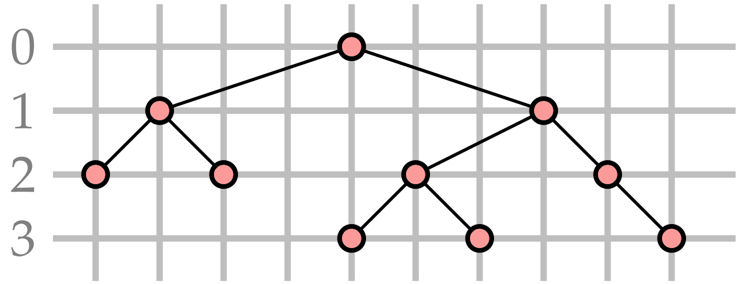
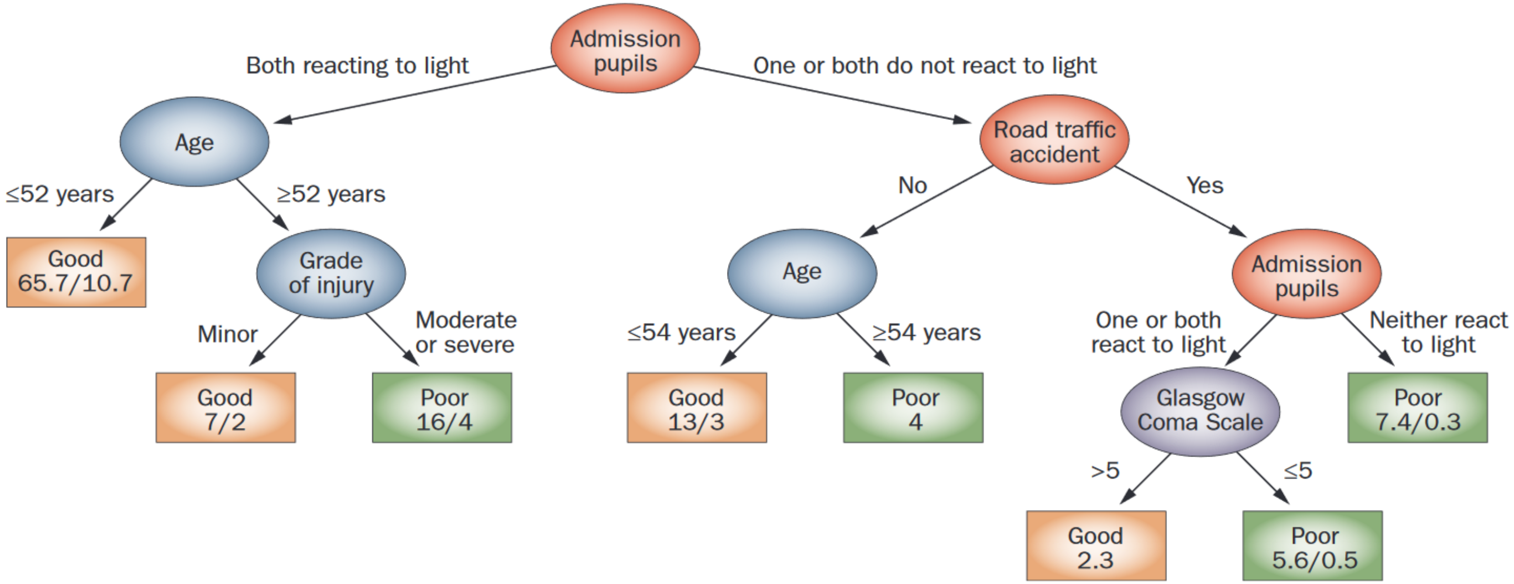
Layered Drawings – Drawing Style





Drawing conventions

- Vertices lie on layers and have integer coordinates
- Parent centered above children

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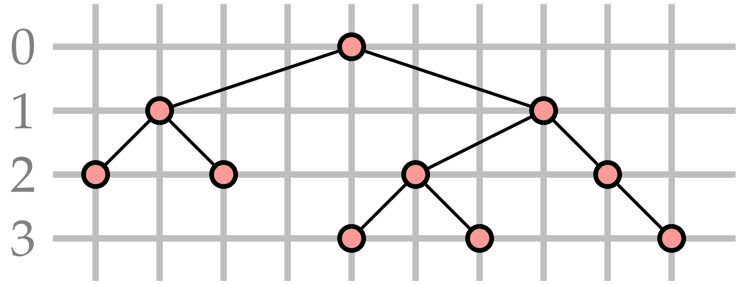
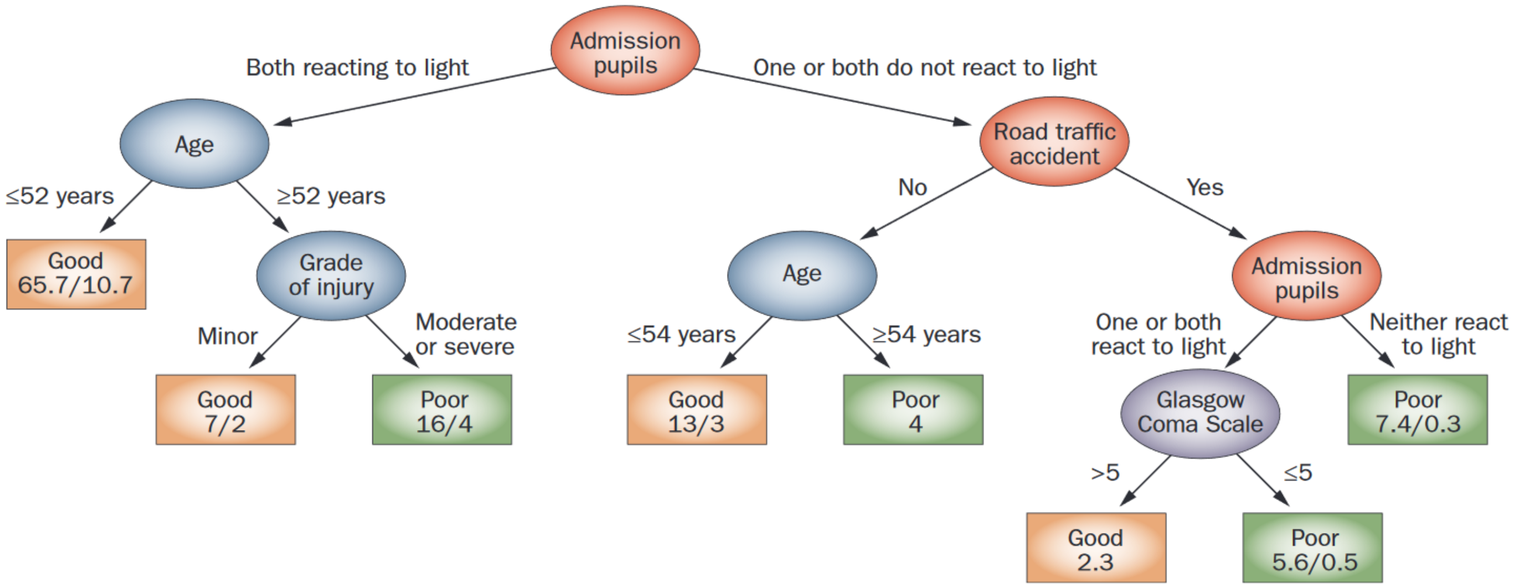
Layered Drawings – Drawing Style





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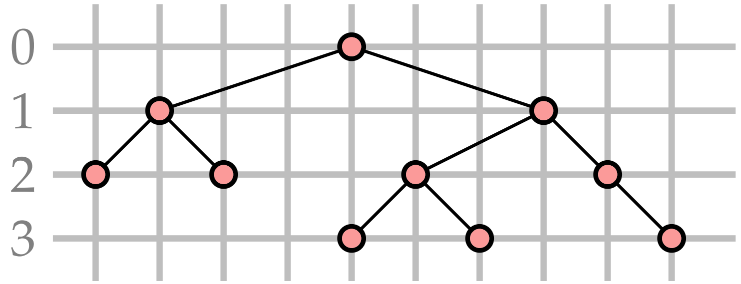
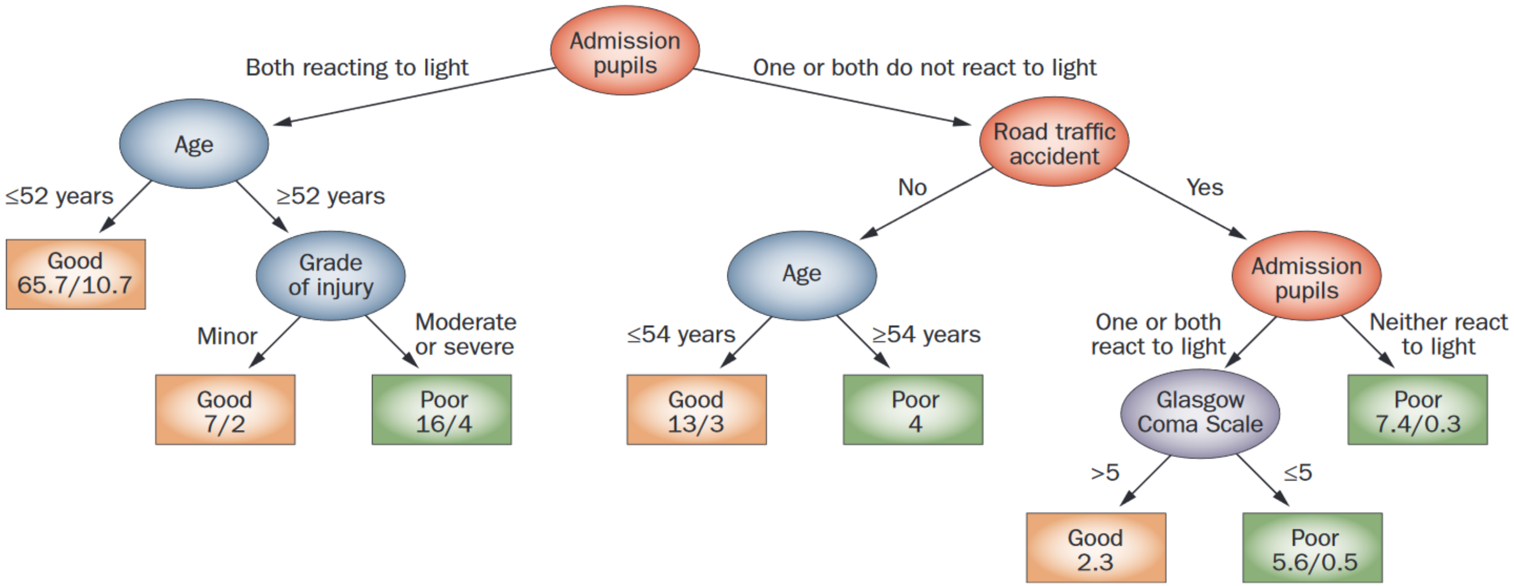
Layered Drawings – Drawing Style





- What are properties of the layout?
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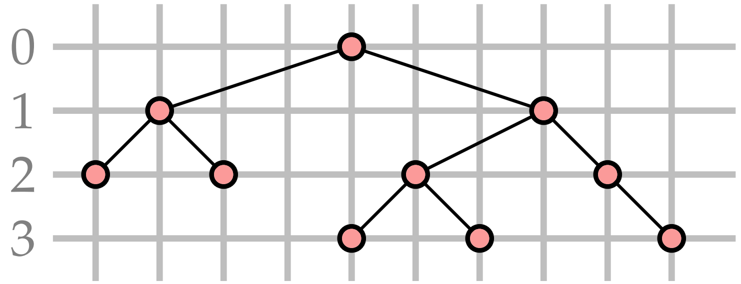
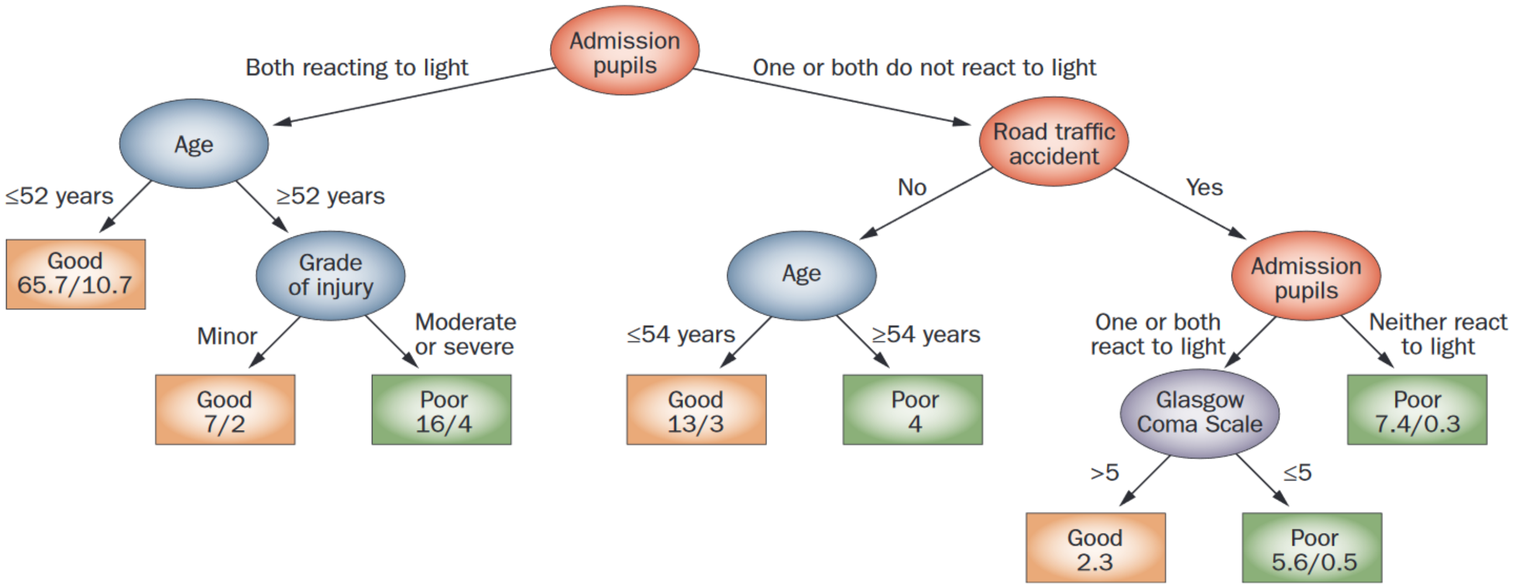
Drawing conventions

- Vertices lie on layers and have integer coordinates
- Parent centered above children
- Edges are straight-line segments
- Isomorphic subtrees have identical drawings

Layered Drawings – Drawing Style





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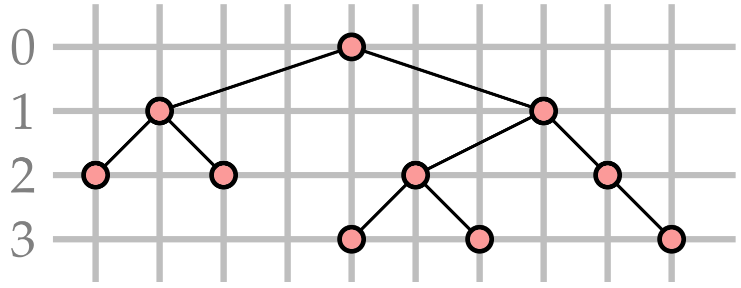

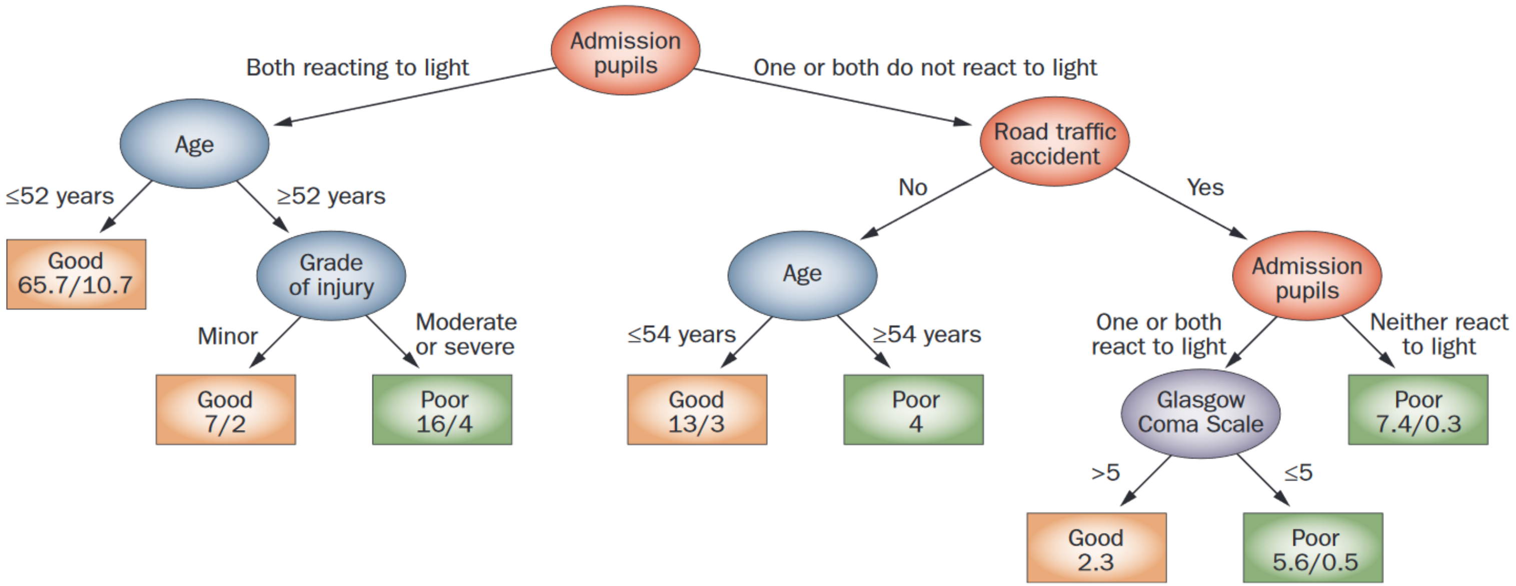

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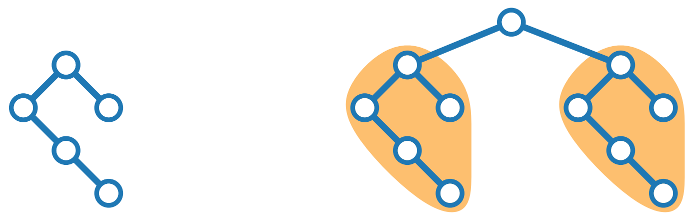
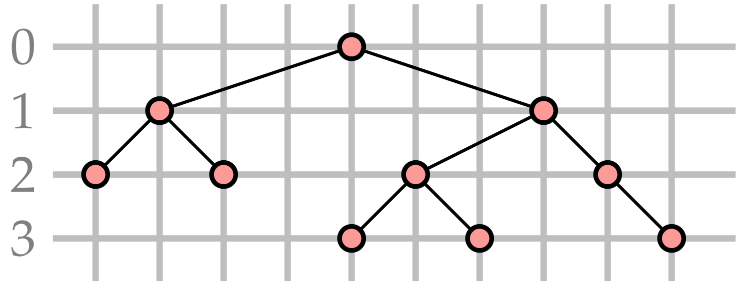
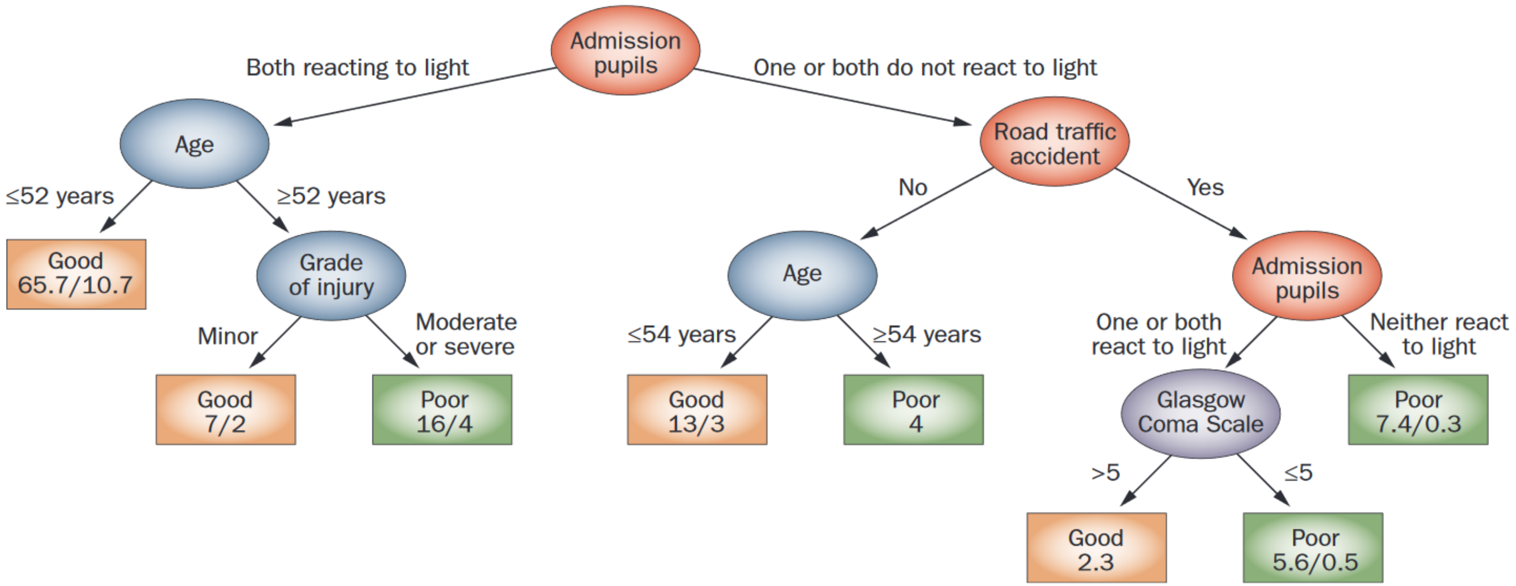


Layered Drawings – Drawing Style





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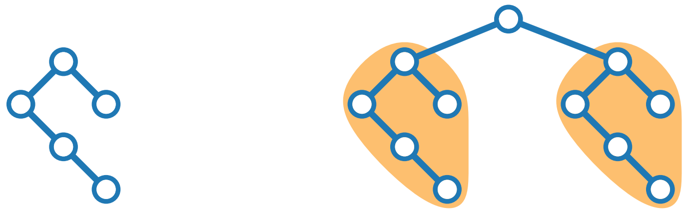

Drawing conventions

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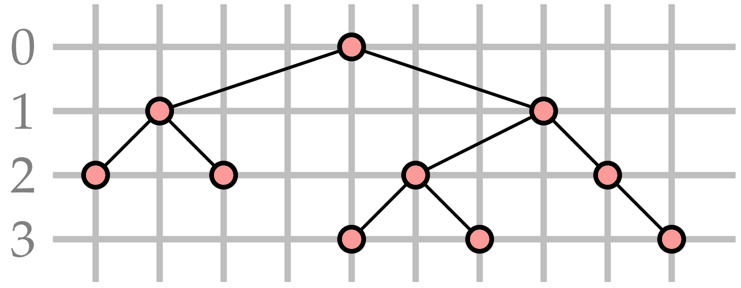
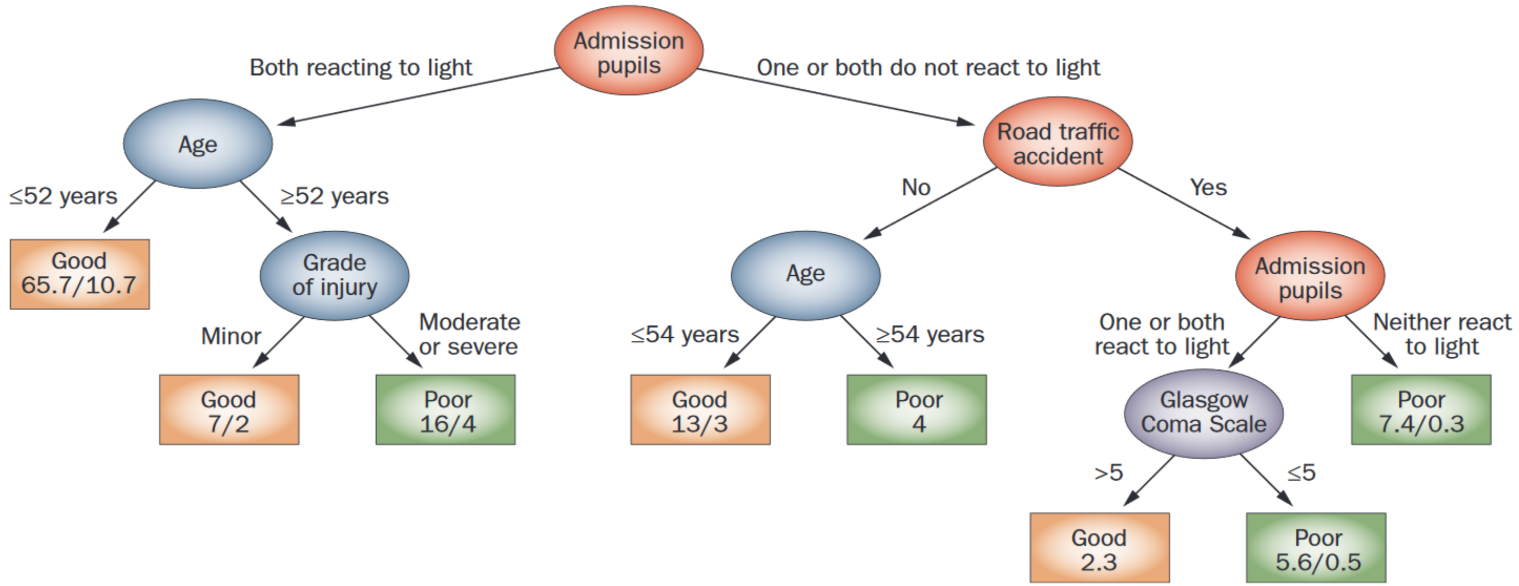
Drawing aesthetics

Layered Drawings – Drawing Style





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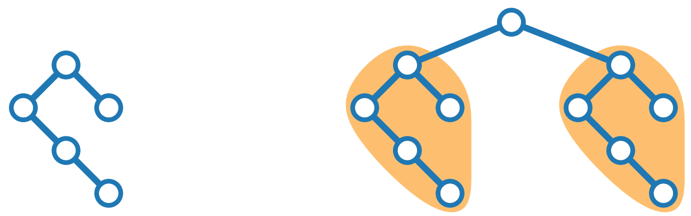

Drawing conventions

- Vertices lie on layers and have integer coordinates
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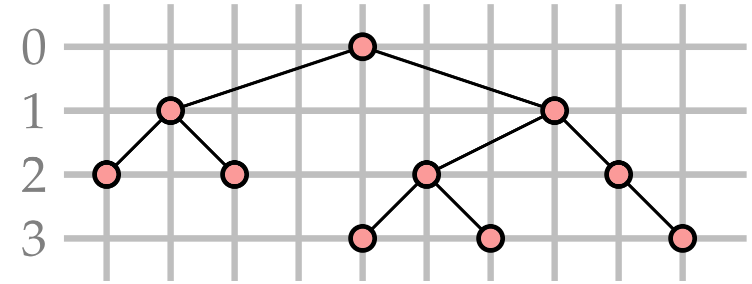
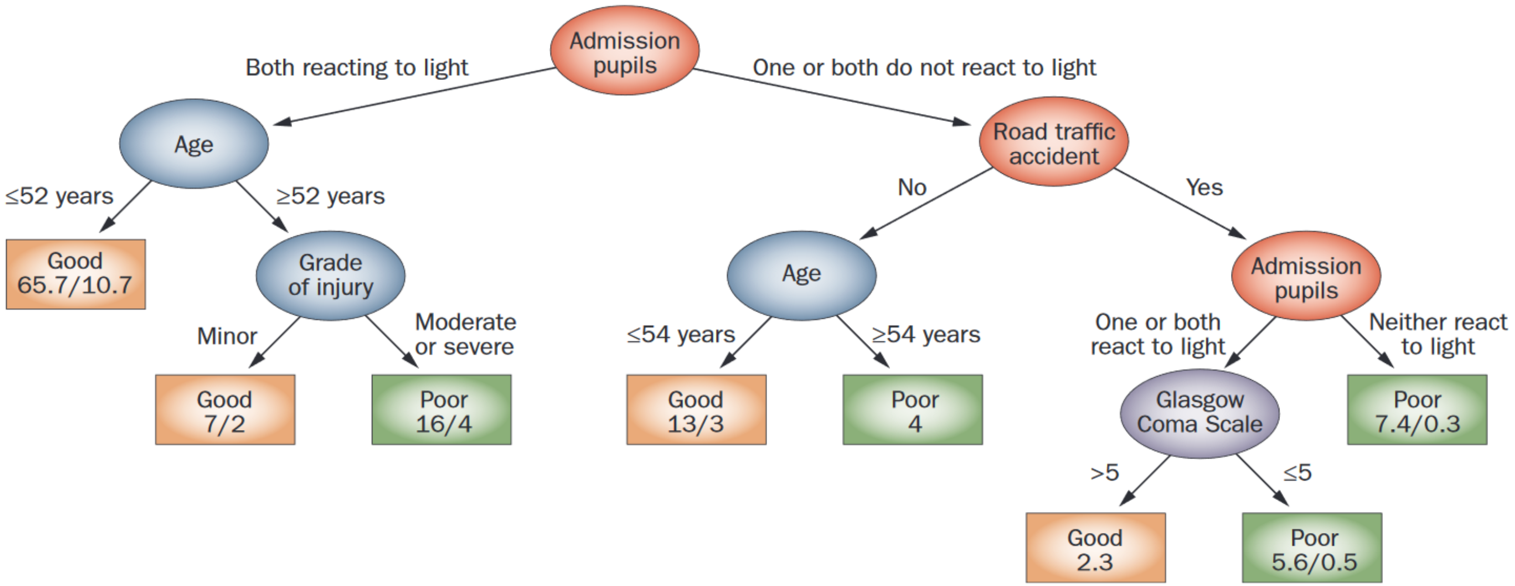
- Isomorphic subtrees have identical drawings

Drawing aesthetics

- Area

Layered Drawings – Drawing Style





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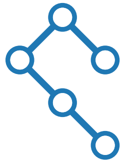

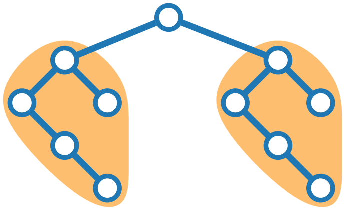

Drawing conventions

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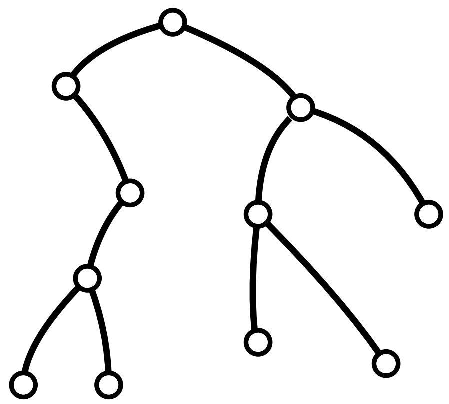
Drawing aesthetics

- Area
- Symmetries

Layered Drawings – Algorithm

Input: A binary tree T
Output: A layered drawing of T



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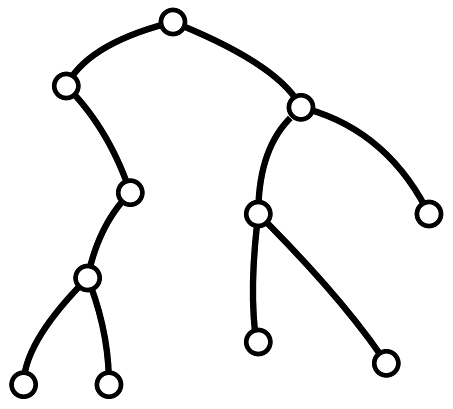
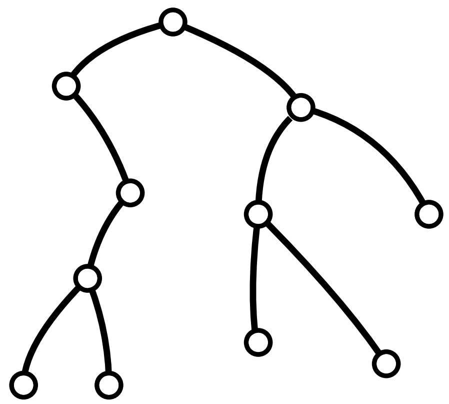


Base case:
Divide:

Conquer:

Layered Drawings – Algorithm

Input: A binary tree T
Output: A layered drawing of T



Base case: A single vertex 
Divide:

Conquer:

Layered Drawings – Algorithm

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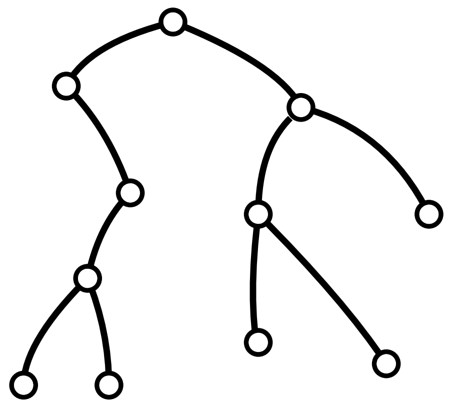
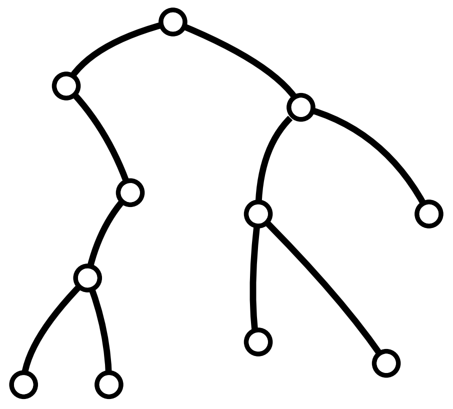


Base case: A single vertex 
Divide: Recursively apply the algorithm to draw the left and right subtrees

Conquer:

Layered Drawings – Algorithm

Input: A binary tree T
Output: A layered drawing of T



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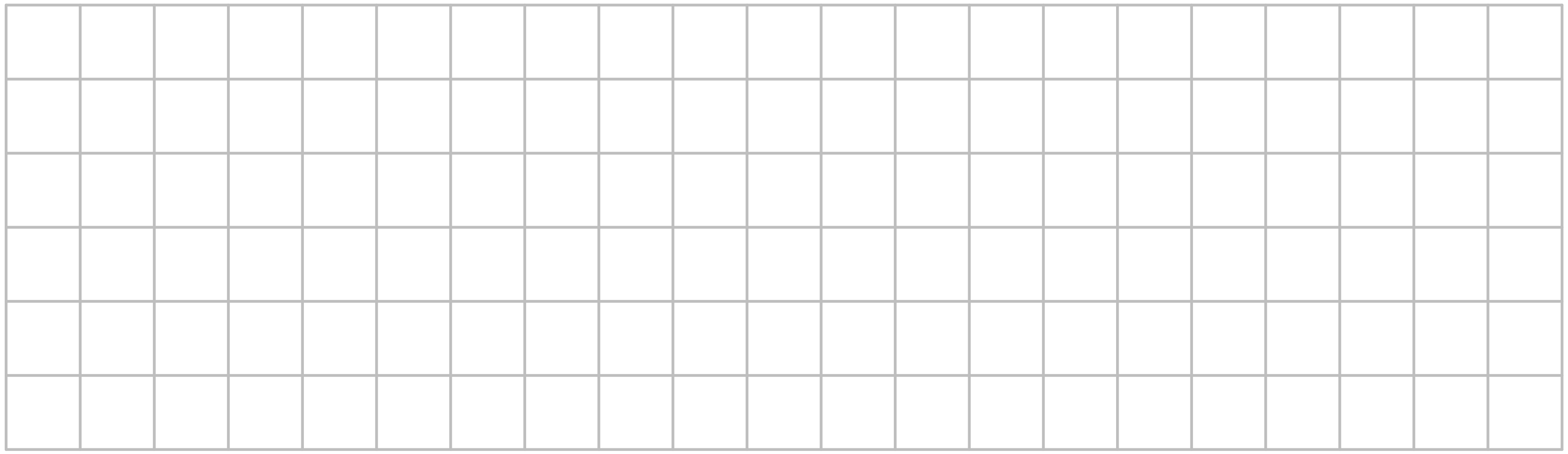

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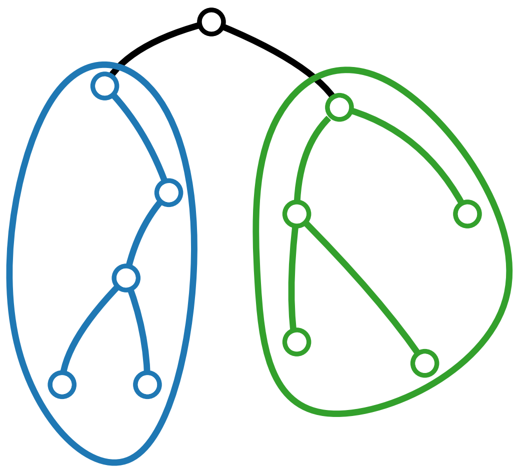

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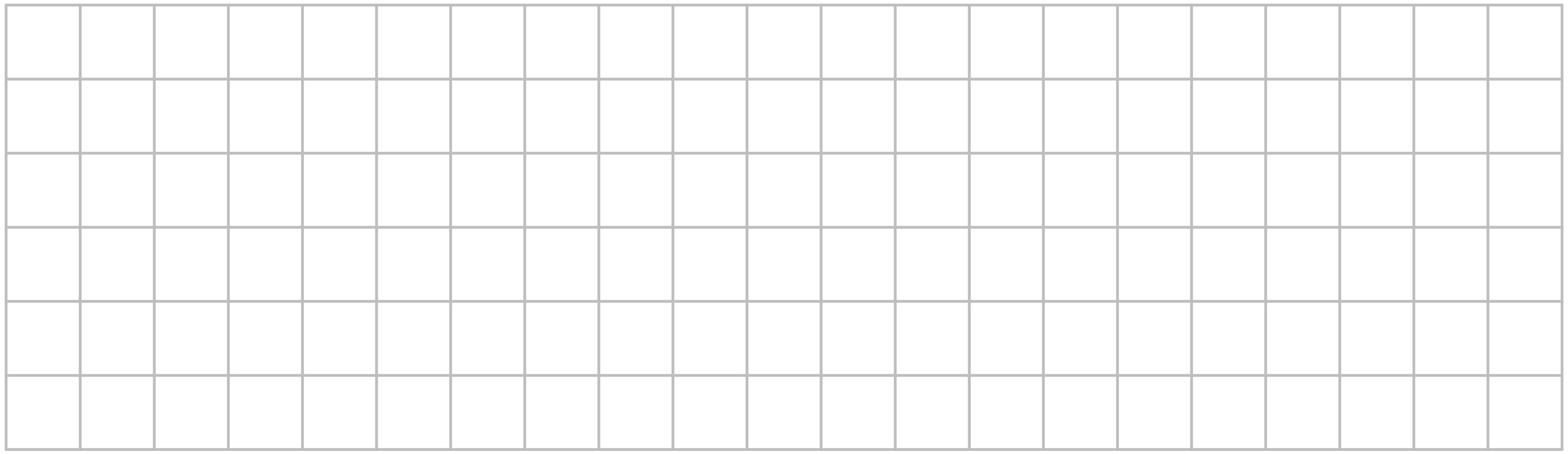

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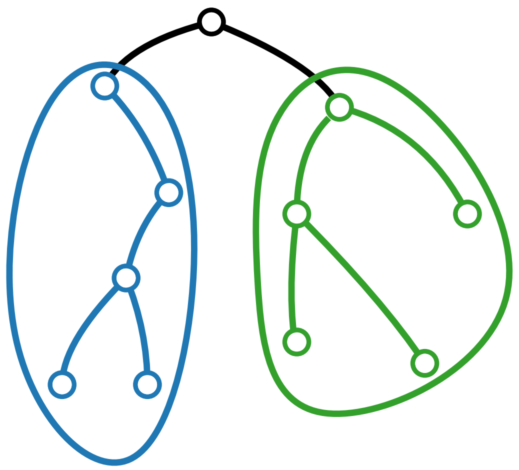

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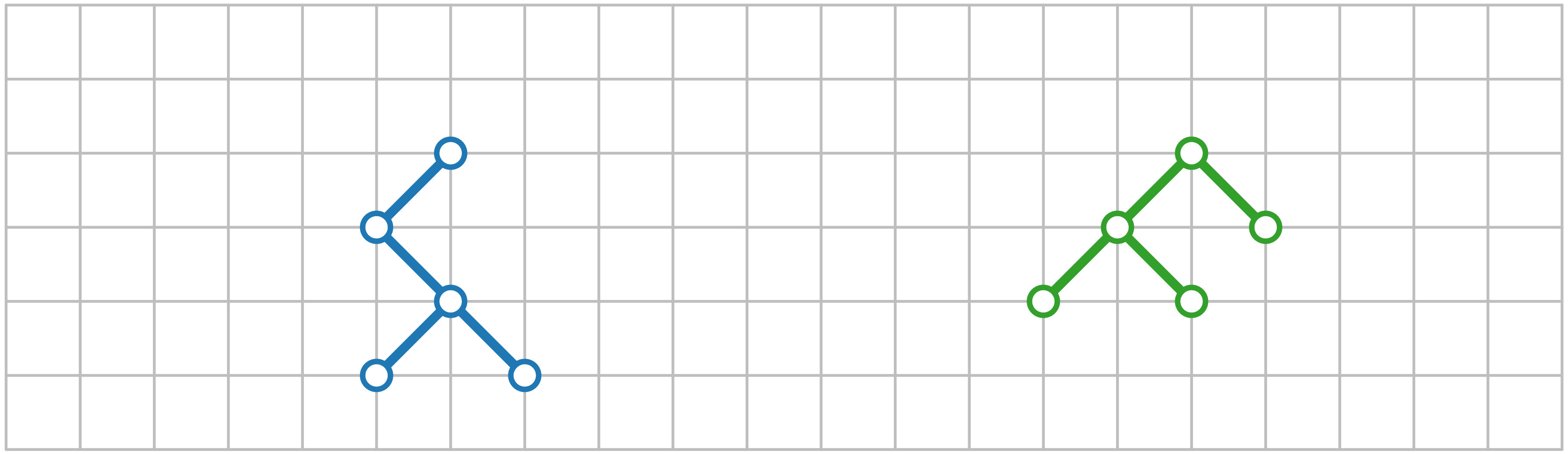

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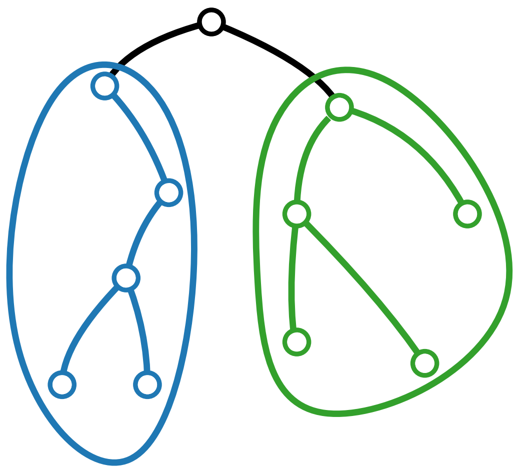

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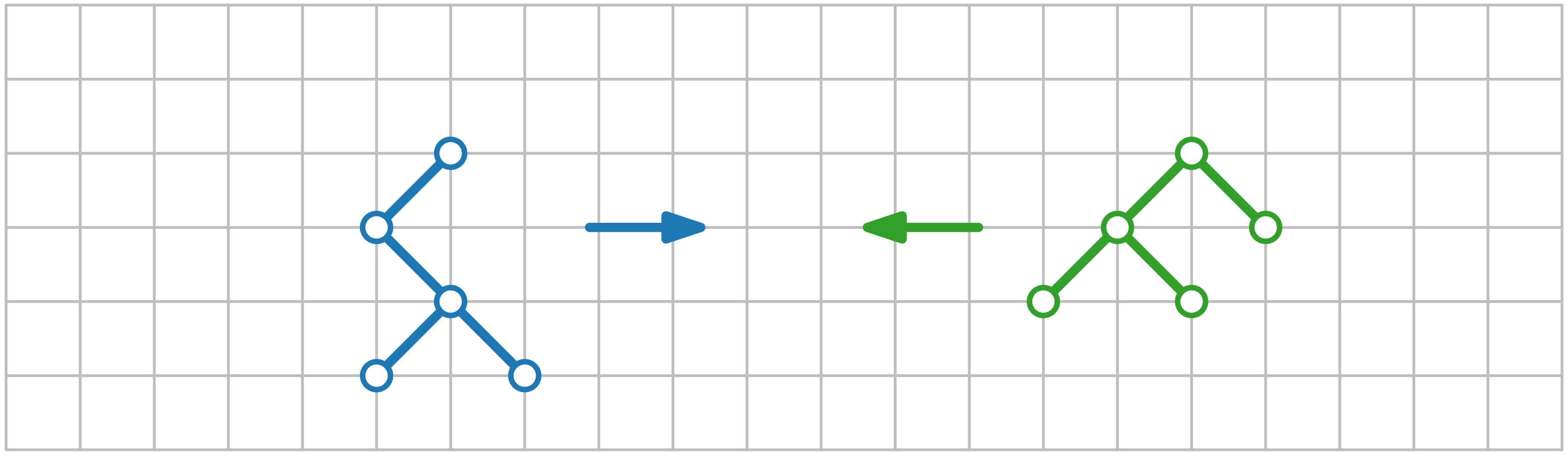

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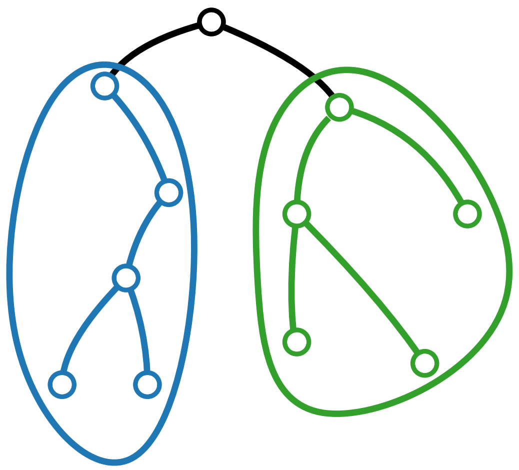
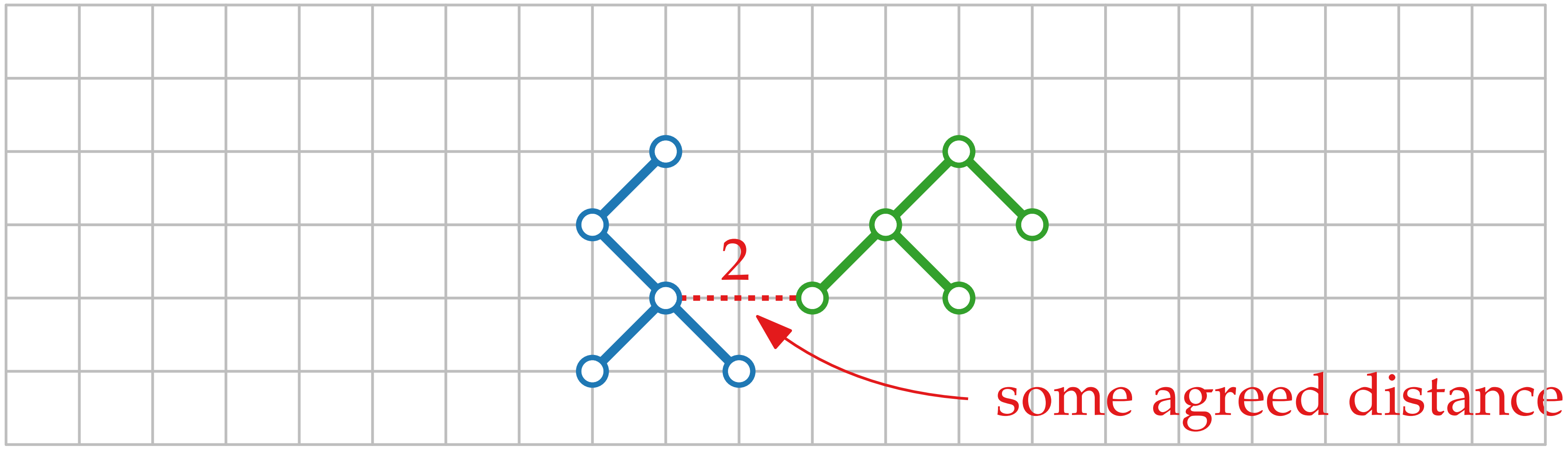


Base case: A single vertex ○
Divide: Recursively apply the algorithm to draw the left and right subtrees

Conquer:



Layered Drawings – Algorithm

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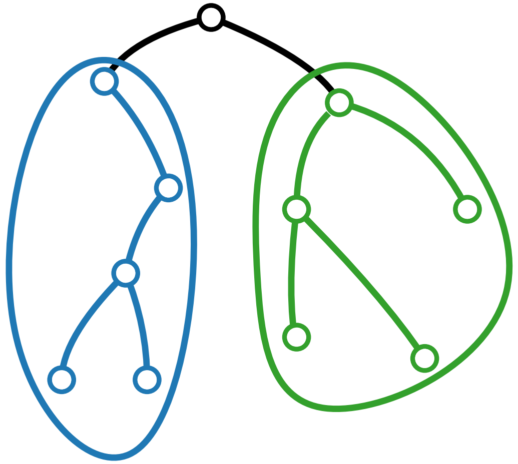

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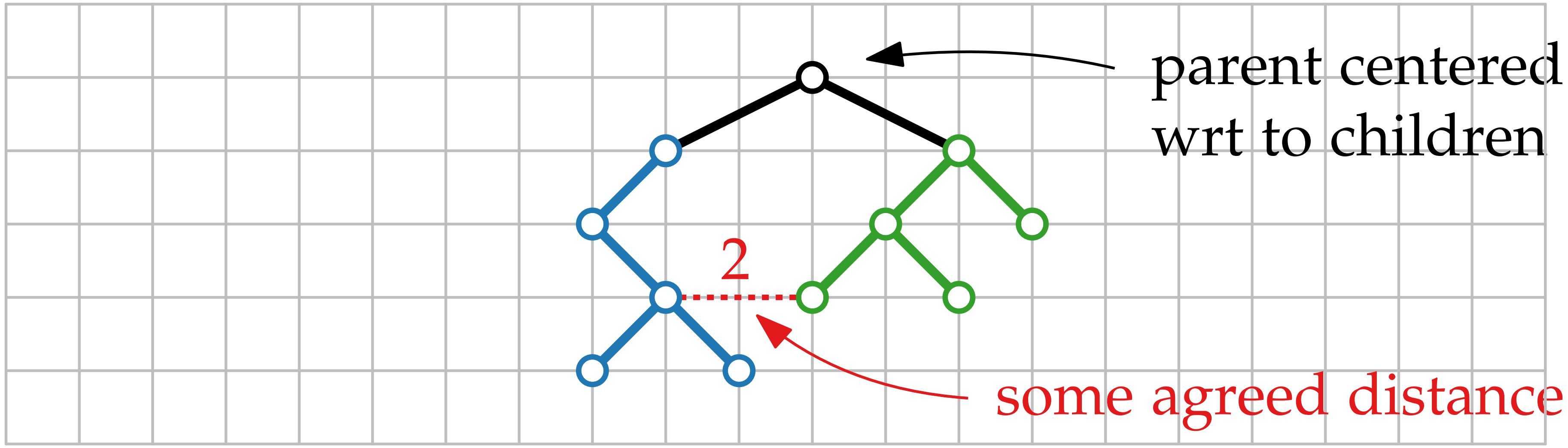

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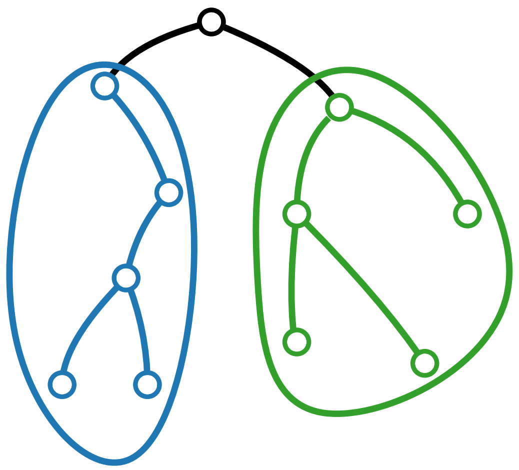

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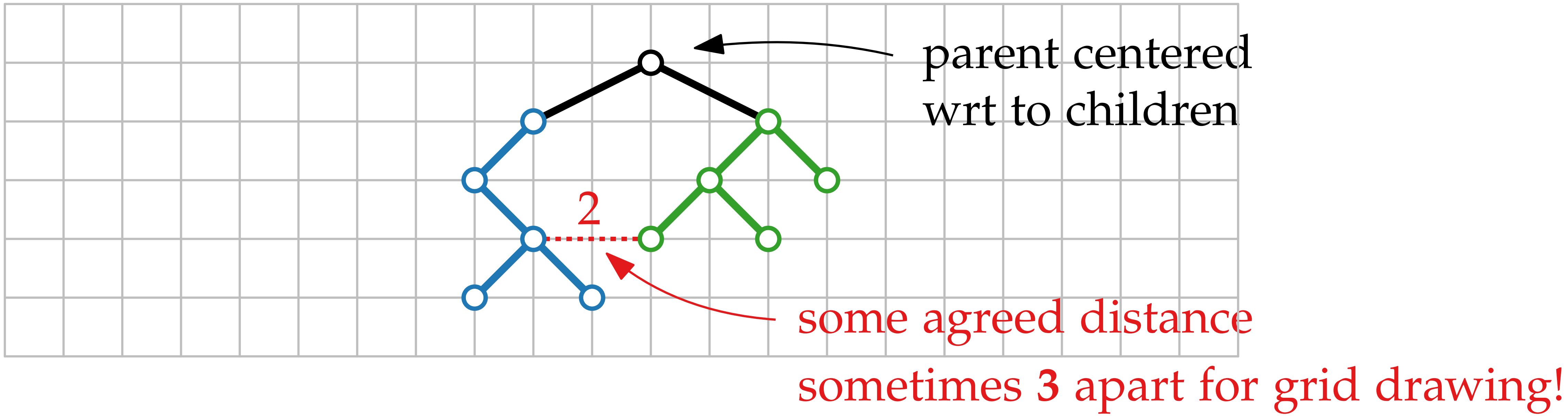

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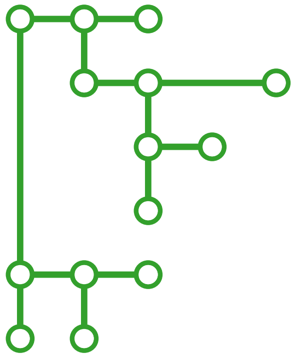
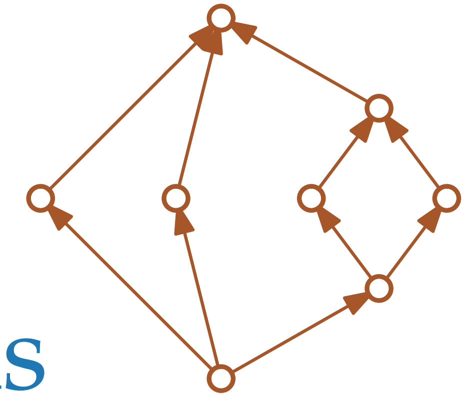
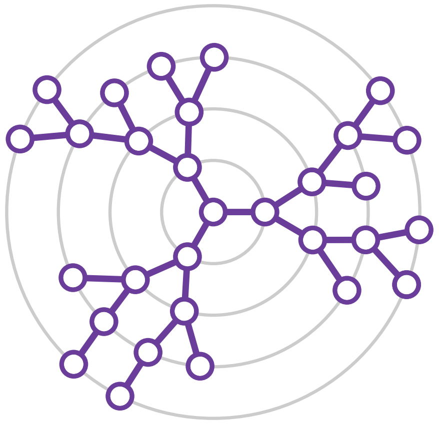


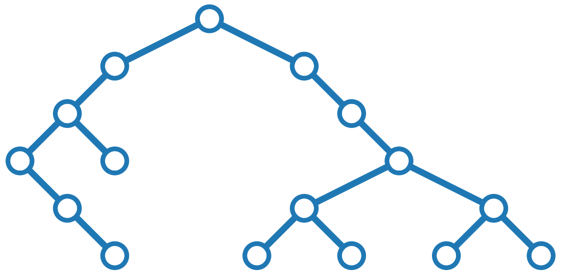
Visualization of Graphs



Lecture 2:
Drawing Trees and Series-Parallel Graphs



Part II:
Layered Drawings – Algorithmic Details

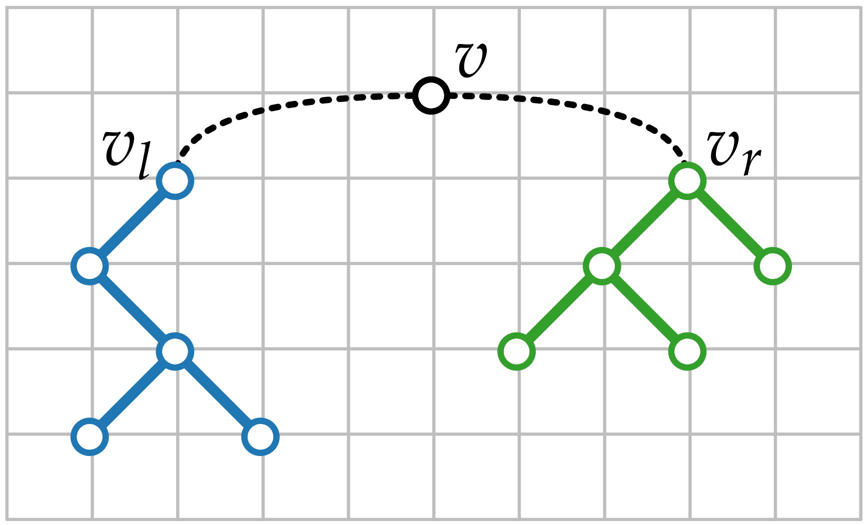


Philipp Kindermann

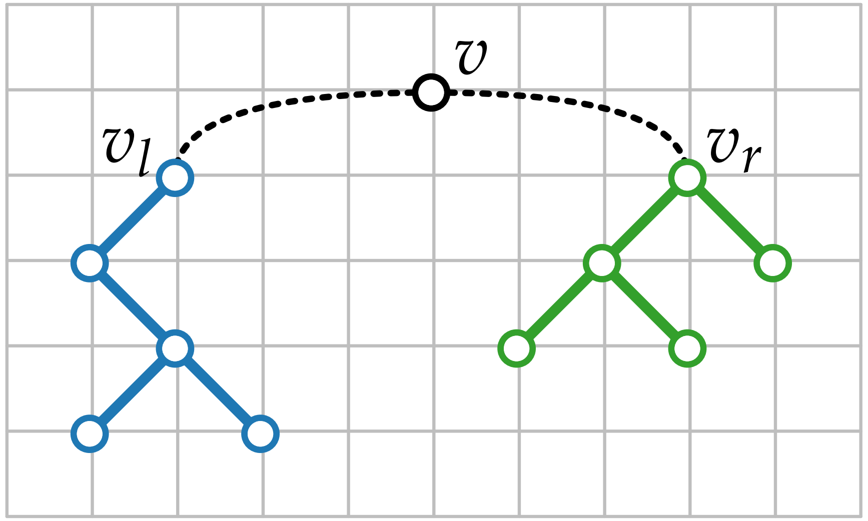
Layered Drawings – Algorithm Details

Phase 1 – postorder traversal:

- For each vertex compute horizontal displacement of left and right child



Layered Drawings – Algorithm Details



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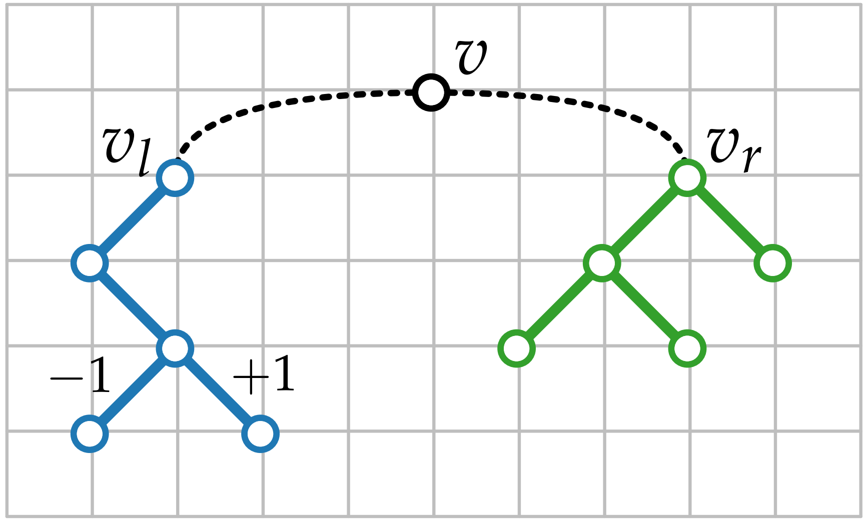
Phase 2 – preorder traversal:

- Compute x- and y-coordinates

Layered Drawings – Algorithm Details



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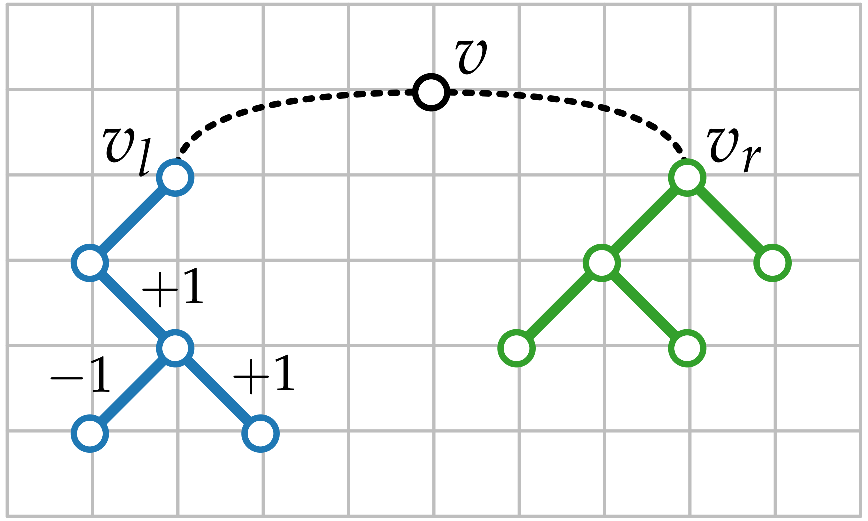

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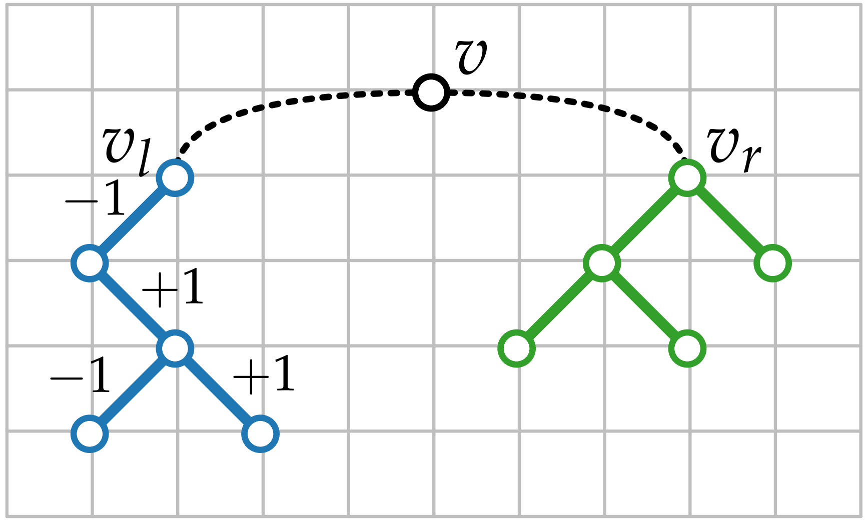

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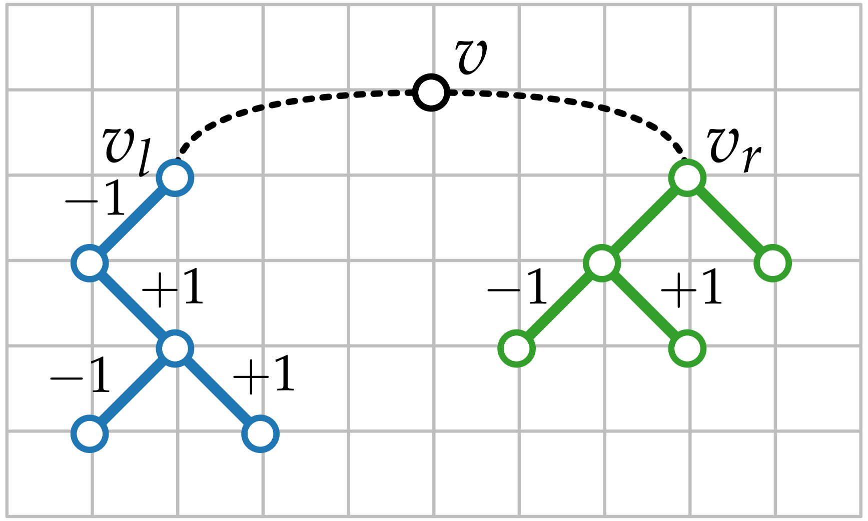

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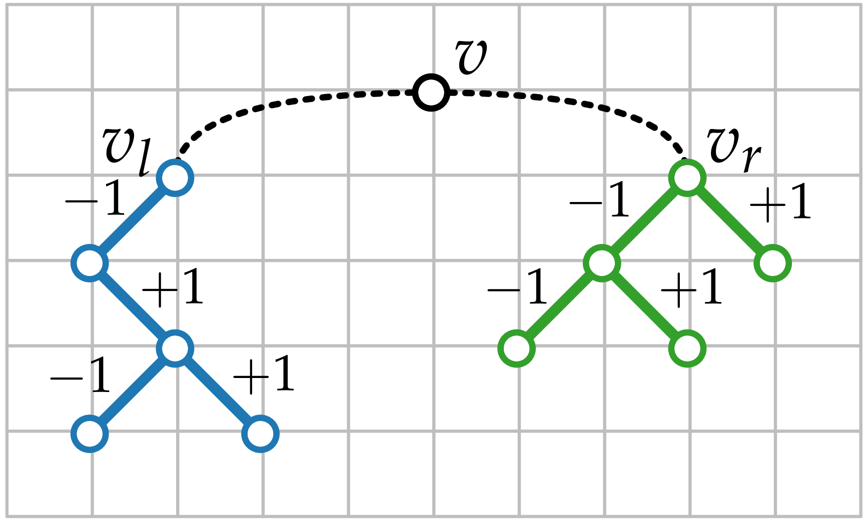

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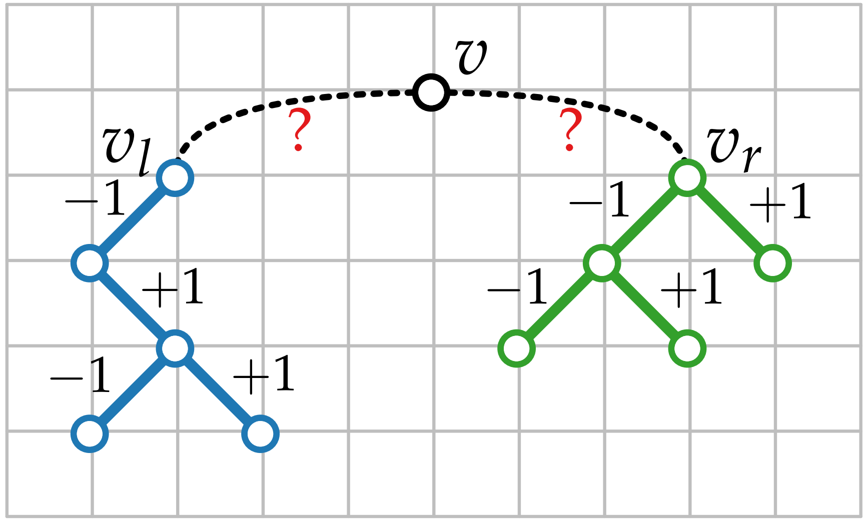

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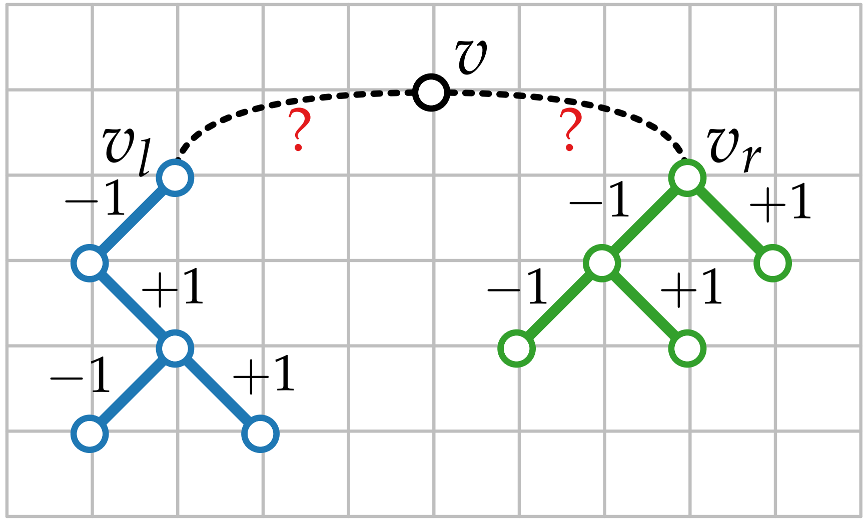
Layered Drawings – Algorithm Details



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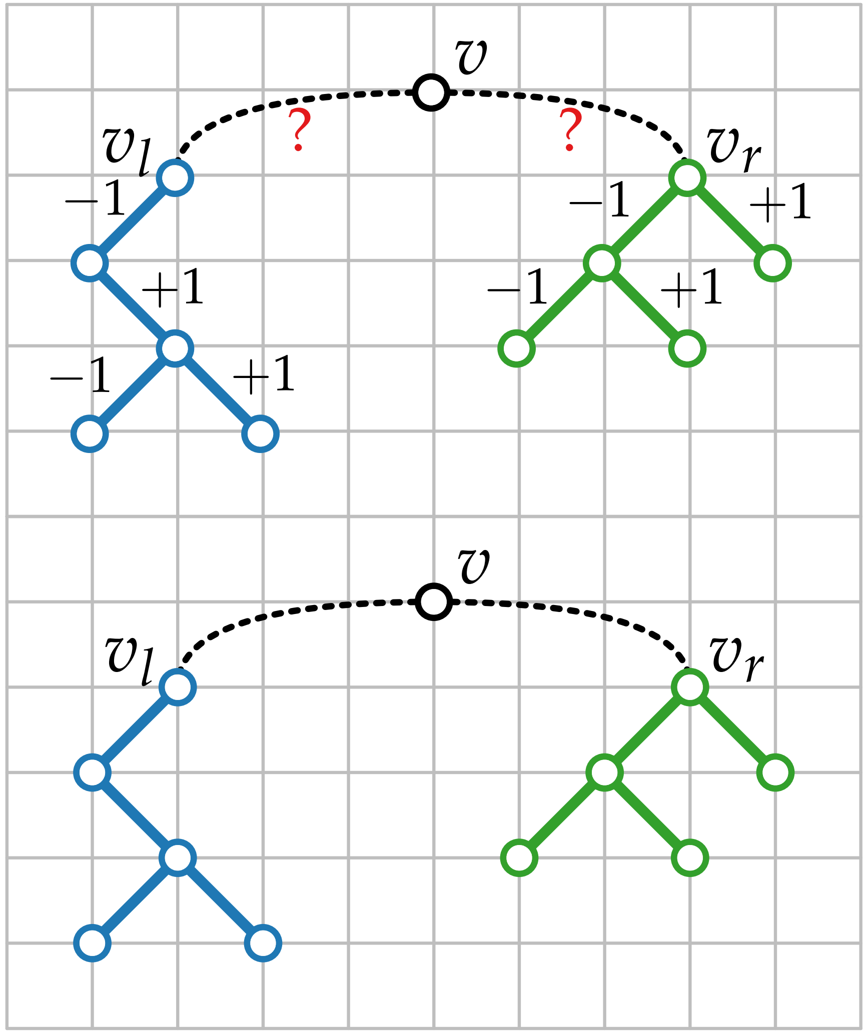

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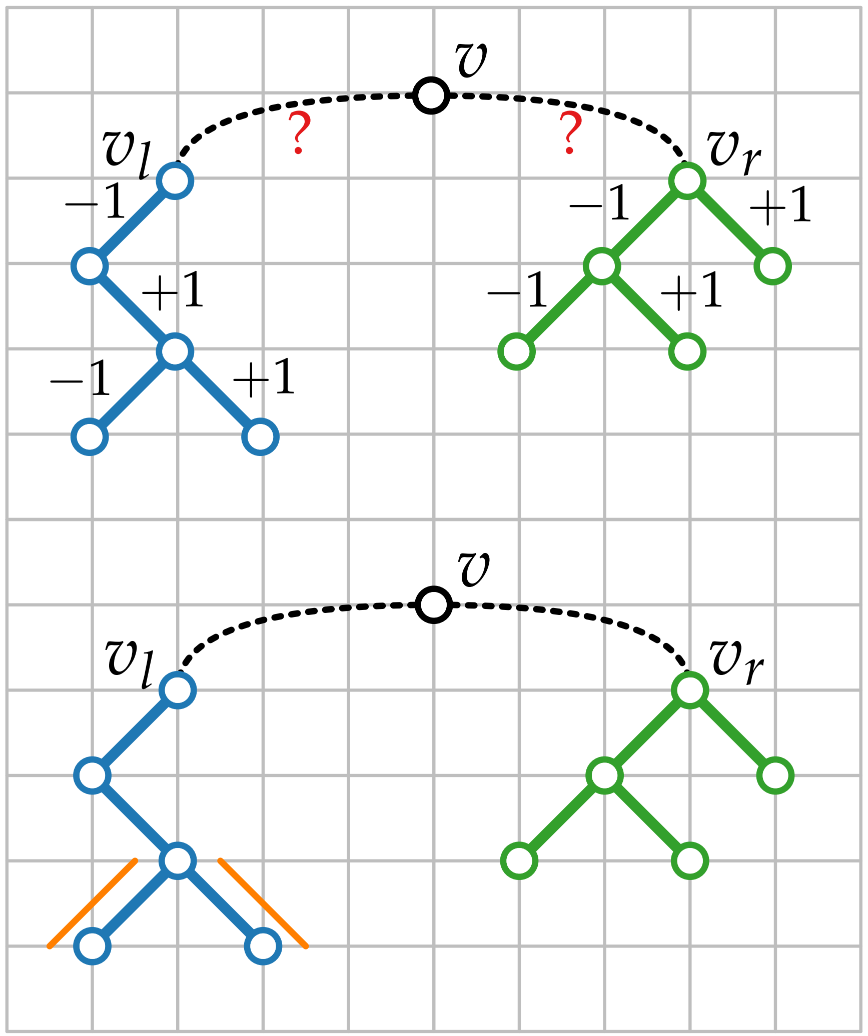

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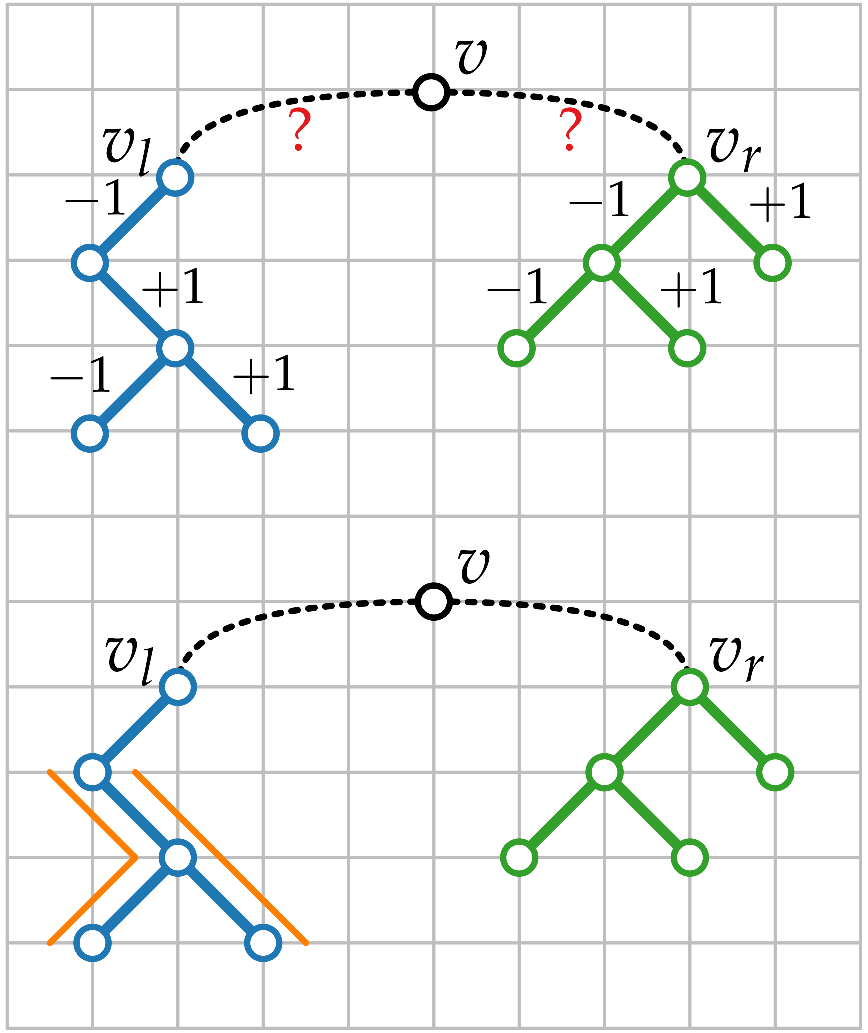

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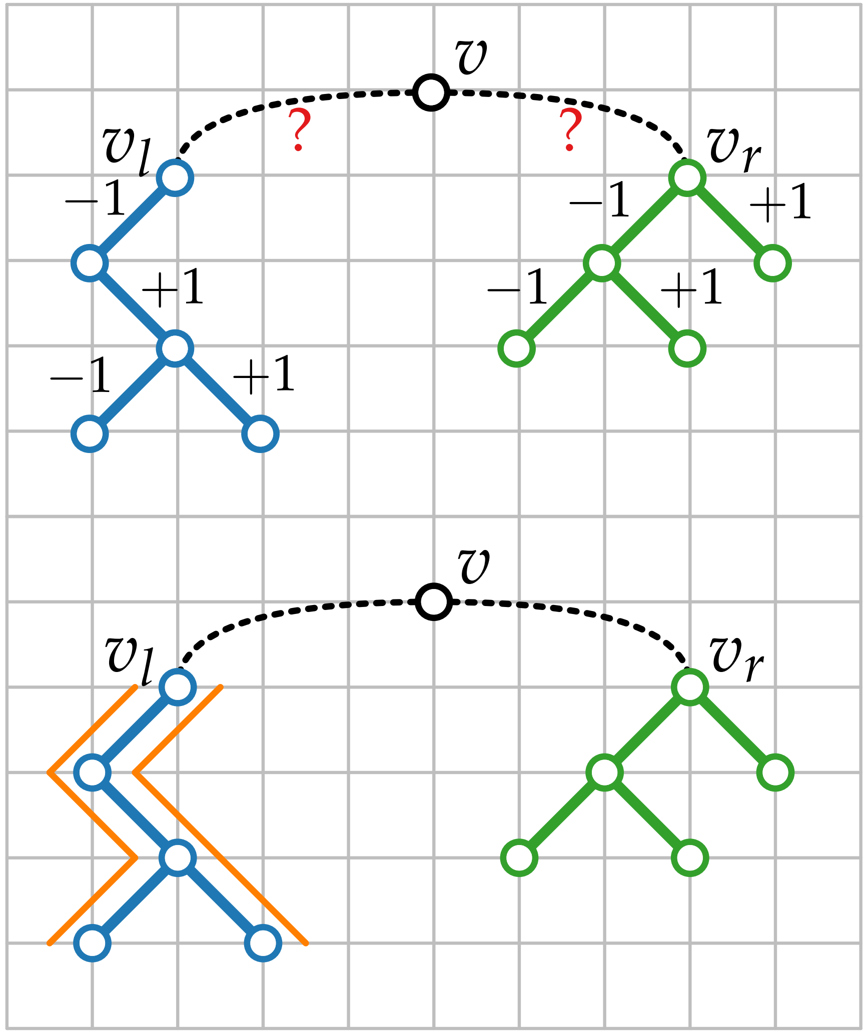

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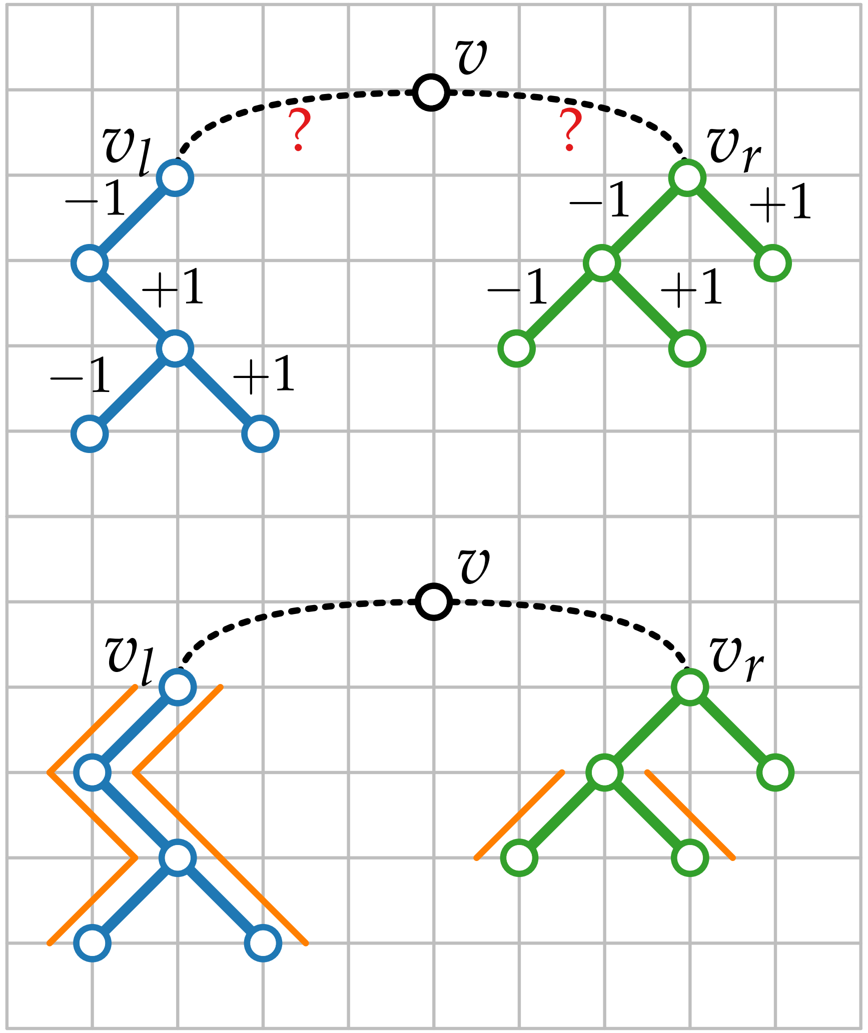

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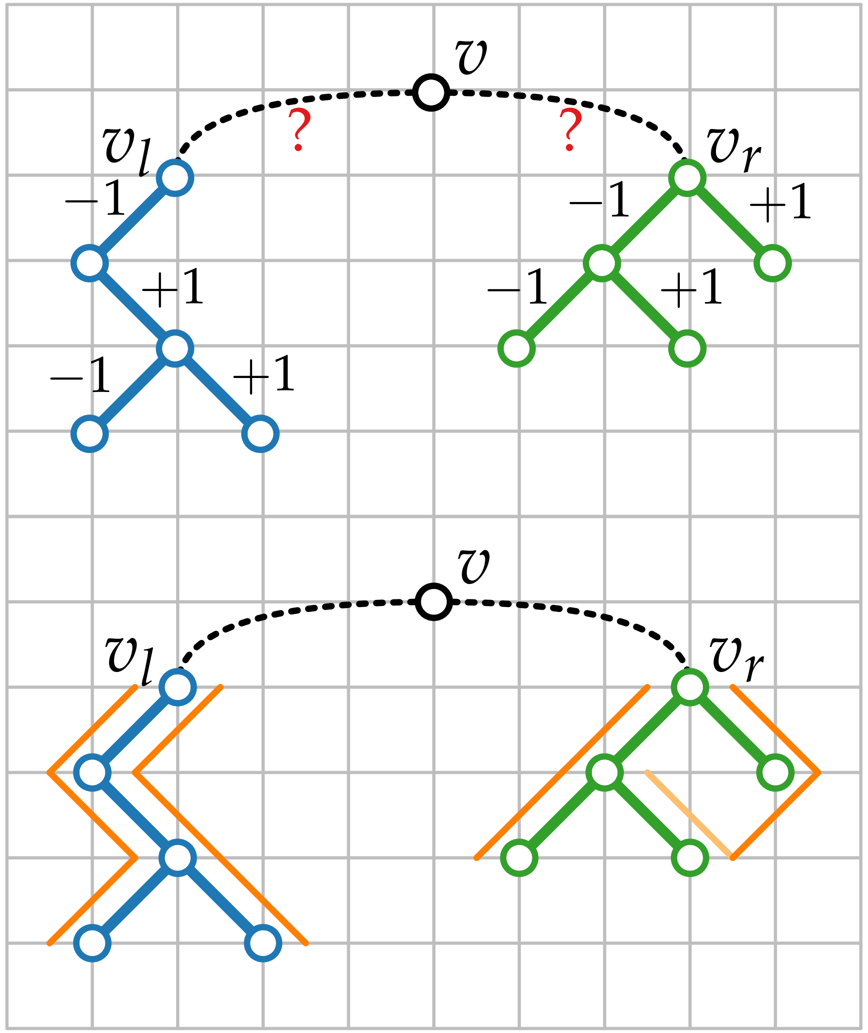

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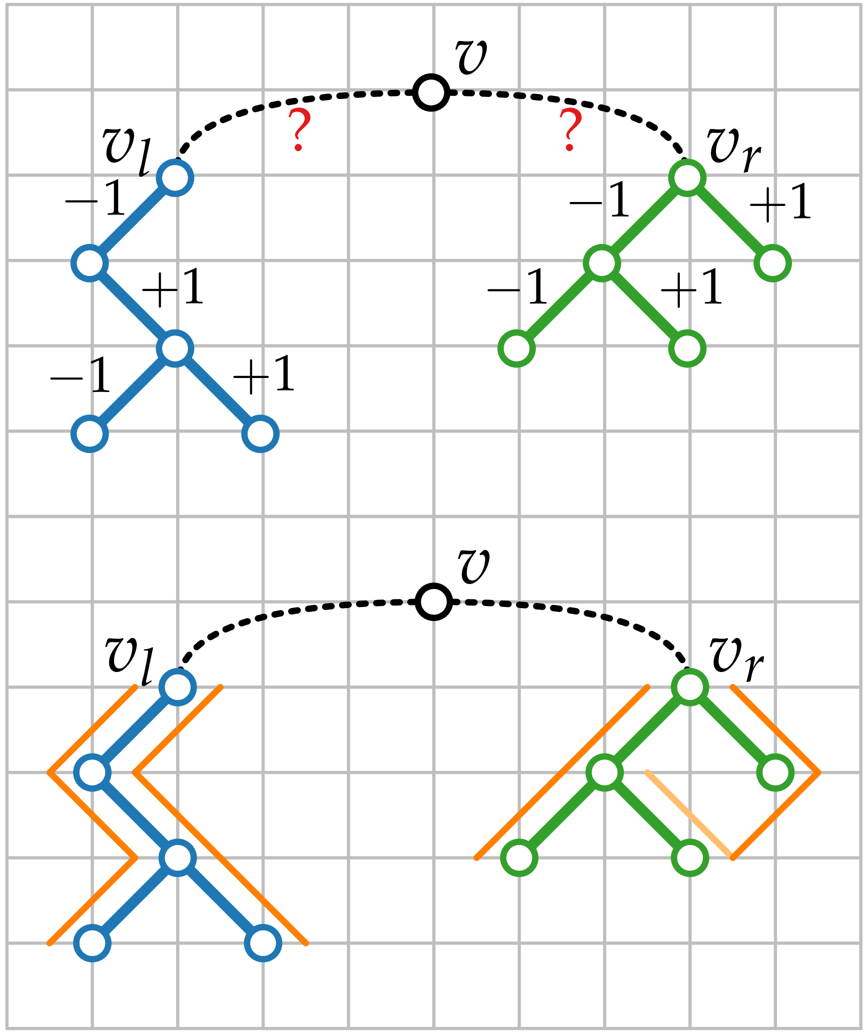

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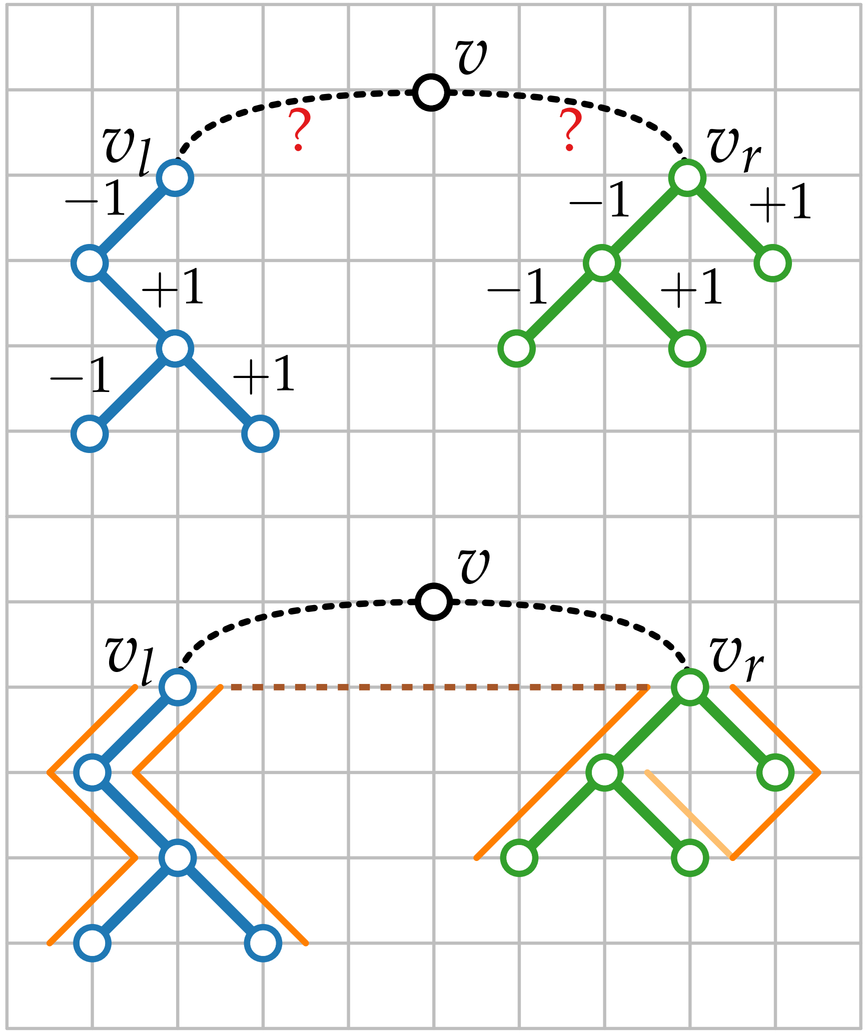

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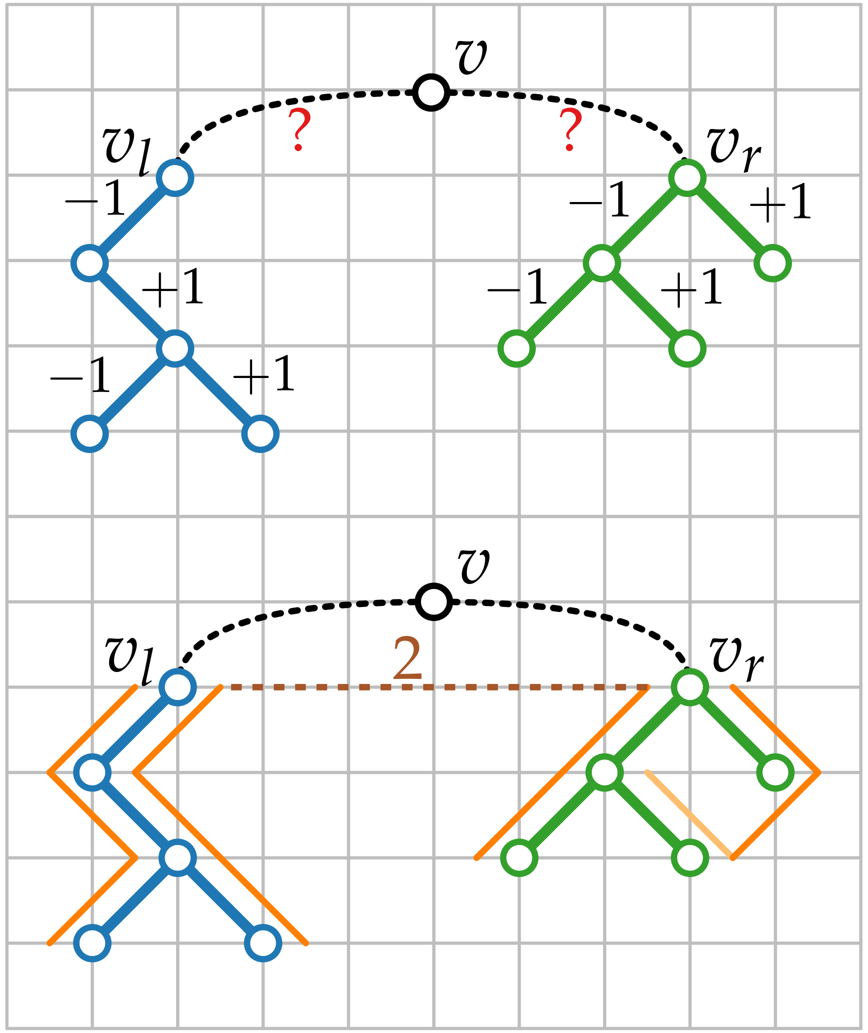

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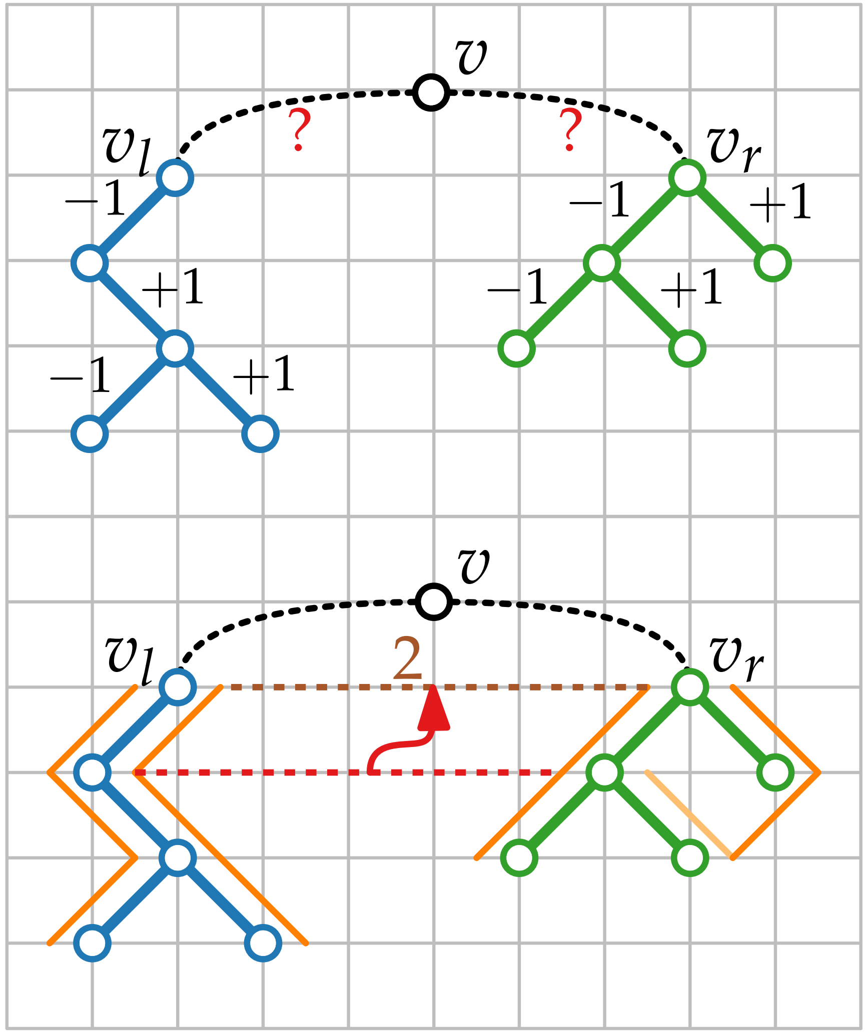

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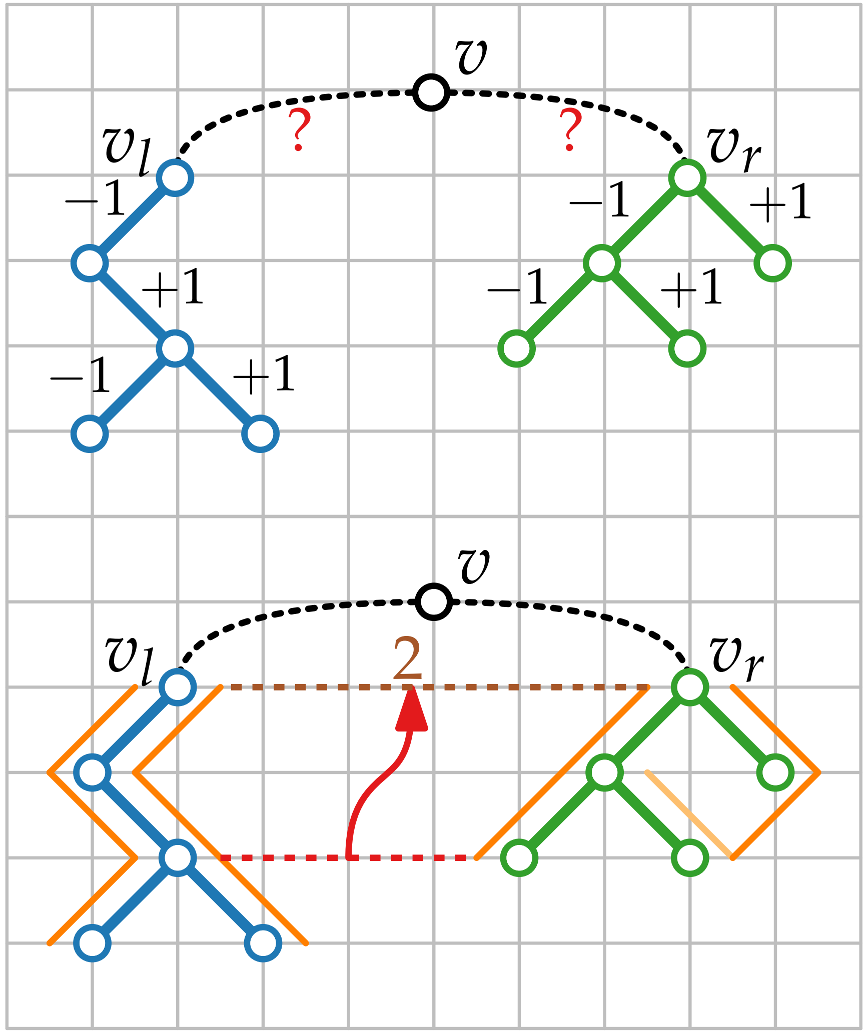

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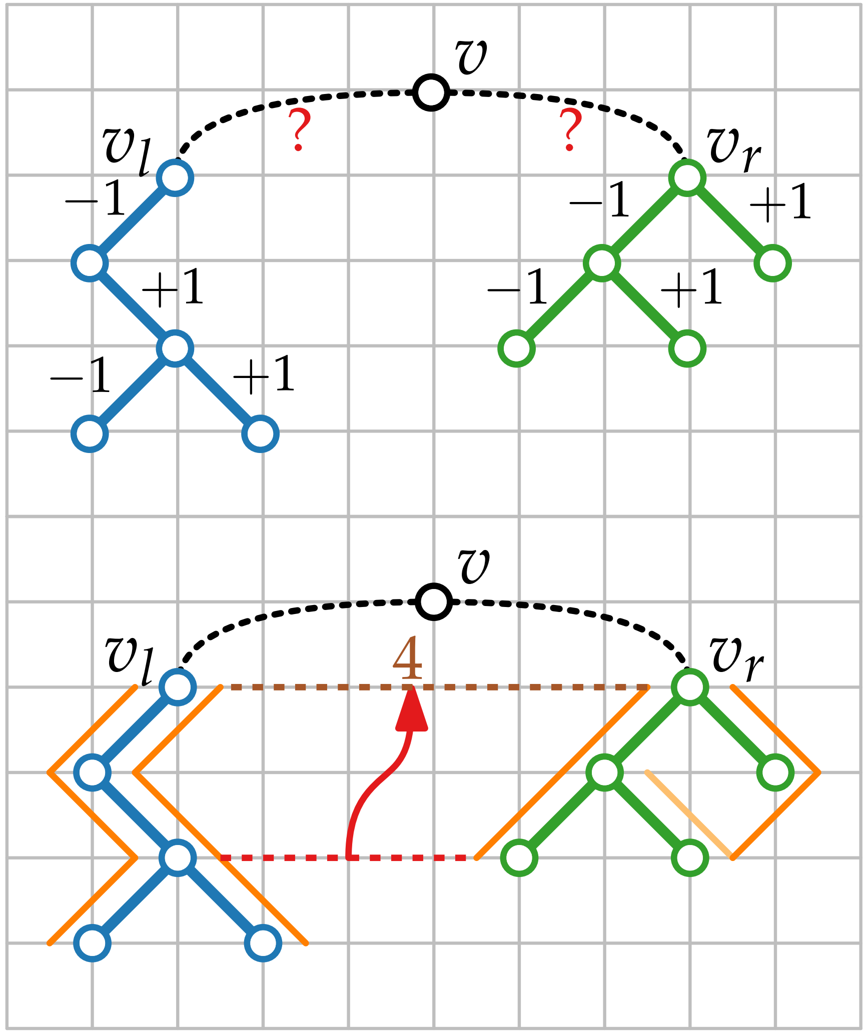

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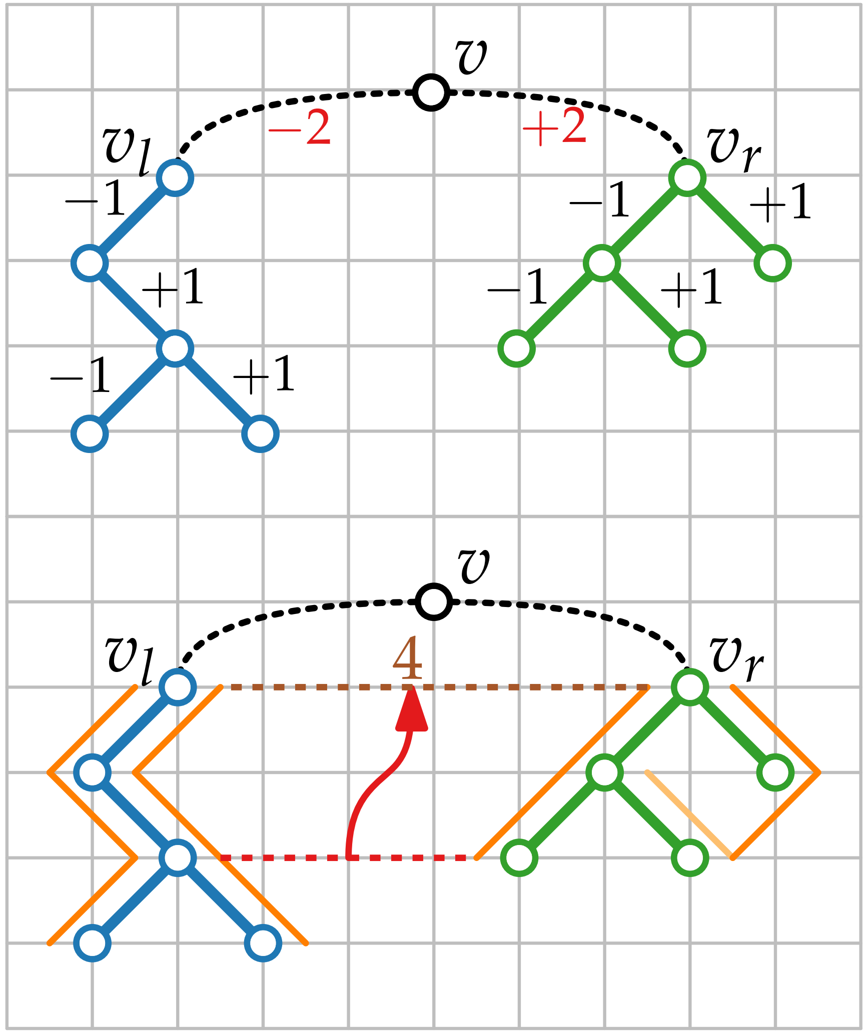

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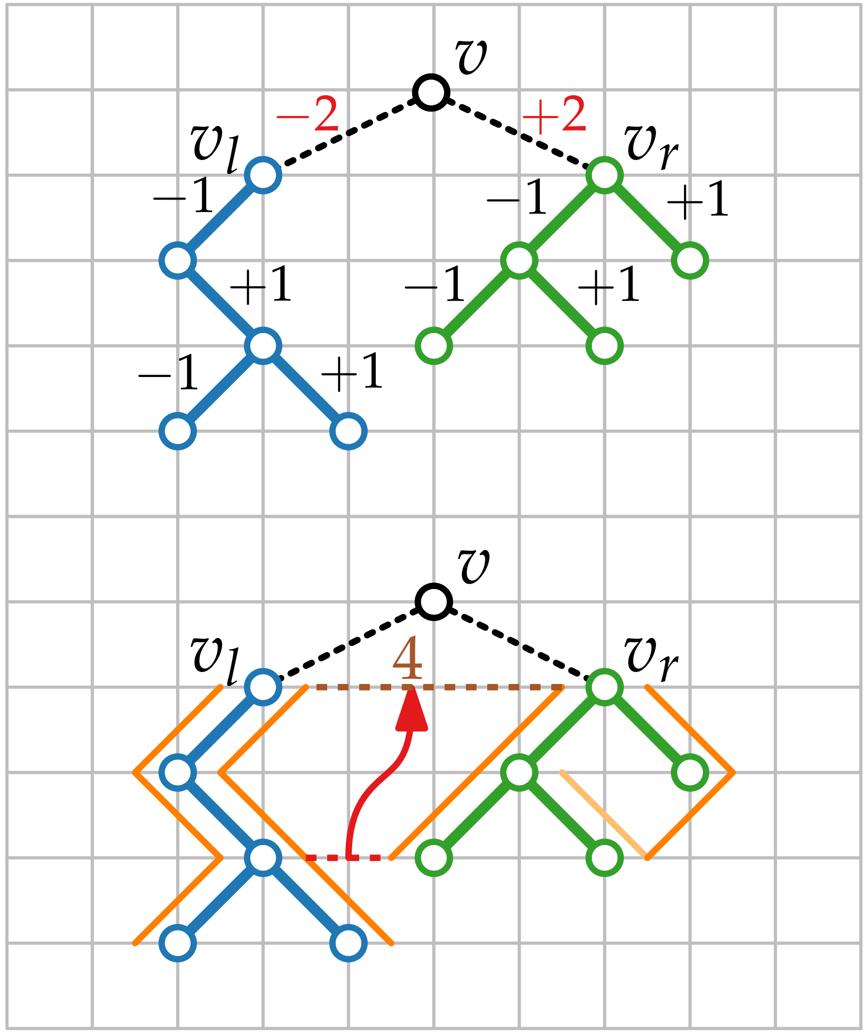

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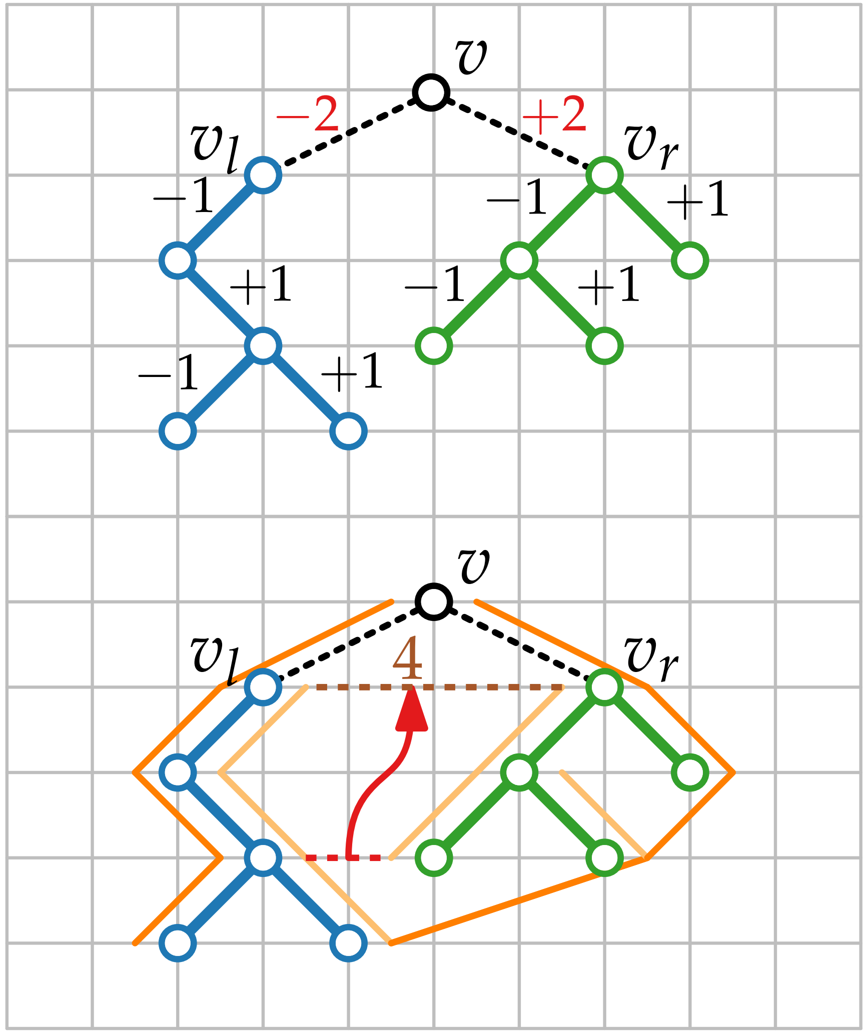

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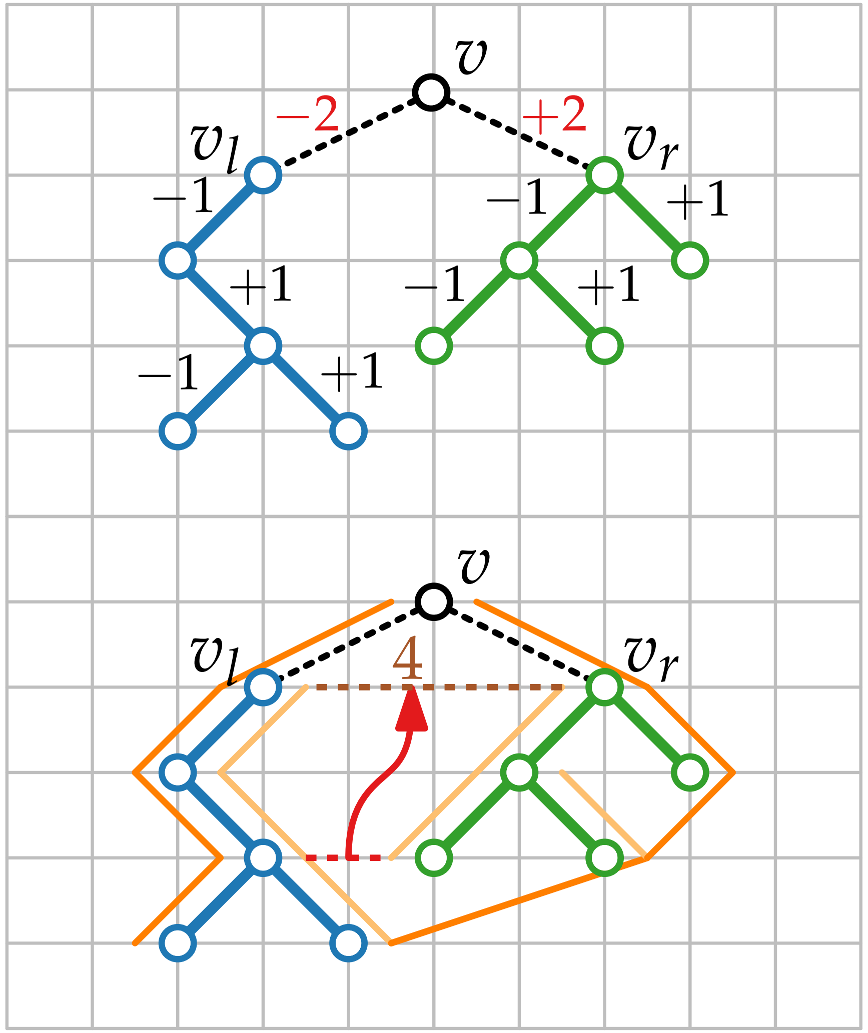

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Runtime?

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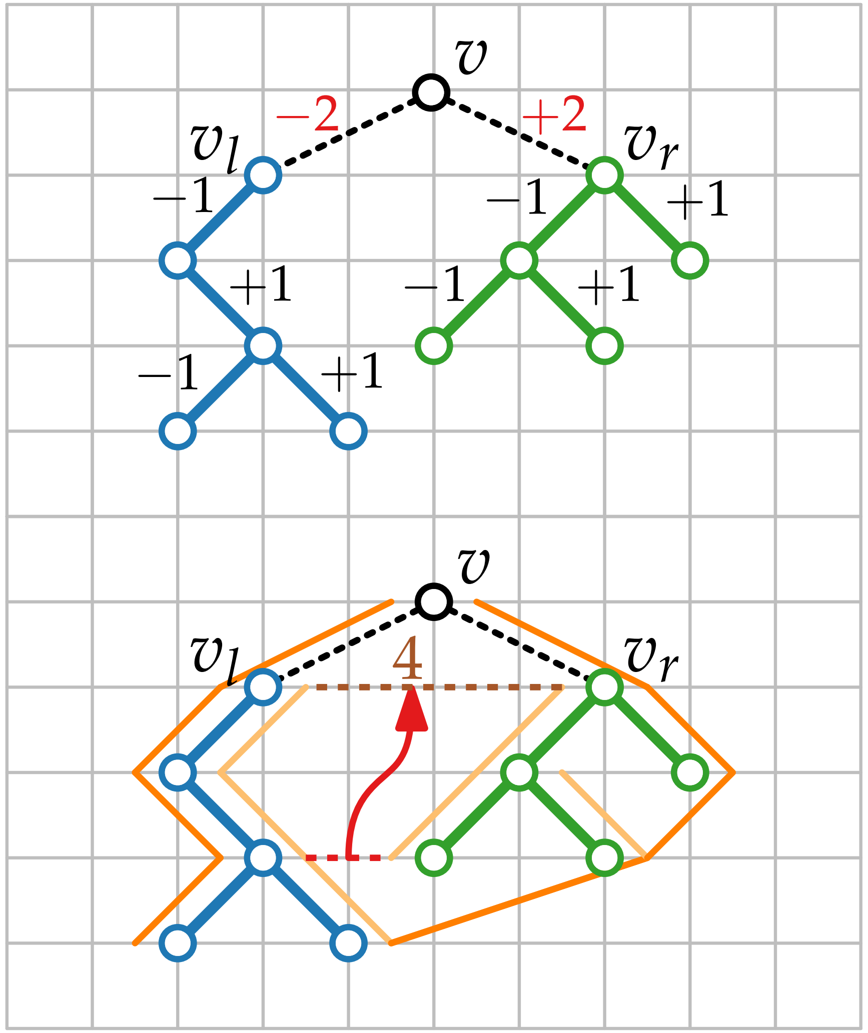

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Runtime?

- How often do we have to walk along a contour?

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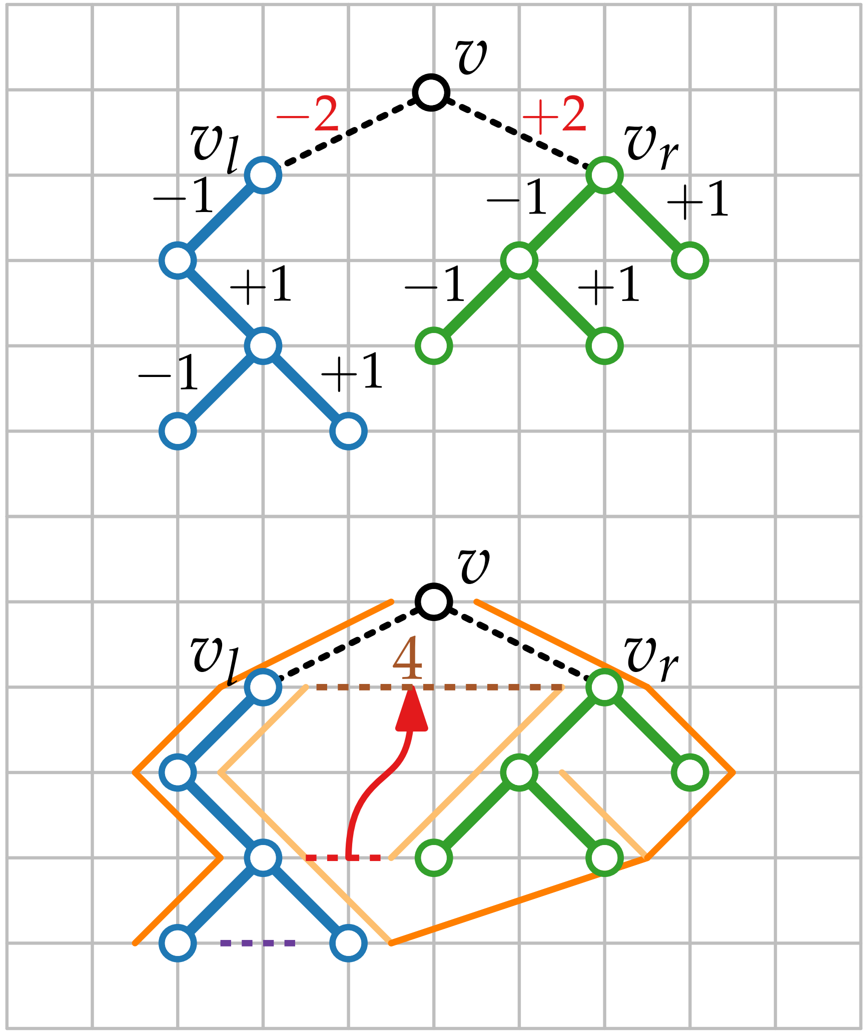

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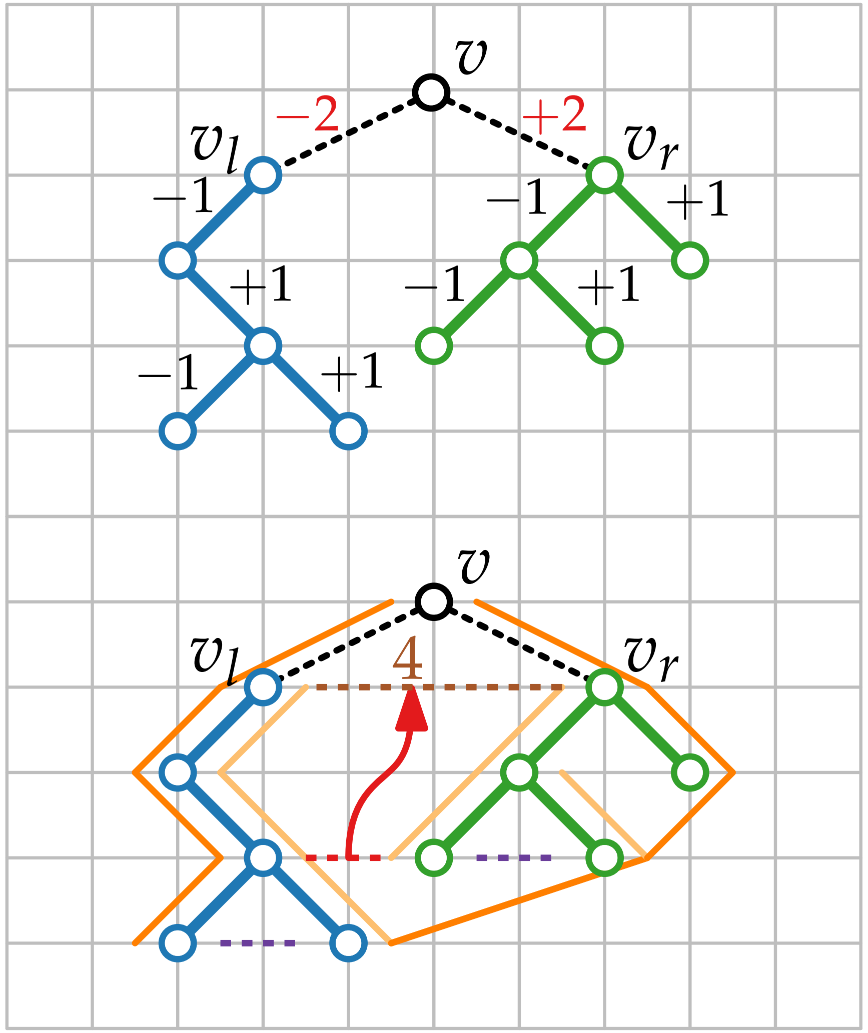

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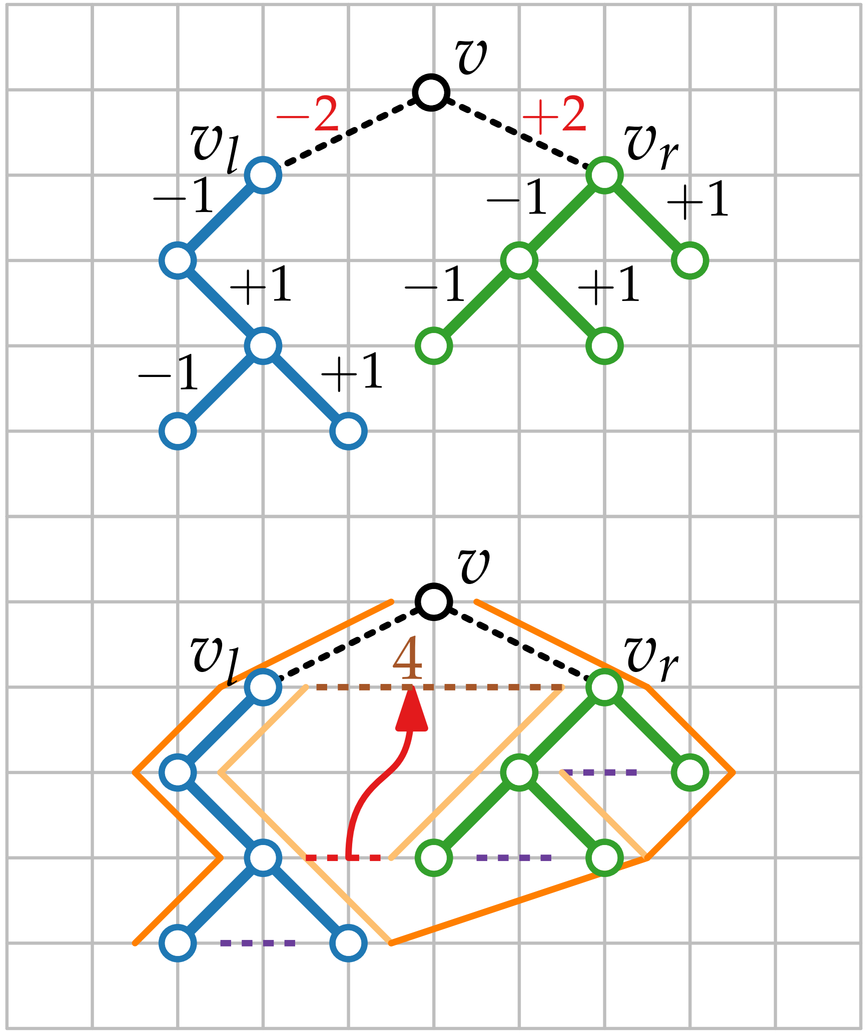

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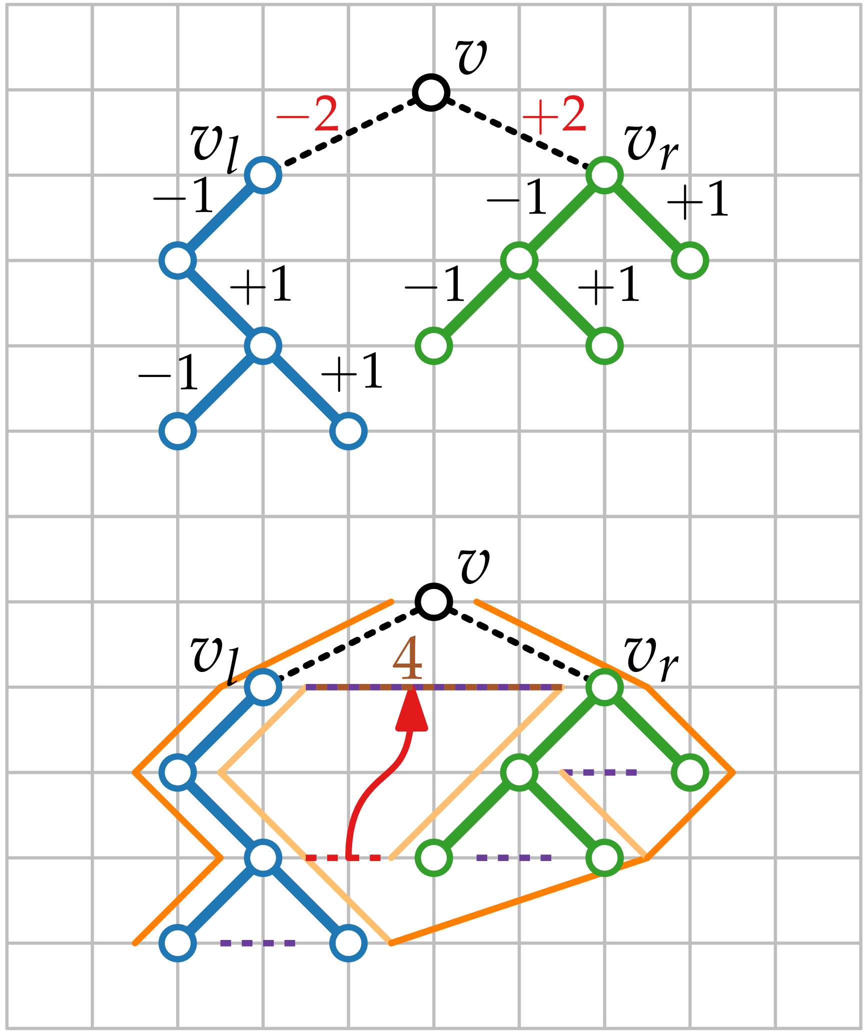

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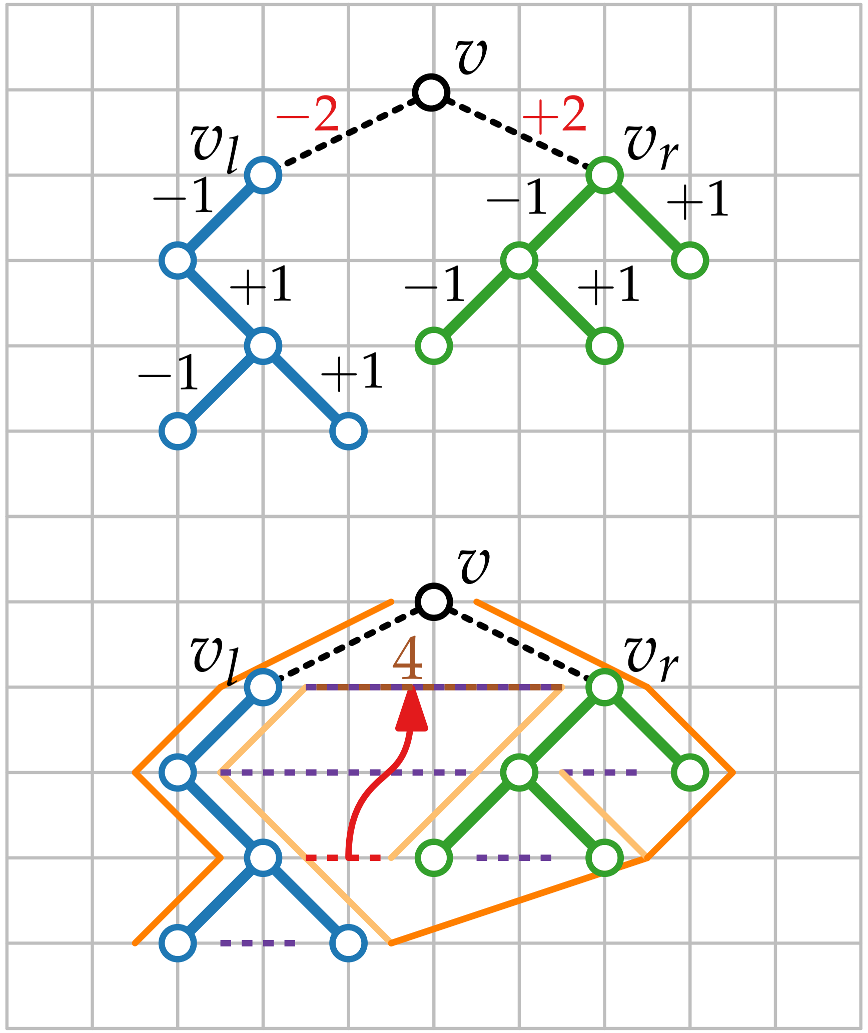

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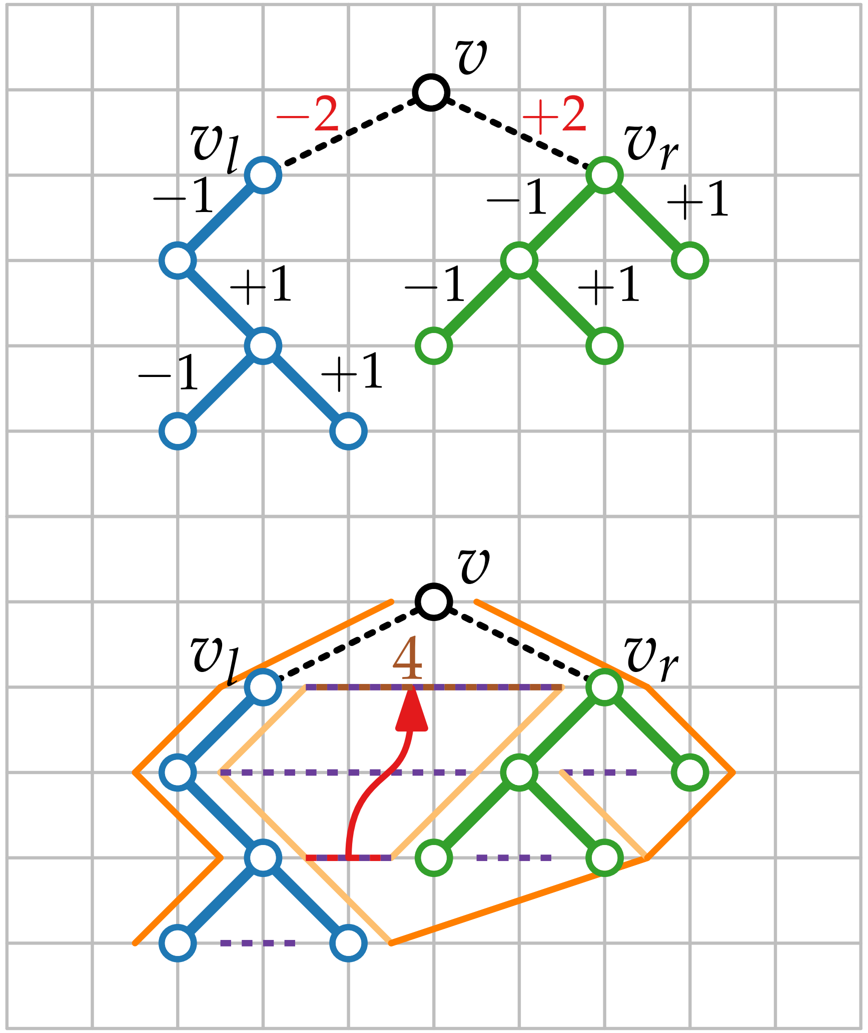


Layered Drawings – Algorithm Details

Phase 1 – postorder traversal:

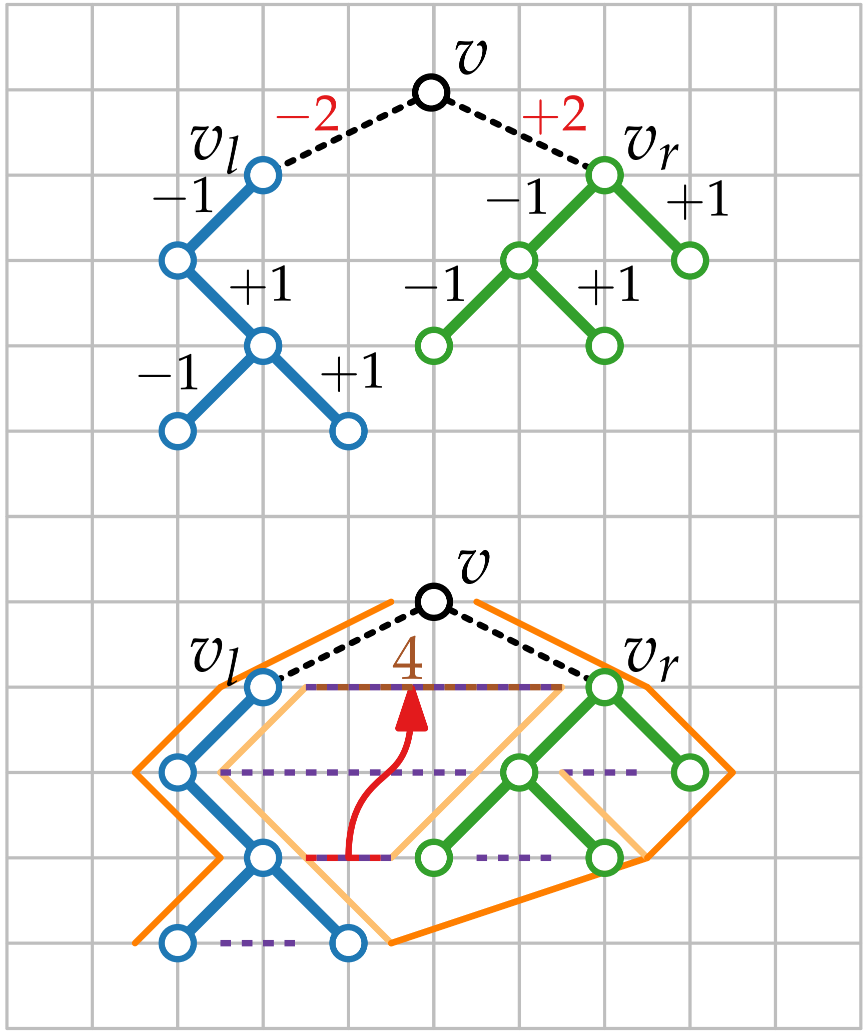
- For each vertex compute horizontal displacement of left and right child
- x-offset$(v_l) = -\lceil \frac{d_v}{2} \rceil$, x-offset$(v_r) = \lceil \frac{d_v}{2} \rceil$
- At vertex u (below v) store left and right contour of subtree $T(u)$
- Contour is linked list of vertex coordinates/offsets
- Find d_v = min. horiz. distance between v_l and v_r

Phase 2 – preorder traversal:

- Compute x- and y-coordinates

Runtime?

- How often do we have to walk along a contour? $\Rightarrow \mathcal{O}(n)$



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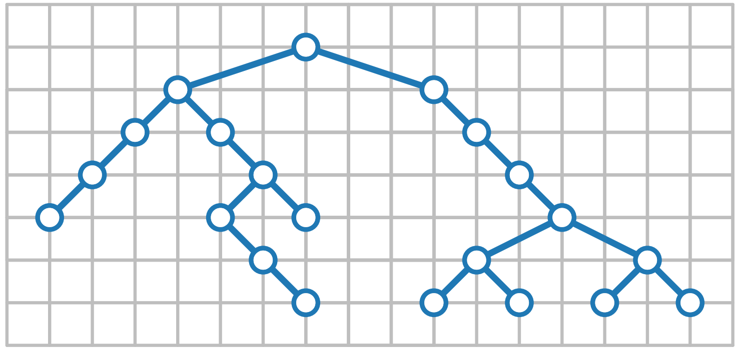

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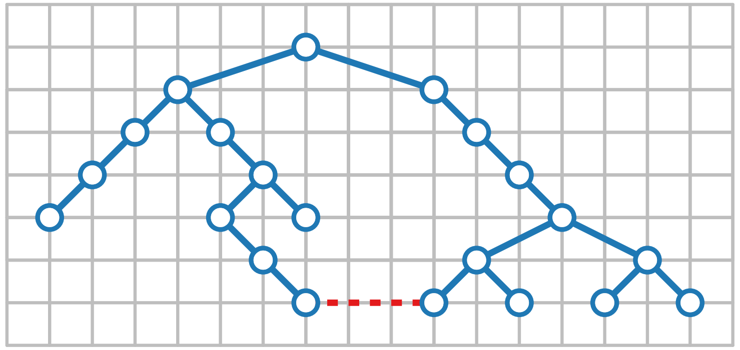

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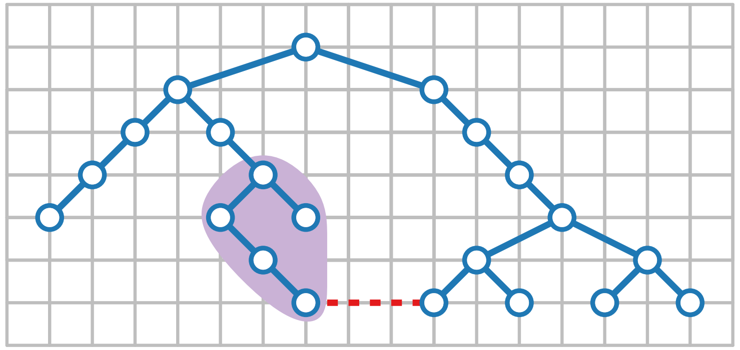

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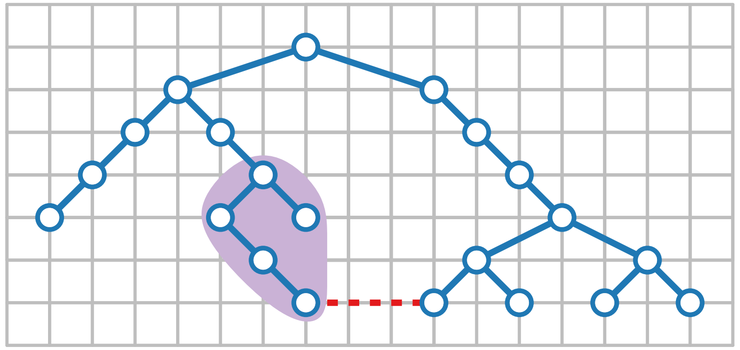

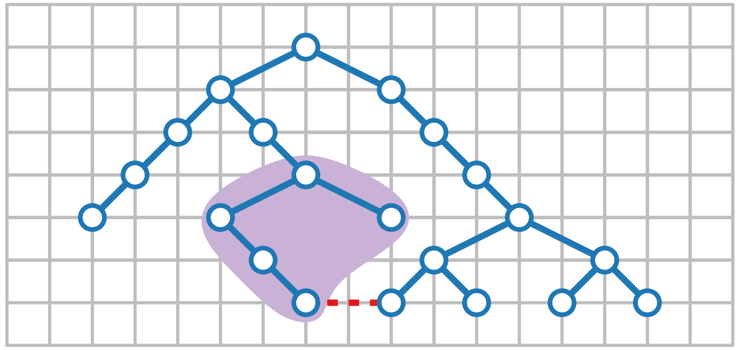

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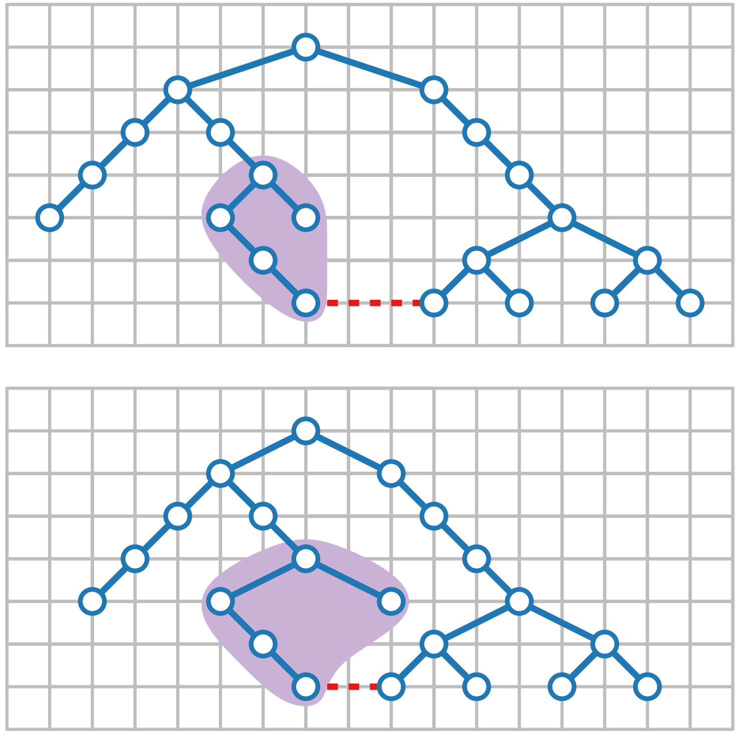

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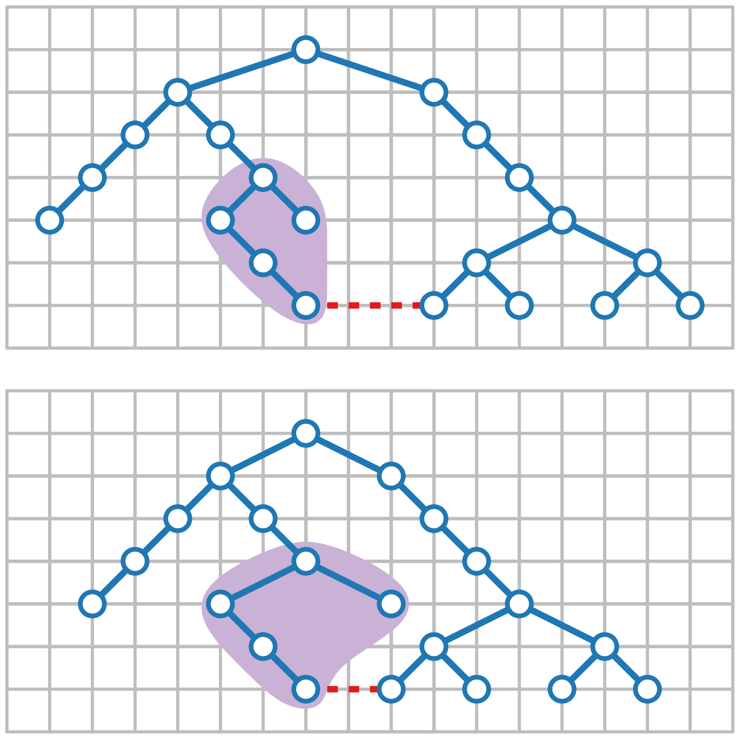

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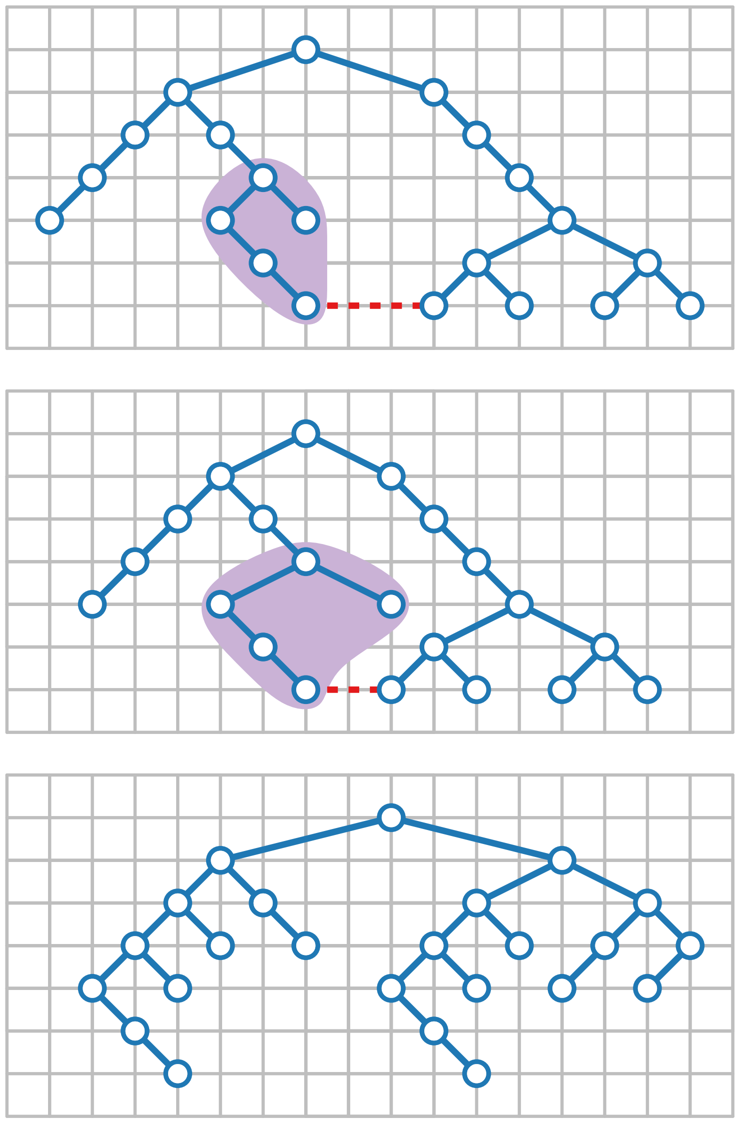

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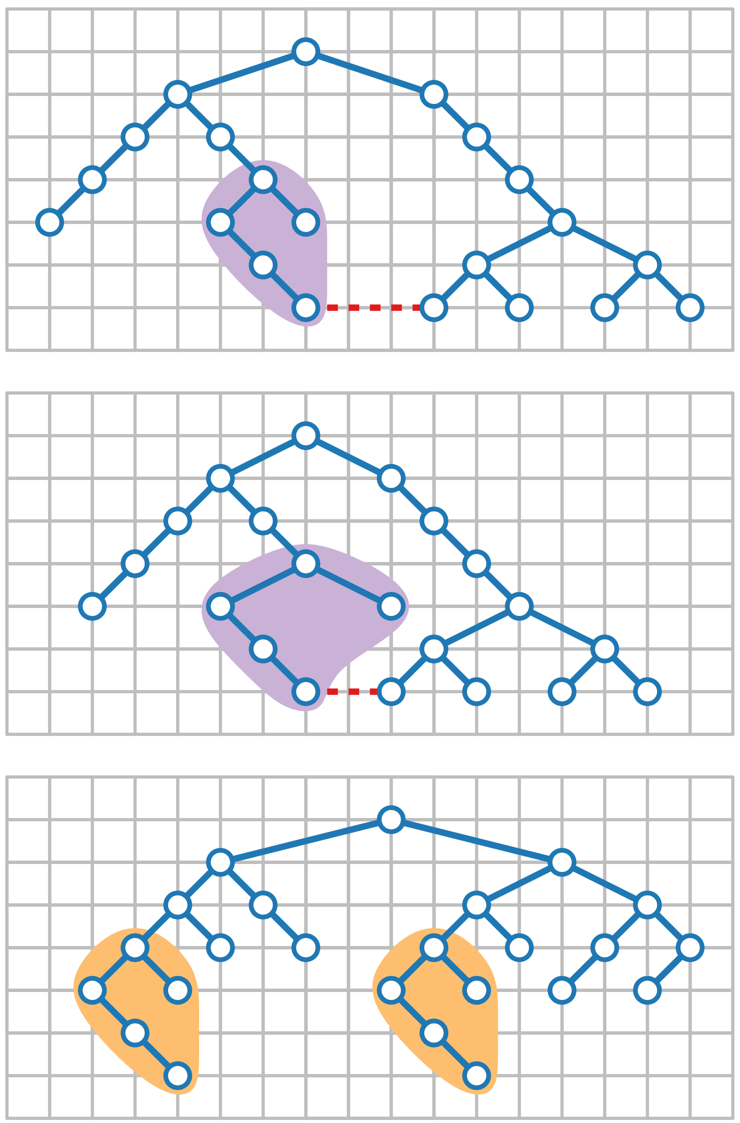

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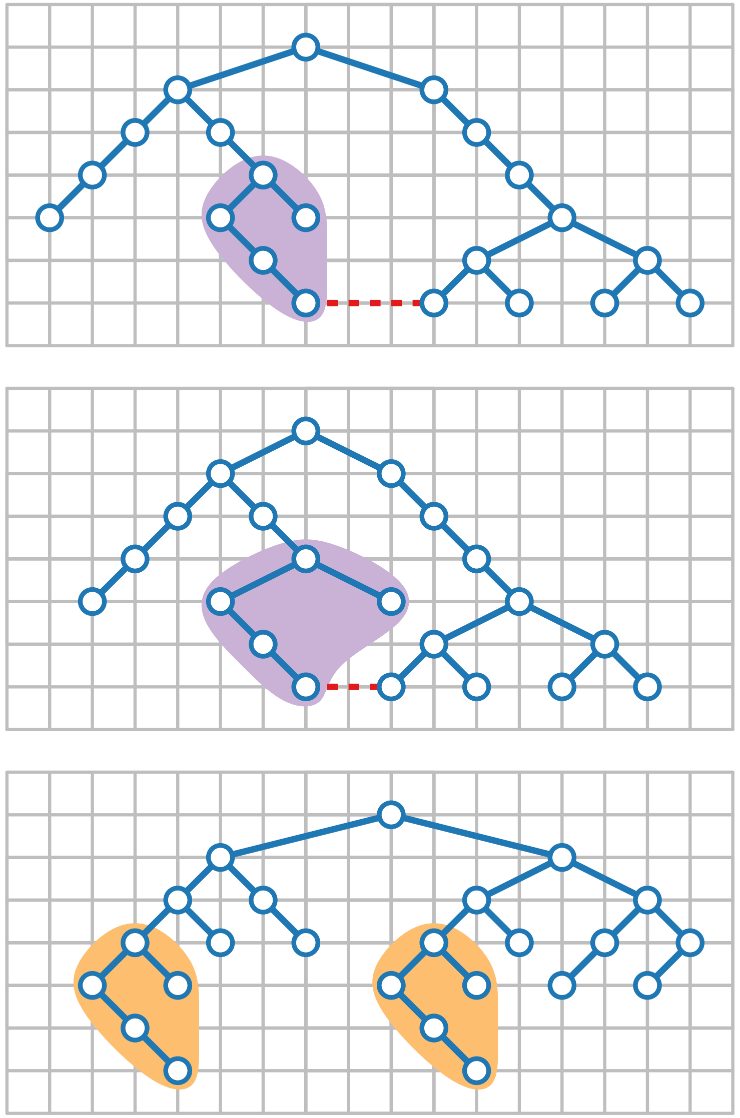

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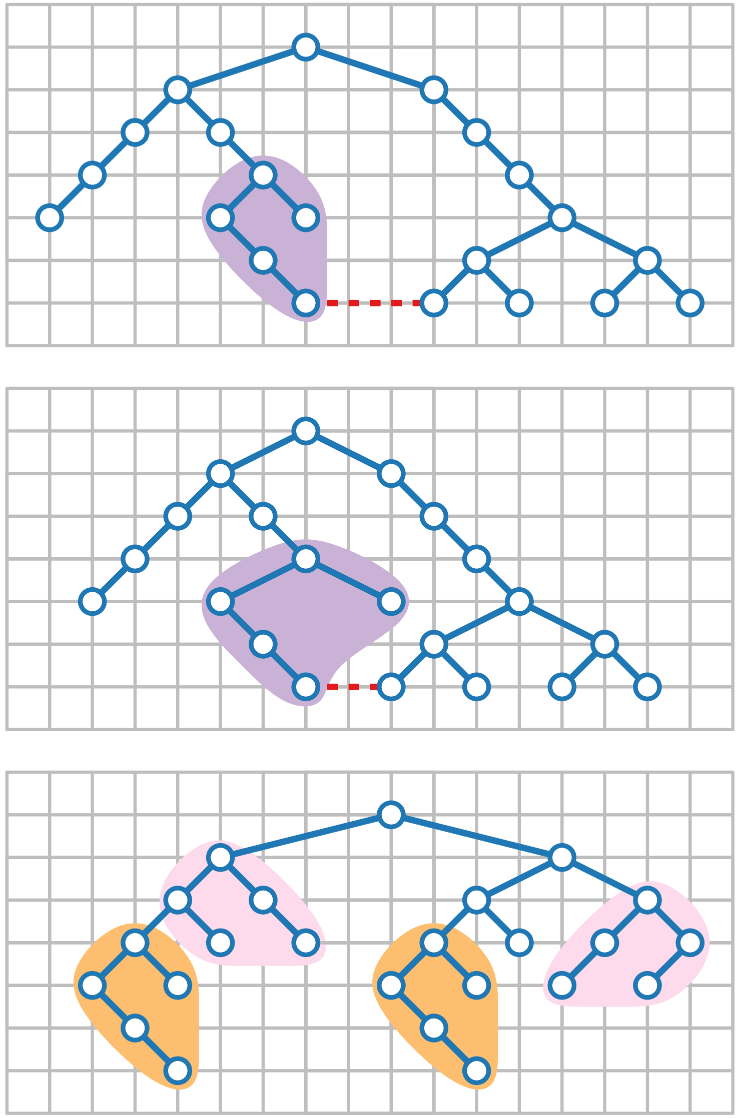

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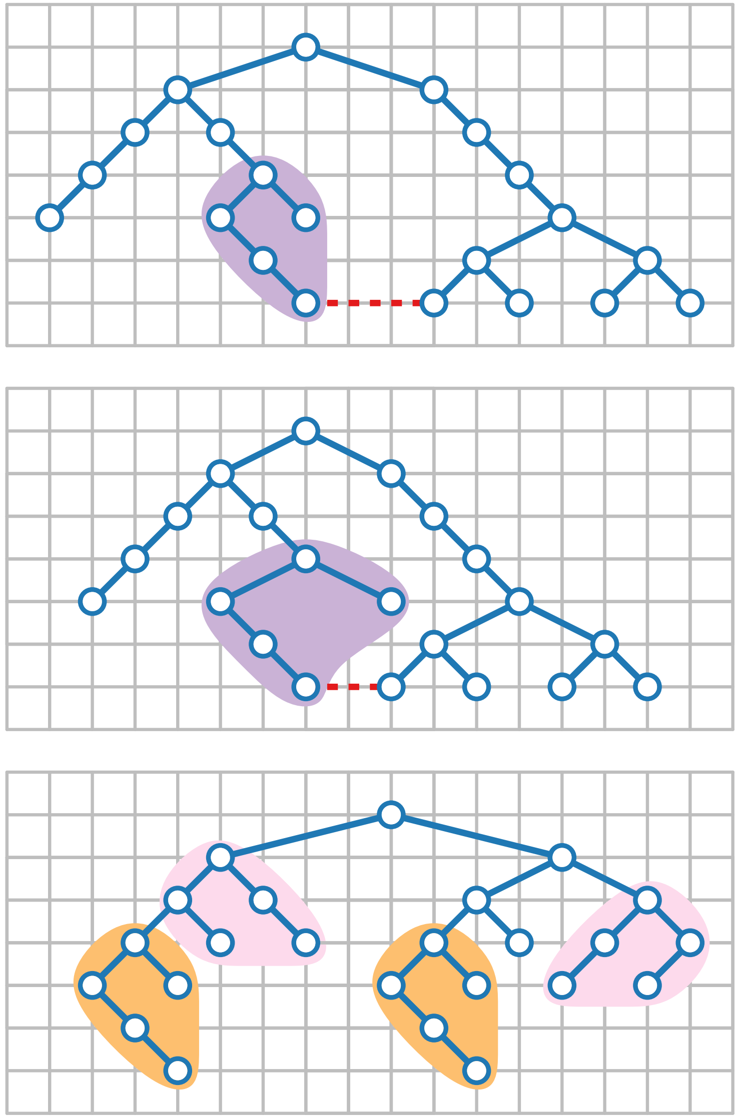

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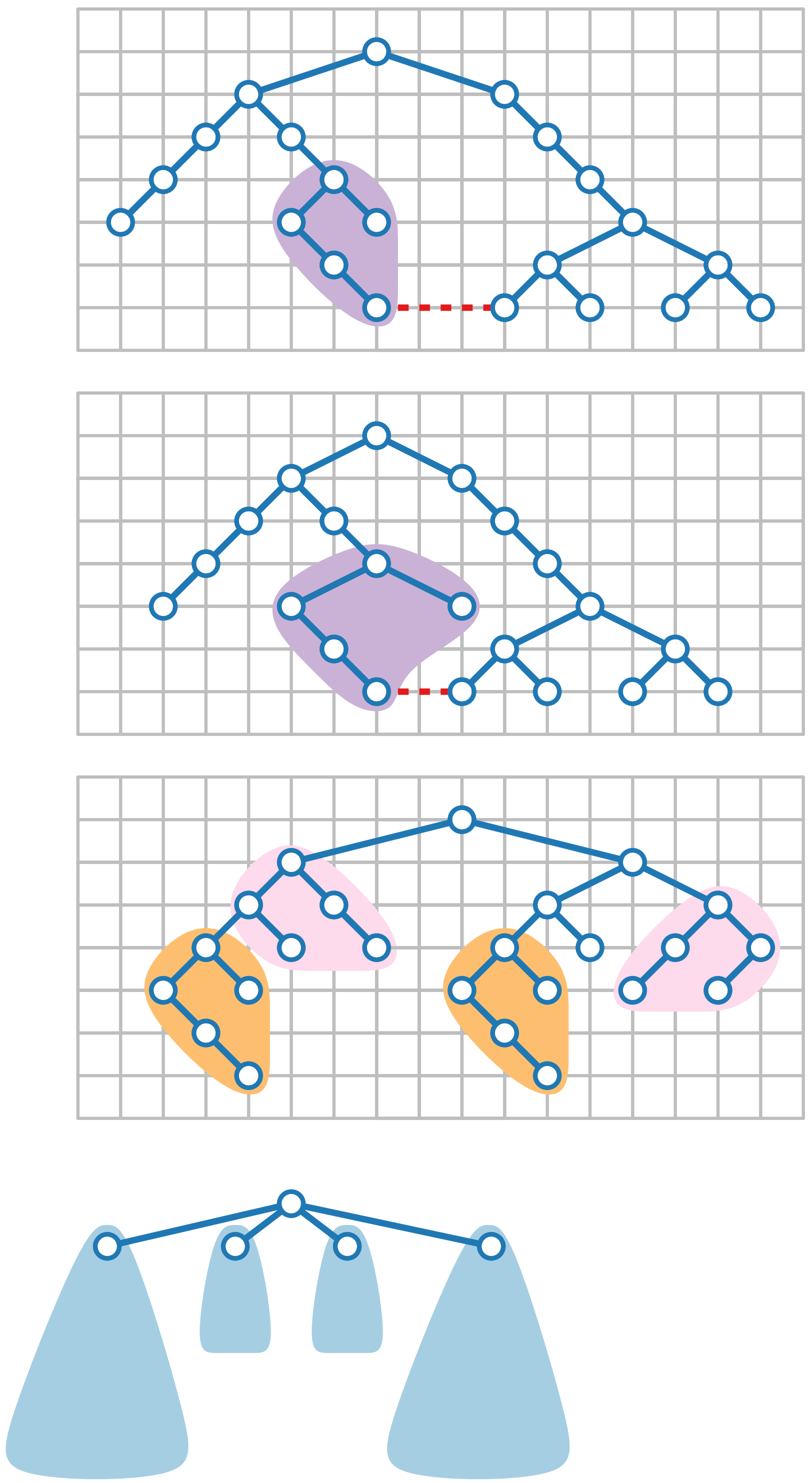

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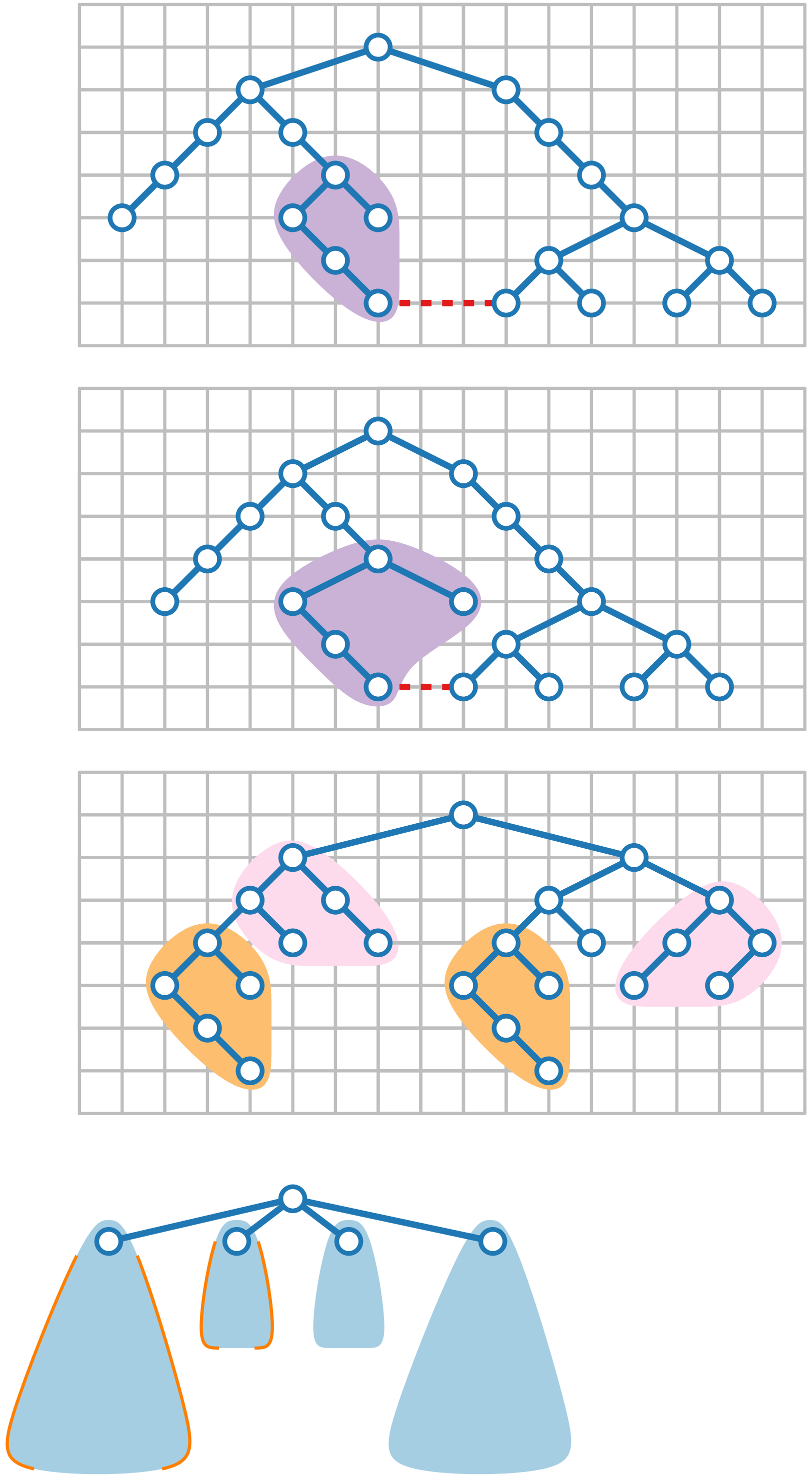

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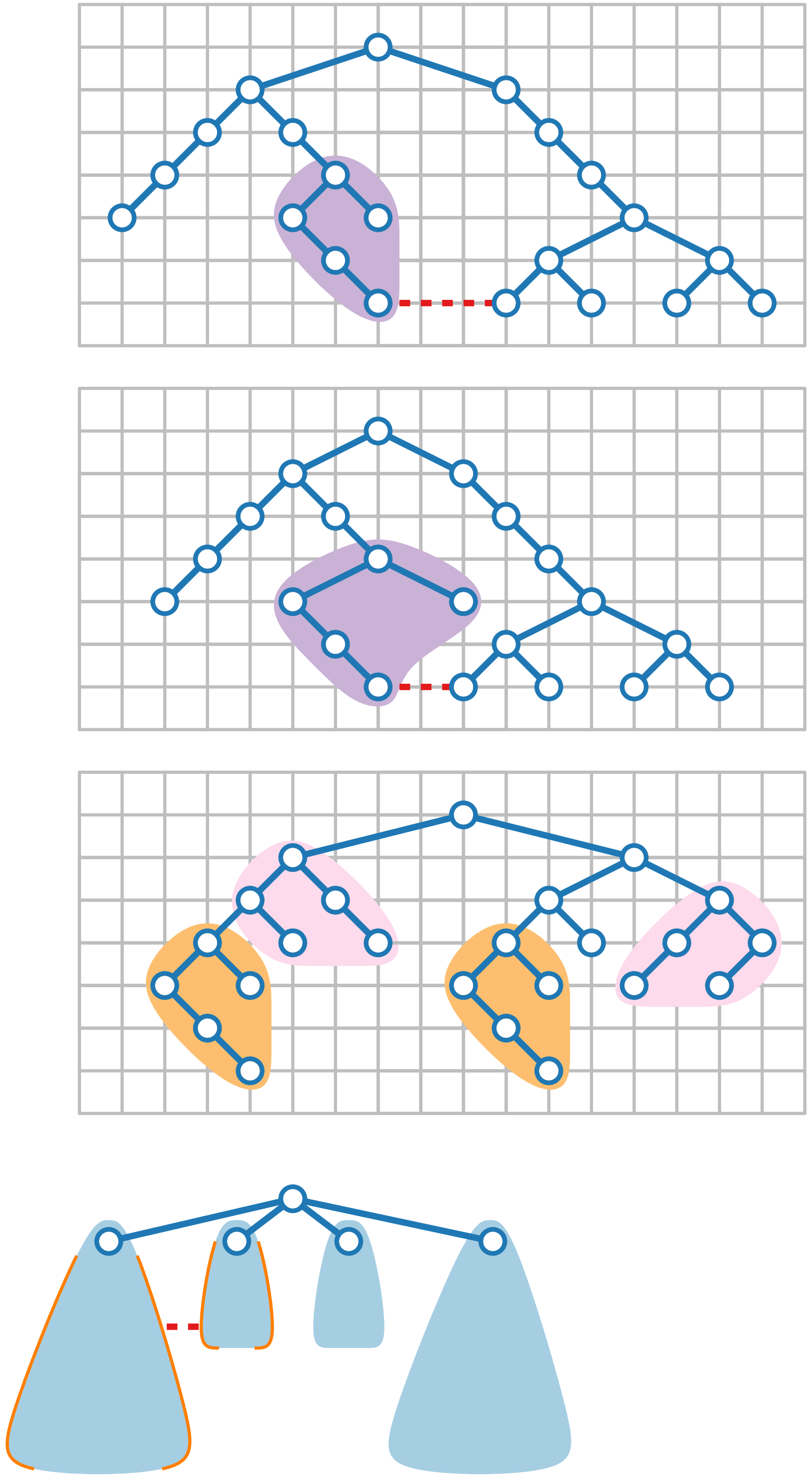

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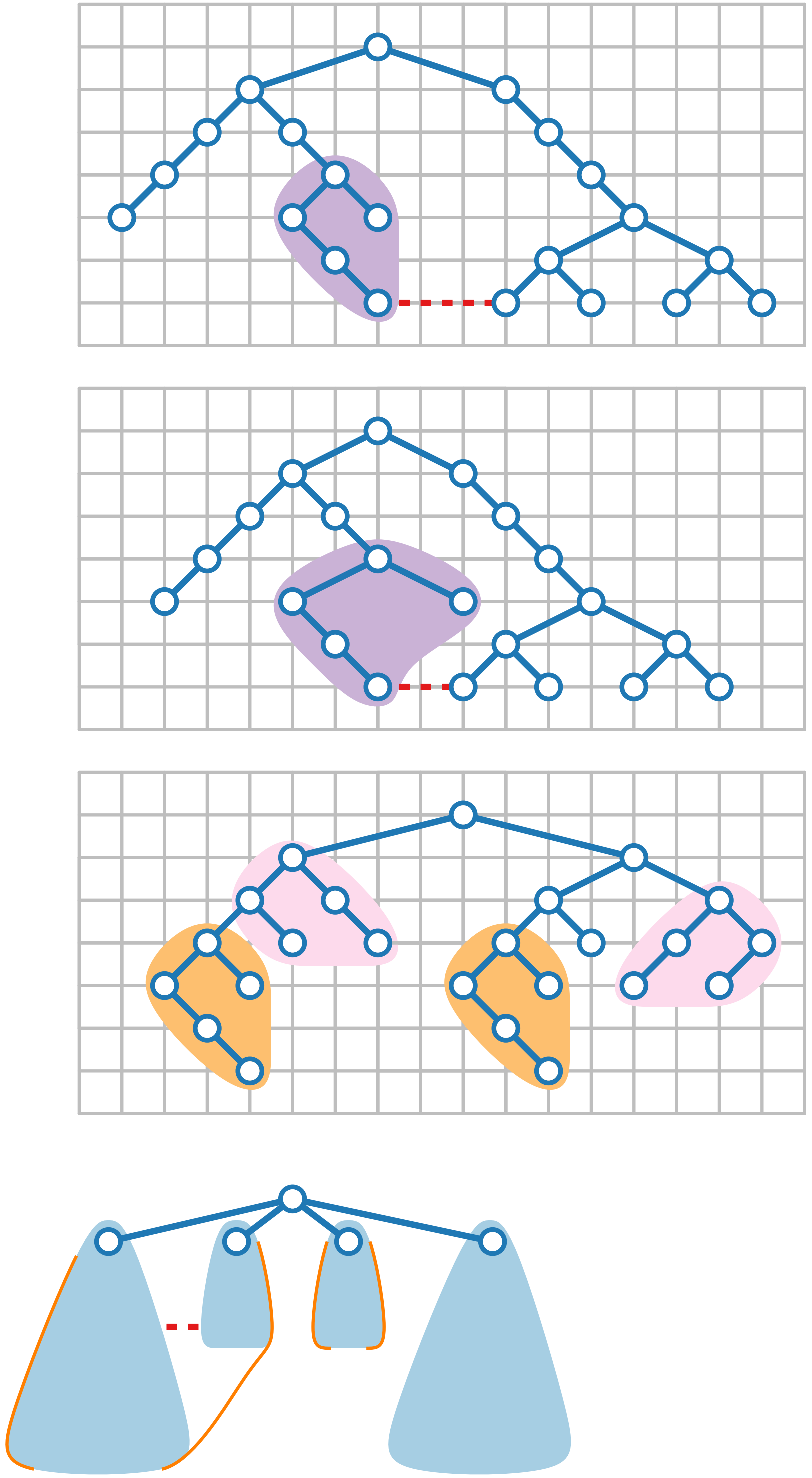

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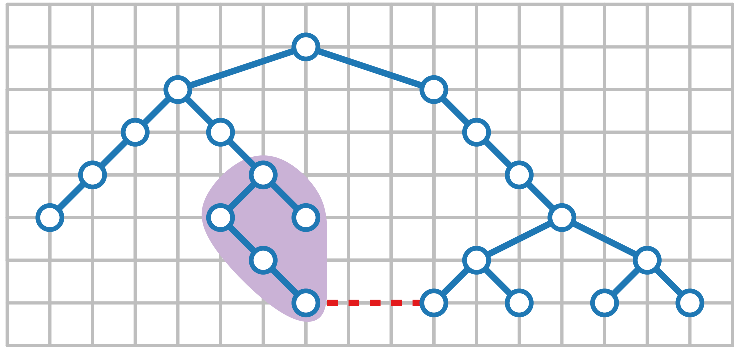

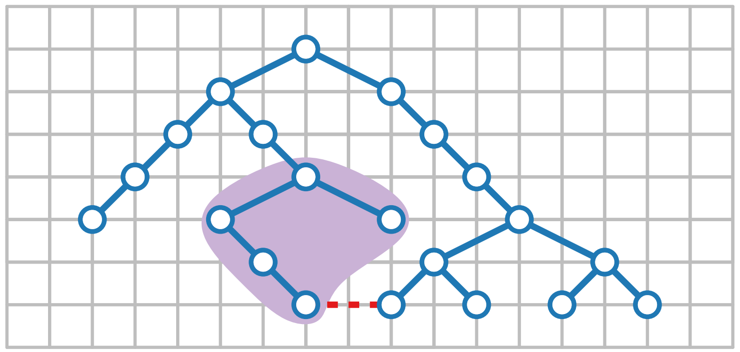

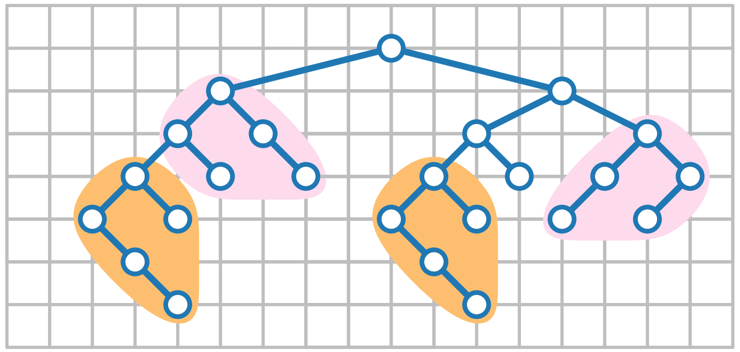

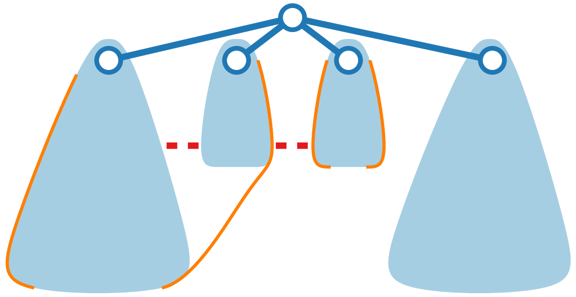

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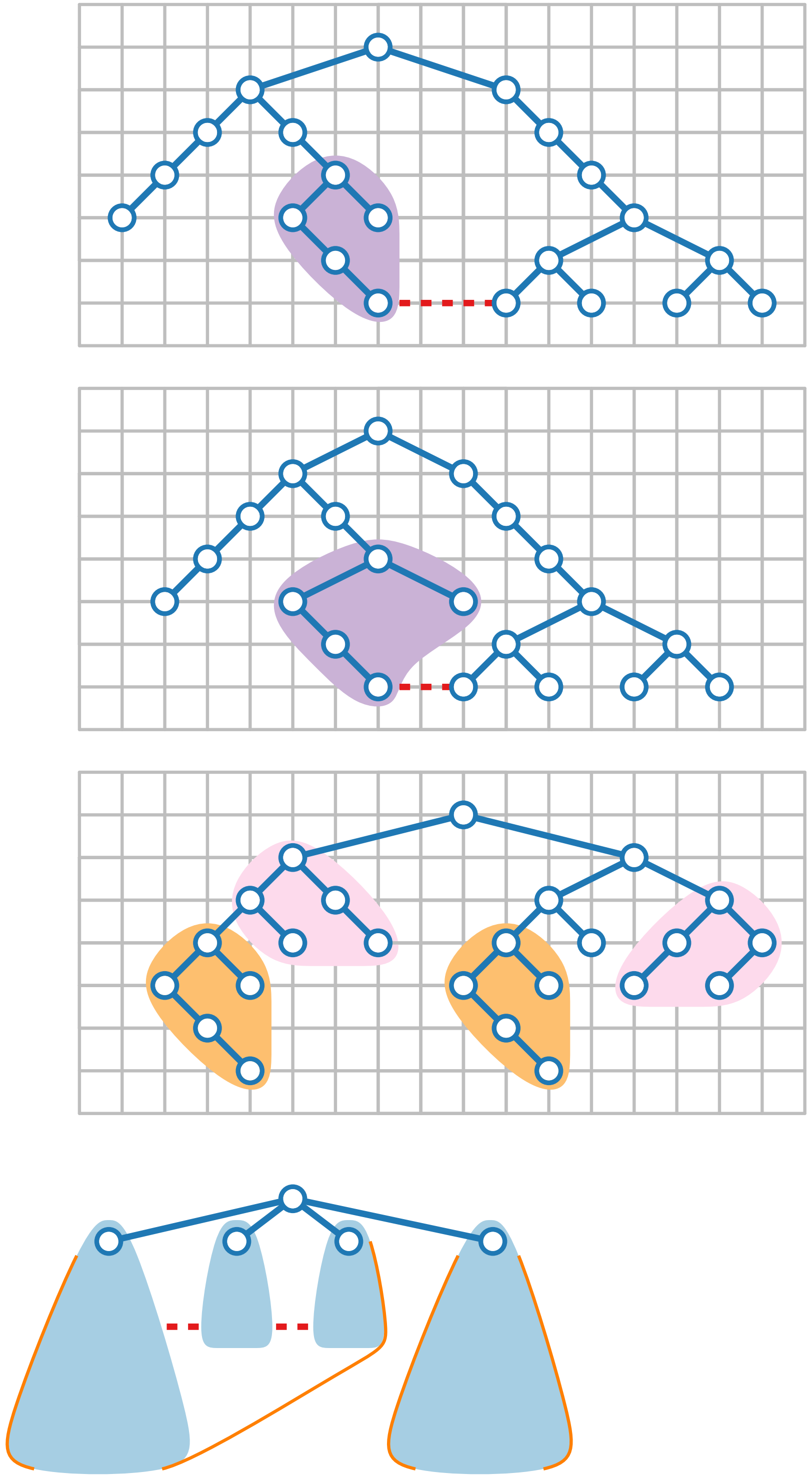

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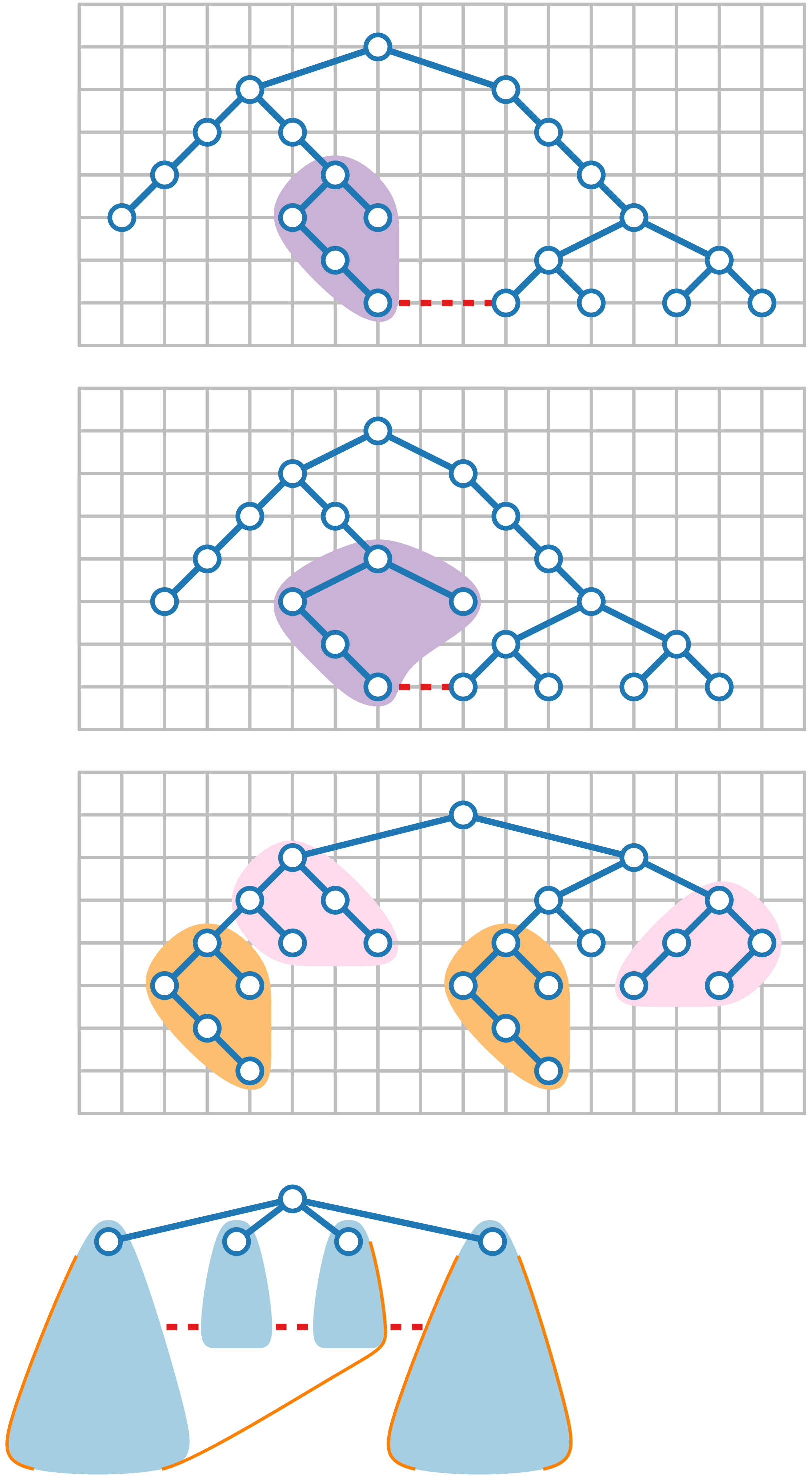

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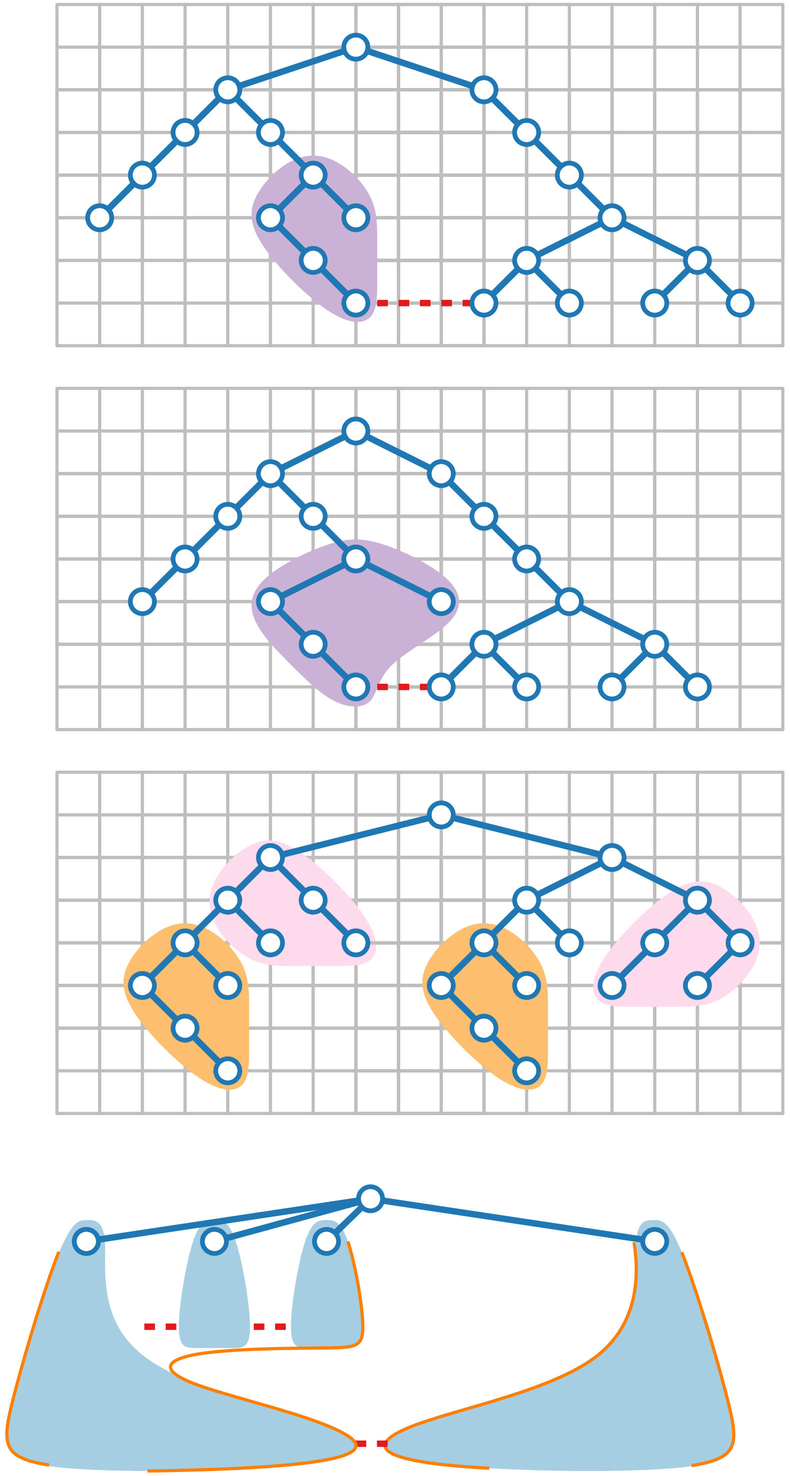

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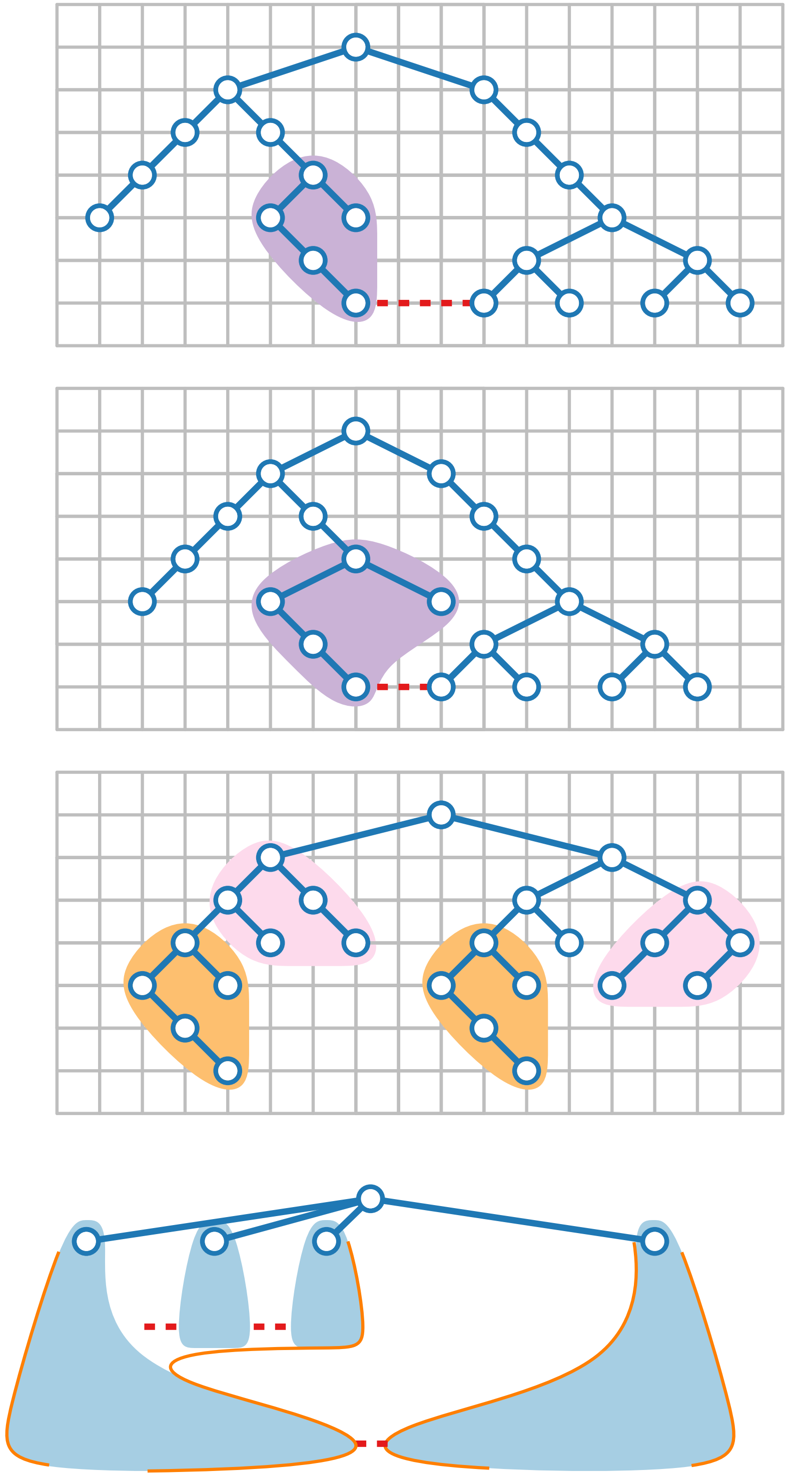

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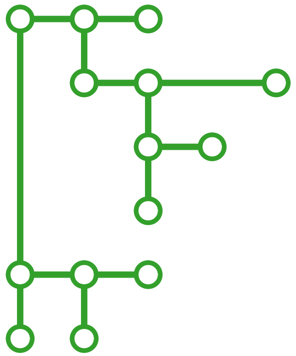
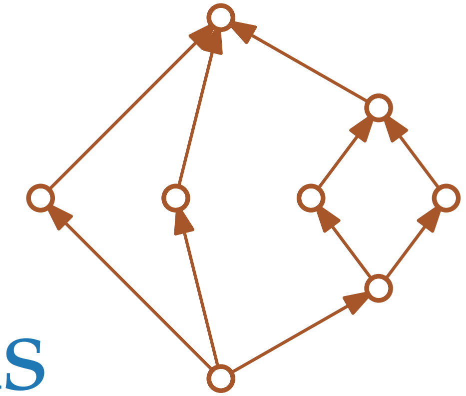
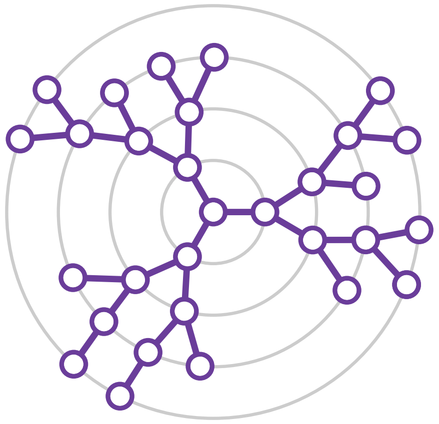


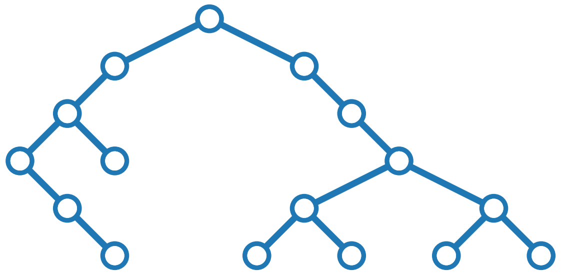
Visualization of Graphs



Lecture 2:
Drawing Trees and Series-Parallel Graphs

Part III:
HV-Drawings





Philipp Kindermann

HV-Drawings – Drawing Style

Applications

- Cons cell diagram in LISP

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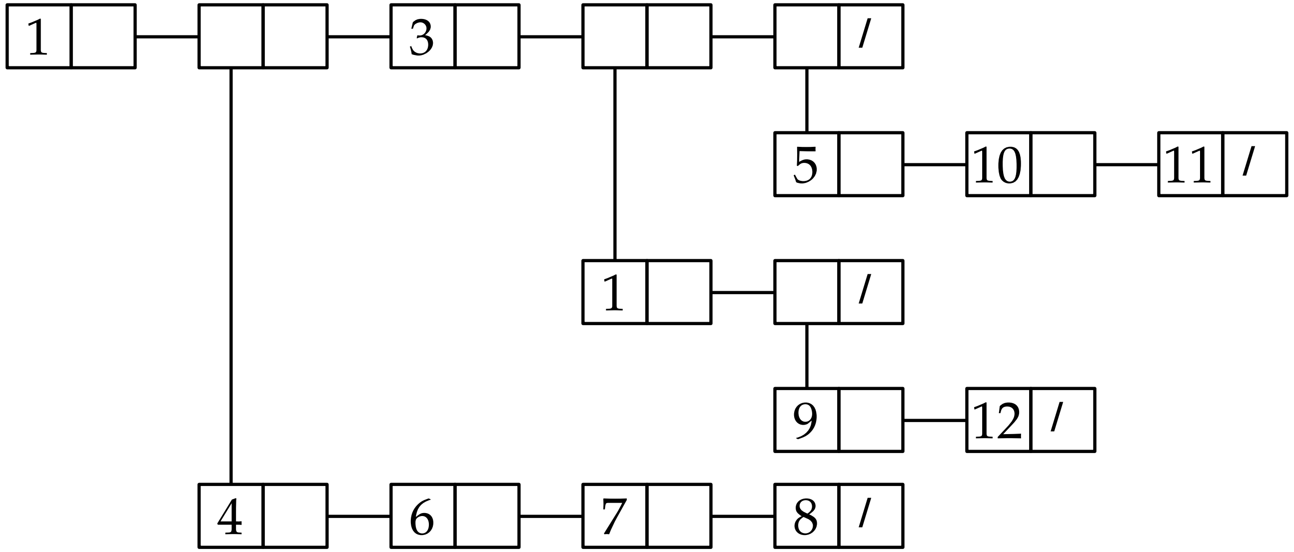

Source: after `gajon.org/trees-linked-lists-common-lisp/`

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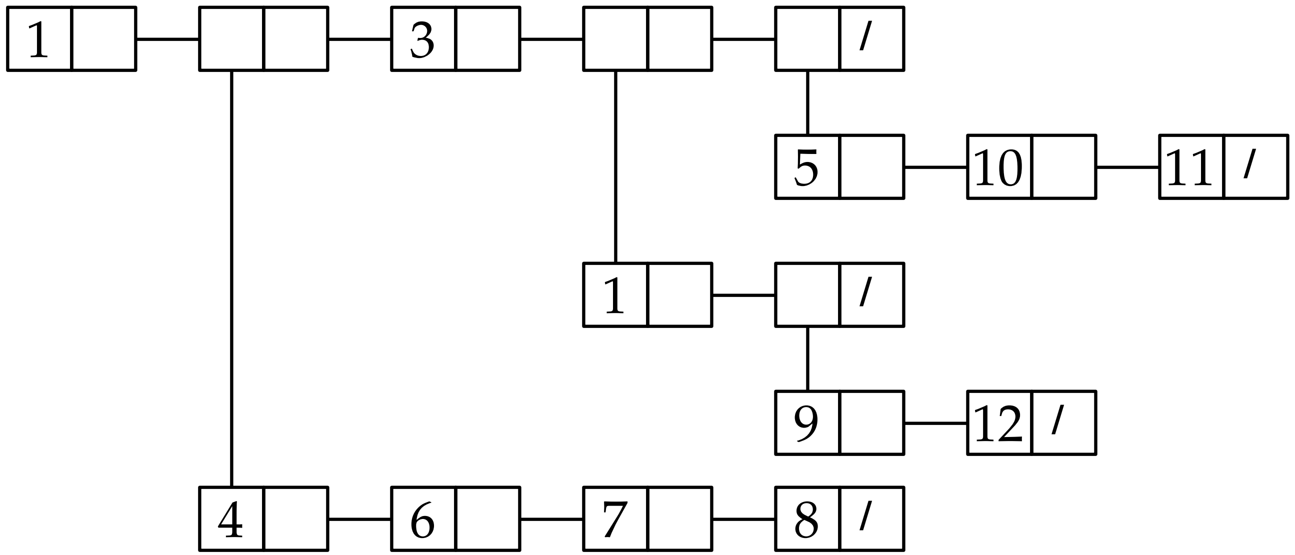

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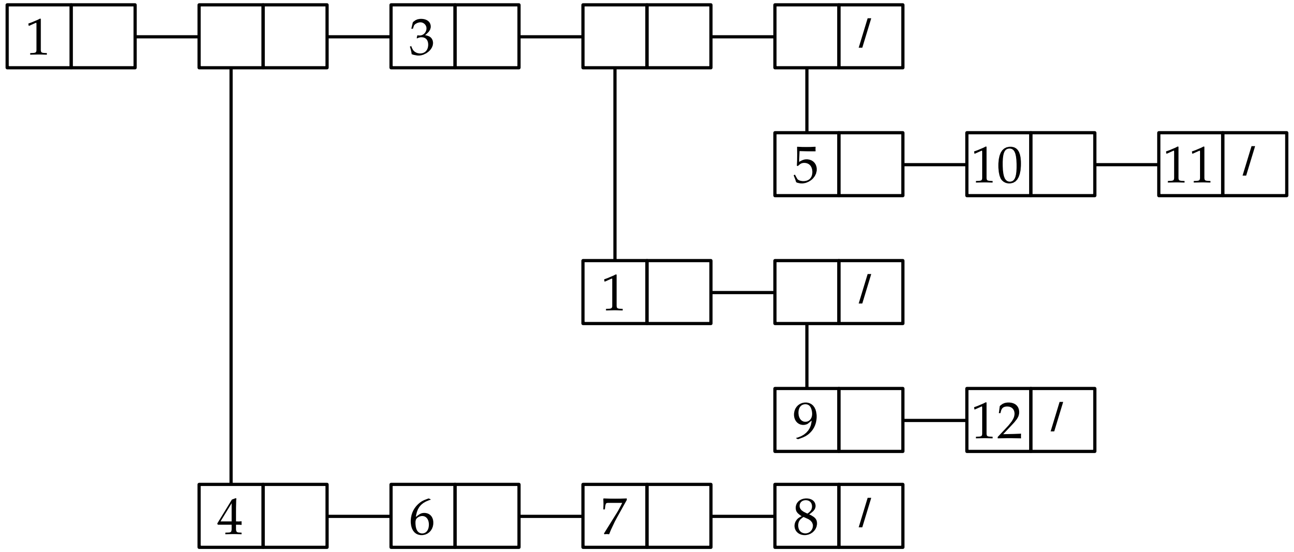

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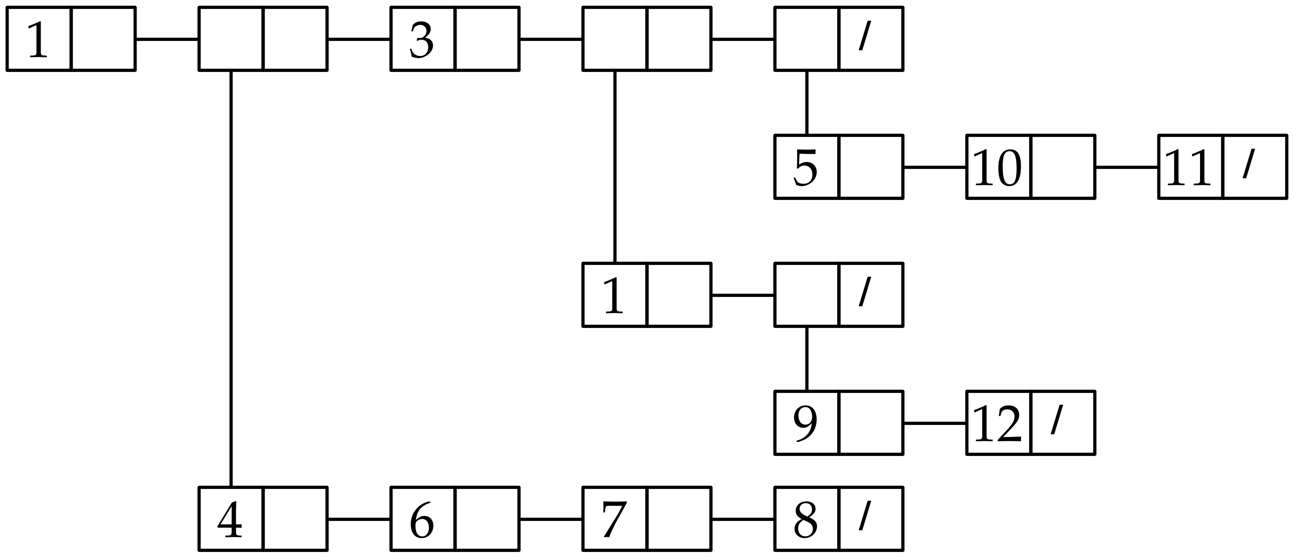

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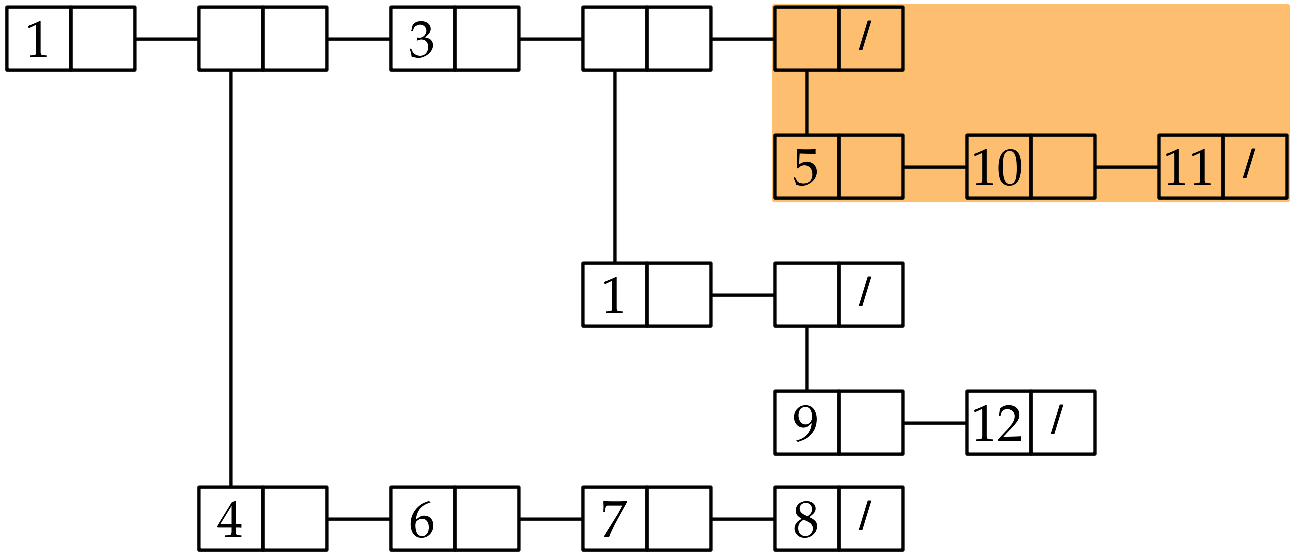
- The bounding boxes of the subtrees of the children are disjoint

Drawing aesthetics

HV-Drawings – Drawing Style

Applications

- Cons cell diagram in LISP
- *Cons*(constructs) are memory objects which hold two values or pointers to values



Source: after gajon.org/trees-linked-lists-common-lisp/

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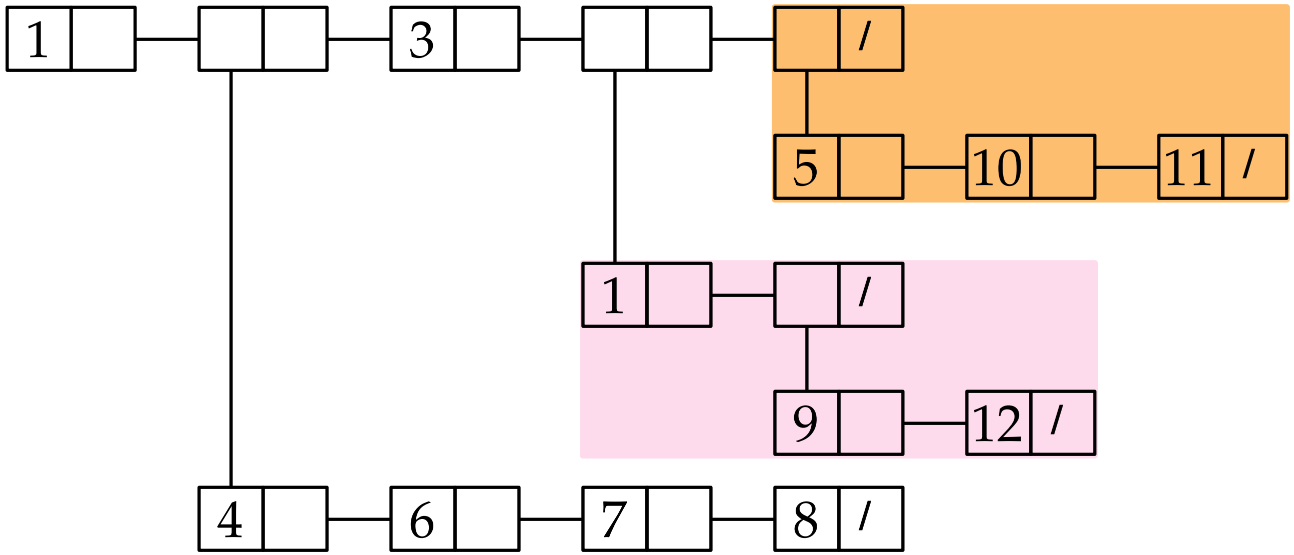

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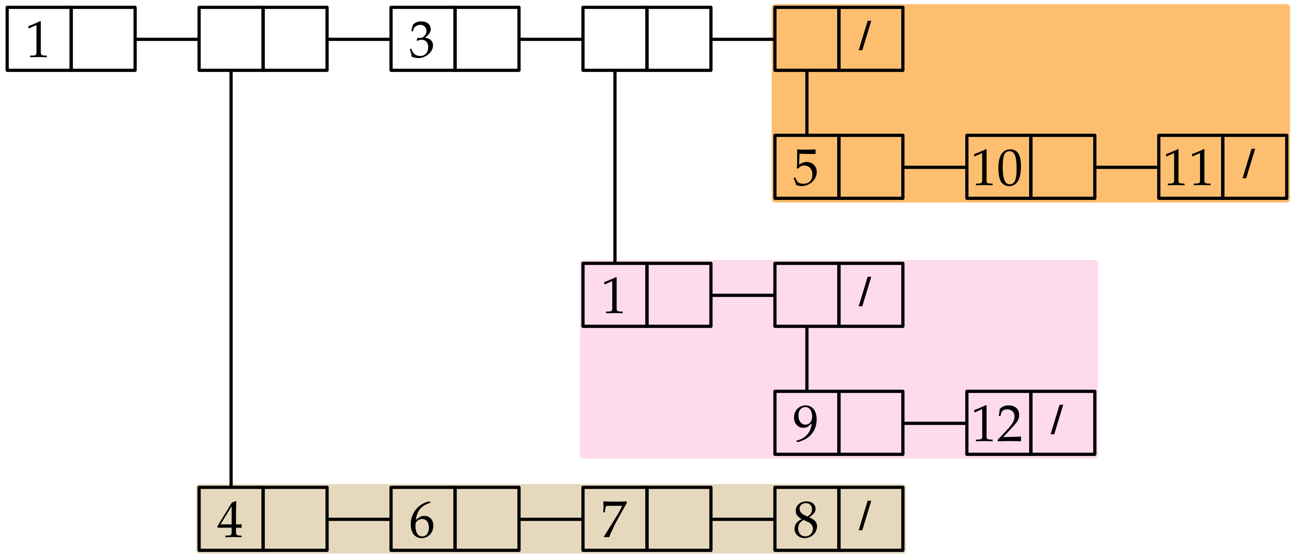

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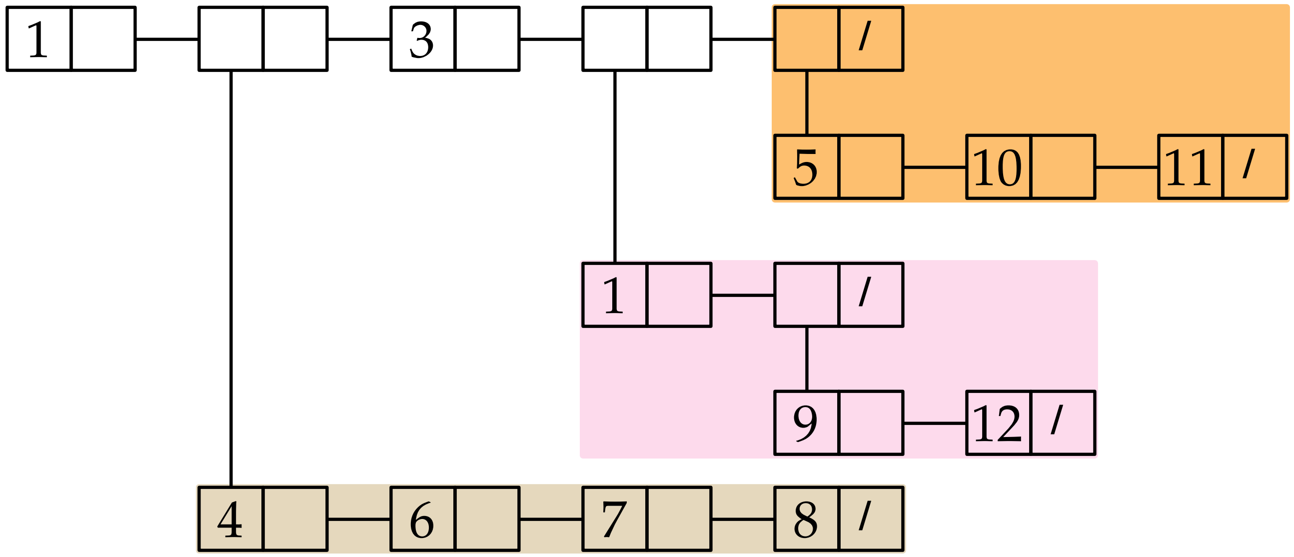

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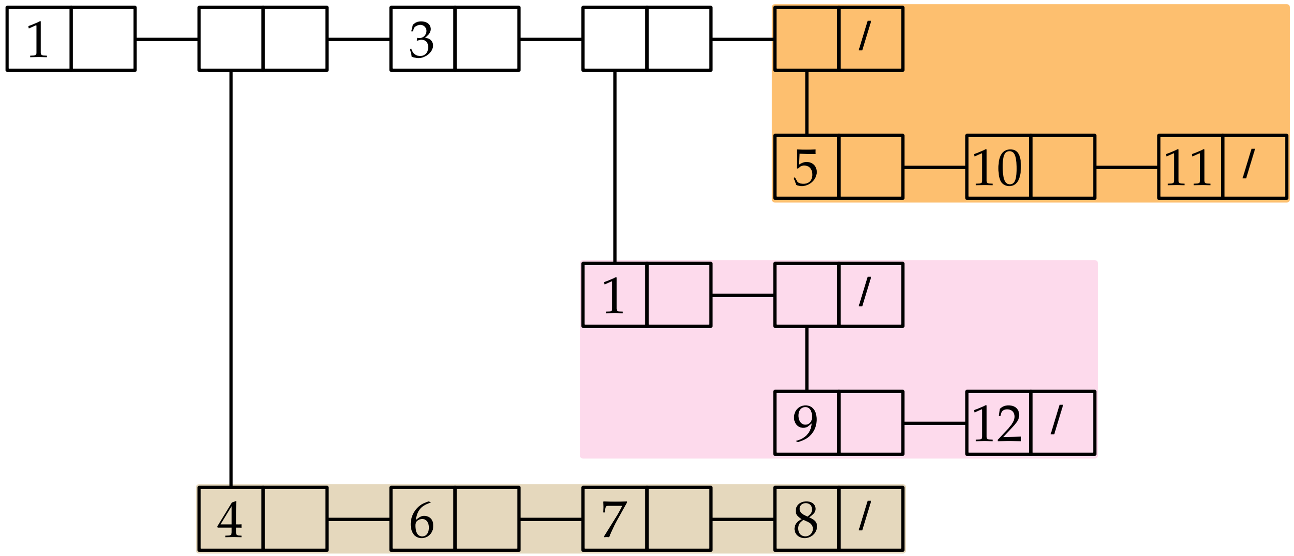

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Drawing conventions

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Drawing aesthetics

- Height, width, area

HV-Drawings – Algorithm

Input: A binary tree T
Output: An HV-drawing of T

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Divide: Recursively apply the algorithm to draw the left and right subtrees

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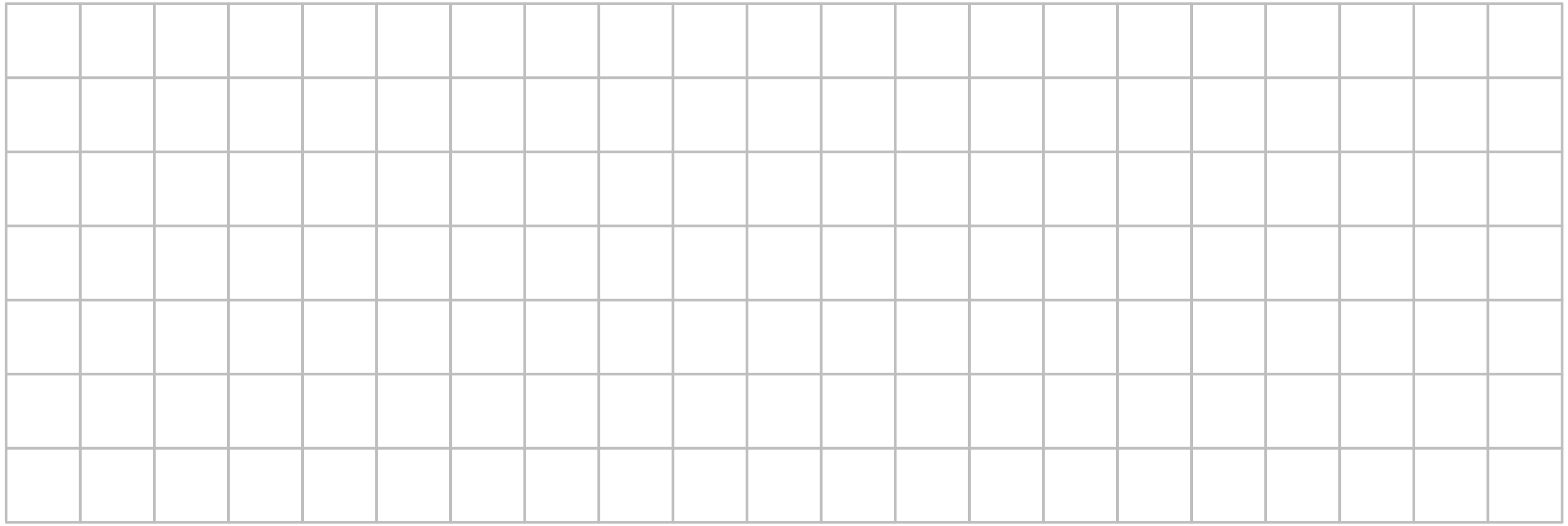
Conquer:



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Input: A binary tree T
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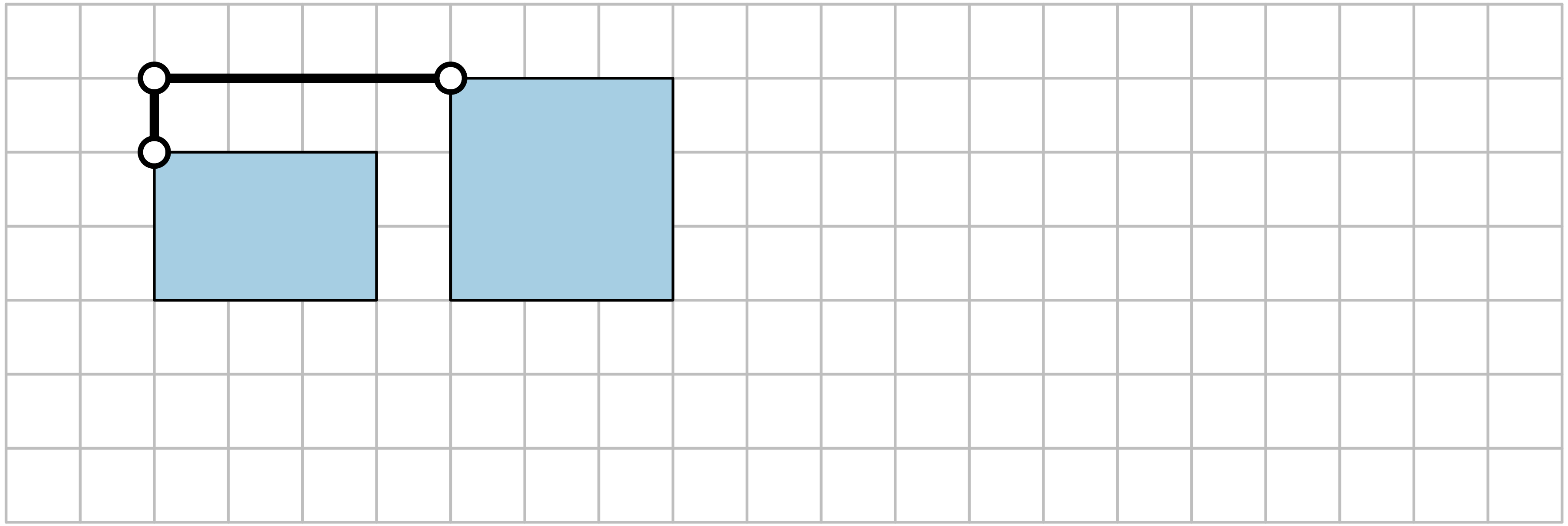
Conquer: horizontal combination



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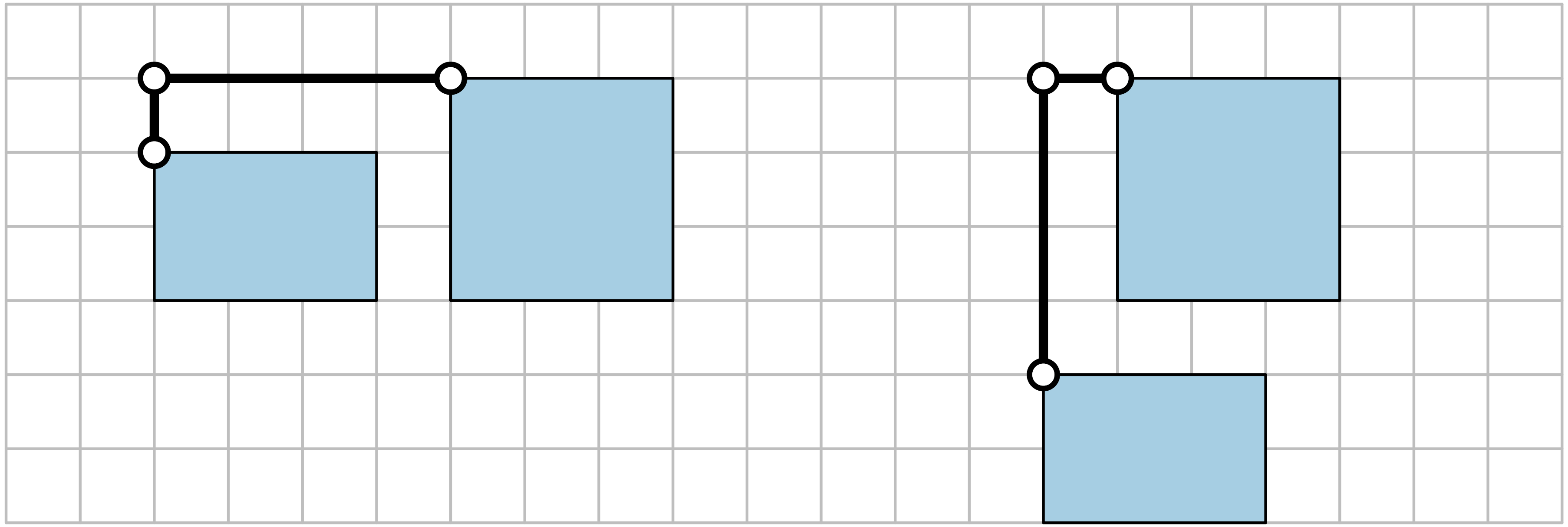

HV-Drawings – Right-Heavy HV-Layout

Right-heavy approach

- Always apply horizontal combination

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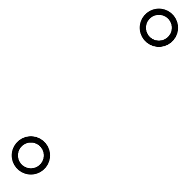

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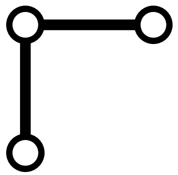

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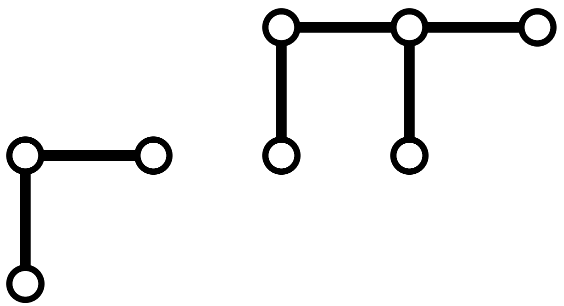

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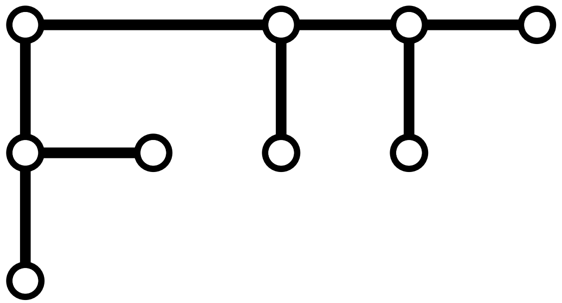

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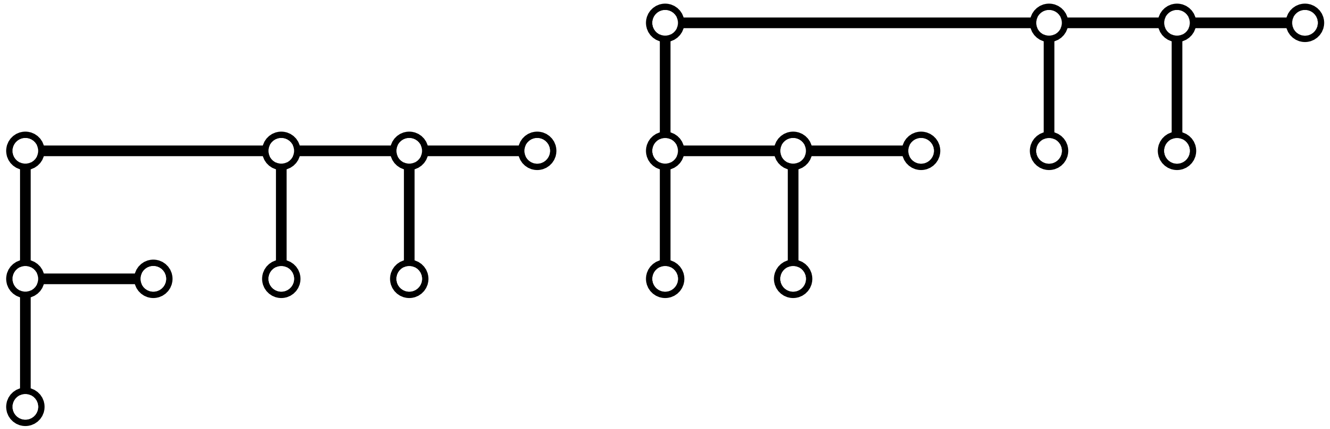

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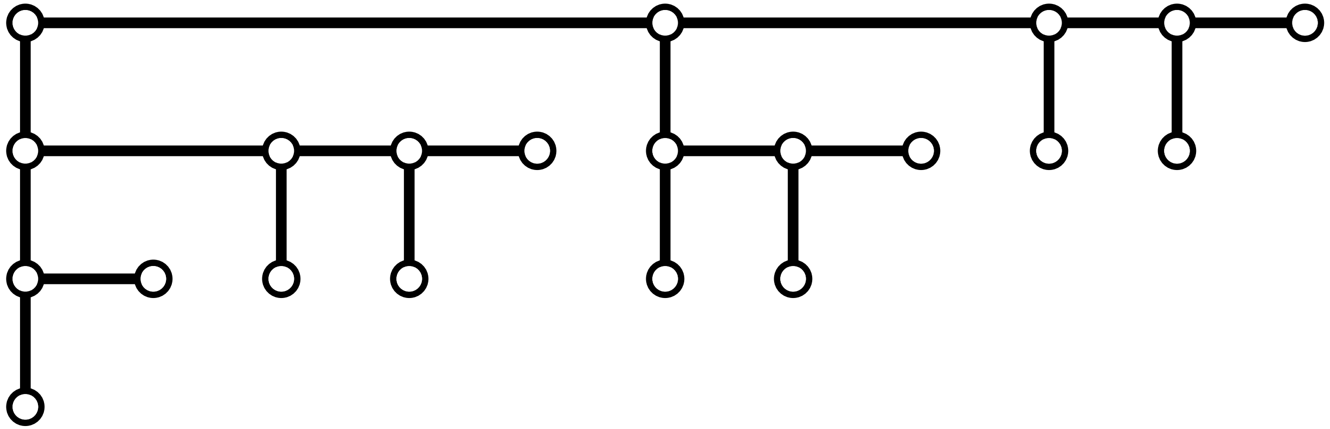

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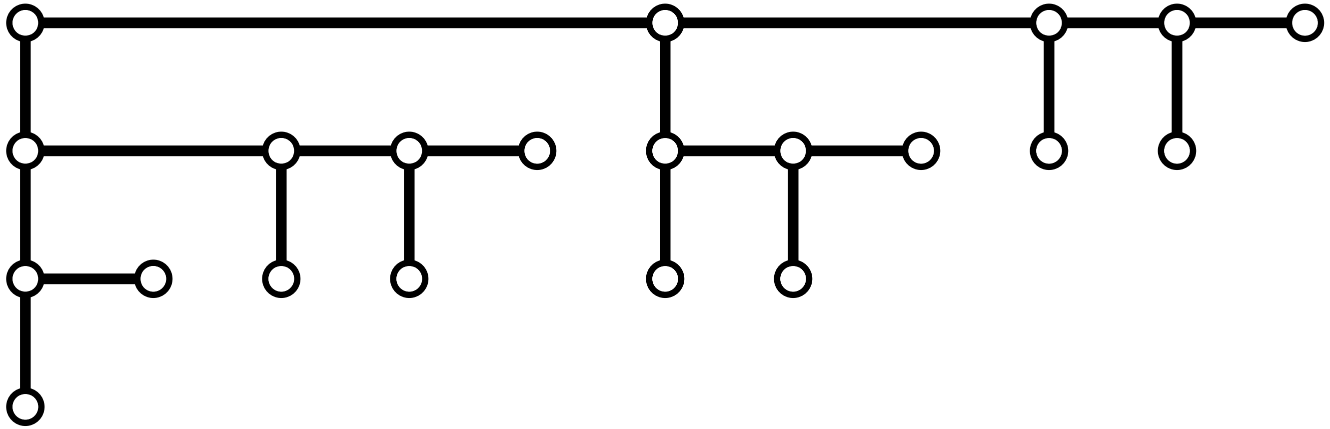

Lemma. Let T be a binary tree. The drawing constructed by the right-heavy approach has

- width at most   and
- height at most

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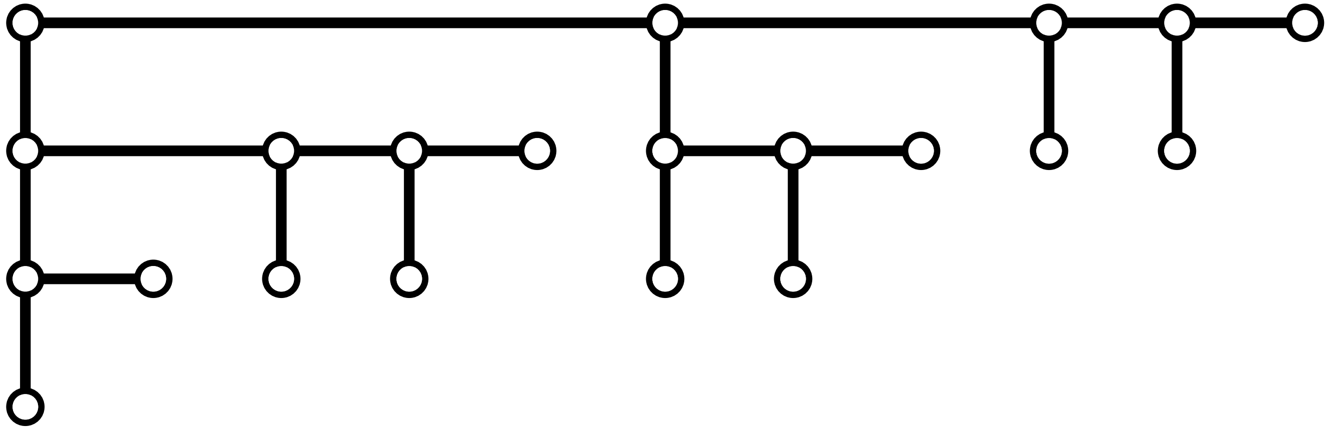

Lemma. Let T be a binary tree. The drawing constructed by the right-heavy approach has

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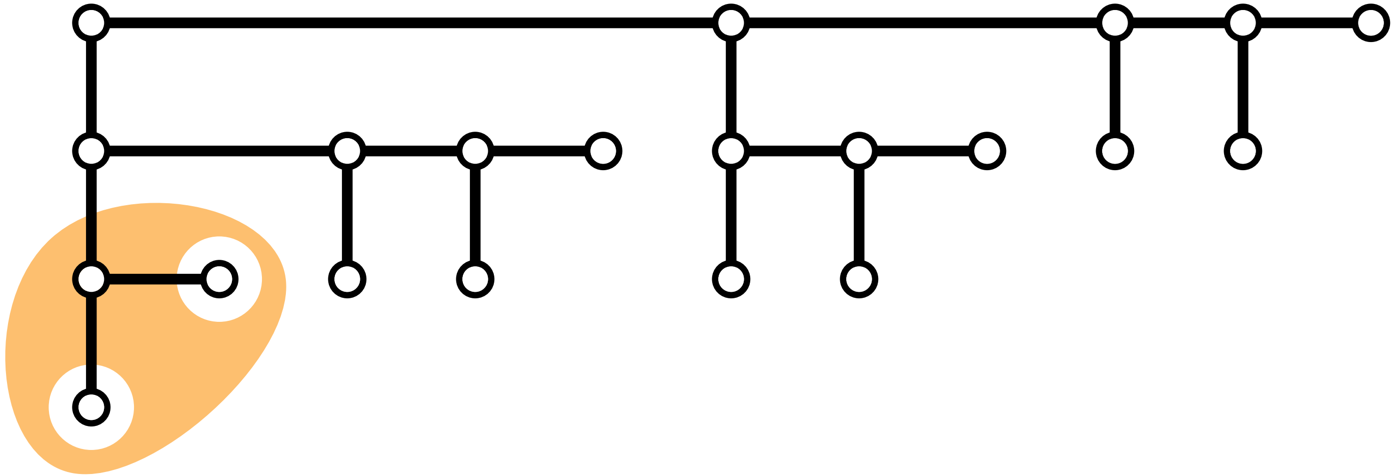

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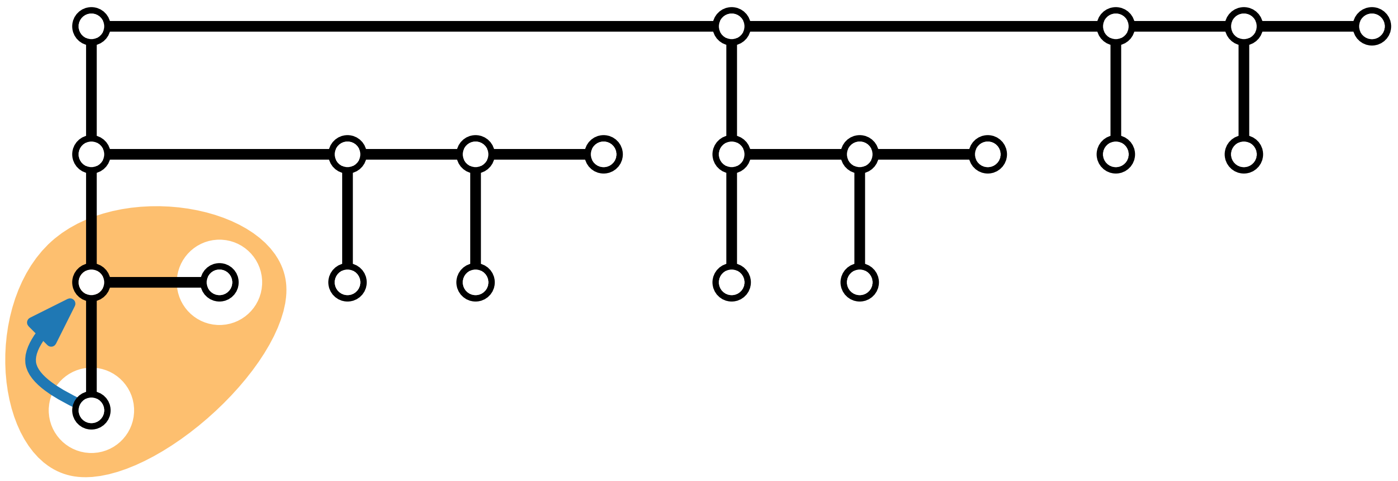

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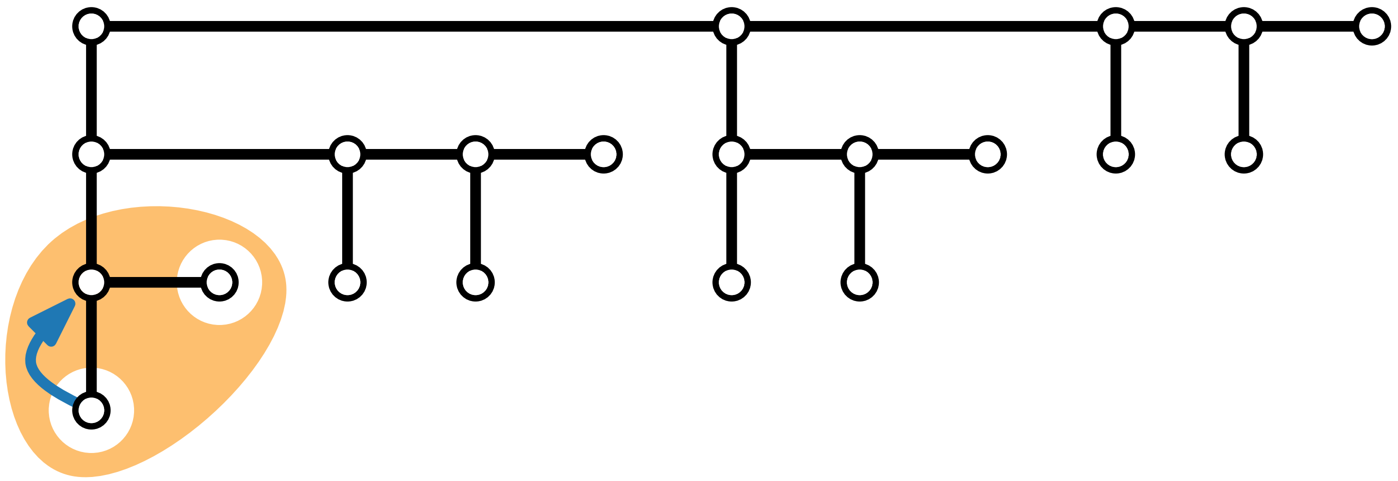
at least ·2



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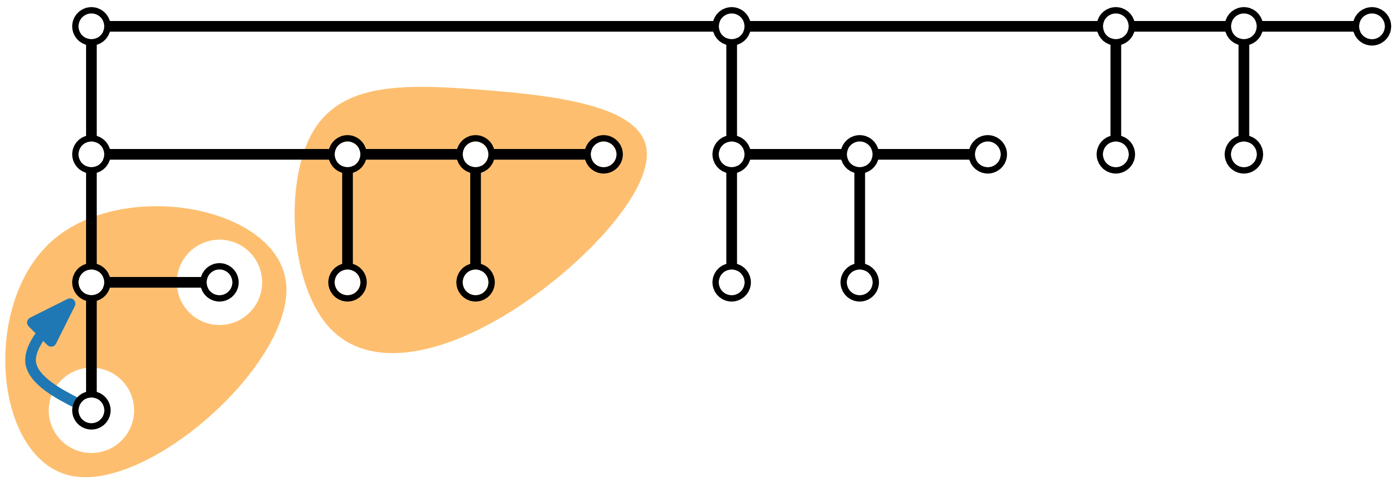

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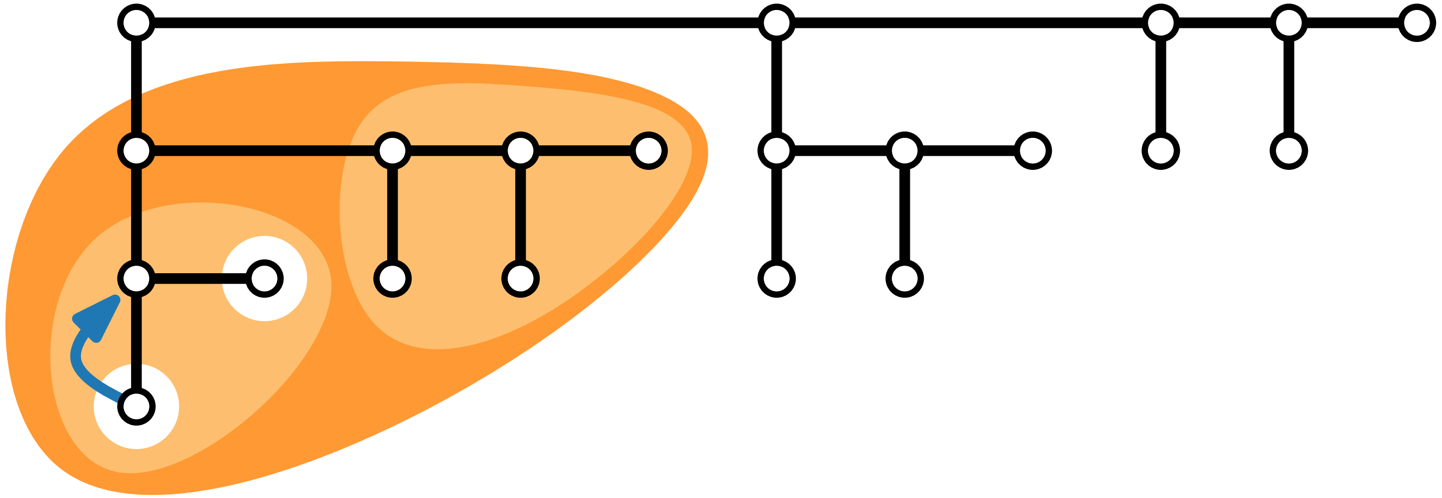

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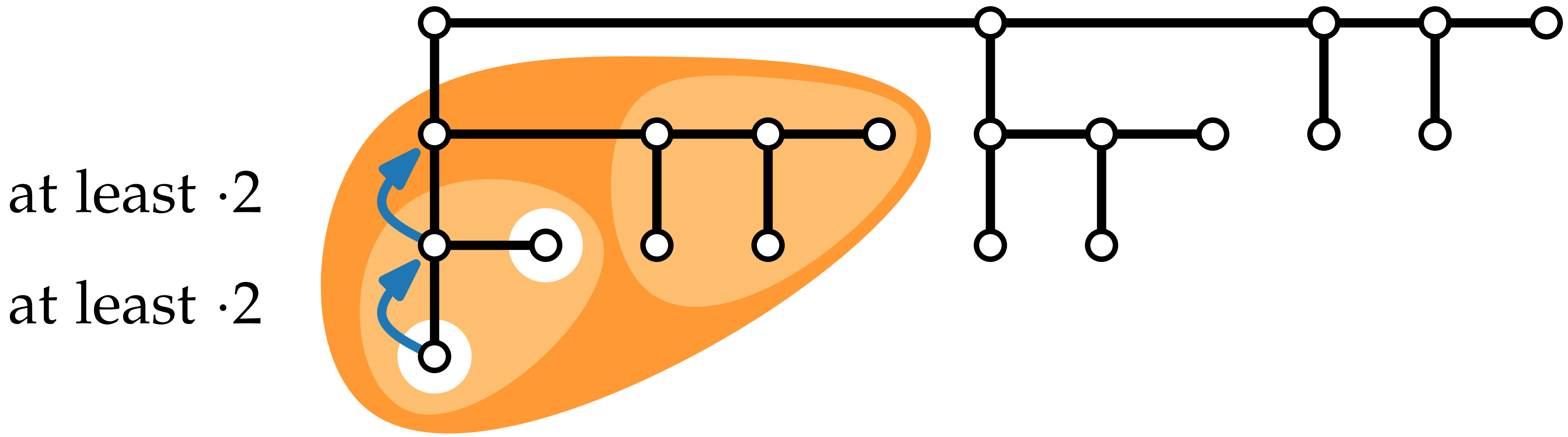

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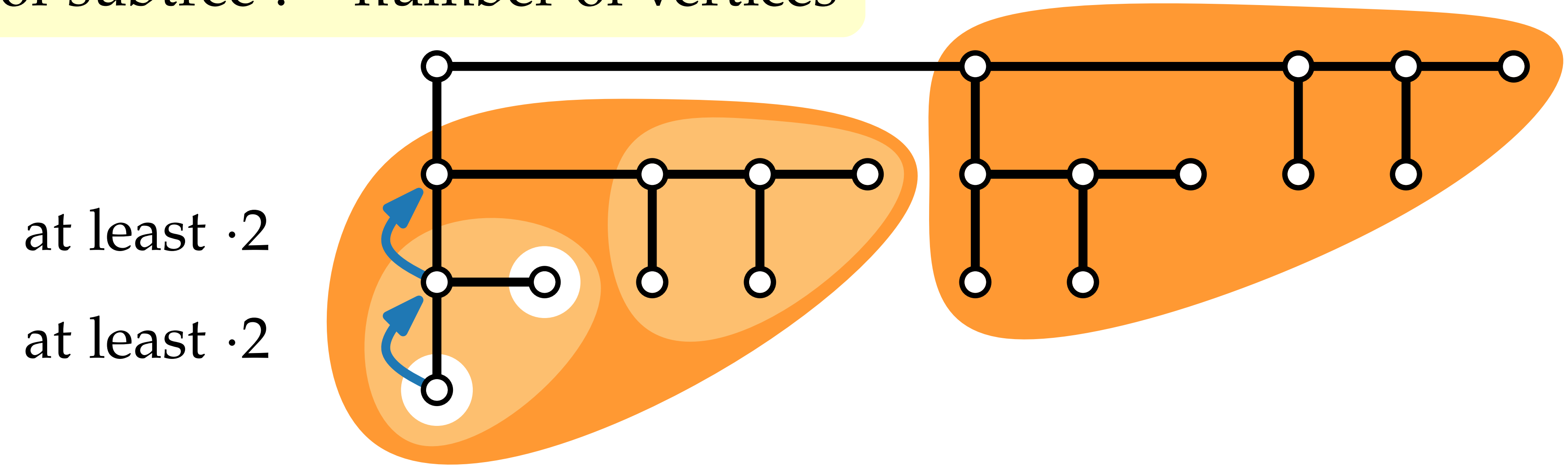

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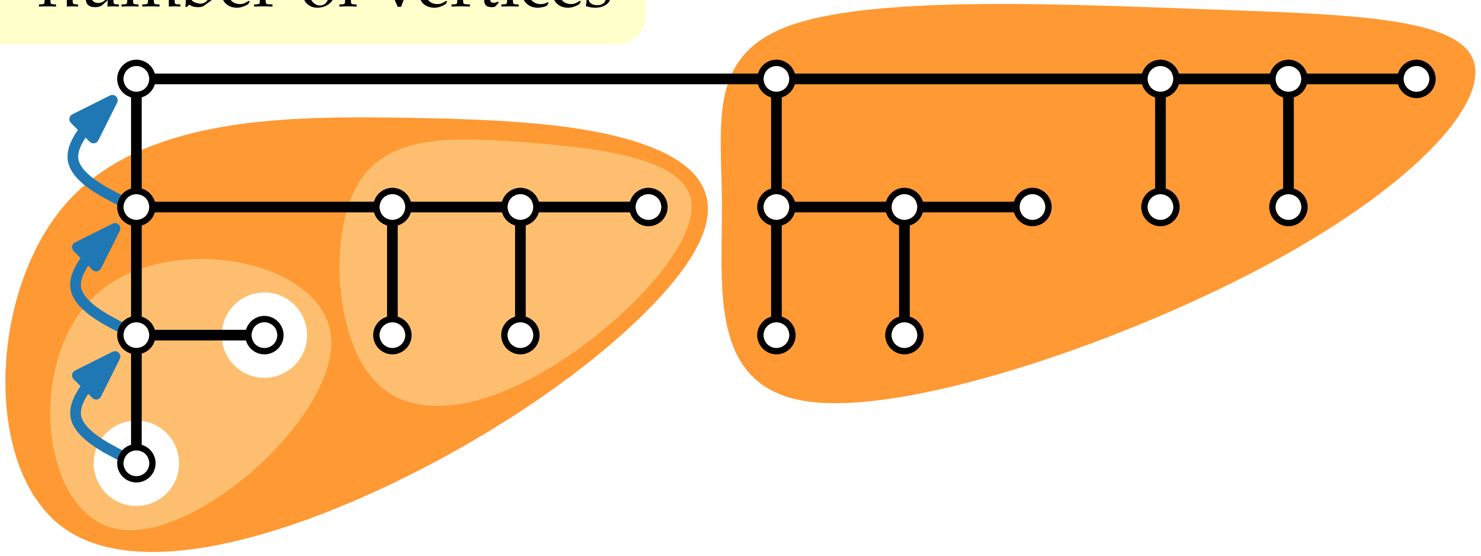

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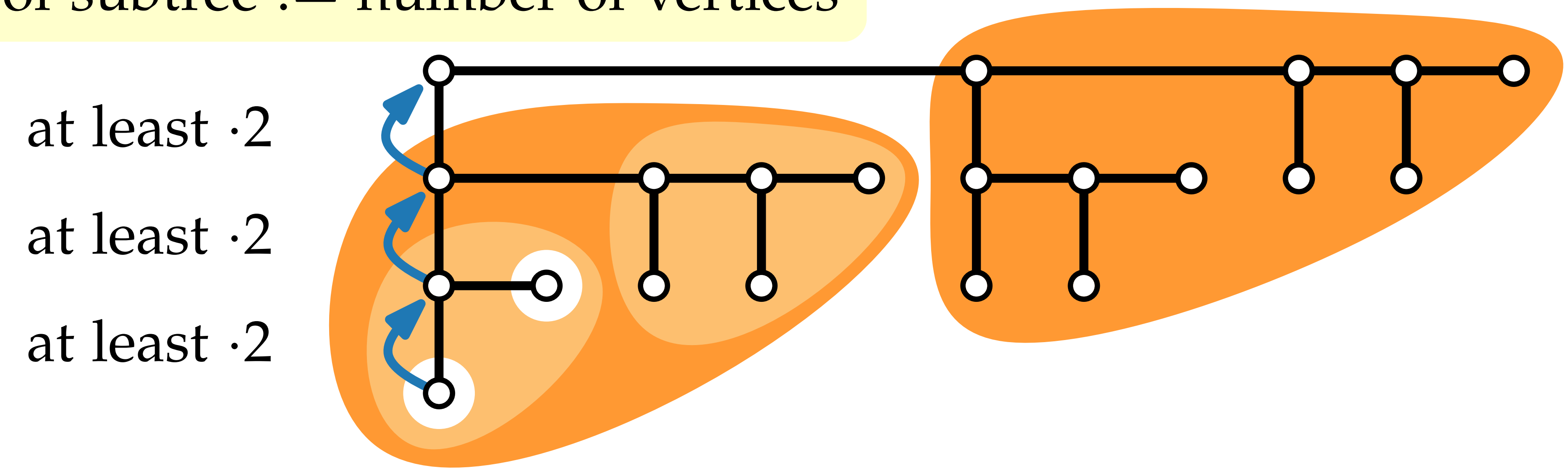

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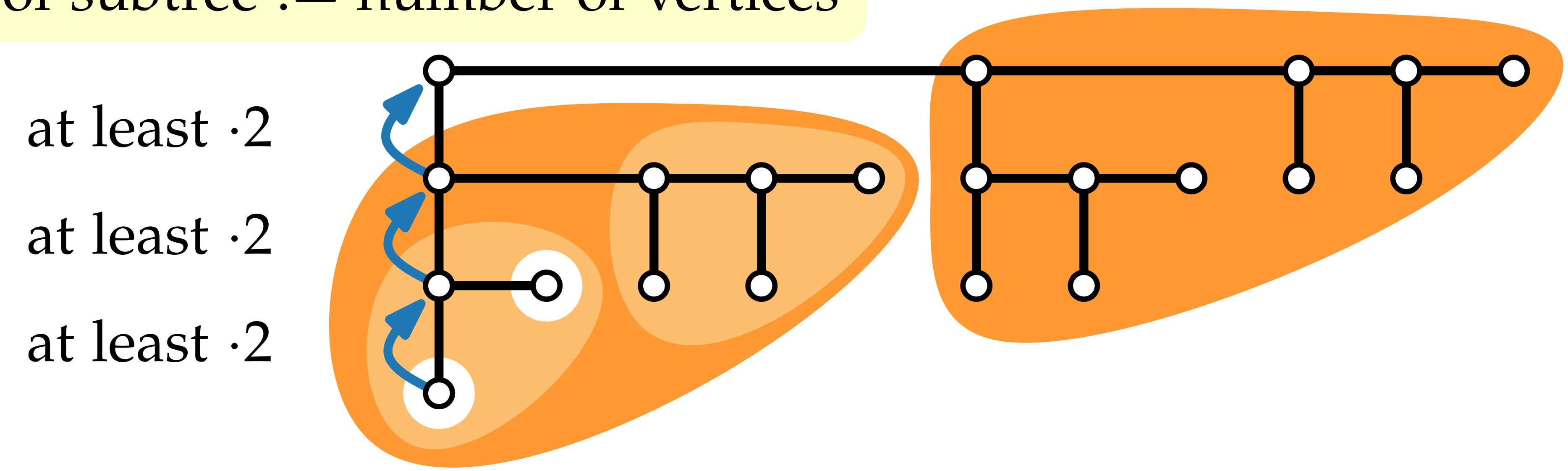
How to implement this in linear time?



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Theorem.
Let T be a binary tree with n vertices. The right-heavy algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

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General rooted tree

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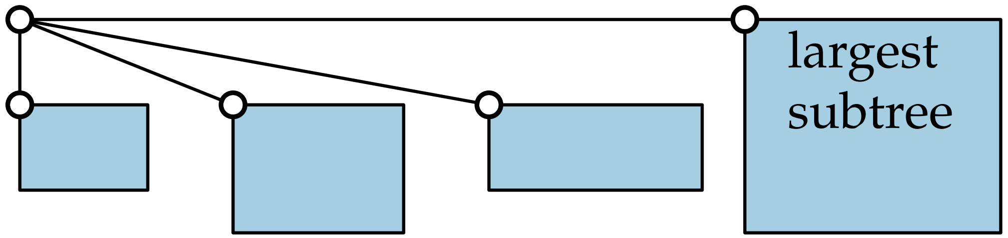

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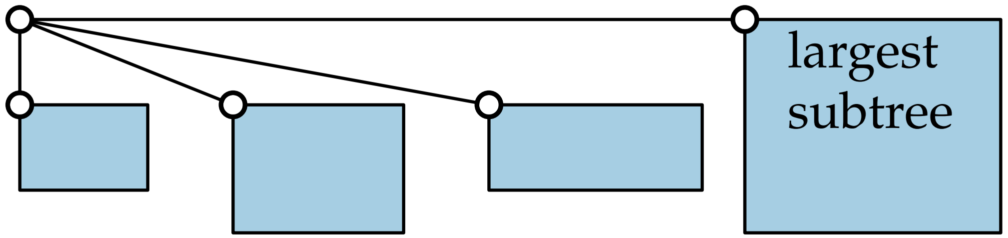

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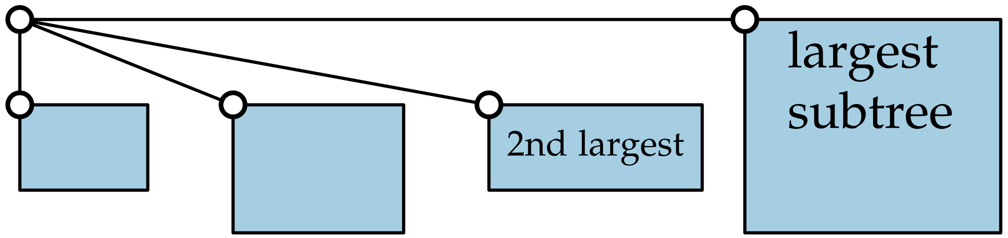

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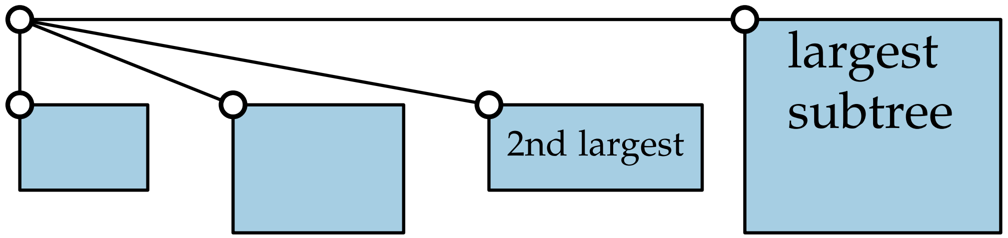

Optimal area?

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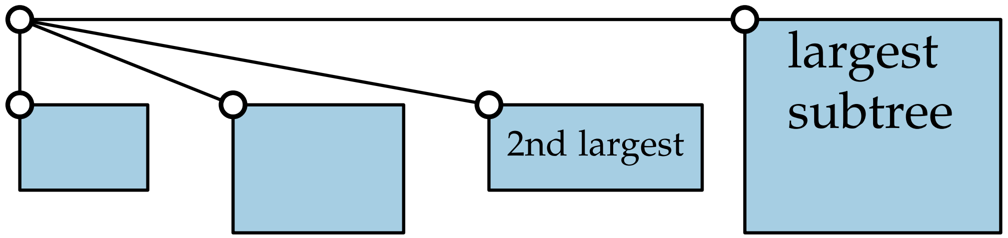
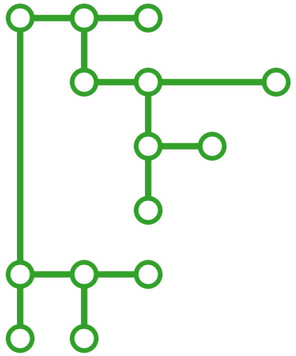


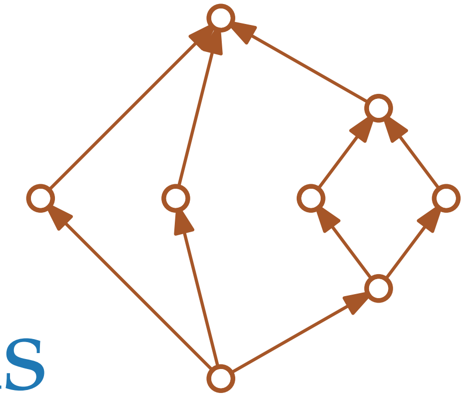
Optimal area?

Not with divide & conquer approach, but can be computed with Dynamic Programming.

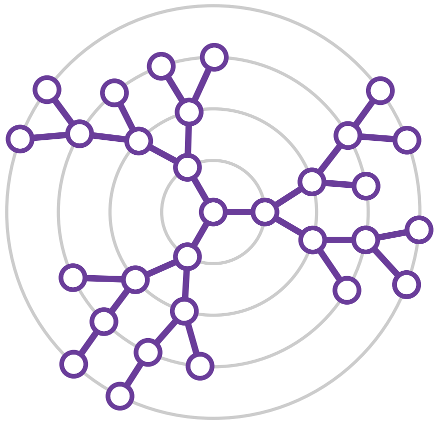
Universität Trier



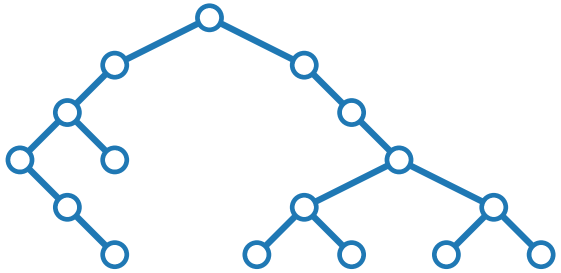
Visualization of Graphs



Lecture 2:
Drawing Trees and Series-Parallel Graphs

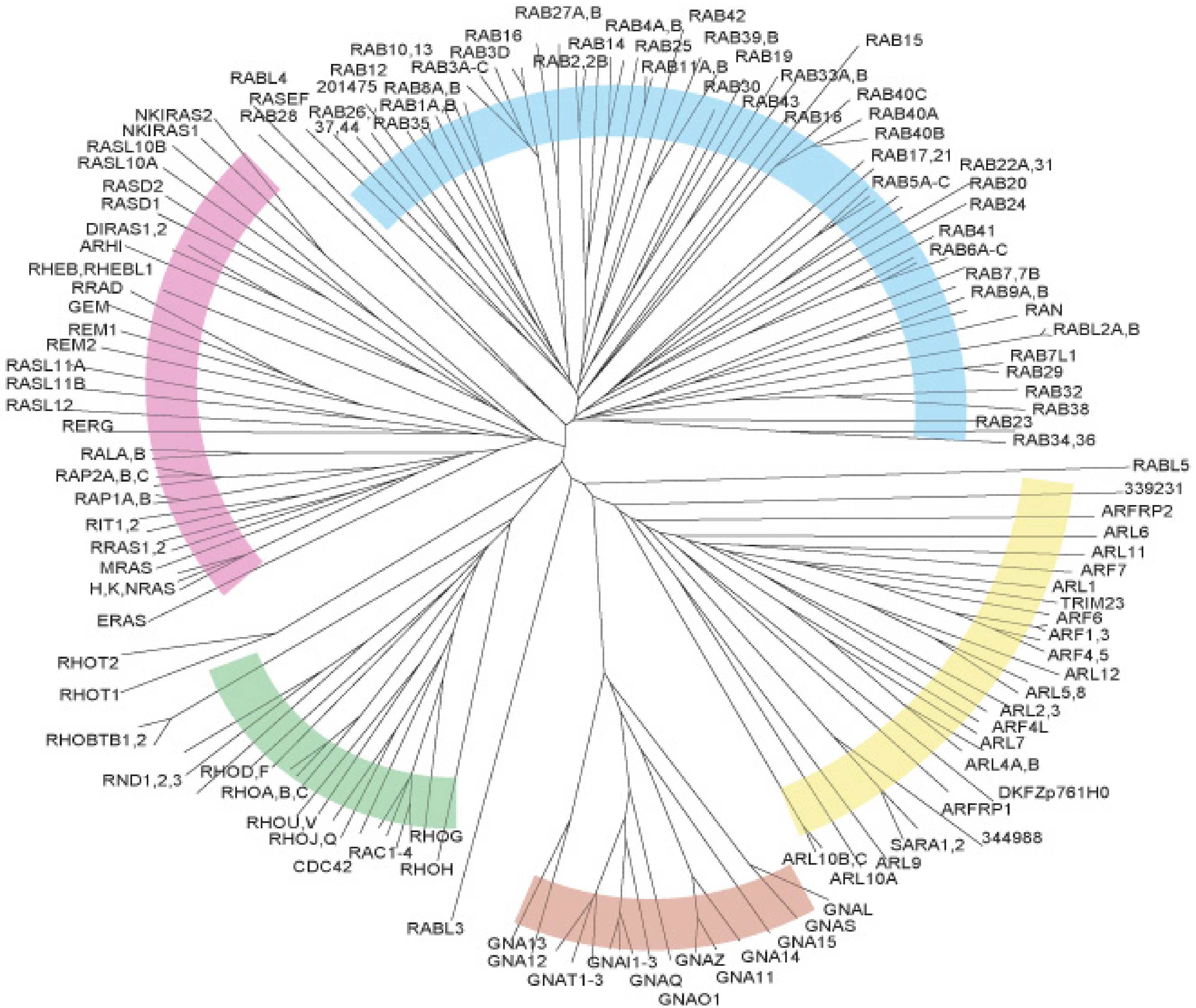


Part IV:
Radial Layouts



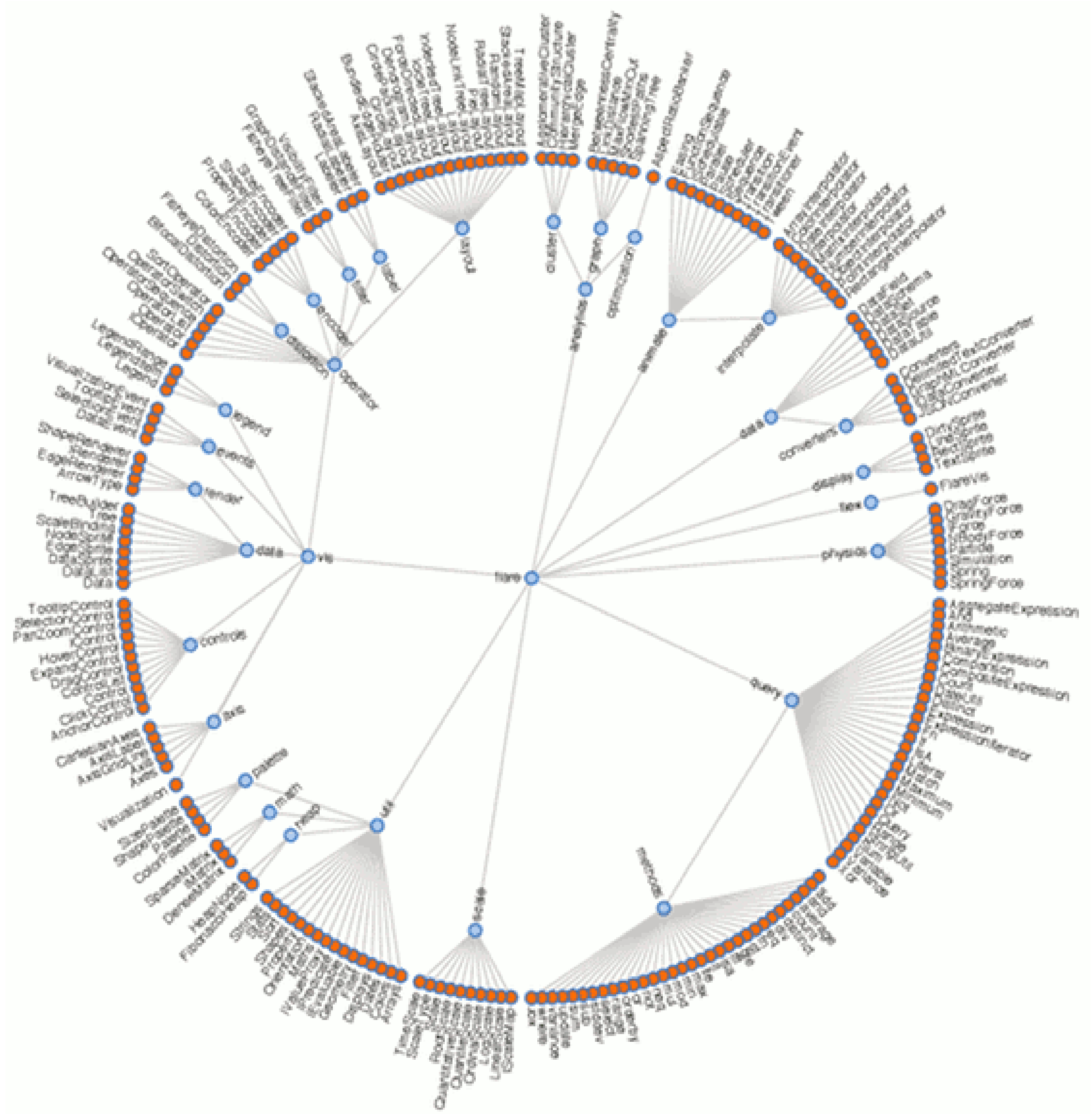
Philipp Kindermann

Radial Layouts – Applications

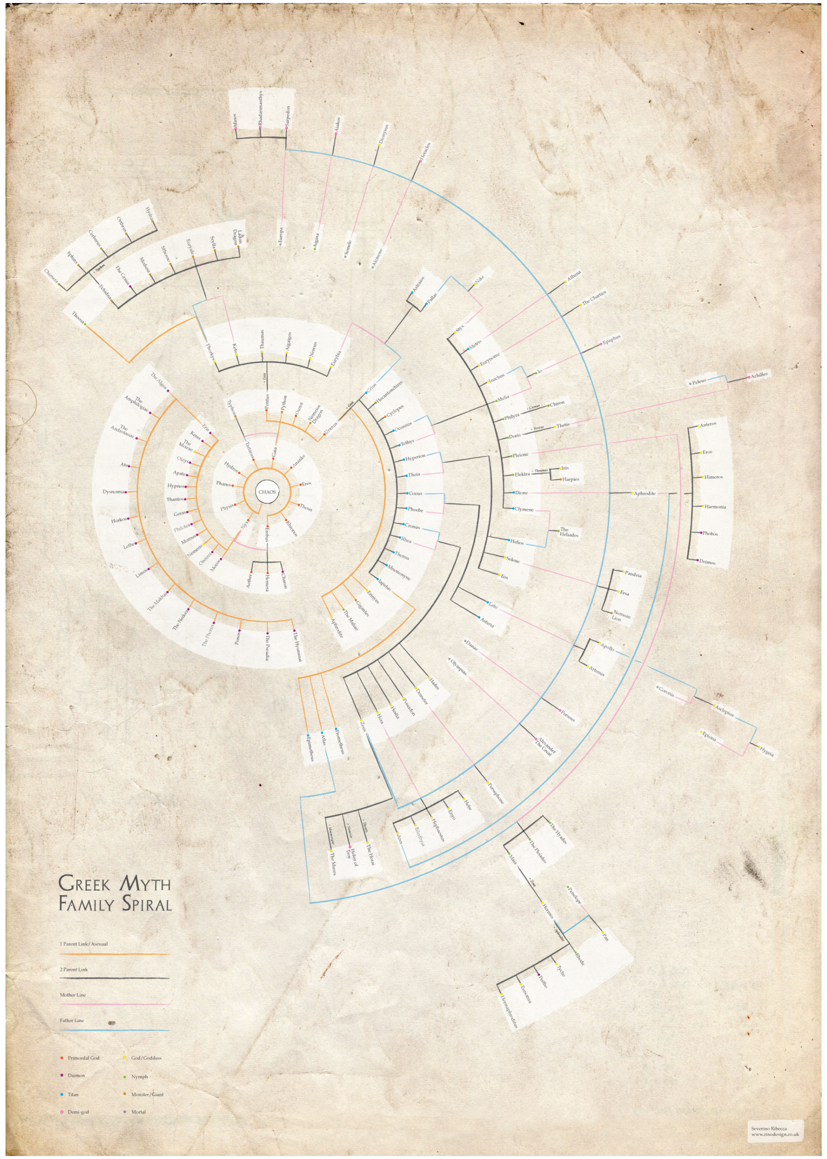


Phylogenetic tree
by Colicelli, ScienceSignaling, 2004

Radial Layouts – Applications

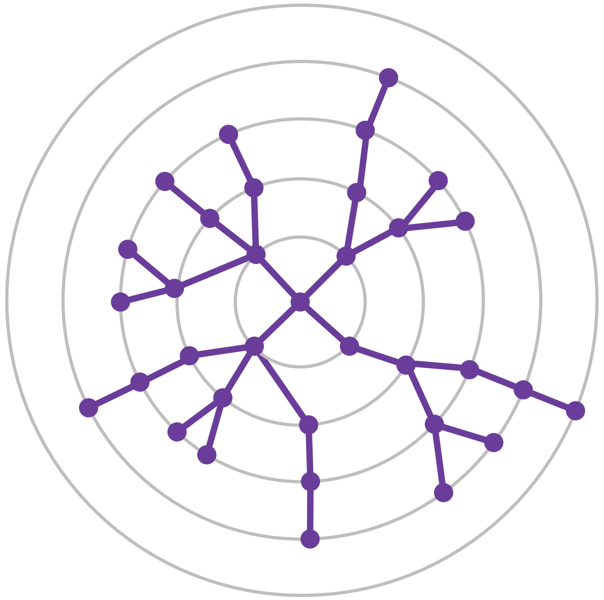


Flare Visualization Toolkit code structure
by Heer, Bostock and Ogievetsky, 2010



Greek Myth Family
by Ribecca, 2011

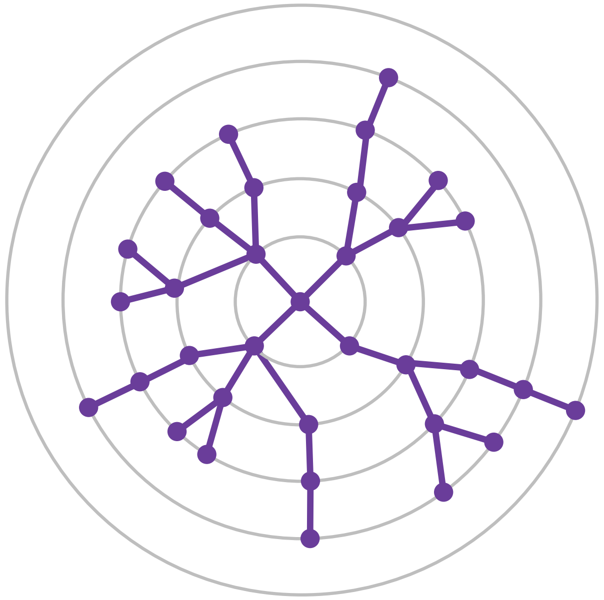
Radial Layouts – Drawing Style



Drawing conventions

Drawing aesthetics

Radial Layouts – Drawing Style



Drawing conventions

- Vertices lie on circular layers according to their depth

Drawing aesthetics

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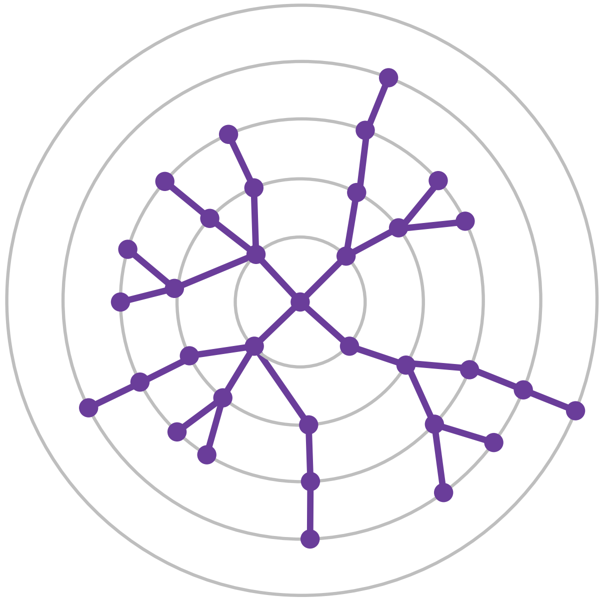

Drawing conventions

- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing aesthetics

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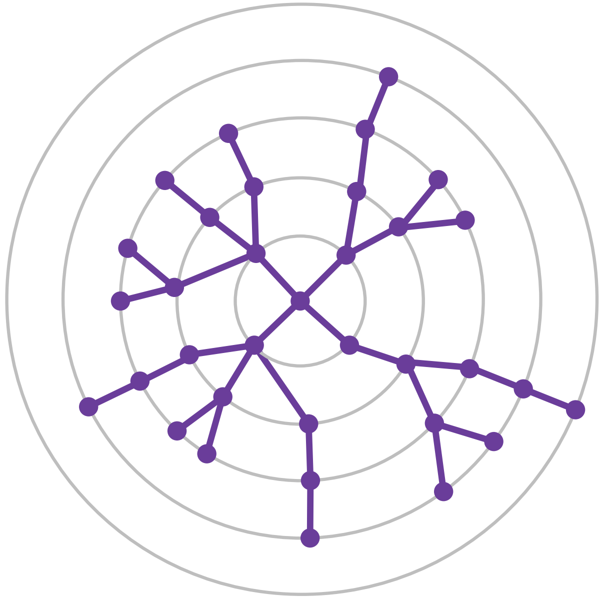

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Drawing aesthetics

- Distribution of the vertices

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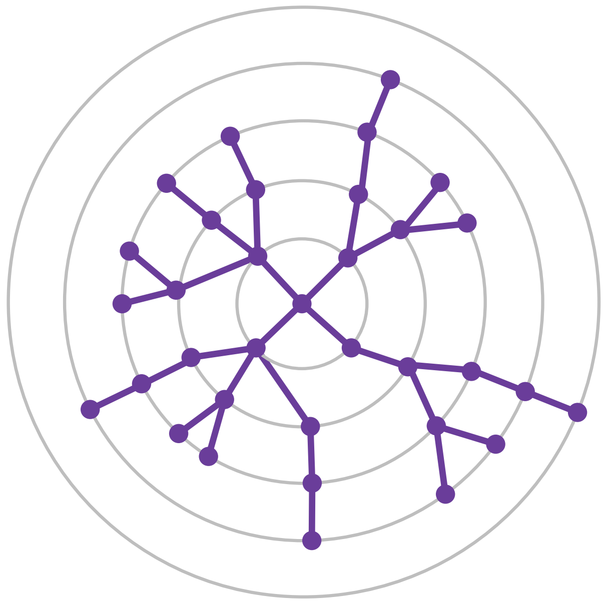

Drawing conventions

- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing aesthetics

- Distribution of the vertices

How can an algorithm optimize the distribution of the vertices?

Radial Layouts – Algorithm Attempt

Idea

- Reserve area corresponding to size $\ell(u)$ of $T(u)$:

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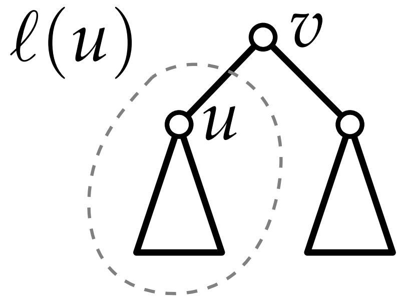

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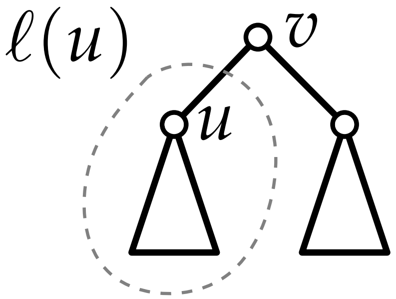

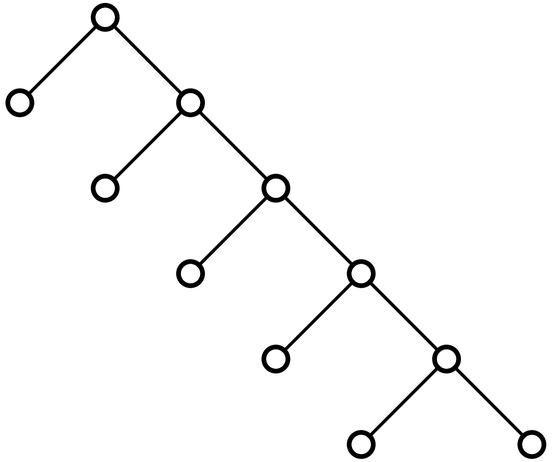

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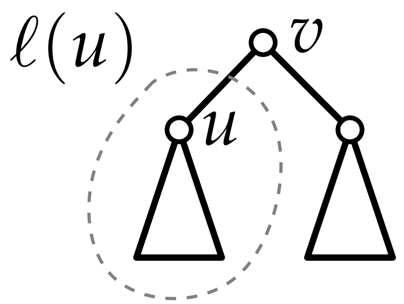

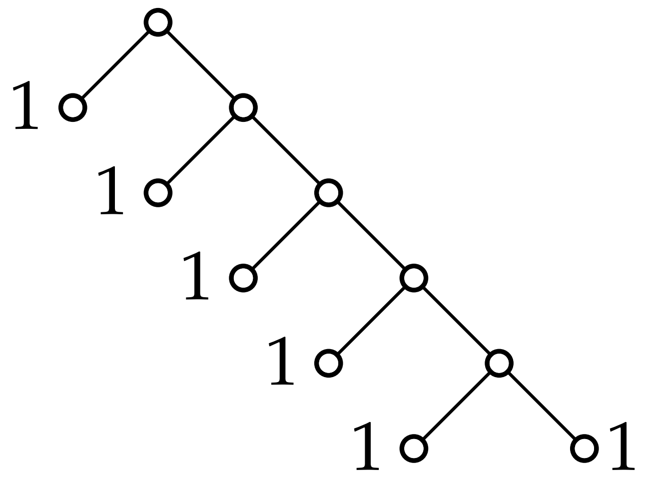

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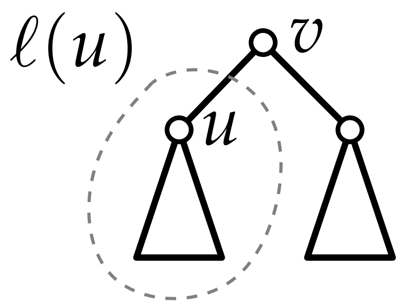

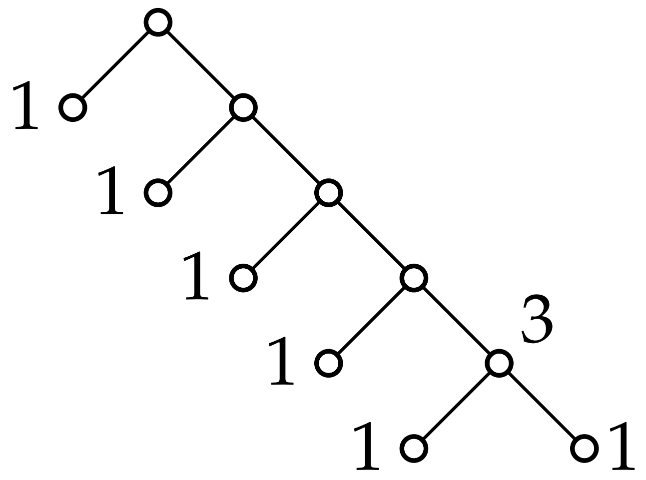

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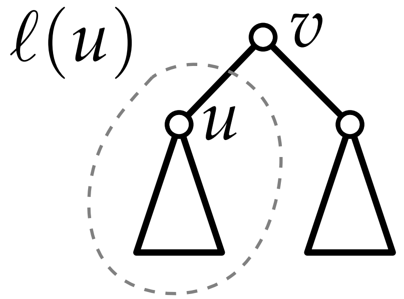

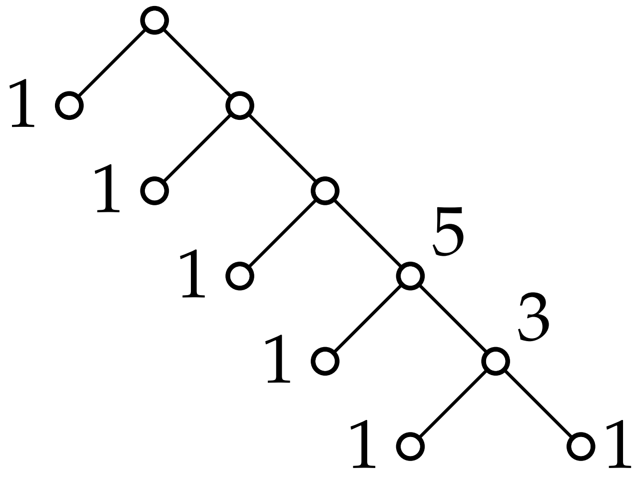

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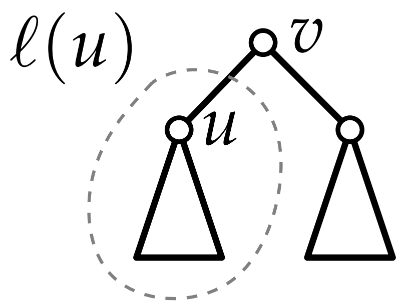

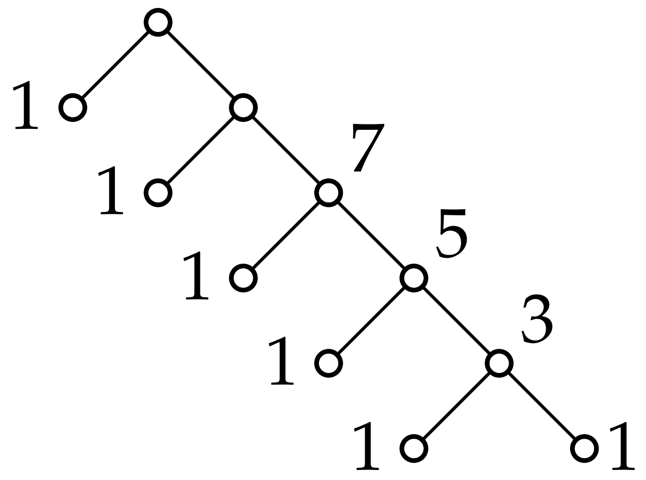

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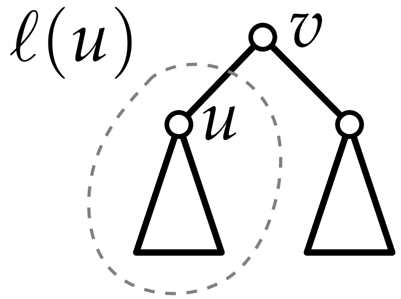

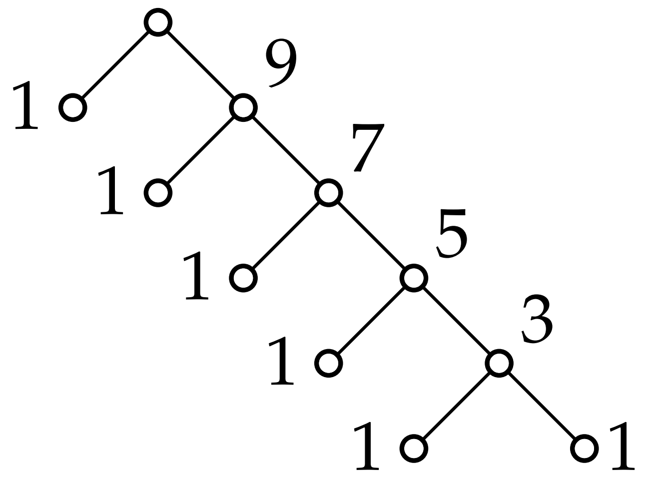

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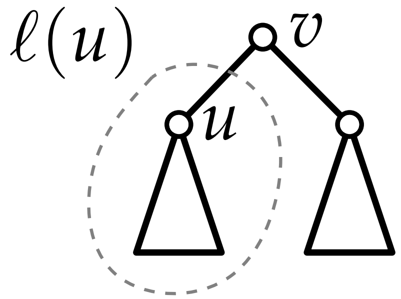

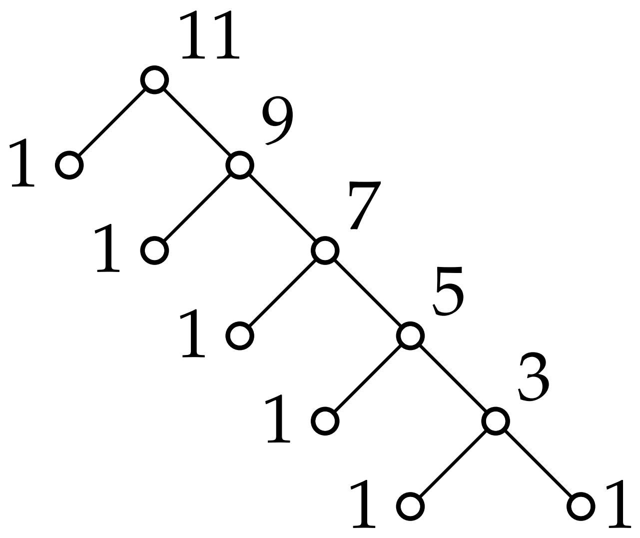

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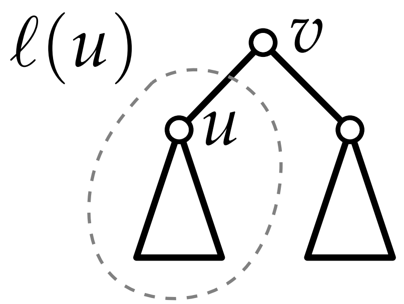
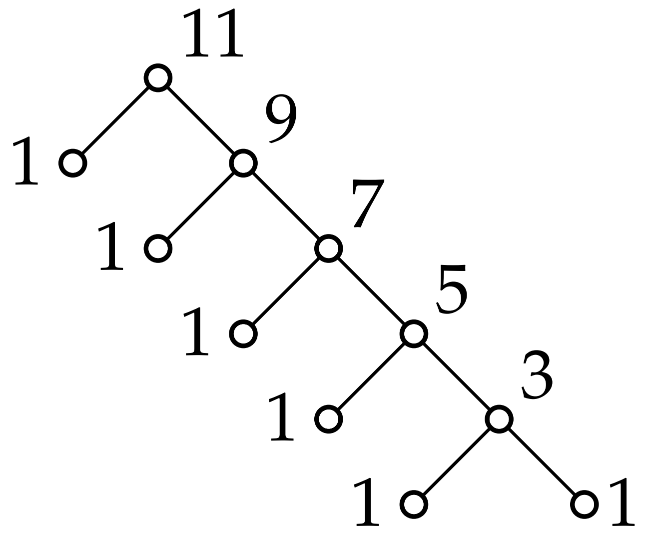


$$\tau_u = \frac{\ell(u)}{\ell(v) - 1}$$



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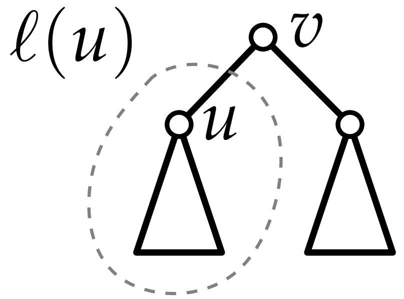

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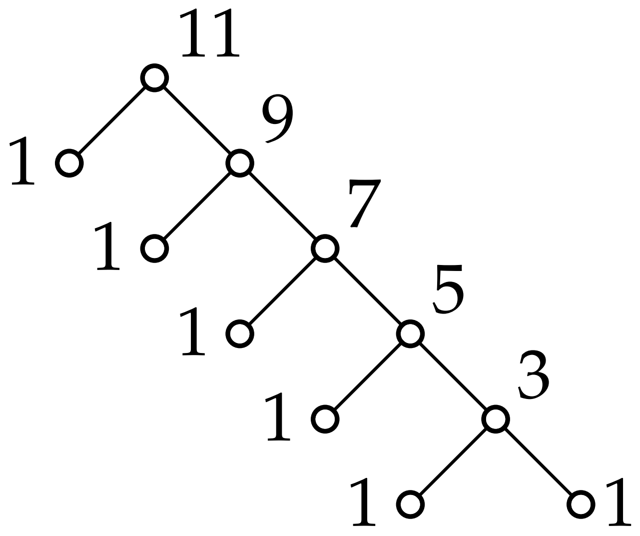

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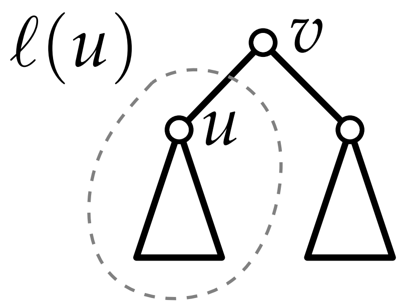

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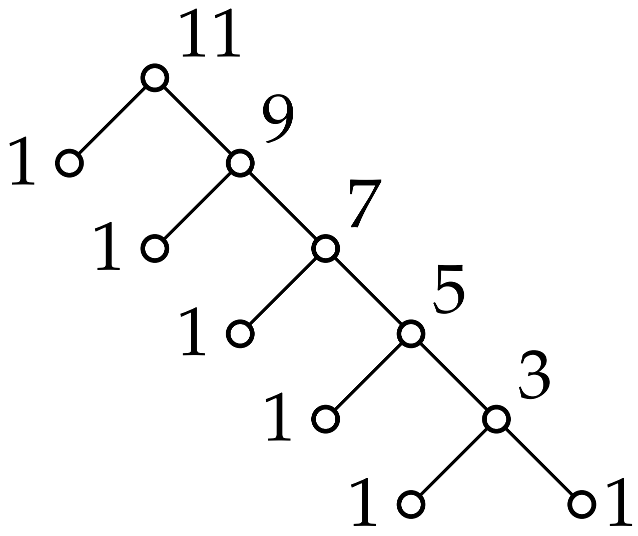

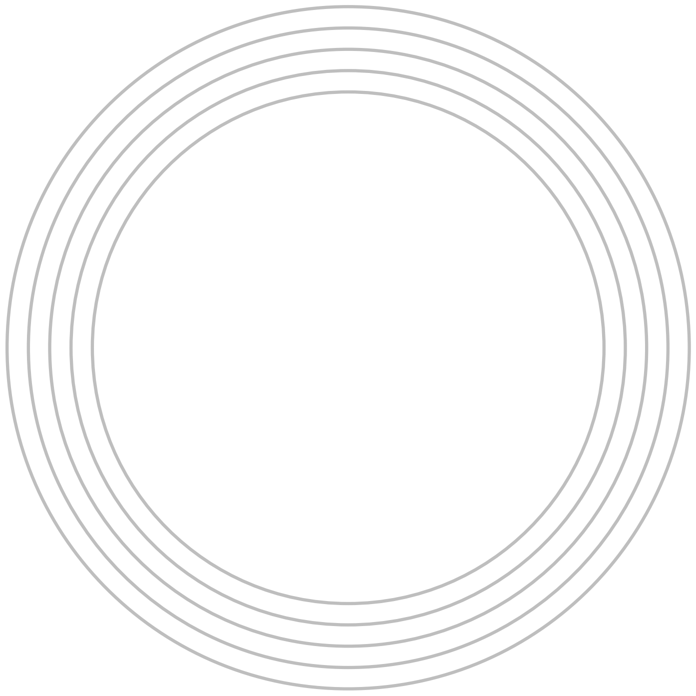

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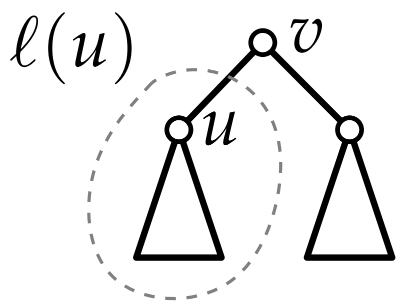

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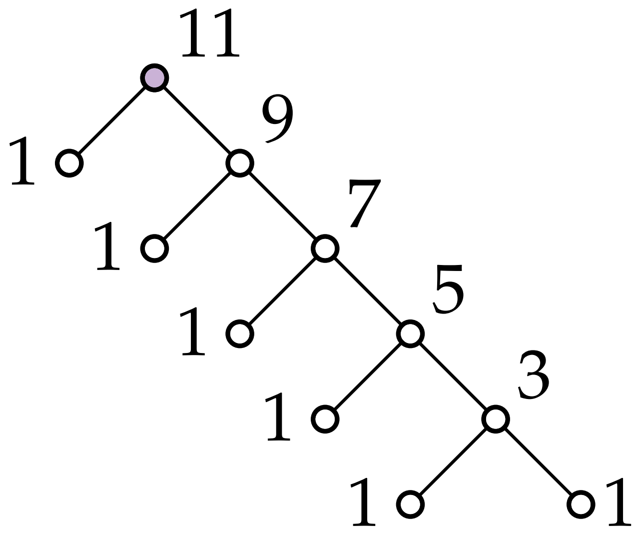

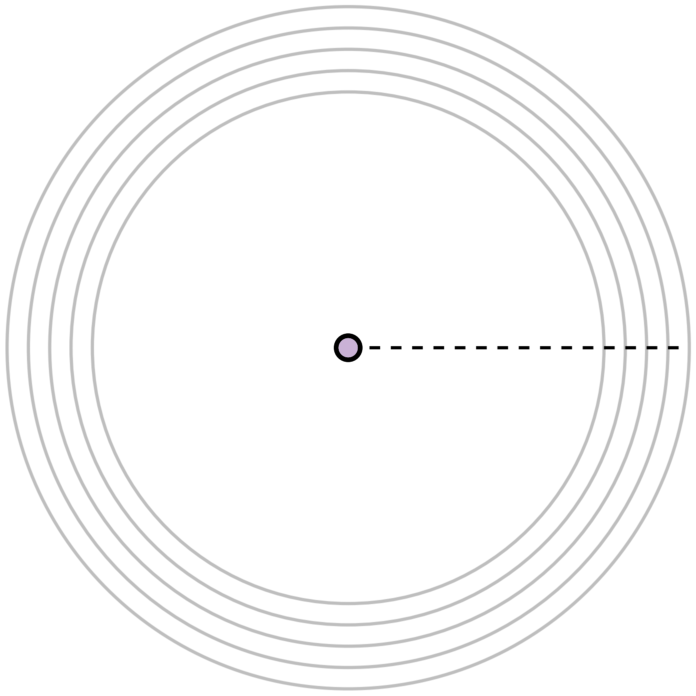

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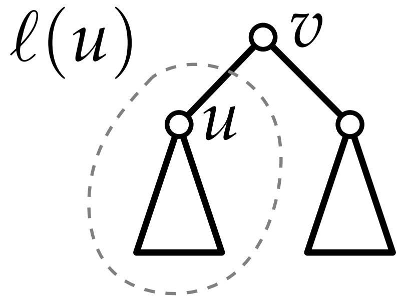

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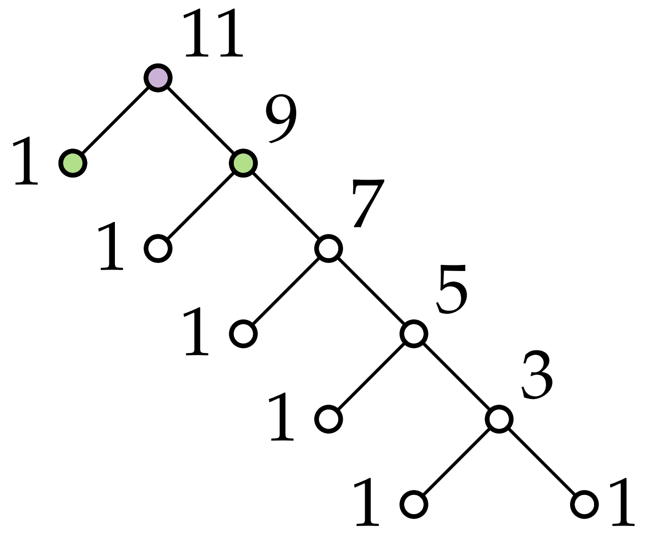

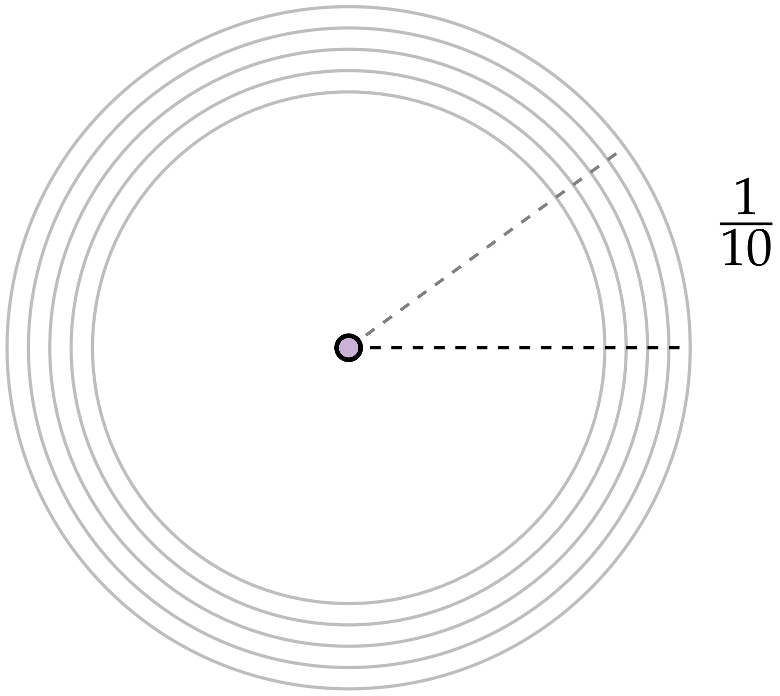

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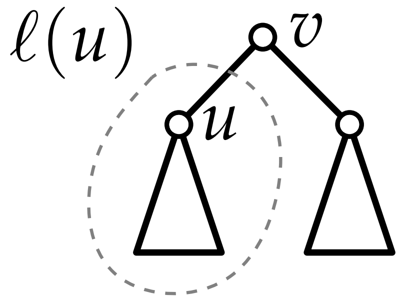

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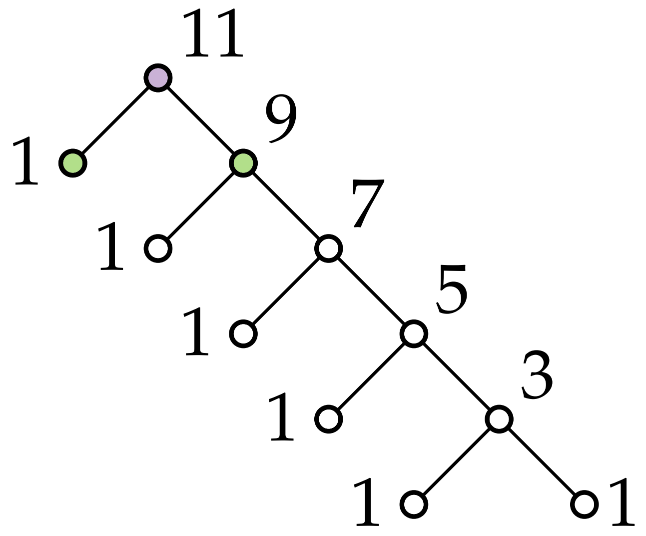

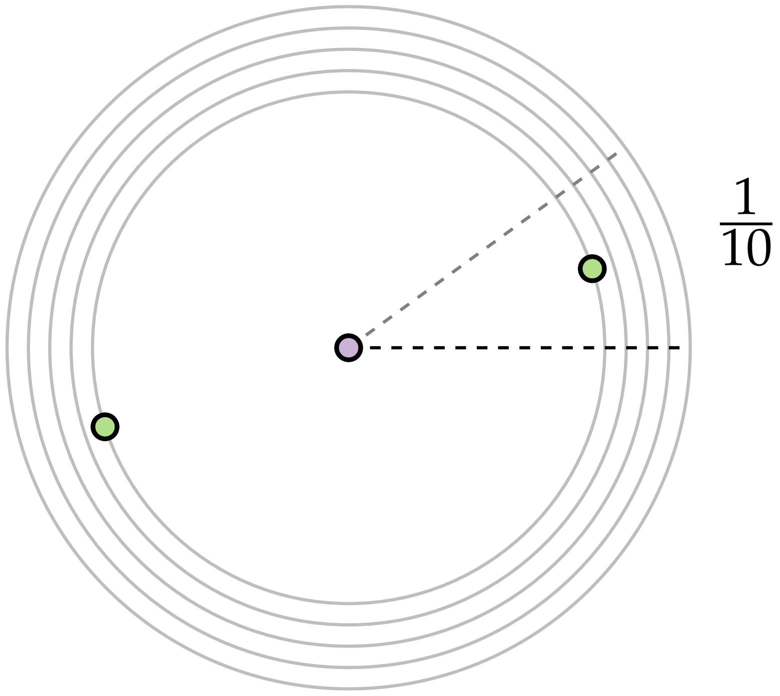

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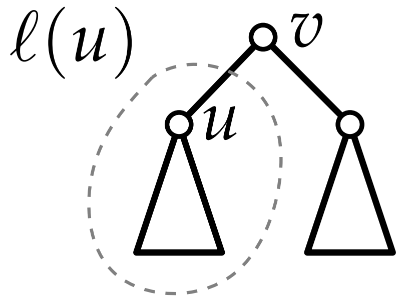

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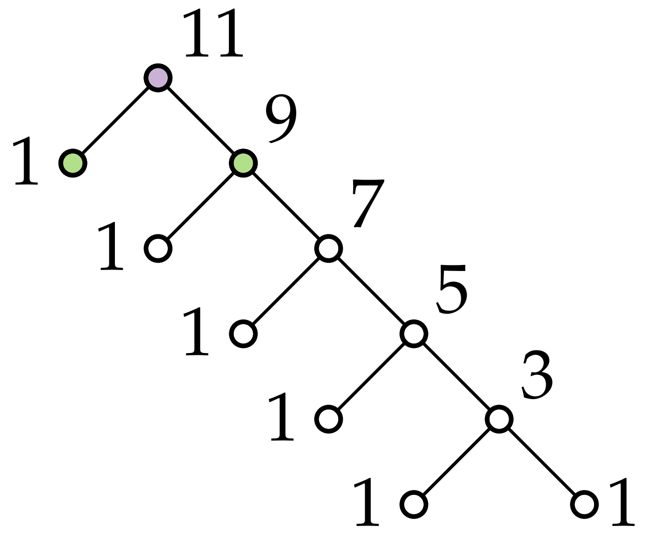

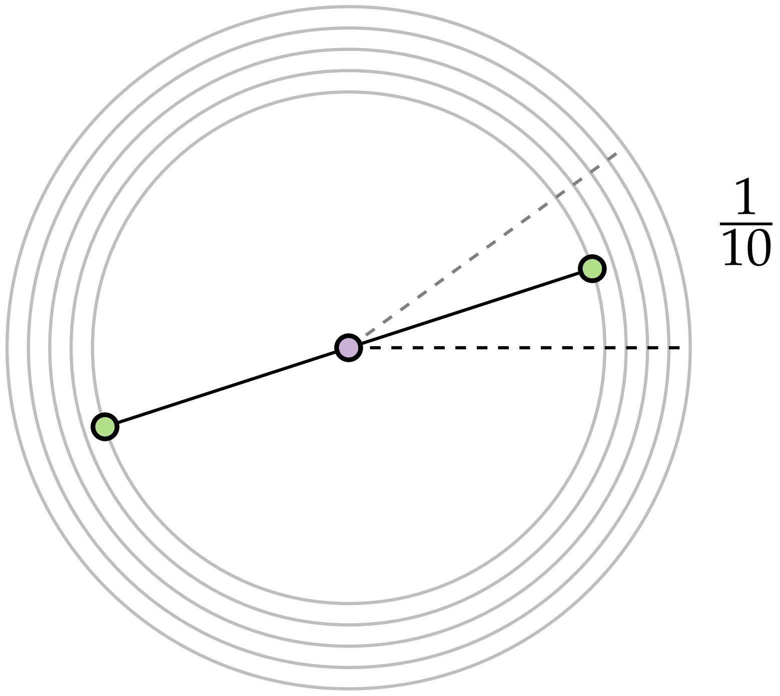

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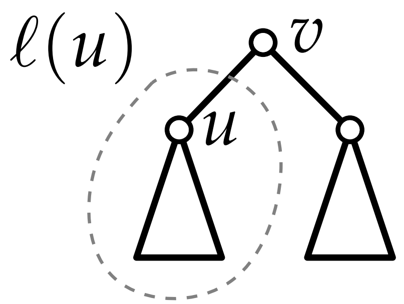

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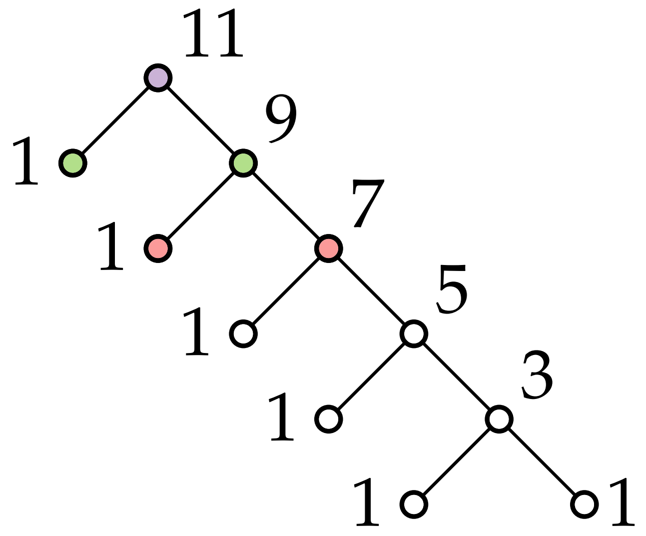

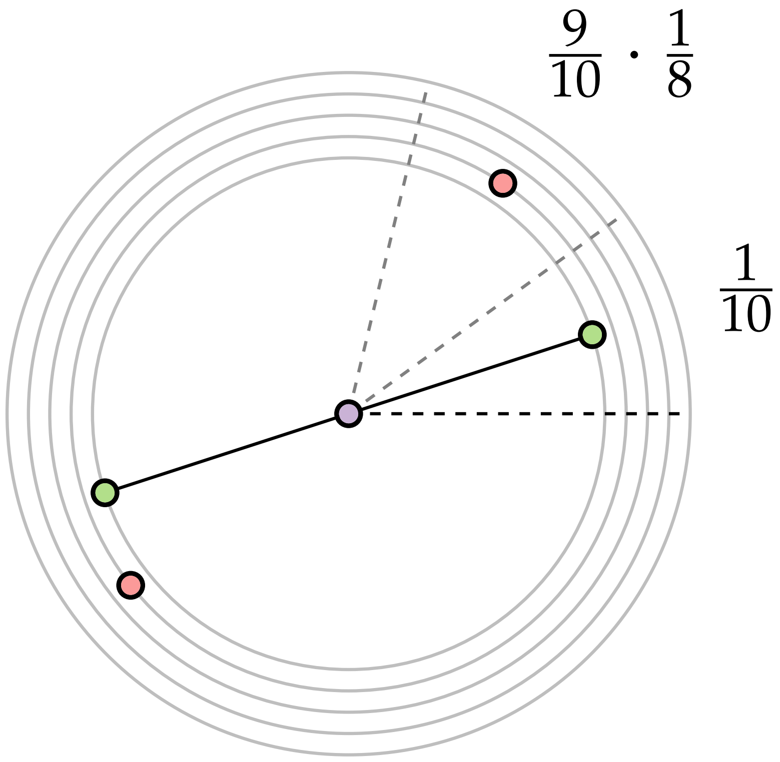

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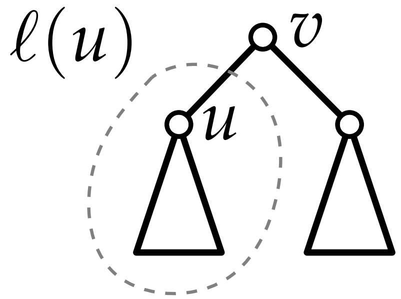

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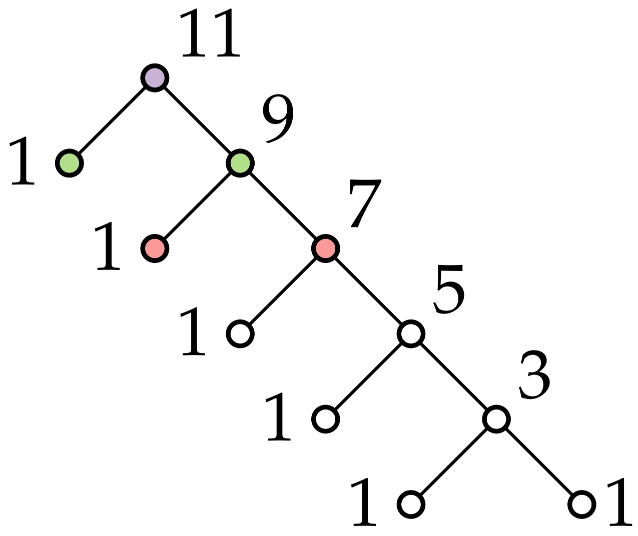

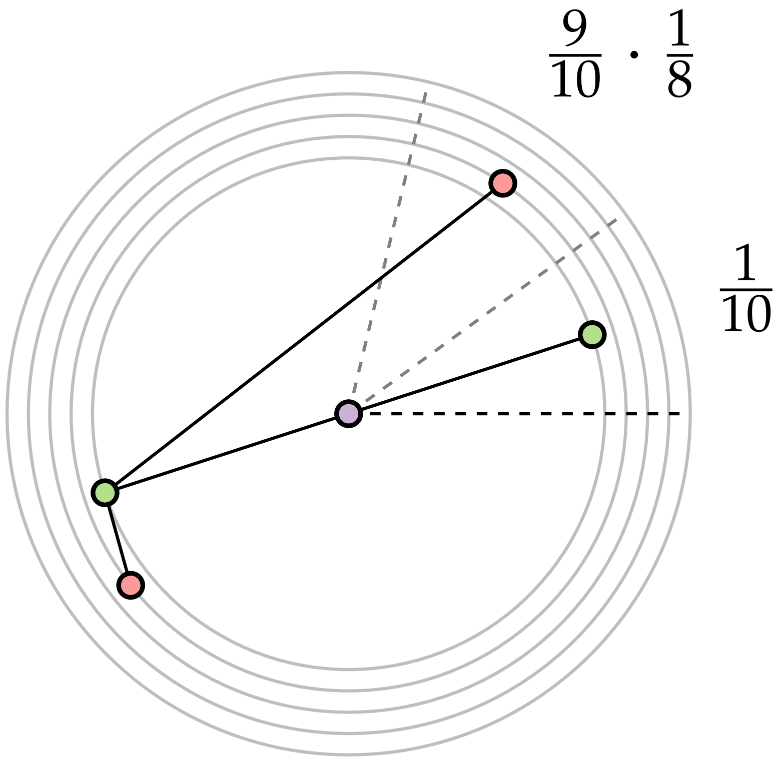

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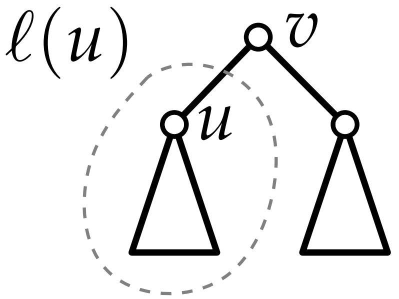

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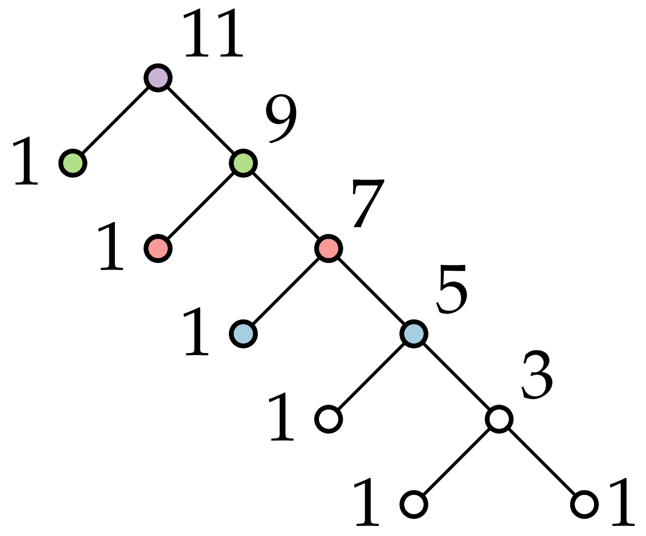

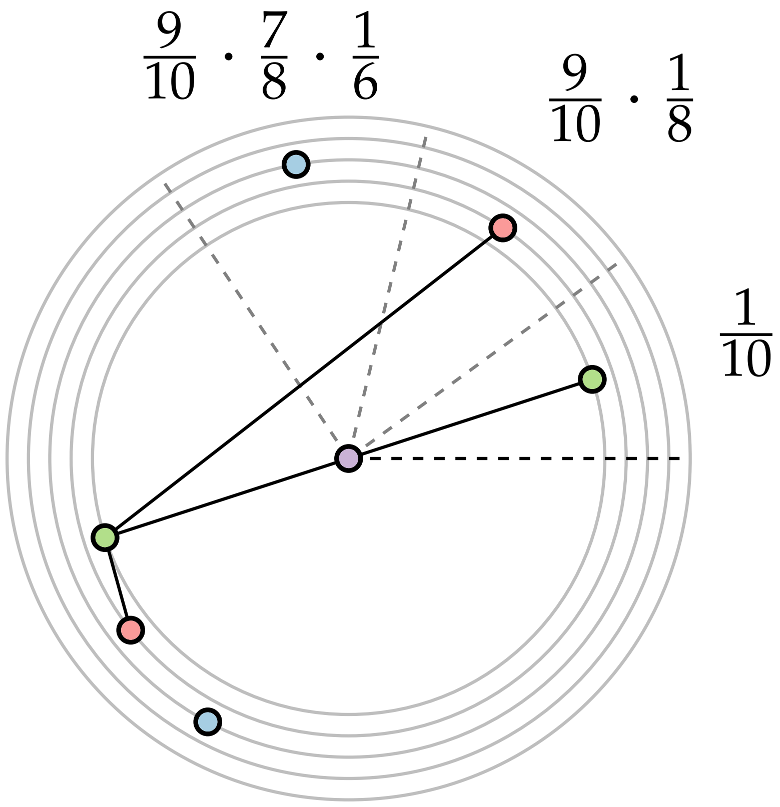

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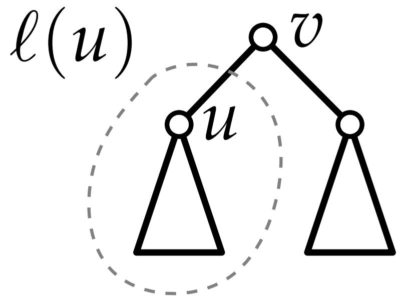

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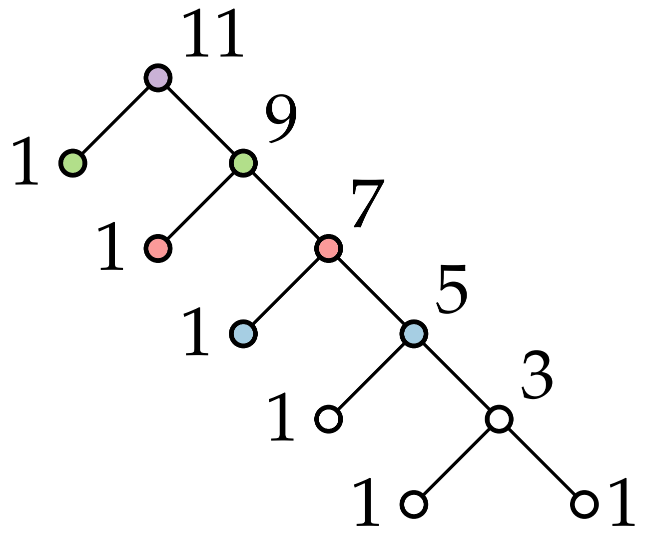
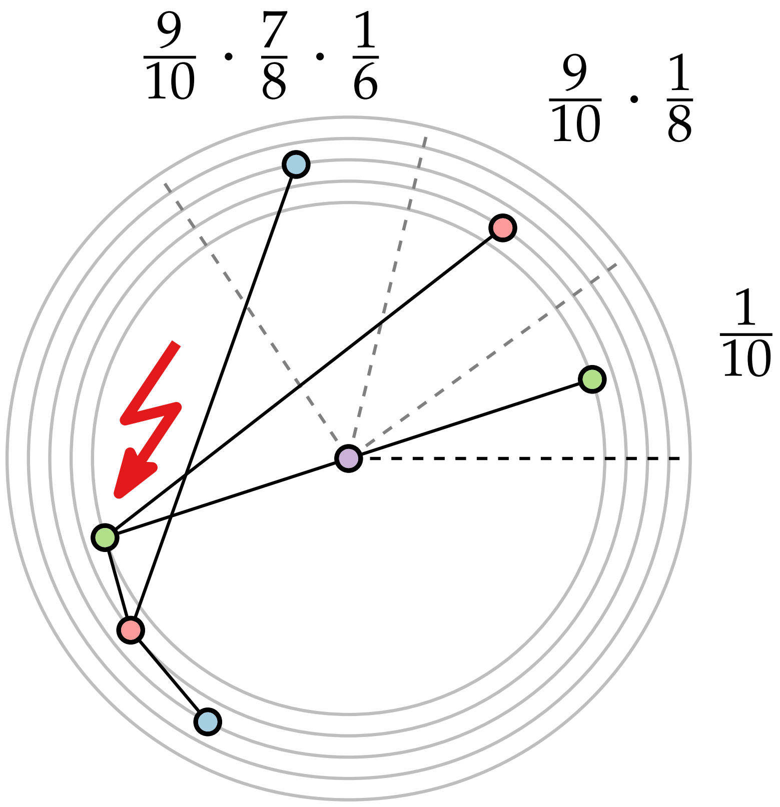
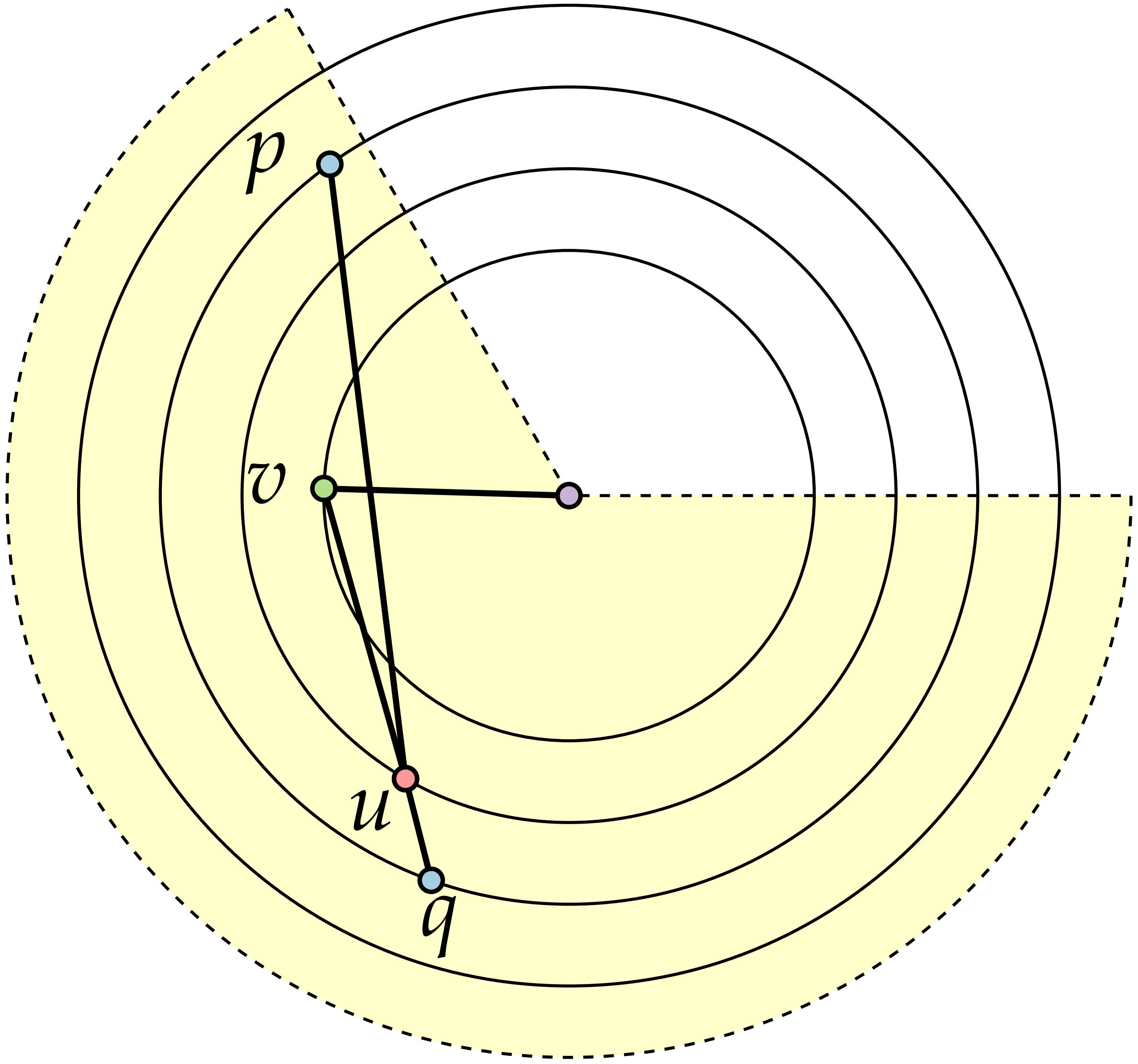




Radial Layouts – How To Avoid Crossings



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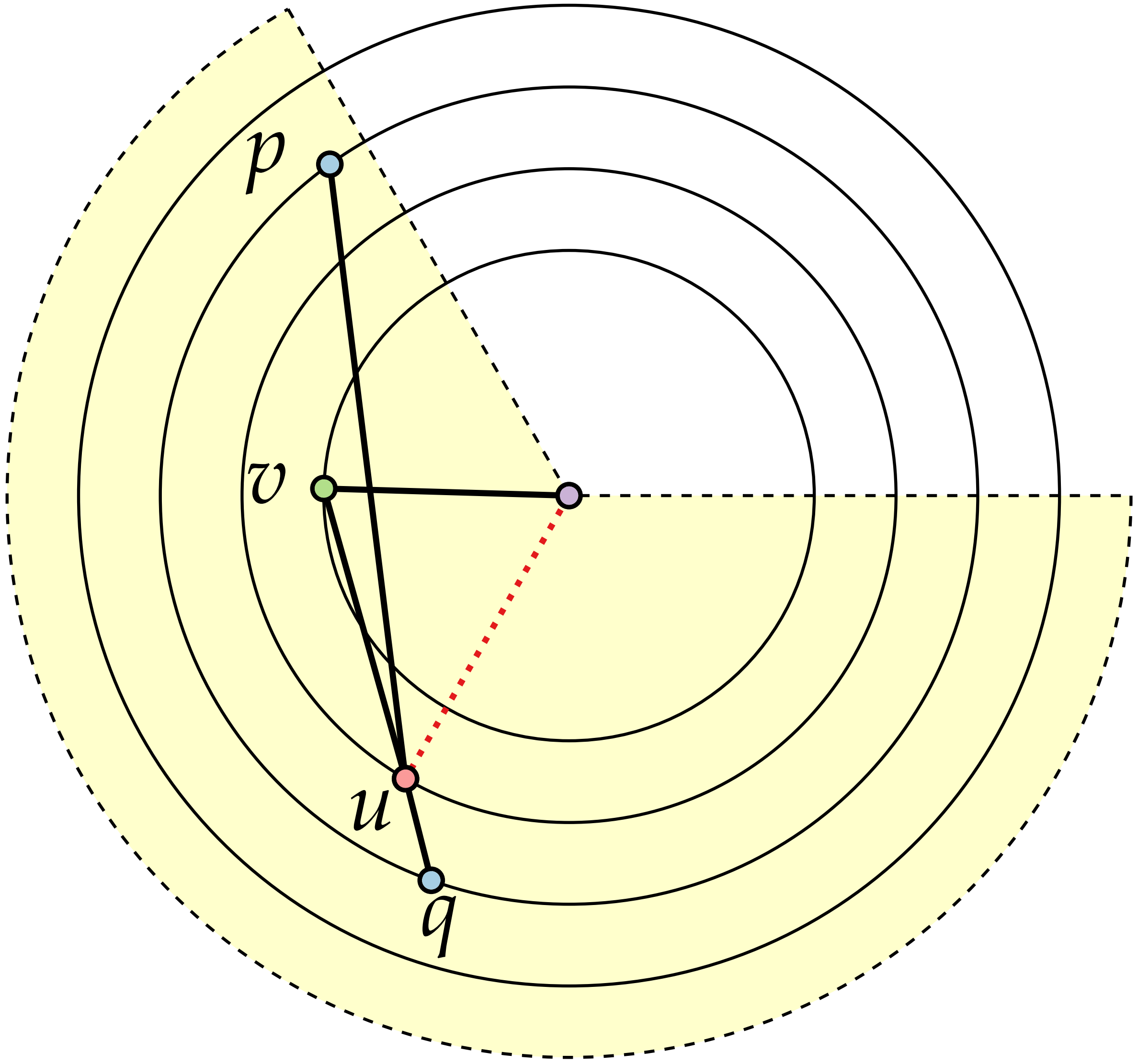

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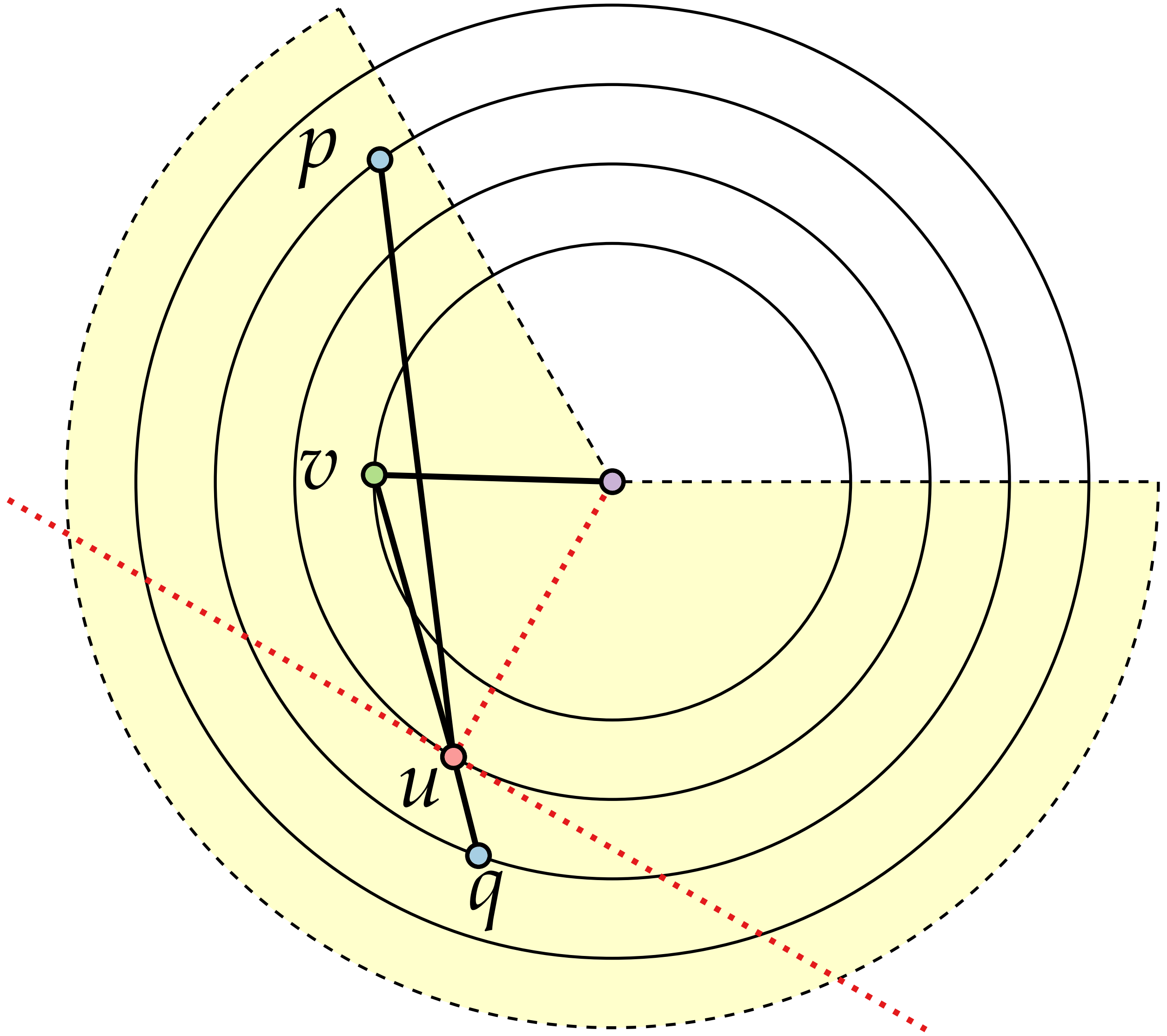

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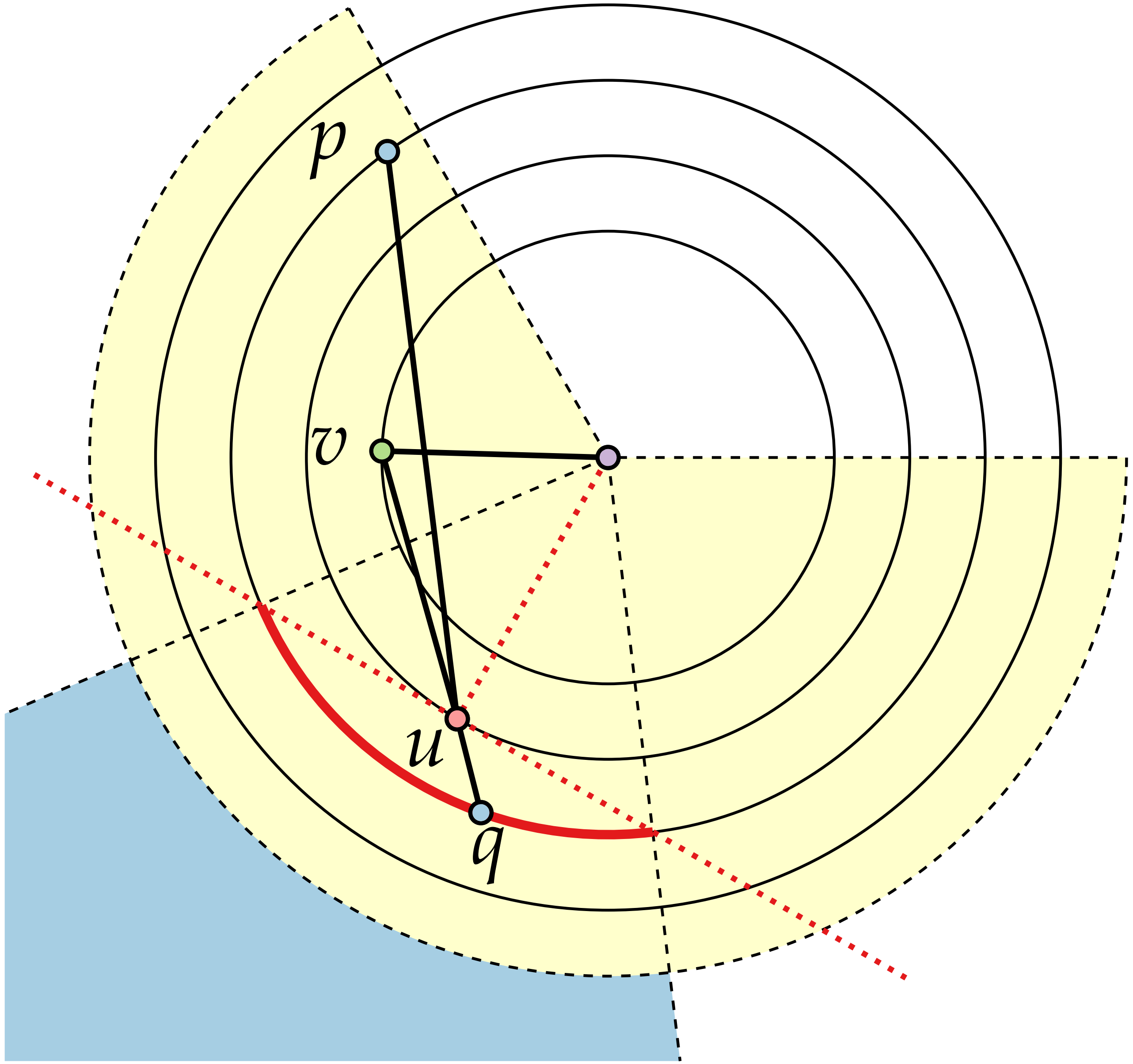

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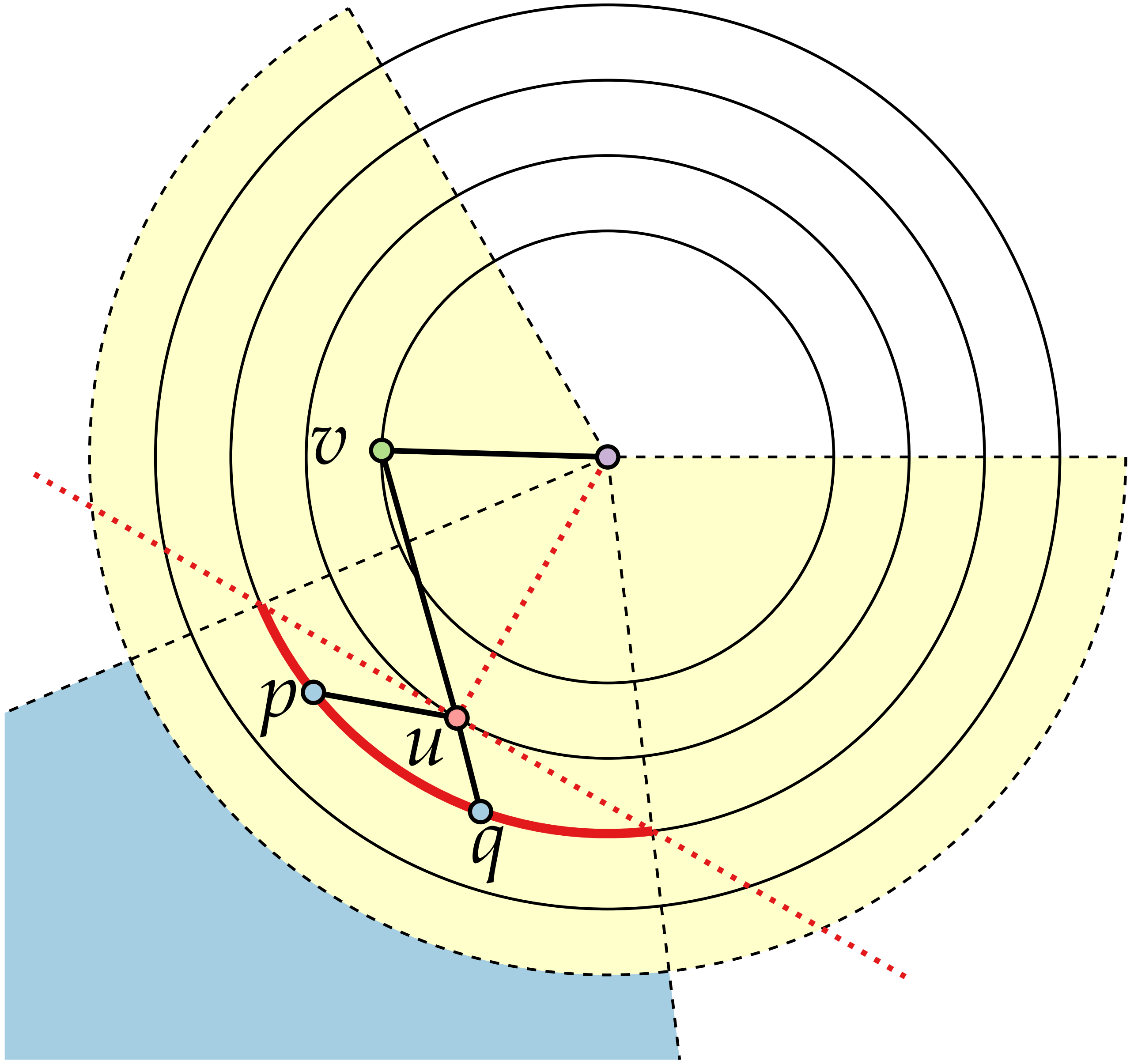

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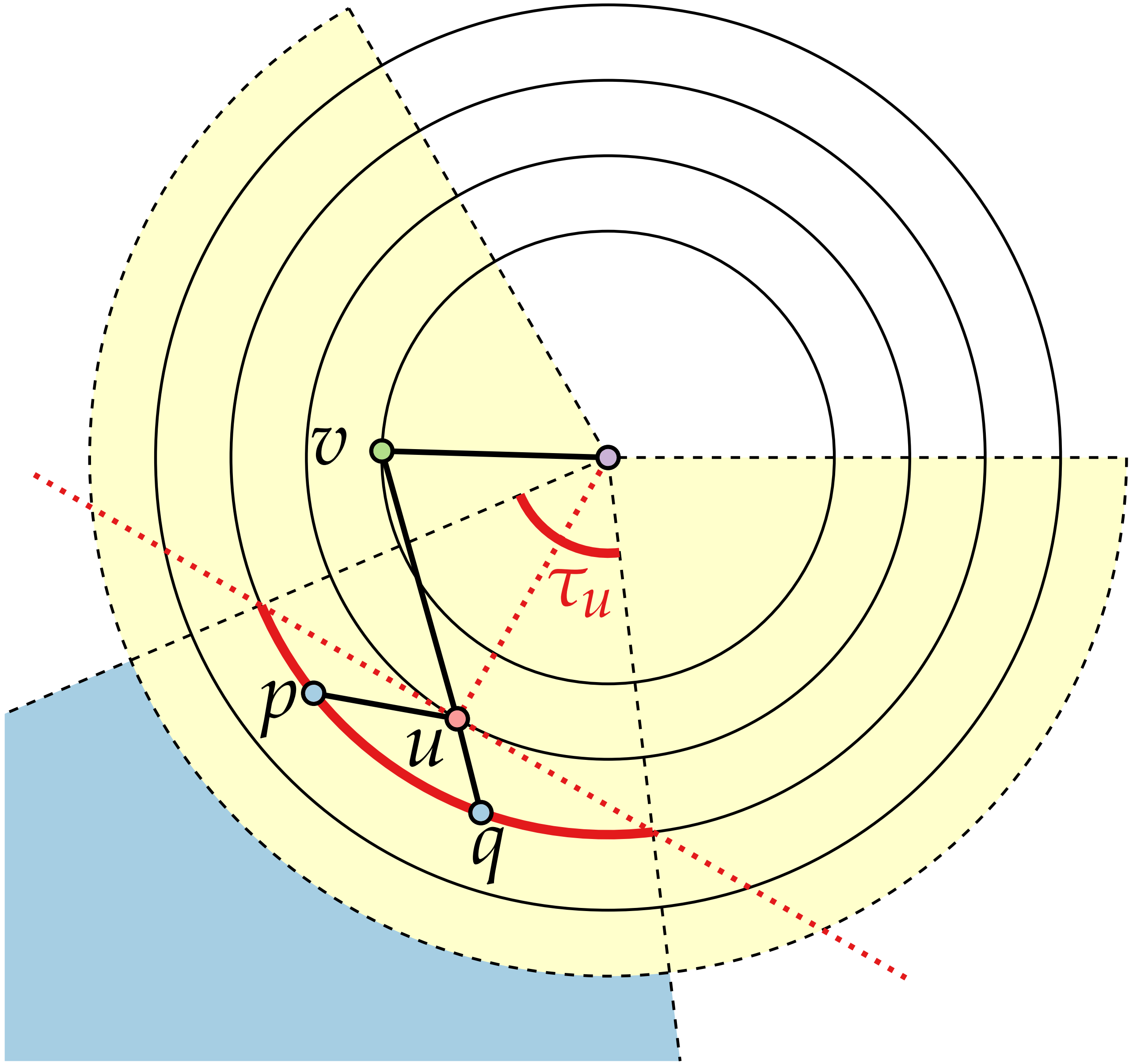

- τ_u – angle of the wedge corresponding to vertex u

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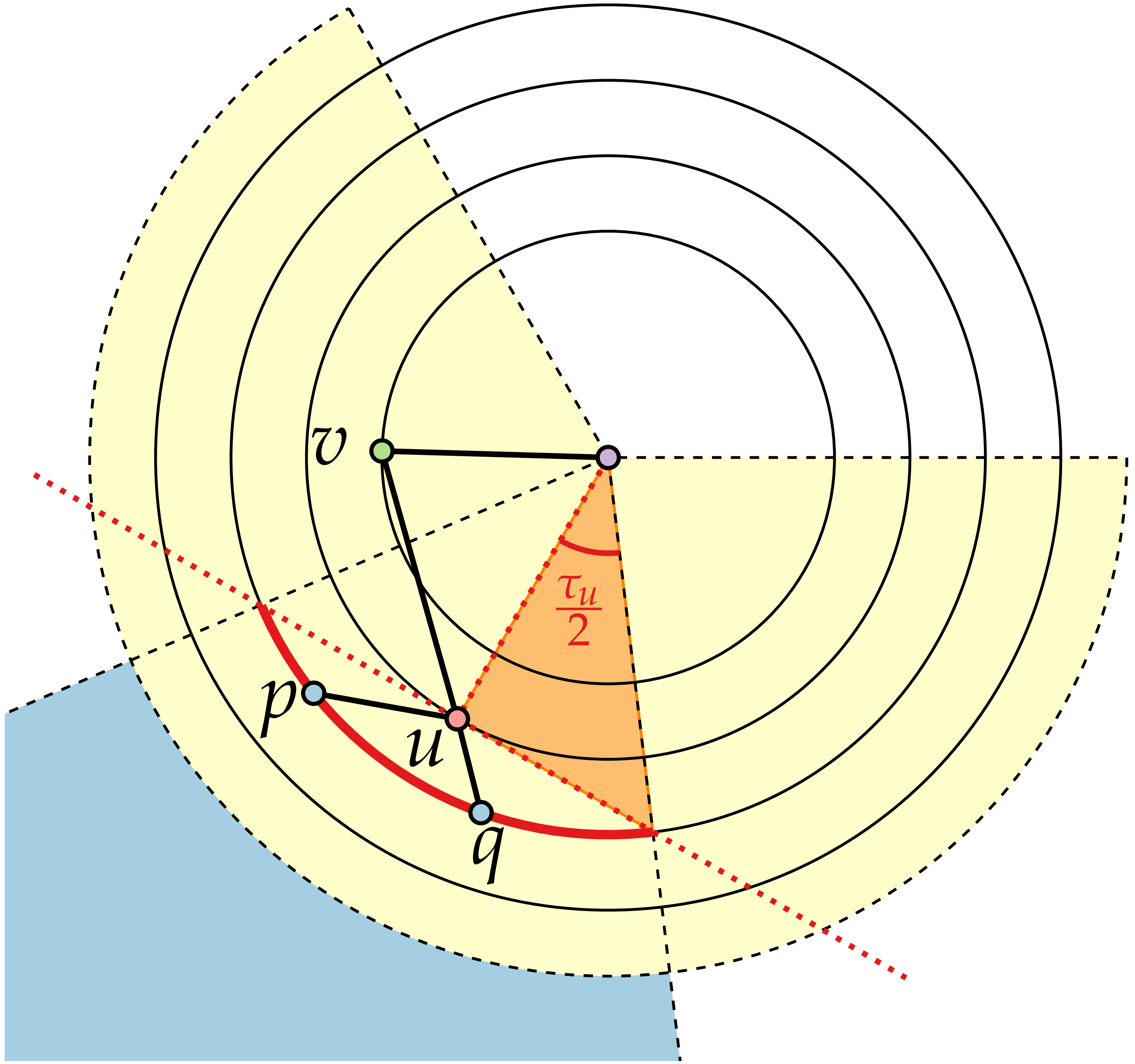

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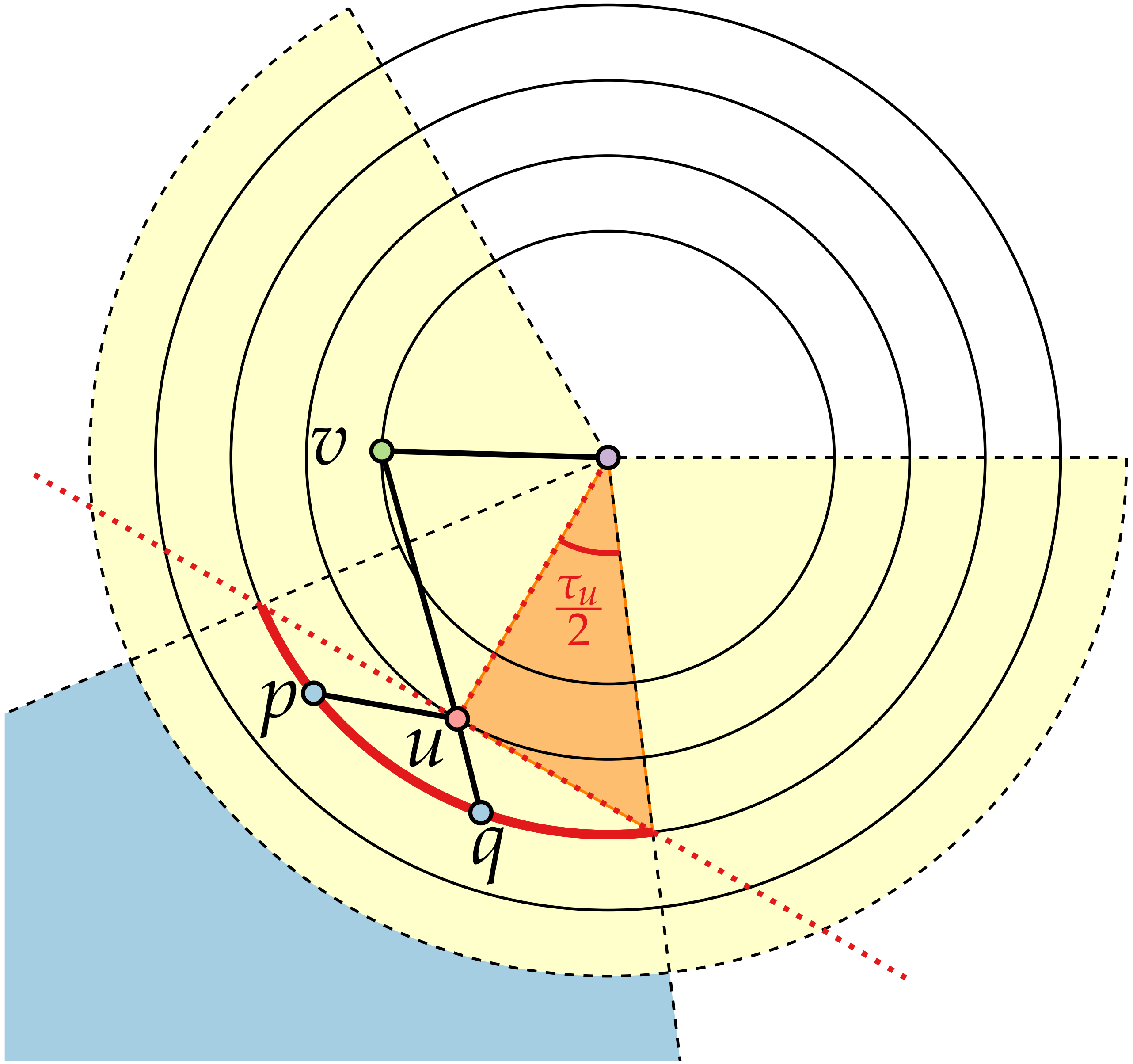

- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u

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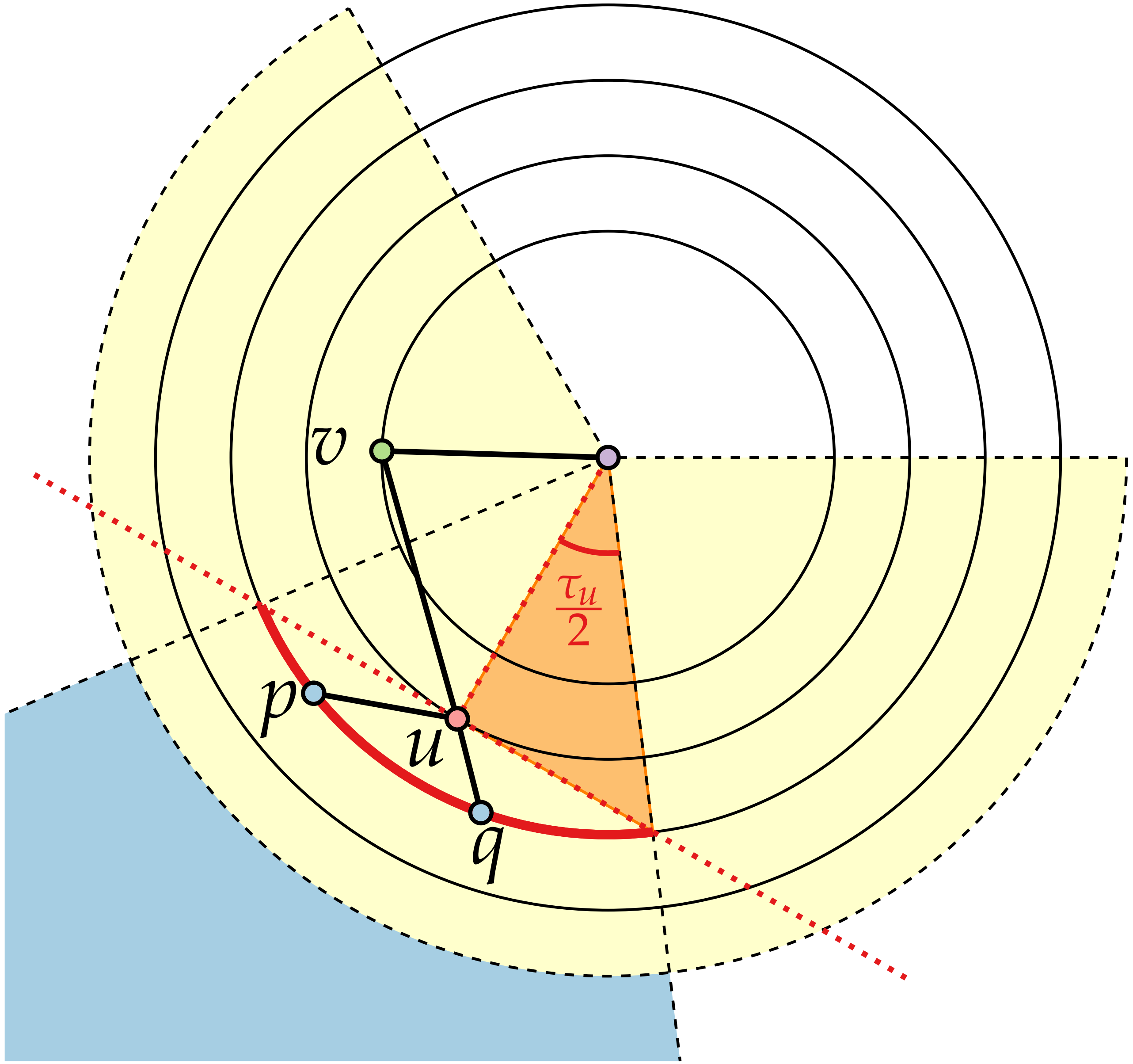

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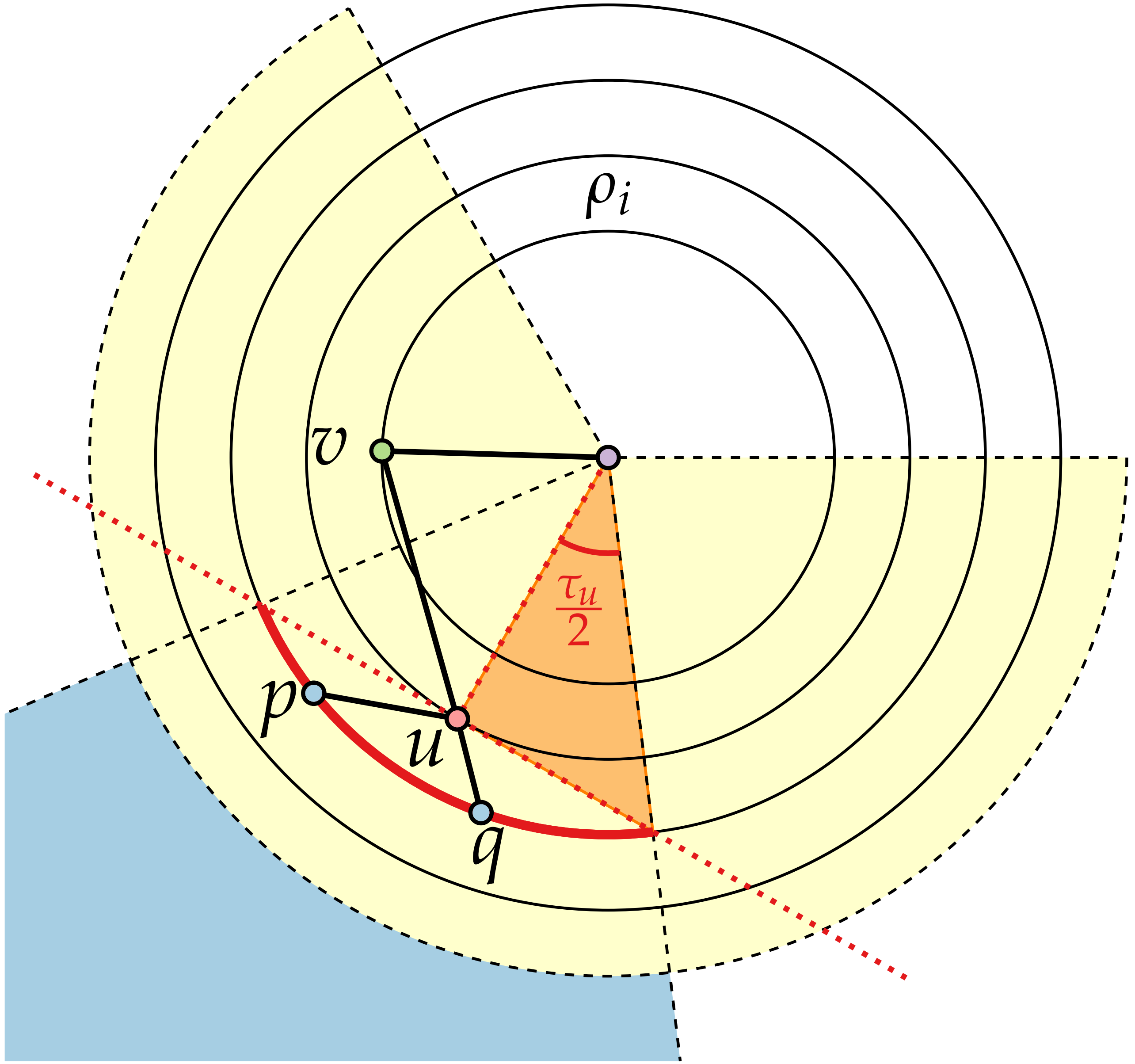

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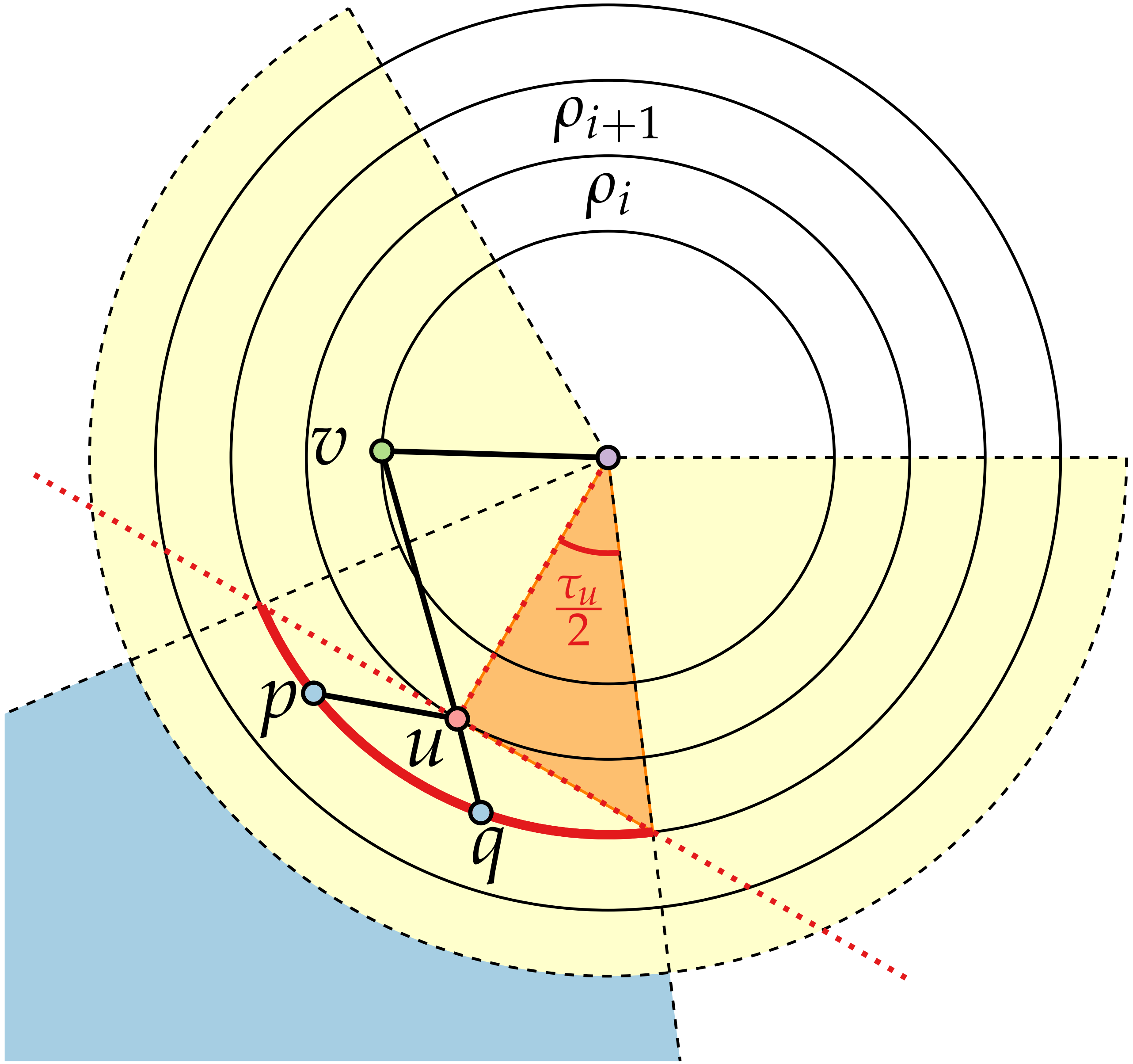

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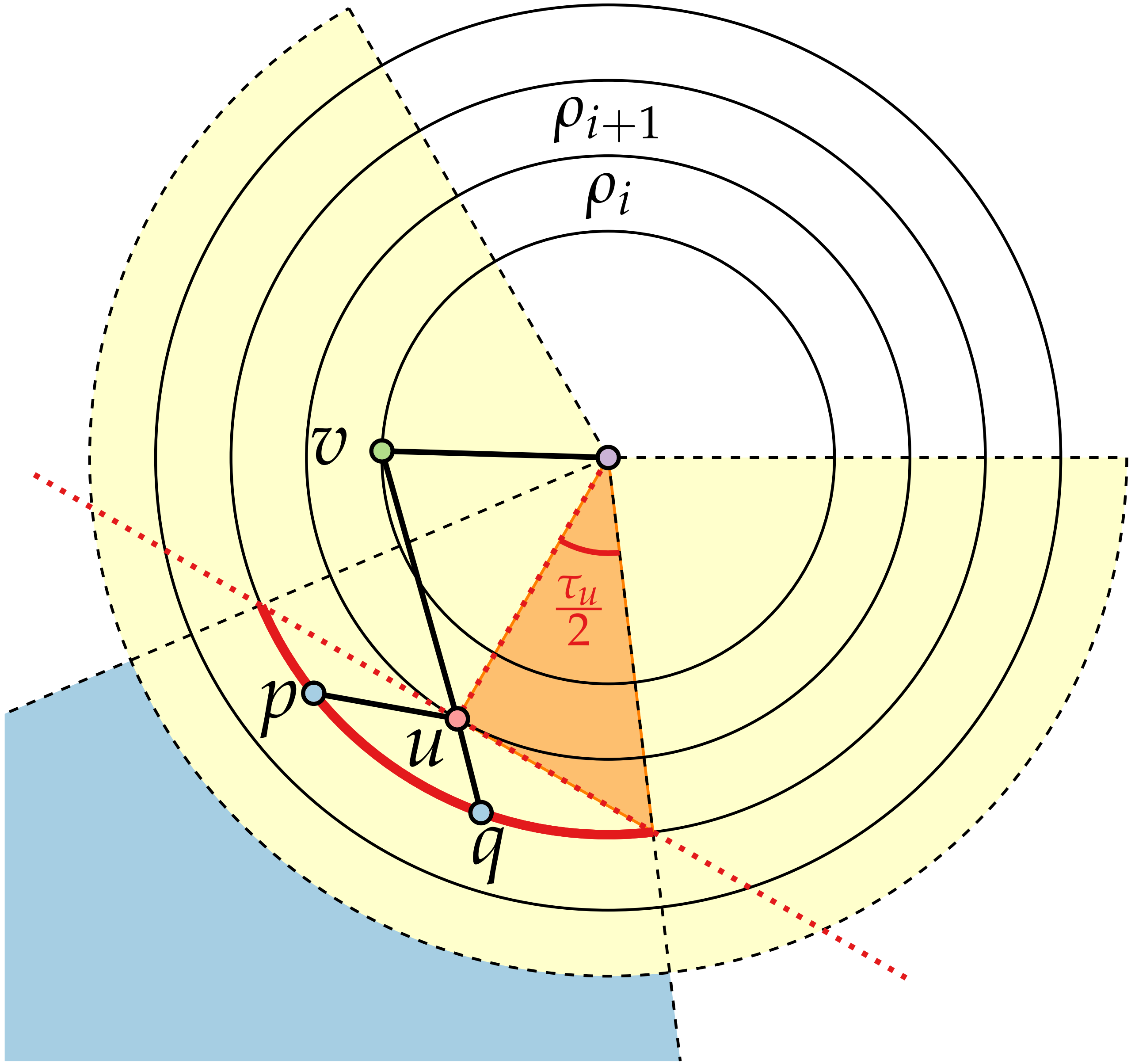
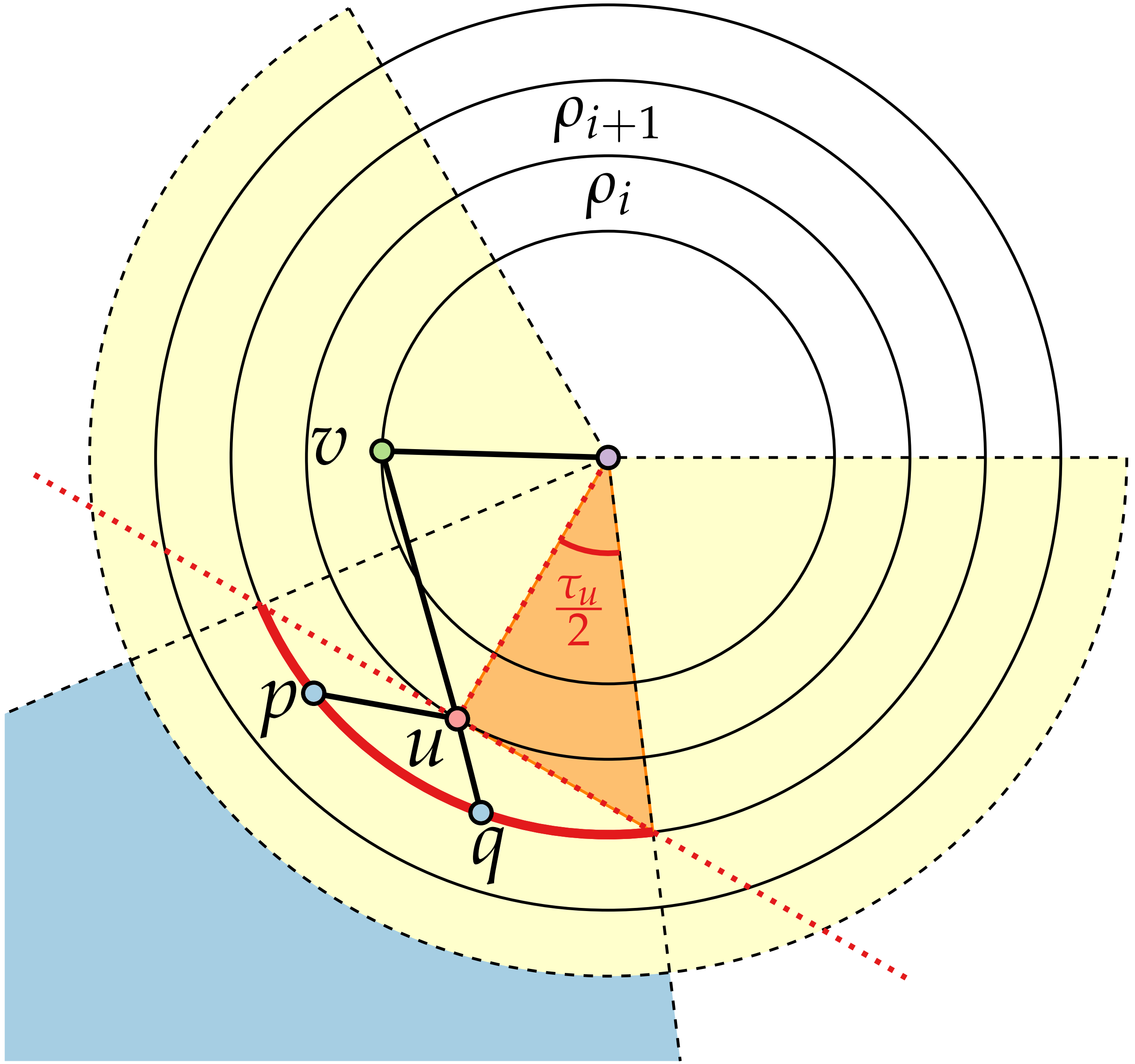


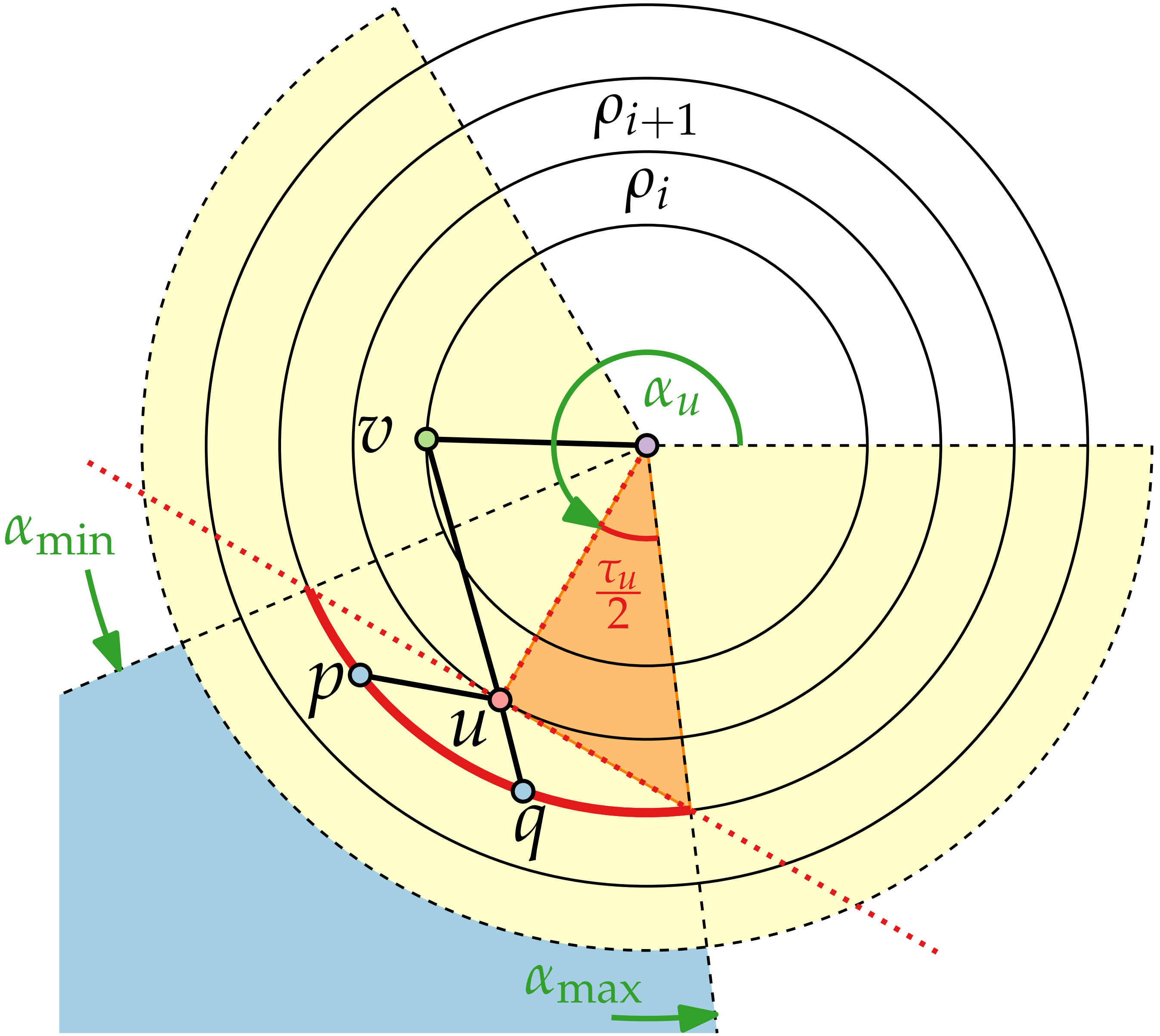
- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u
- ρ_i – radius of layer i
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$

Radial Layouts – How To Avoid Crossings



- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u
- ρ_i – radius of layer i
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$
- $\tau_u = \min\{\frac{\ell(u)}{\ell(v)-1}, 2\arccos \frac{\rho_i}{\rho_{i+1}}\}$

Radial Layouts – How To Avoid Crossings



- τ_u – angle of the wedge corresponding to vertex u
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- ρ_i – radius of layer i
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$
- $\tau_u = \min\{\frac{\ell(u)}{\ell(v)-1}, 2 \arccos \frac{\rho_i}{\rho_{i+1}}\}$
- Alternative:

$$\alpha_{\min} = \alpha_u - \arccos \frac{\rho_i}{\rho_{i+1}}$$

$$\alpha_{\max} = \alpha_u + \arccos \frac{\rho_i}{\rho_{i+1}}$$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$\quad$ *postorder*(r)

$\quad$ *preorder*$(r, 0, 0, 2\pi)$

$\quad$ **return** $(d_v, \alpha_v)_{v \in V(T)}$

$\quad$ // vertex pos./polar coord.

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

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postorder(r)

preorder$(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

// vertex pos./polar coord.

postorder(vertex v)

$\ell(v) \leftarrow 1$

foreach child w of v **do**

calculate the size of the subtree recursively

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

```
begin
    postorder(r)
    preorder(r, 0, 0, 2π)
    return (d_v, α_v)_{v∈V(T)}
    // vertex pos./polar coord.
```

postorder(vertex v)

```
    ℓ(v) ← 1
    foreach child w of v do
        postorder(w)
        ℓ(v) ← ℓ(v) + ℓ(w)
```

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

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    ℓ(v) ← 1
    foreach child w of v do
        postorder(w)
        ℓ(v) ← ℓ(v) + ℓ(w)
```

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

Radial Layouts – Pseudocode

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  postorder(r)

  preorder$(r, 0, 0, 2\pi)$

  return $(d_v, \alpha_v)_{v \in V(T)}$

  // vertex pos./polar coord.

postorder(vertex v)

  $\ell(v) \leftarrow 1$

  foreach child w of v **do**

    postorder(w)

    $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

  $d_v \leftarrow \rho_t$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

  postorder(r)

  preorder$(r, 0, 0, 2\pi)$

  return $(d_v, \alpha_v)_{v \in V(T)}$

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  foreach child w of v **do**

    postorder(w)

    $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

  $d_v \leftarrow \rho_t$

  $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

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  $d_v \leftarrow \rho_t$
  $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ *//output*

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \dots < \rho_k$)

begin
- *postorder*(r)
- *preorder*$(r, 0, 0, 2\pi)$
- **return** $(d_v, \alpha_v)_{v \in V(T)}$
- // vertex pos./polar coord.

postorder(vertex v)
- $\ell(v) \leftarrow 1$
- **foreach** child w of v **do**
  - *postorder*(w)
  - $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)
- $d_v \leftarrow \rho_t$
- $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ //*output*
- **if** $t > 0$ **then**

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin
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  preorder$(r, 0, 0, 2\pi)$
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postorder(vertex v)
  $\ell(v) \leftarrow 1$
  foreach child w of v **do**
    postorder(w)
    $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)
  $d_v \leftarrow \rho_t$
  $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$
  //output
  if $t > 0$ **then**
    $\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$
    $\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

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- **foreach** child w of v **do**
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preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)
- $d_v \leftarrow \rho_t$
- $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ //*output*
- **if** $t > 0$ **then**
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- *left* $\leftarrow \alpha_{\min}$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

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foreach child w of v **do**

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

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    postorder(r)
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```

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

```
    d_v ← ρ_t                                //output
    α_v ← (α_min + α_max)/2
    if t > 0 then
        α_min ← max{α_min, α_v − arccos(ρ_t / ρ_{t+1})}
        α_max ← min{α_max, α_v + arccos(ρ_t / ρ_{t+1})}
    left ← α_min
    foreach child w of v do
        right ← left + ℓ(w)/(ℓ(v)−1) · (α_max − α_min)
```

Radial Layouts – Pseudocode

```
RadialTreeLayout(tree T, root r ∈ T, radii ρ_1 < ··· < ρ_k)
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```

```
preorder(vertex v, t, α_min, α_max)
    d_v ← ρ_t                              //output
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    if t > 0 then
        α_min ← max{α_min, α_v − arccos ρ_t/ρ_{t+1}}
        α_max ← min{α_max, α_v + arccos ρ_t/ρ_{t+1}}
    left ← α_min
    foreach child w of v do
        right ← left + ℓ(w)/(ℓ(v)−1) · (α_max − α_min)
        preorder(w, t + 1, left, right)
```

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

```
begin
    postorder(r)
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    foreach child w of v do
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        preorder(w, t+1, left, right)
        left ← right
```

Radial Layouts – Pseudocode

```
RadialTreeLayout(tree T, root r ∈ T, radii ρ_1 < ··· < ρ_k)
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```

Runtime?

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

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postorder(vertex v)

 $\ell(v) \leftarrow 1$

 foreach child w of v **do**

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preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

 $d_v \leftarrow \rho_t$

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 if $t > 0$ **then**

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 left $\leftarrow \alpha_{\min}$

 foreach child w of v **do**

  right $\leftarrow$ *left* $+ \frac{\ell(w)}{\ell(v)-1} \cdot (\alpha_{\max} - \alpha_{\min})$

  preorder$(w, t+1, \textit{left}, \textit{right})$

  left $\leftarrow$ *right*

Runtime? $\mathcal{O}(n)$

Radial Layouts – Pseudocode

```
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```
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    d_v ← ρ_t                              //output
    α_v ← (α_min + α_max)/2
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        left ← right
```

Runtime? $\mathcal{O}(n)$

Correctness?

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

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  $\ell(v) \leftarrow 1$

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    postorder(w)

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  $d_v \leftarrow \rho_t$

  $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ *//output*

  if $t > 0$ **then**

    $\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$

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  left $\leftarrow \alpha_{\min}$

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    right $\leftarrow$ *left* $+ \frac{\ell(w)}{\ell(v)-1} \cdot (\alpha_{\max} - \alpha_{\min})$

    preorder$(w, t+1, \textit{left}, \textit{right})$

    left $\leftarrow$ *right*

Runtime? $\mathcal{O}(n)$

Correctness? ✓

Radial Layouts – Result

Theorem.
Let T be a tree with n vertices. The RadialTreeLayout algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

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Let T be a tree with n vertices. The RadialTreeLayout algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

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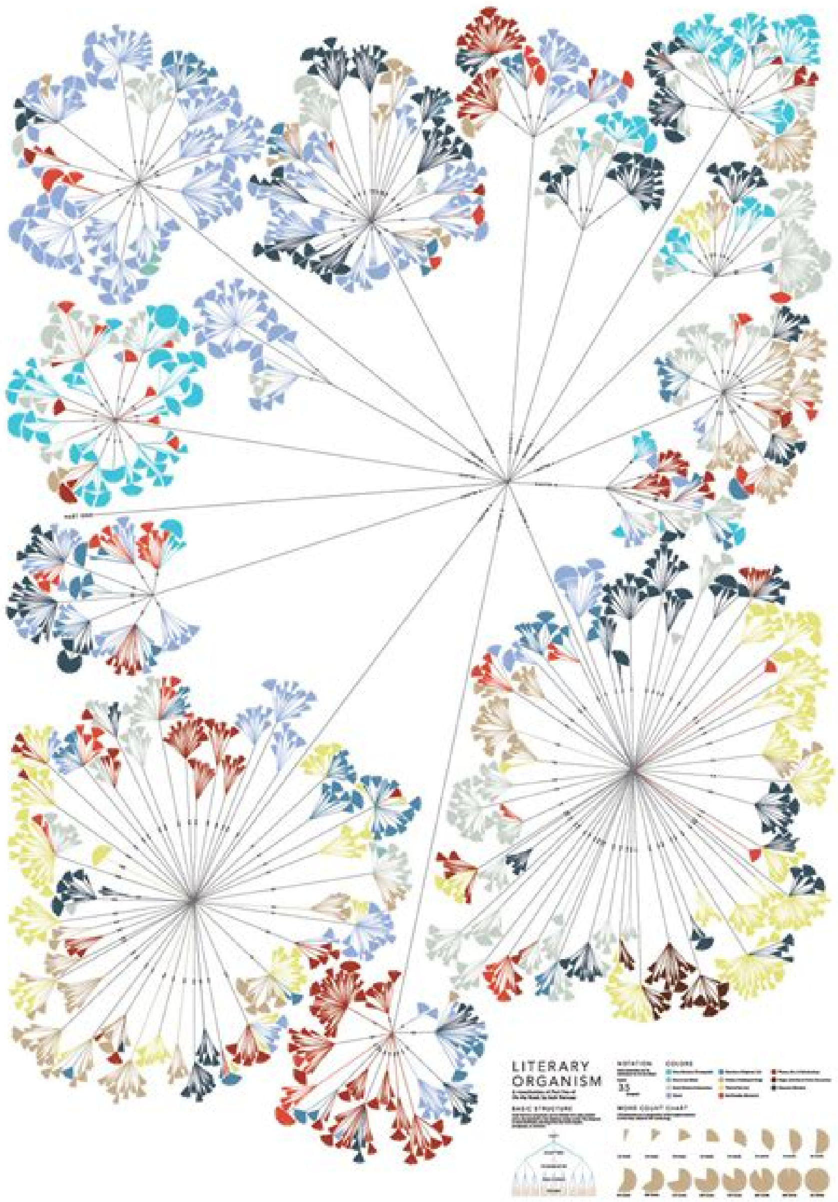
- Vertices lie on circle according to their depth

Radial Layouts – Result

Theorem.
Let T be a tree with n vertices. The RadialTreeLayout algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

- Γ is radial drawing
- Vertices lie on circle according to their depth
- Area quadratic in max degree times height of T
  (see [GD Ch. 3.1.3] if interested)

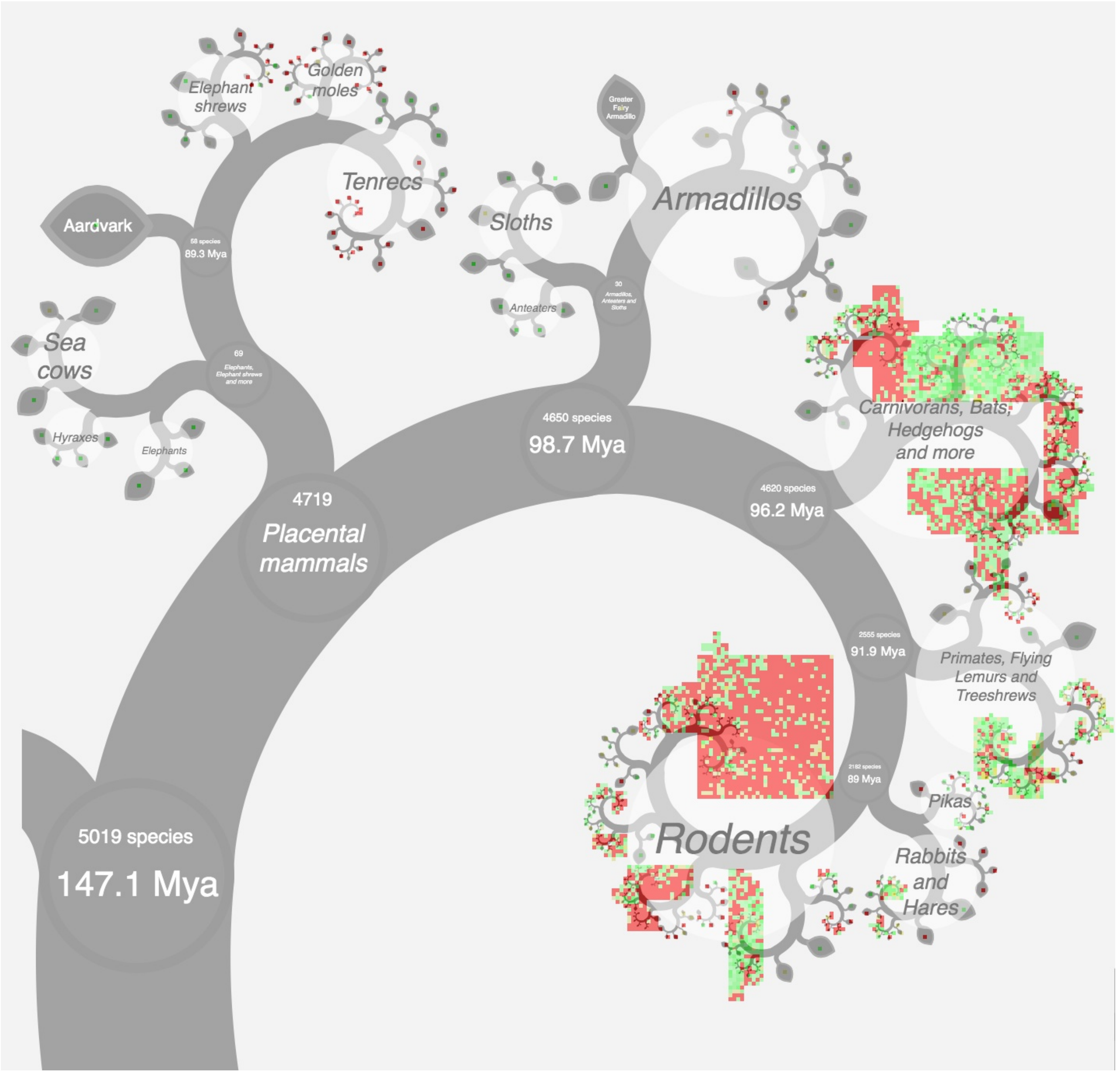
Other Tree Visualization Styles



Writing Without Words:
The project explores methods to visualizes the differences in writing styles of different authors.

Similar to ballon layout

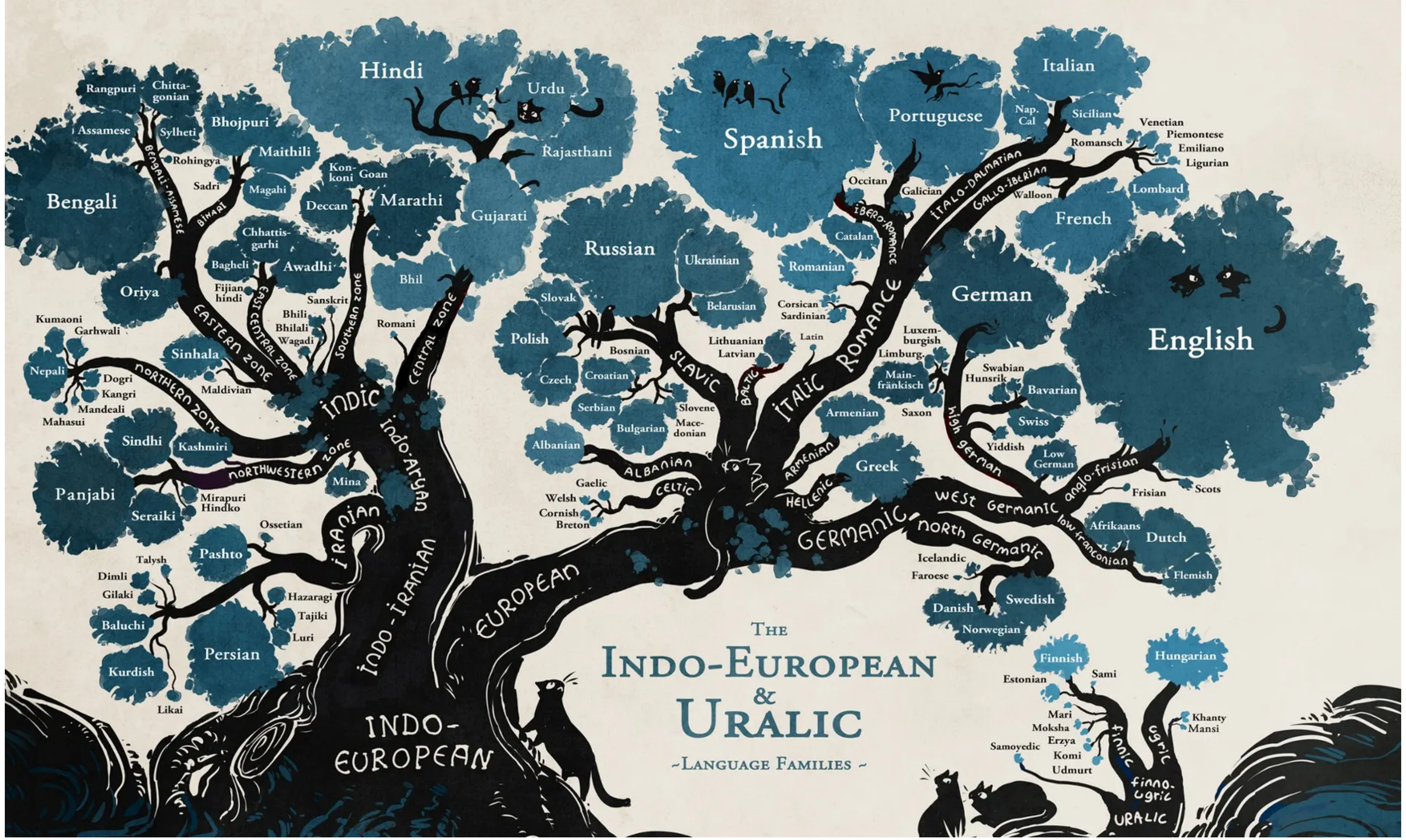
Other Tree Visualization Styles



A phylogenetically organised display of data for all placental mammal species.

Fractal layout

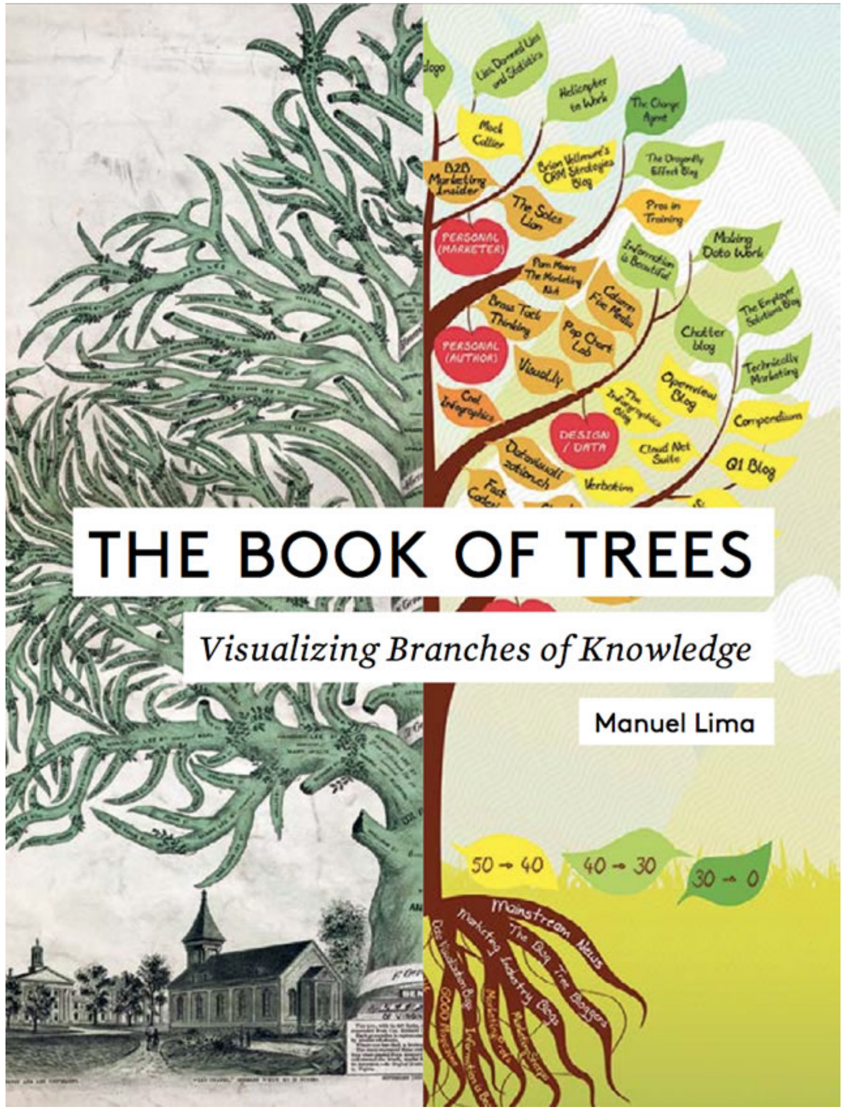
Other Tree Visualization Styles



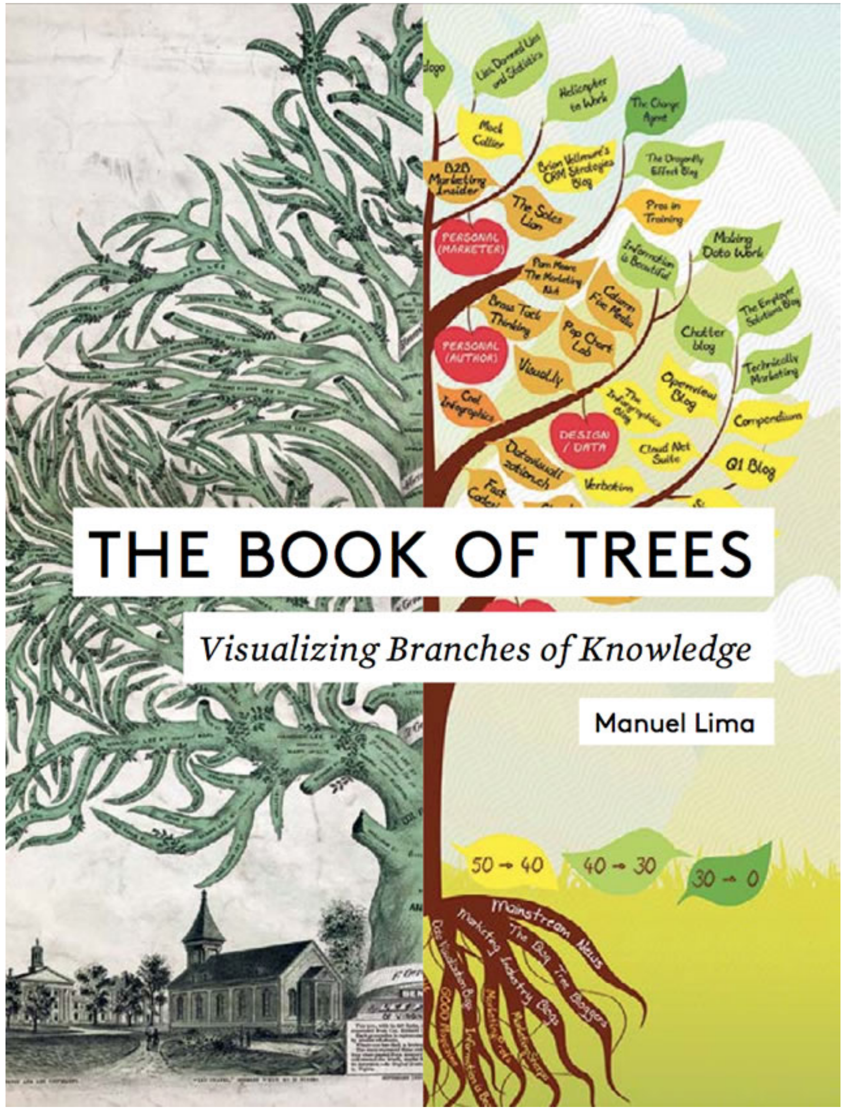
A language family tree – in pictures

Fractal layout

Other Tree Visualization Styles

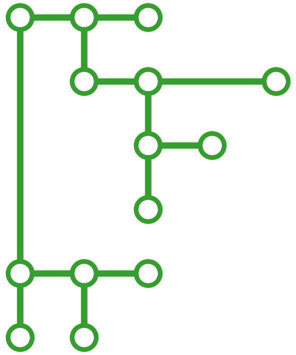


Other Tree Visualization Styles

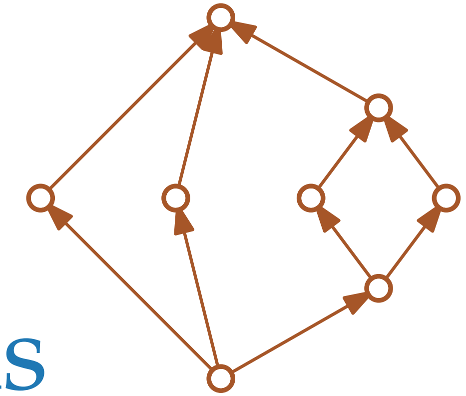


treevis.net

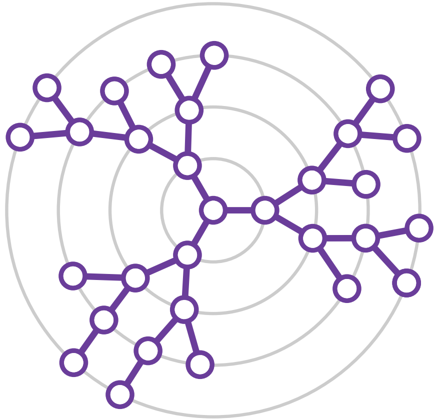
Universität Trier



Visualization of Graphs

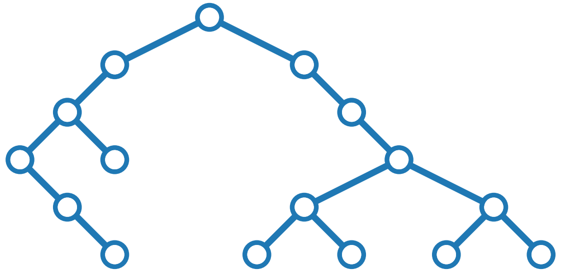


Lecture 2:
Drawing Trees and Series-Parallel Graphs



Part V:
Series-Parallel Graphs

Philipp Kindermann



Series-Parallel Graphs

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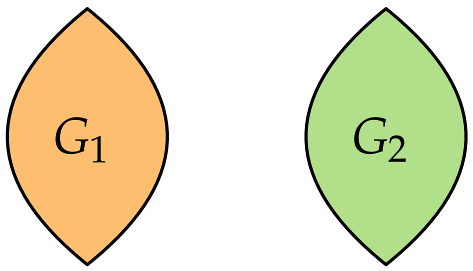
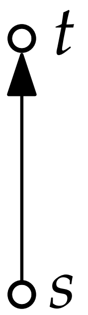
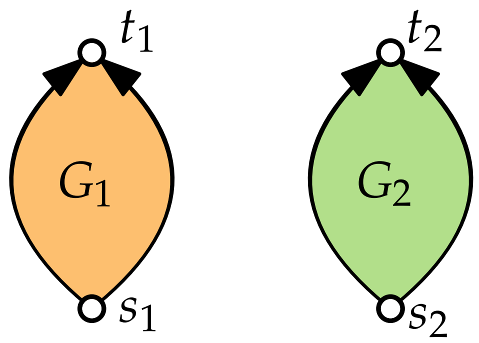


Series-Parallel Graphs

A graph G is **series-parallel**, if

- it contains a single (directed) edge (s, t), or
- it consists of two series-parallel graphs G_1, G_2 with sources s_1, s_2 and sinks t_1, t_2





Series-Parallel Graphs

A graph G is **series-parallel**, if

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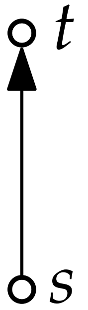
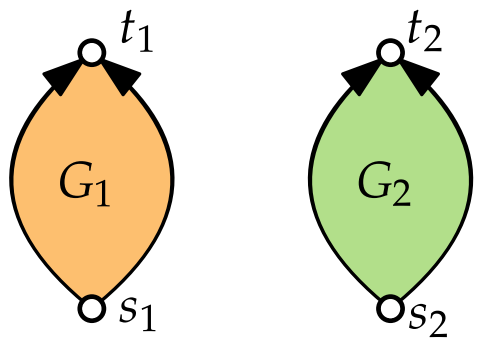
- it consists of two series-parallel graphs G_1, G_2 with sources s_1, s_2 and sinks t_1, t_2 that are combined using one of the following rules:





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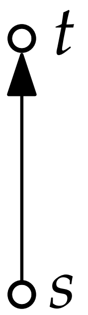
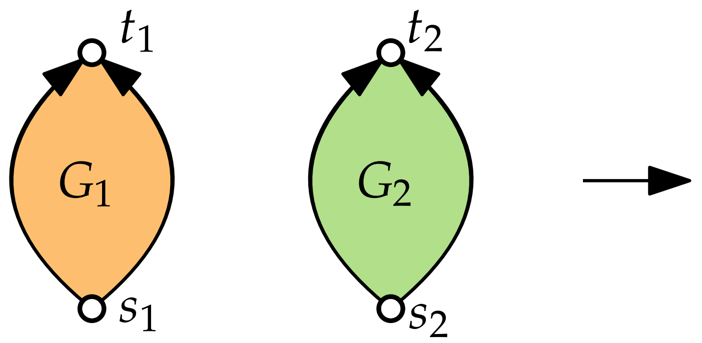


Series composition



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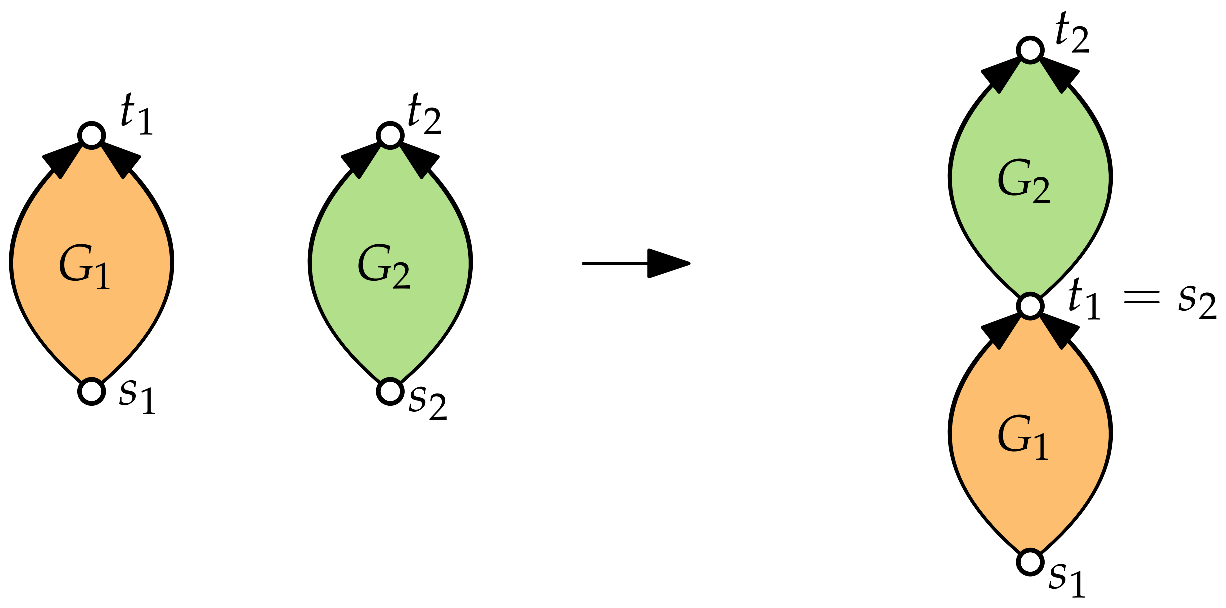

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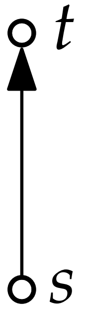
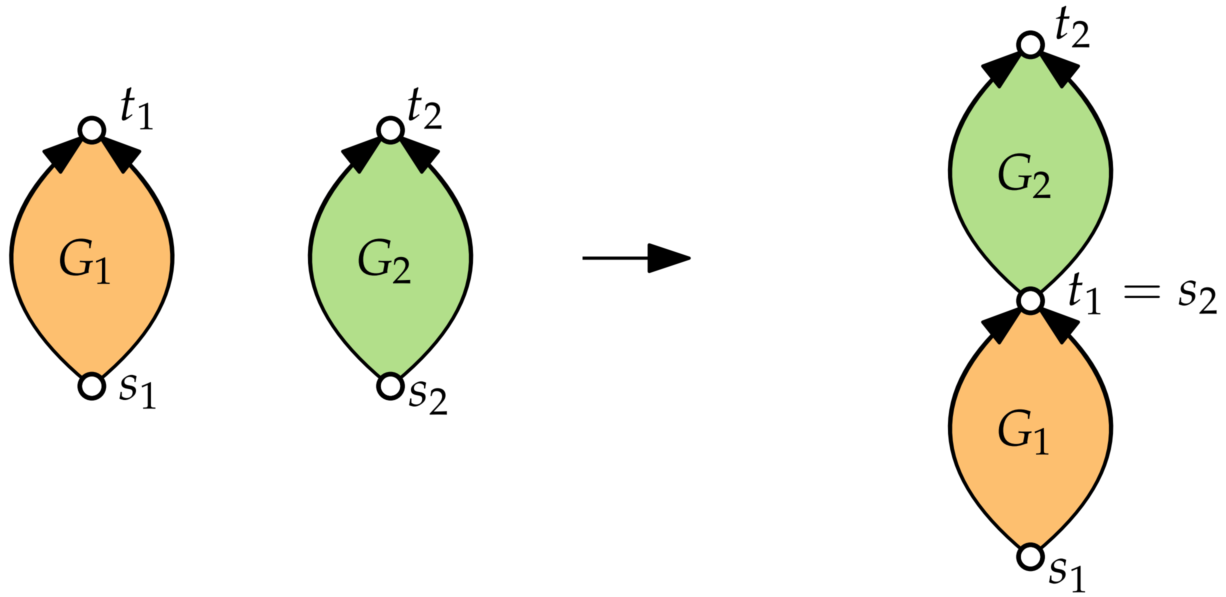


Series composition

Parallel composition



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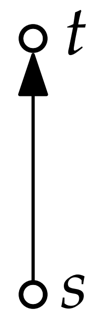
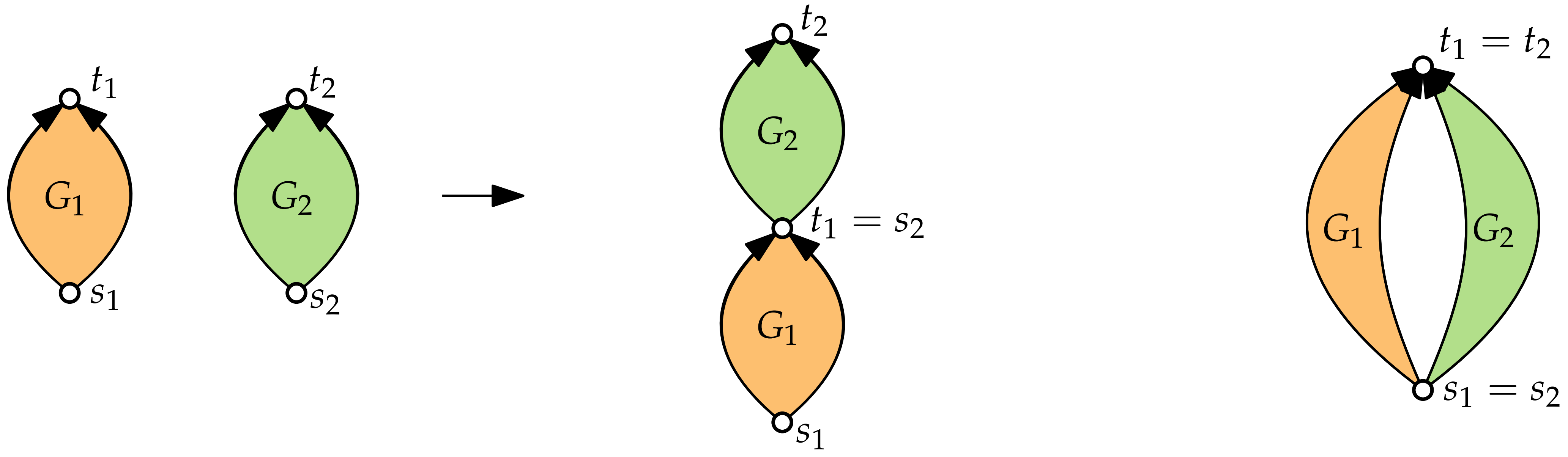


Series composition **Parallel composition**



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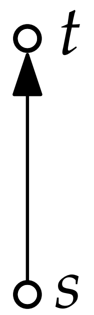
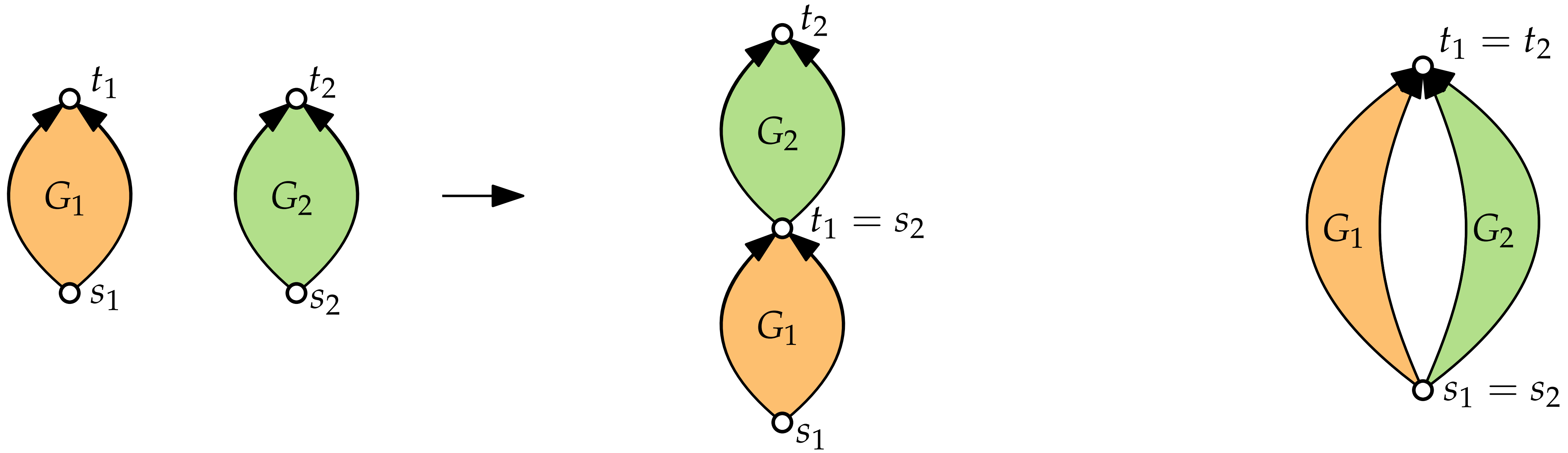


convince yourself that series-parallel graphs are planar

Series composition **Parallel composition**



Series-Parallel Graphs – Decomposition Tree

A **decomposition tree** of G is a binary tree T with nodes of three types: **S**, **P** and **Q**-type

Series-Parallel Graphs – Decomposition Tree

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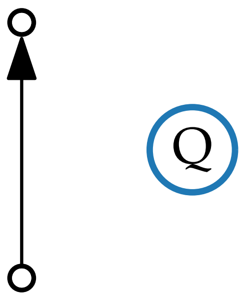

Series-Parallel Graphs – Decomposition Tree

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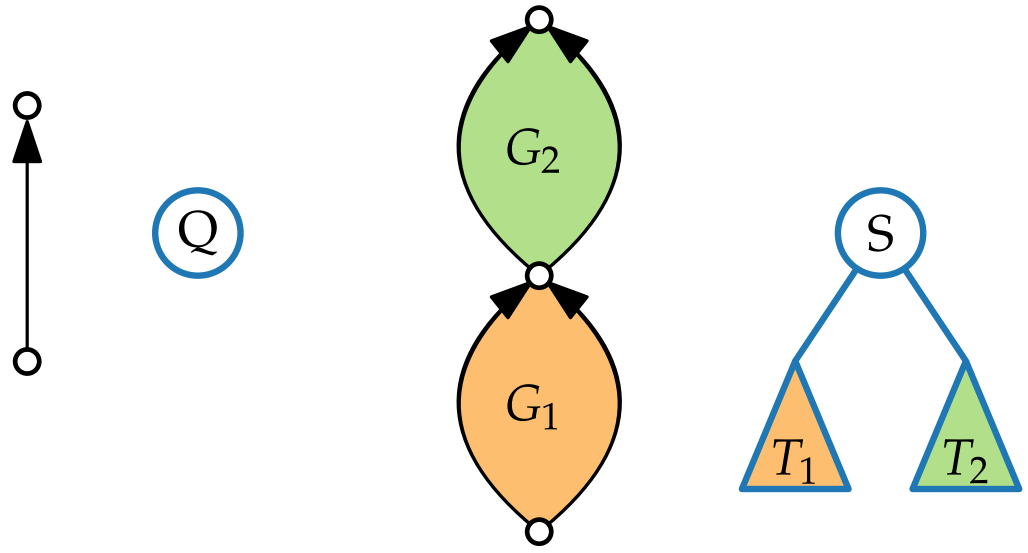

Series-Parallel Graphs – Decomposition Tree

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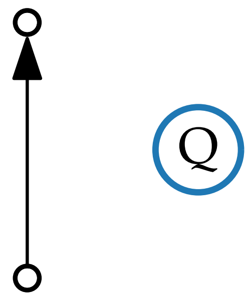
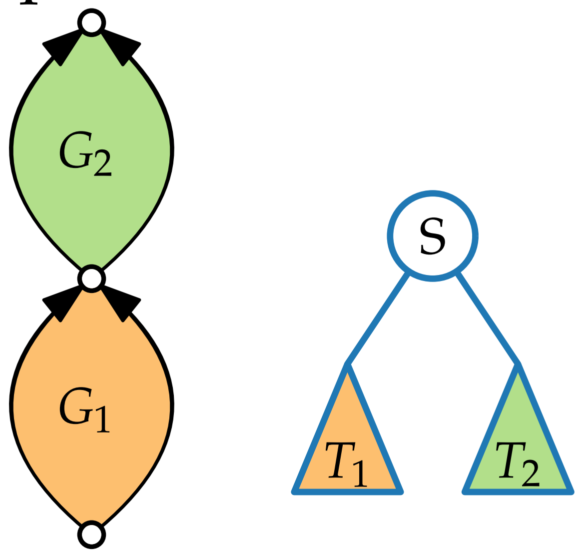
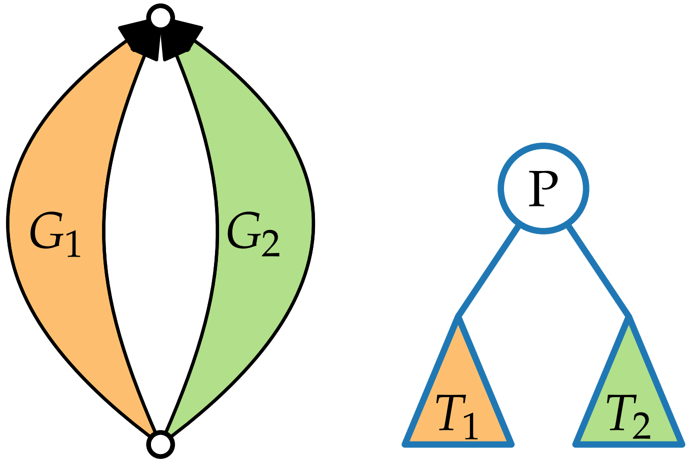
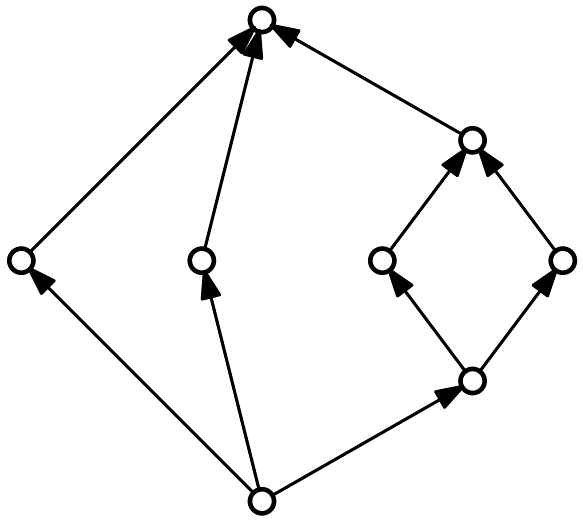






Series-Parallel Graphs – Decomposition Example



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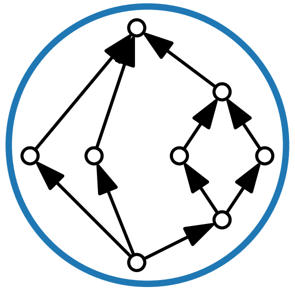

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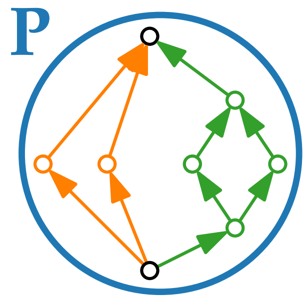

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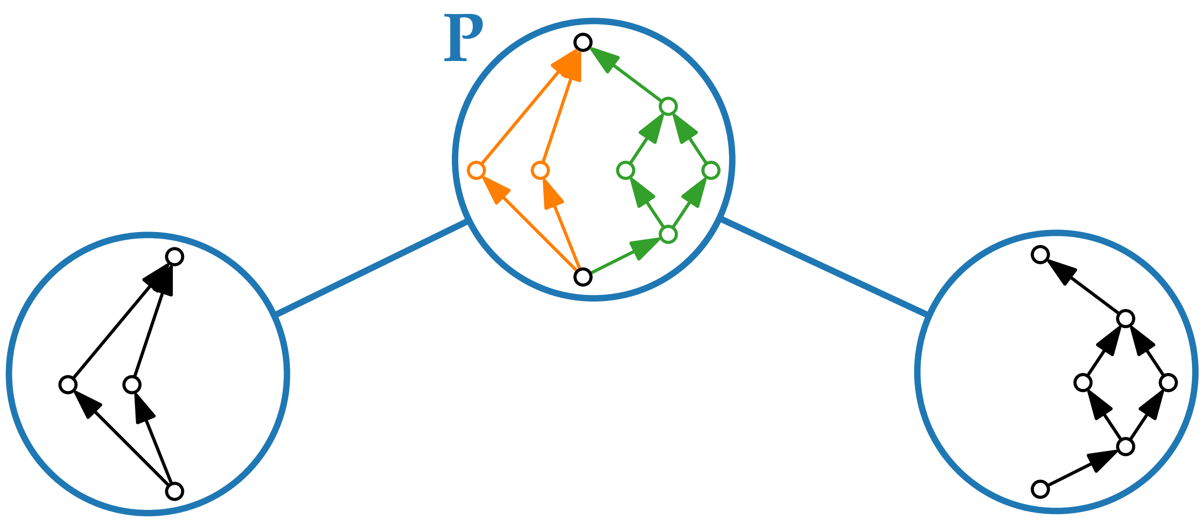

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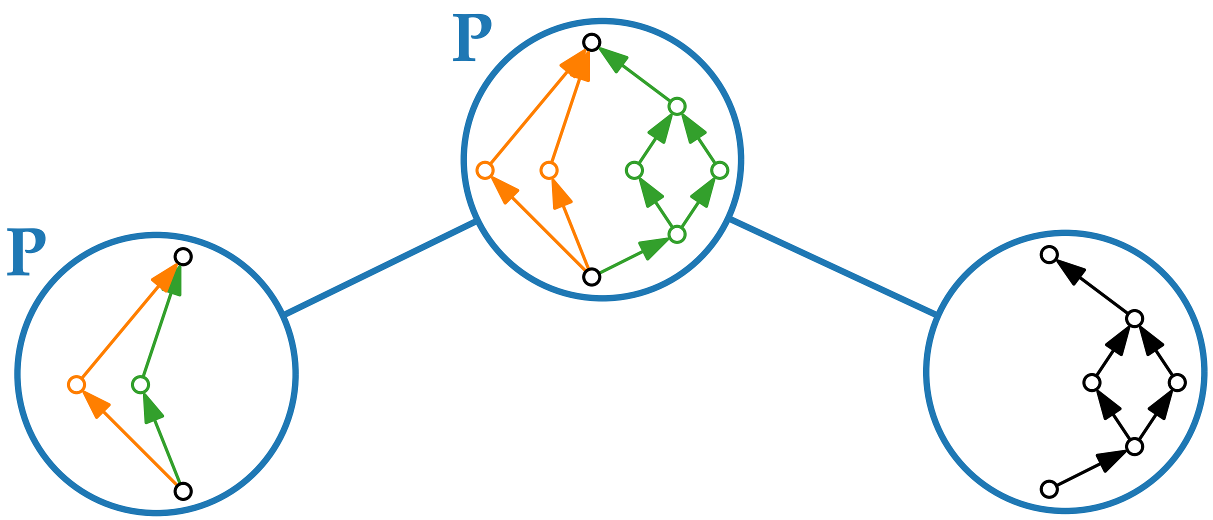

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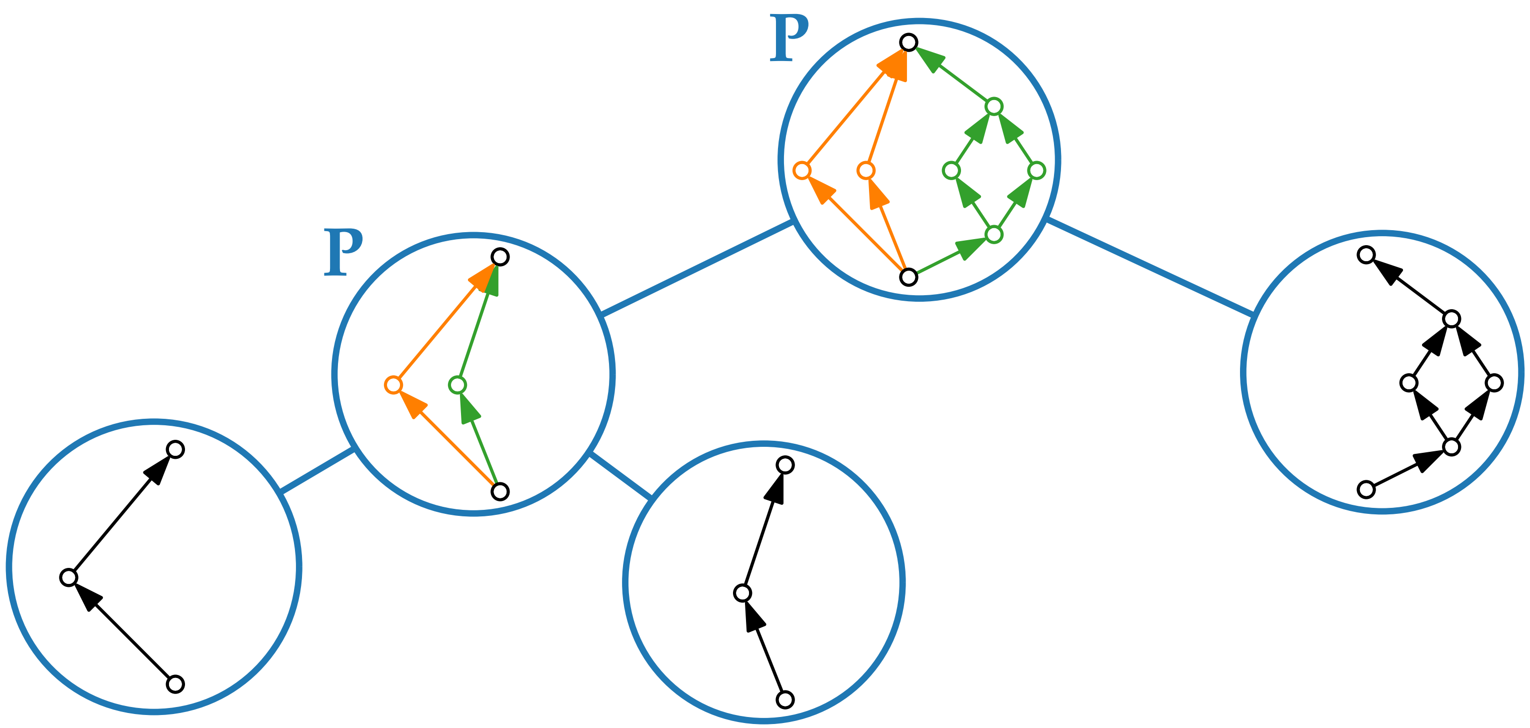

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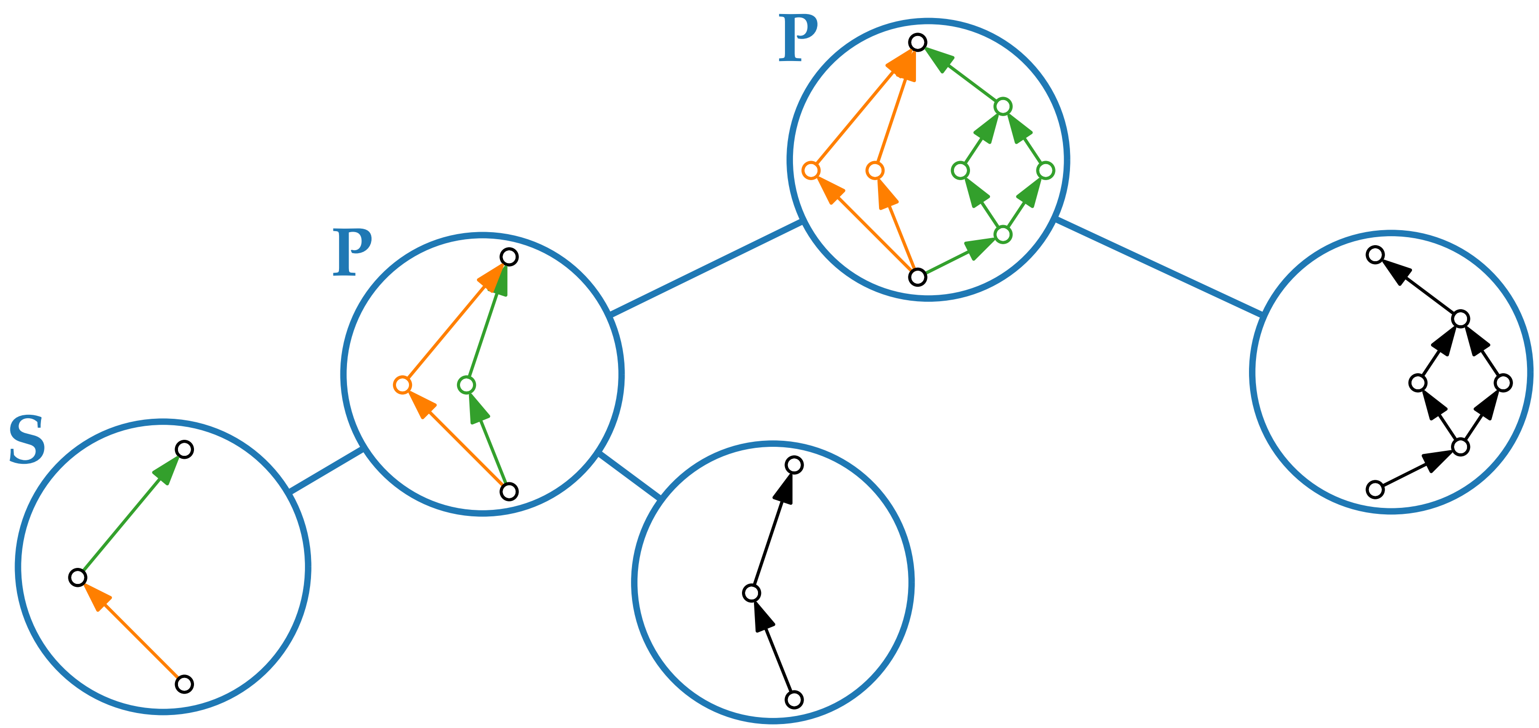

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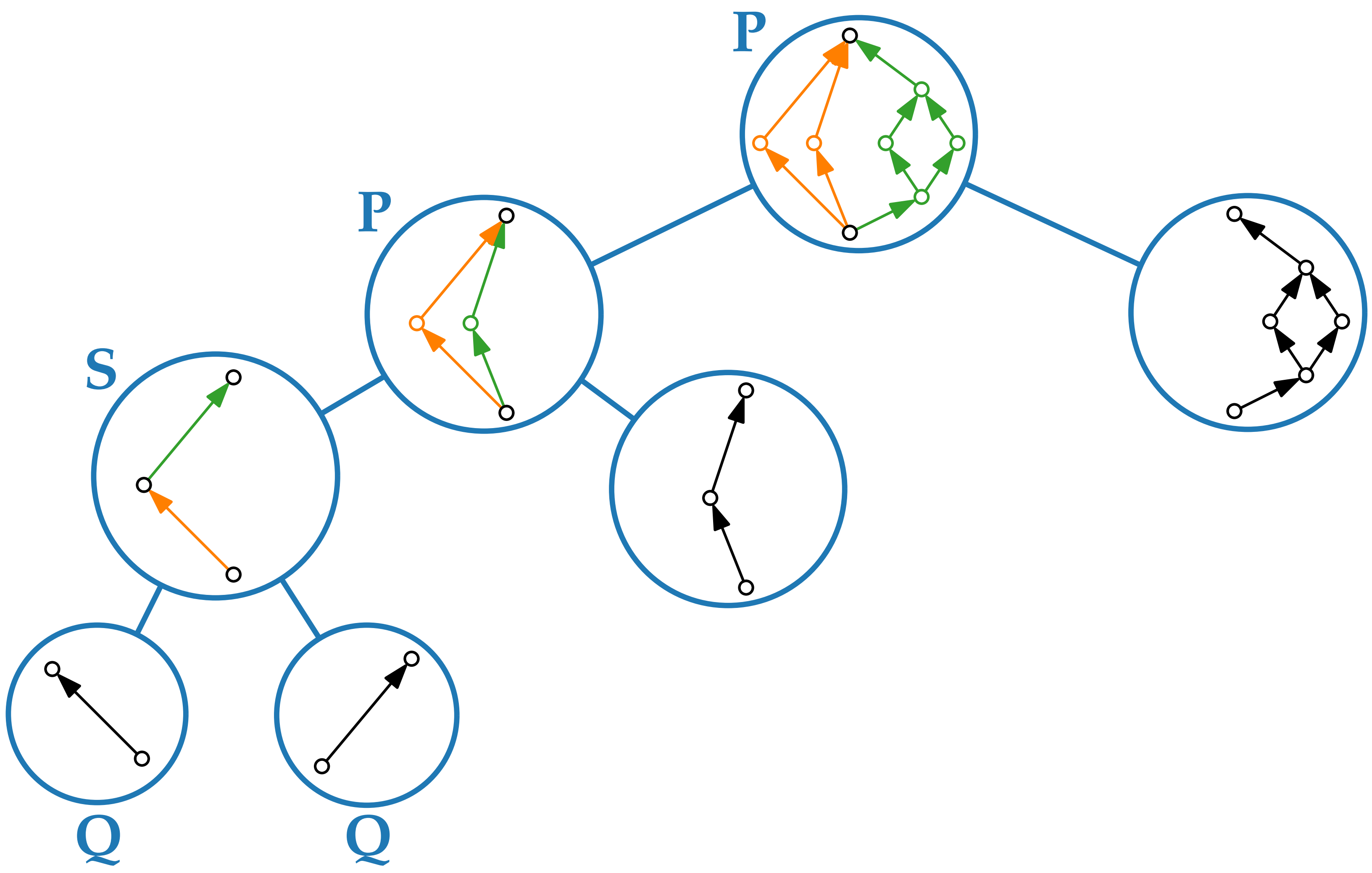

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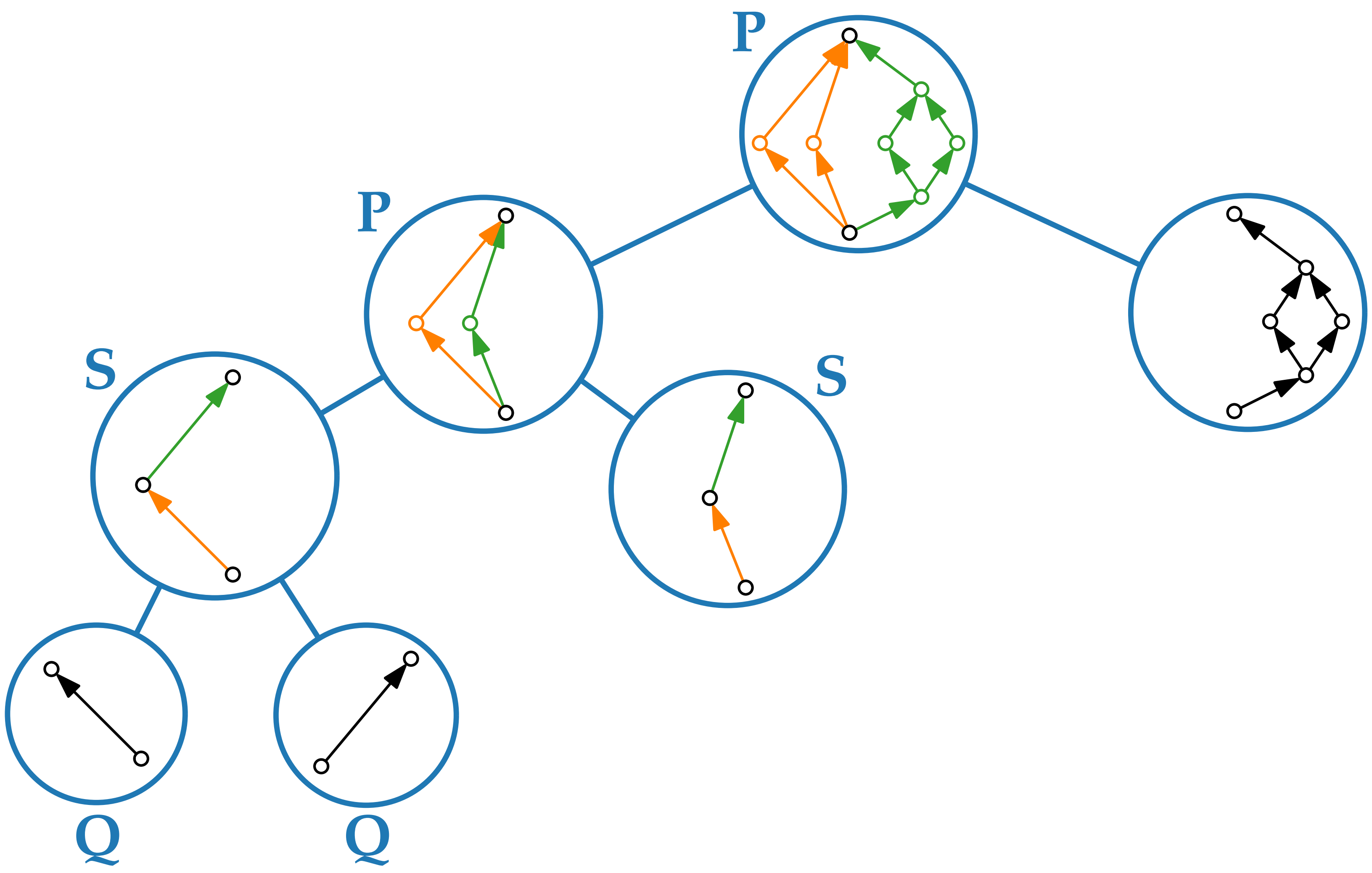

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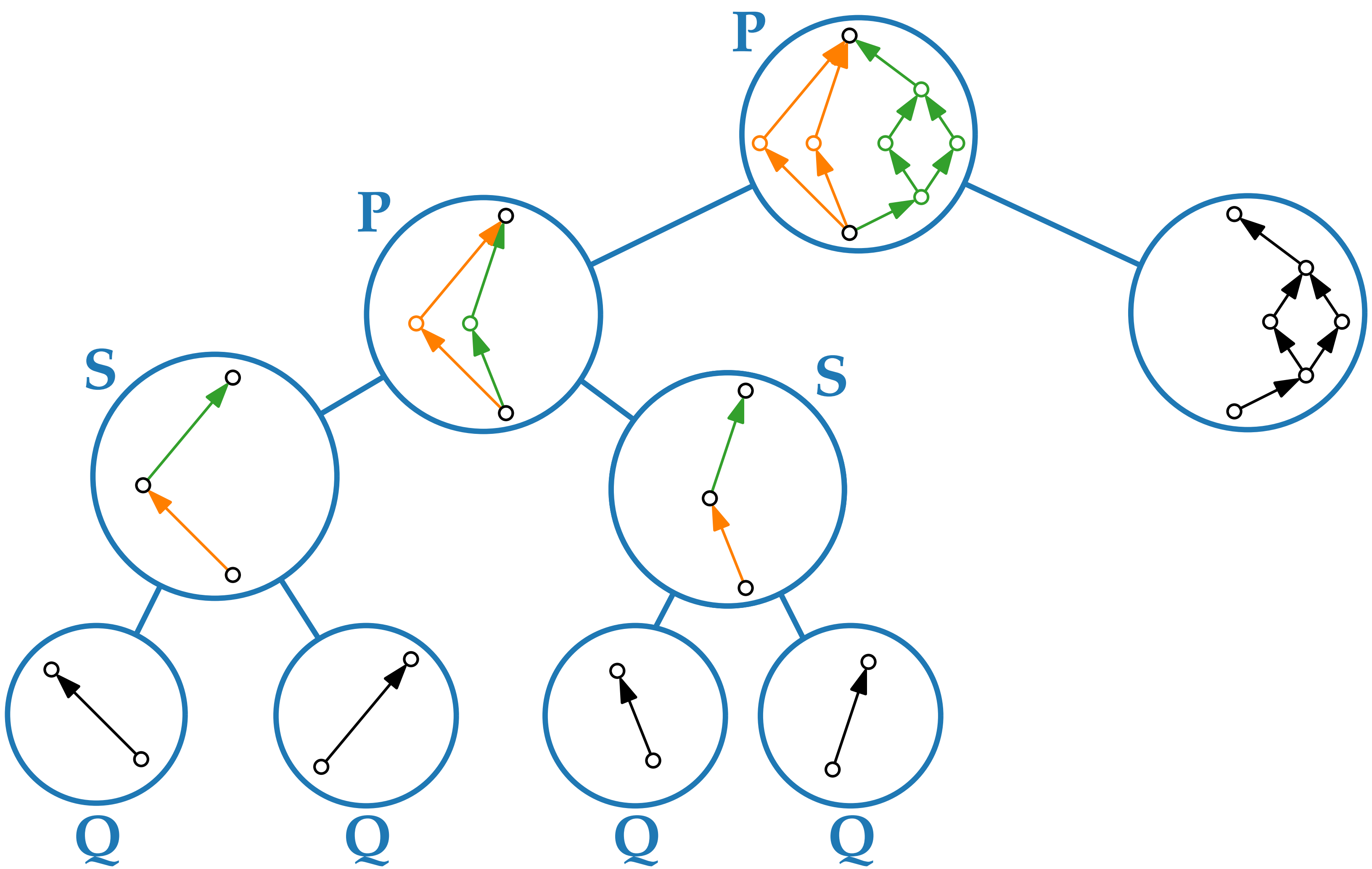

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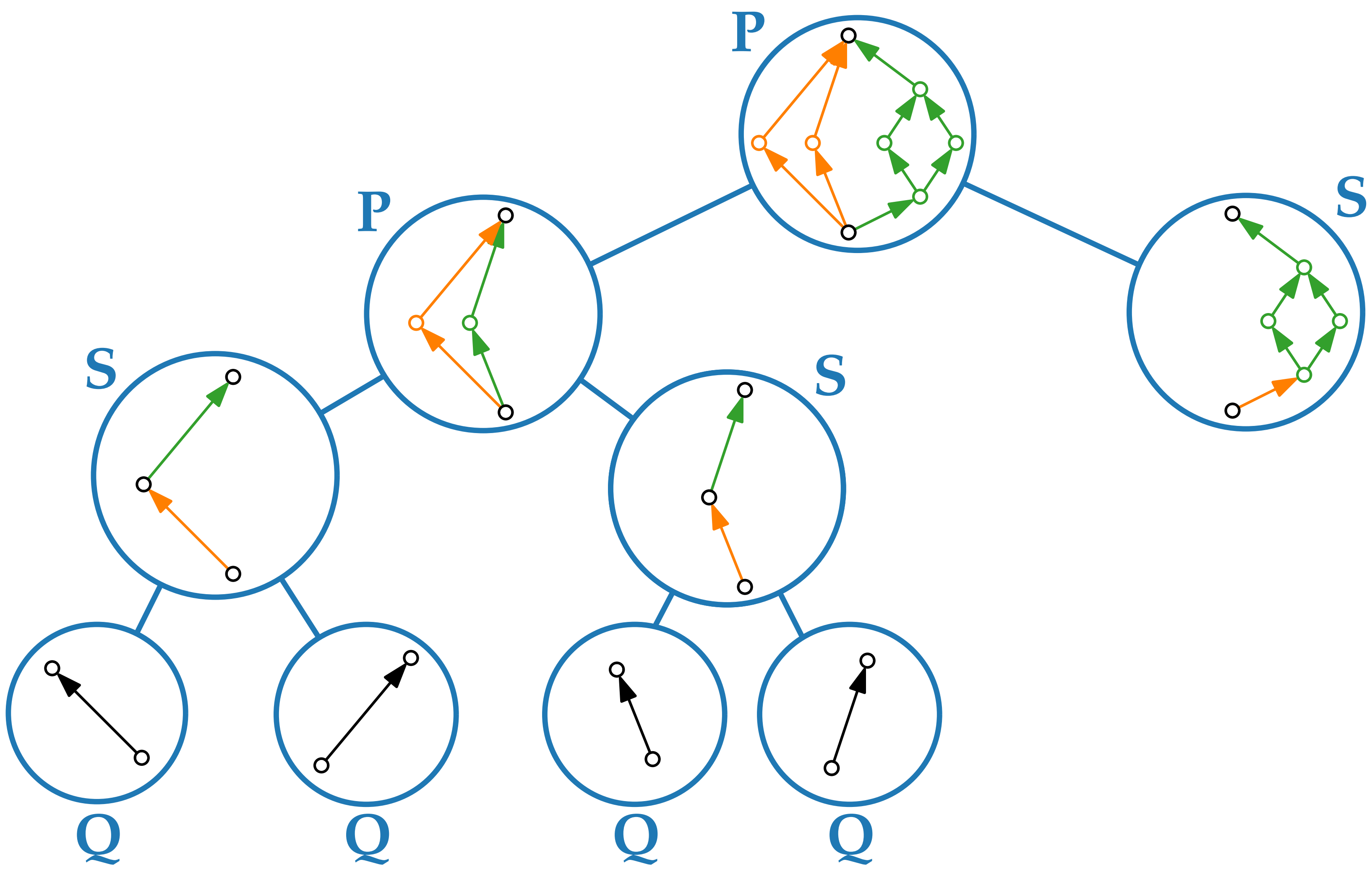

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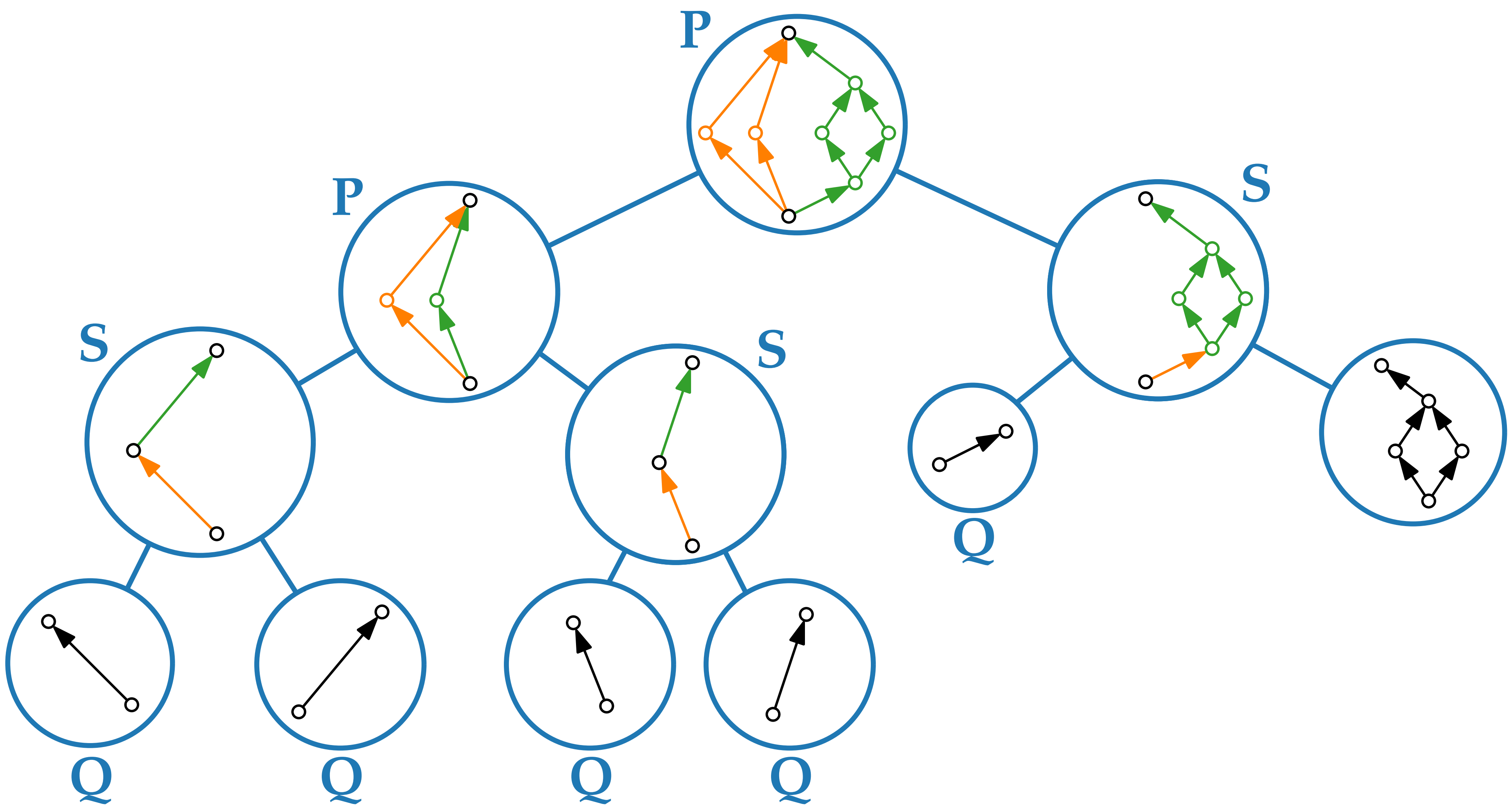

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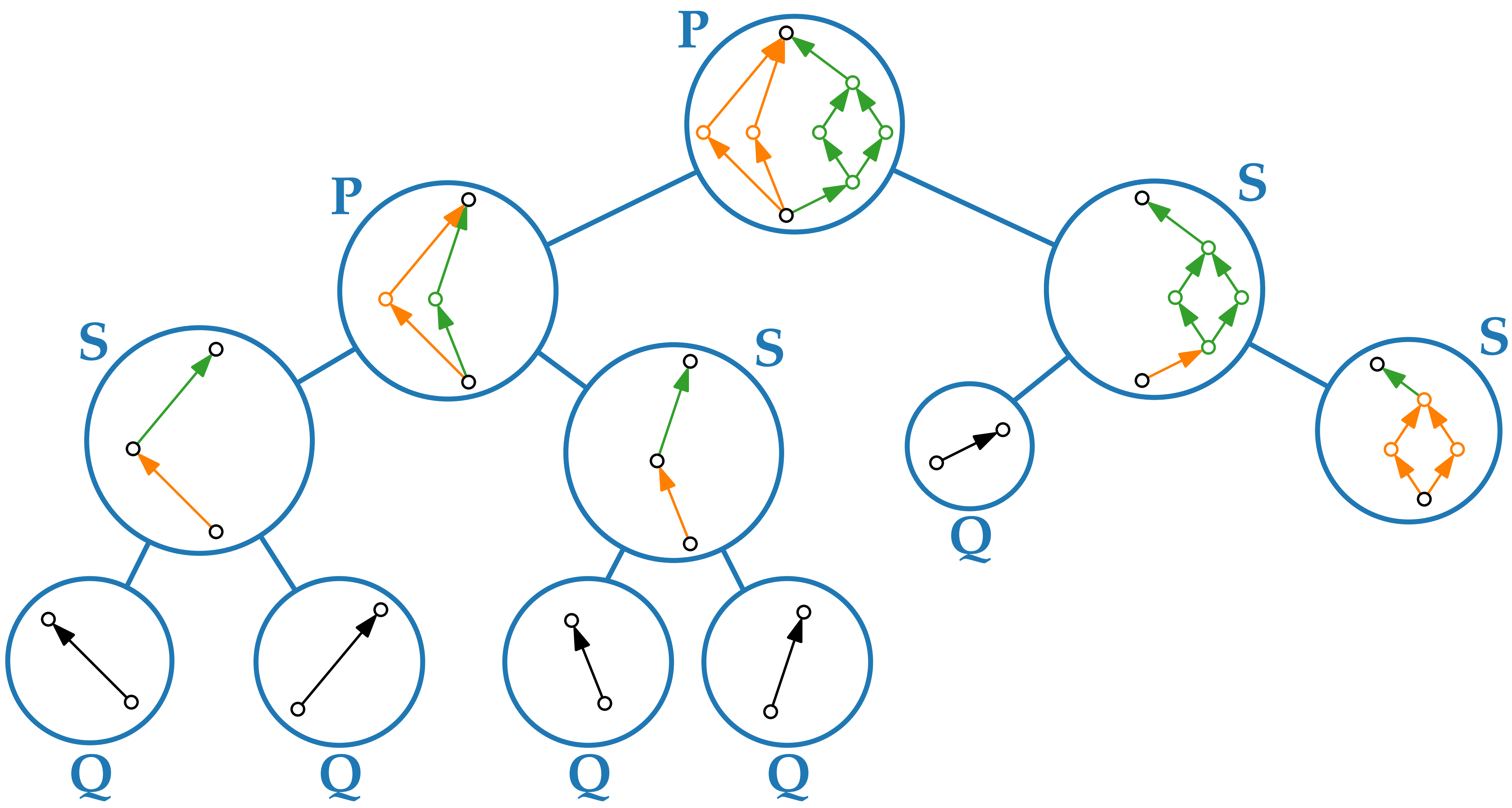

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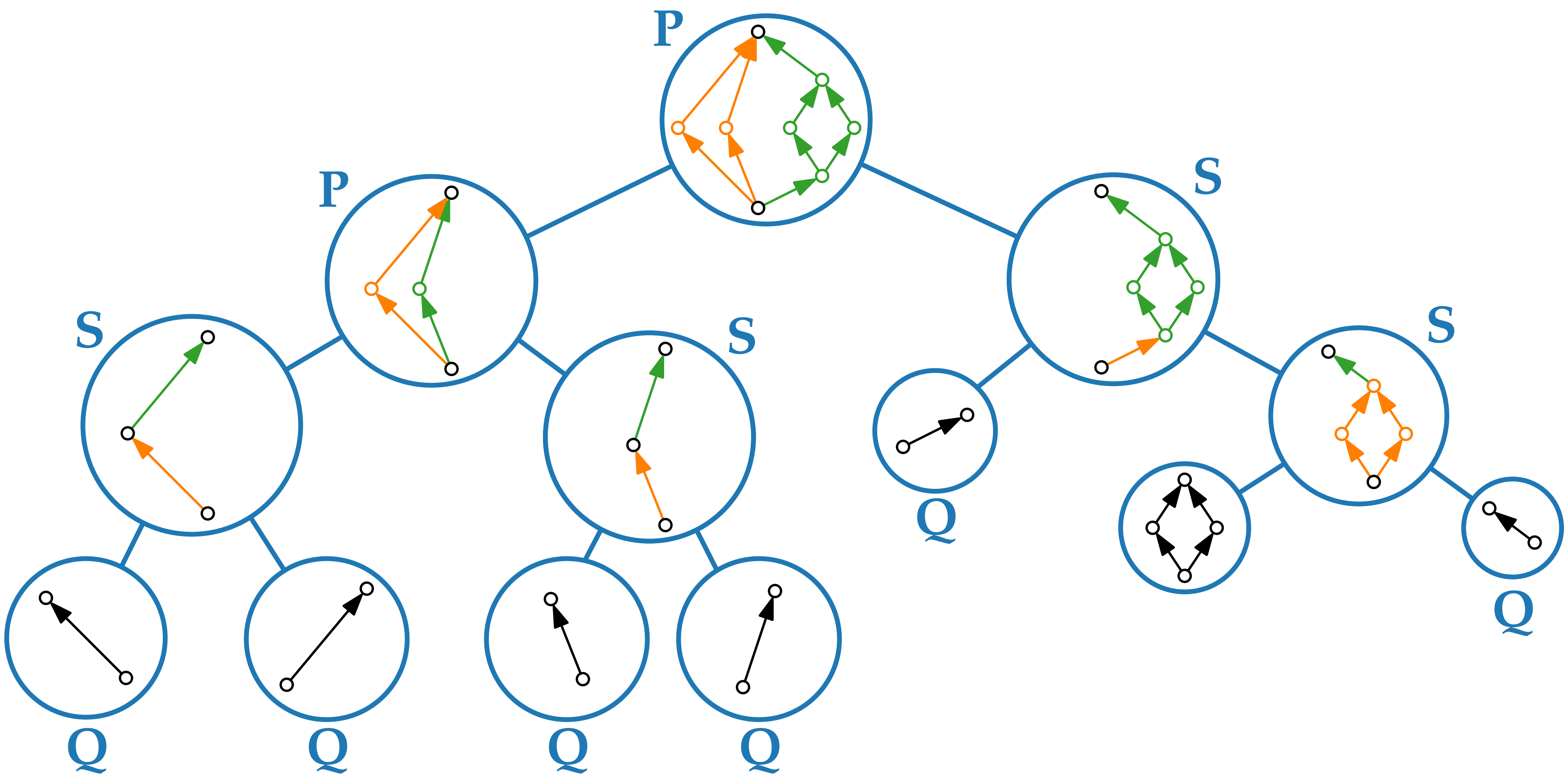

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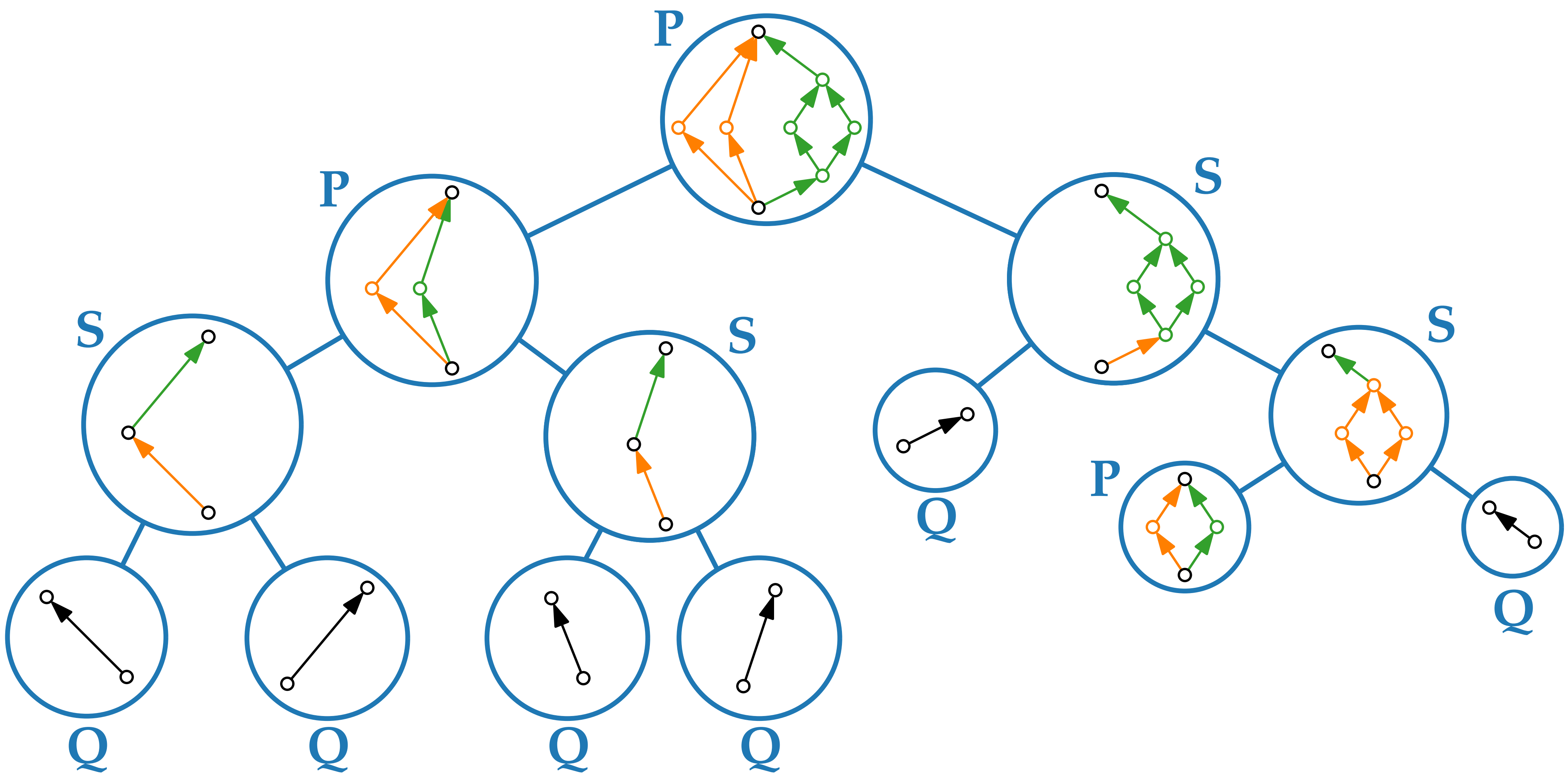

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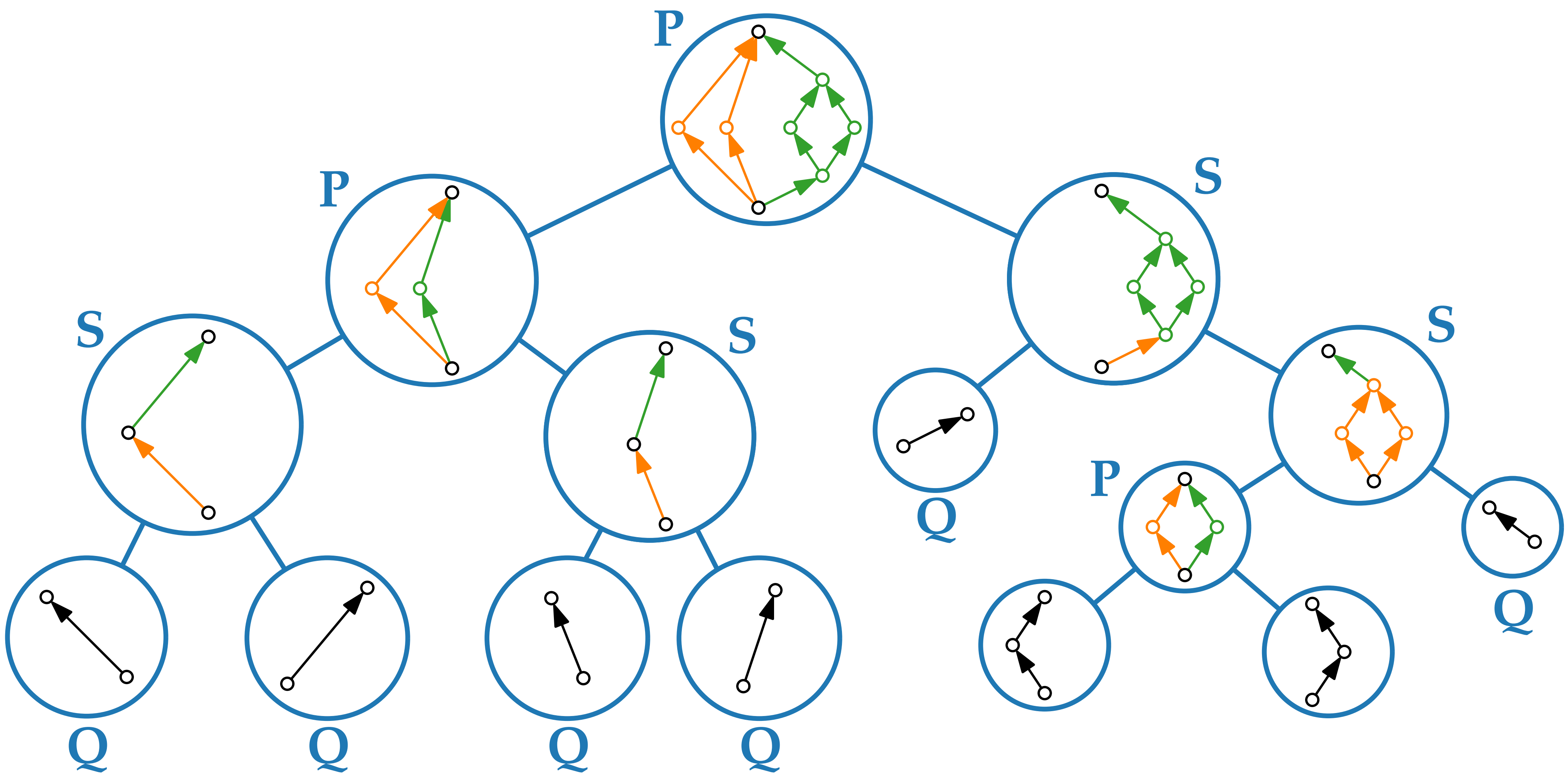

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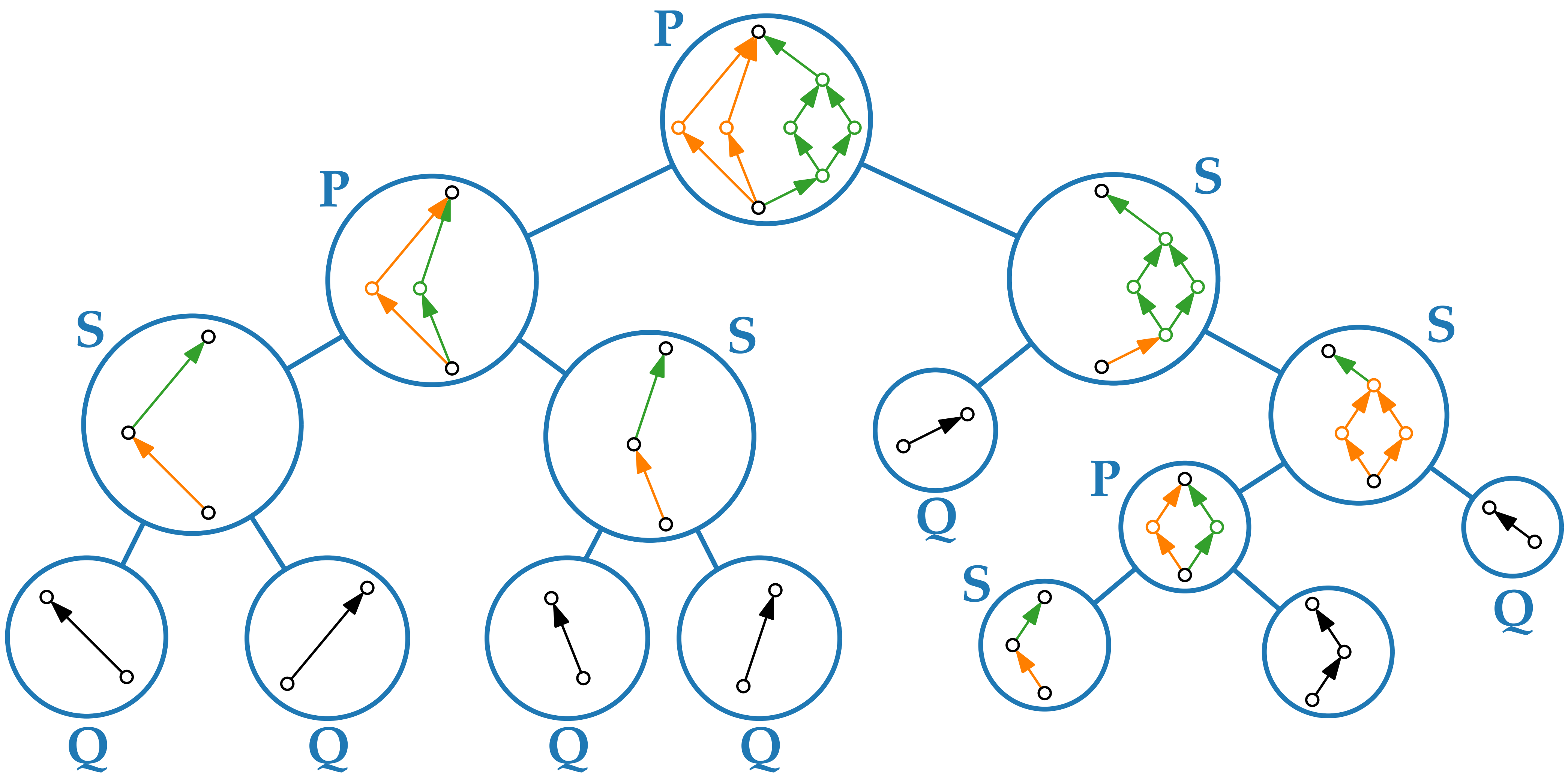

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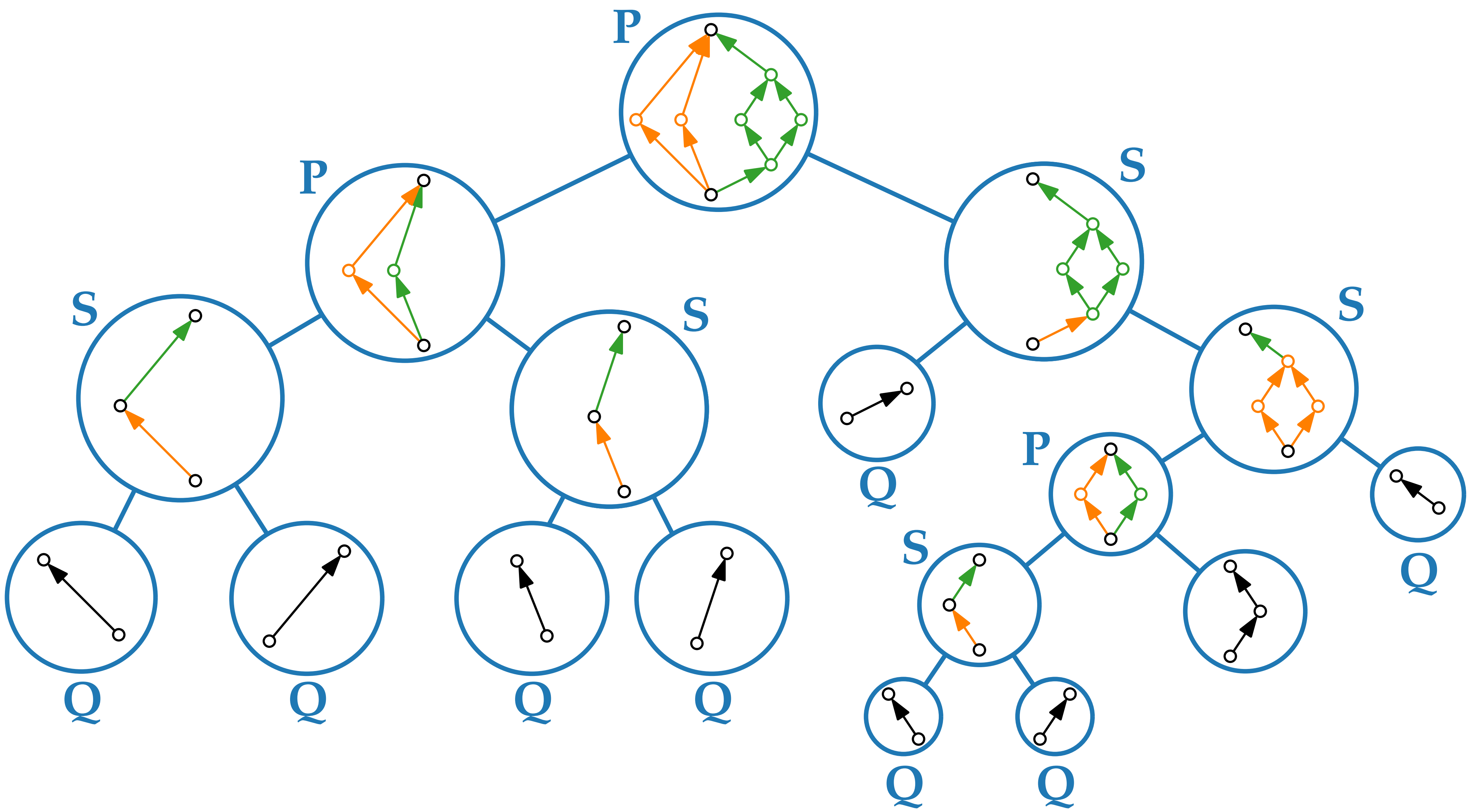

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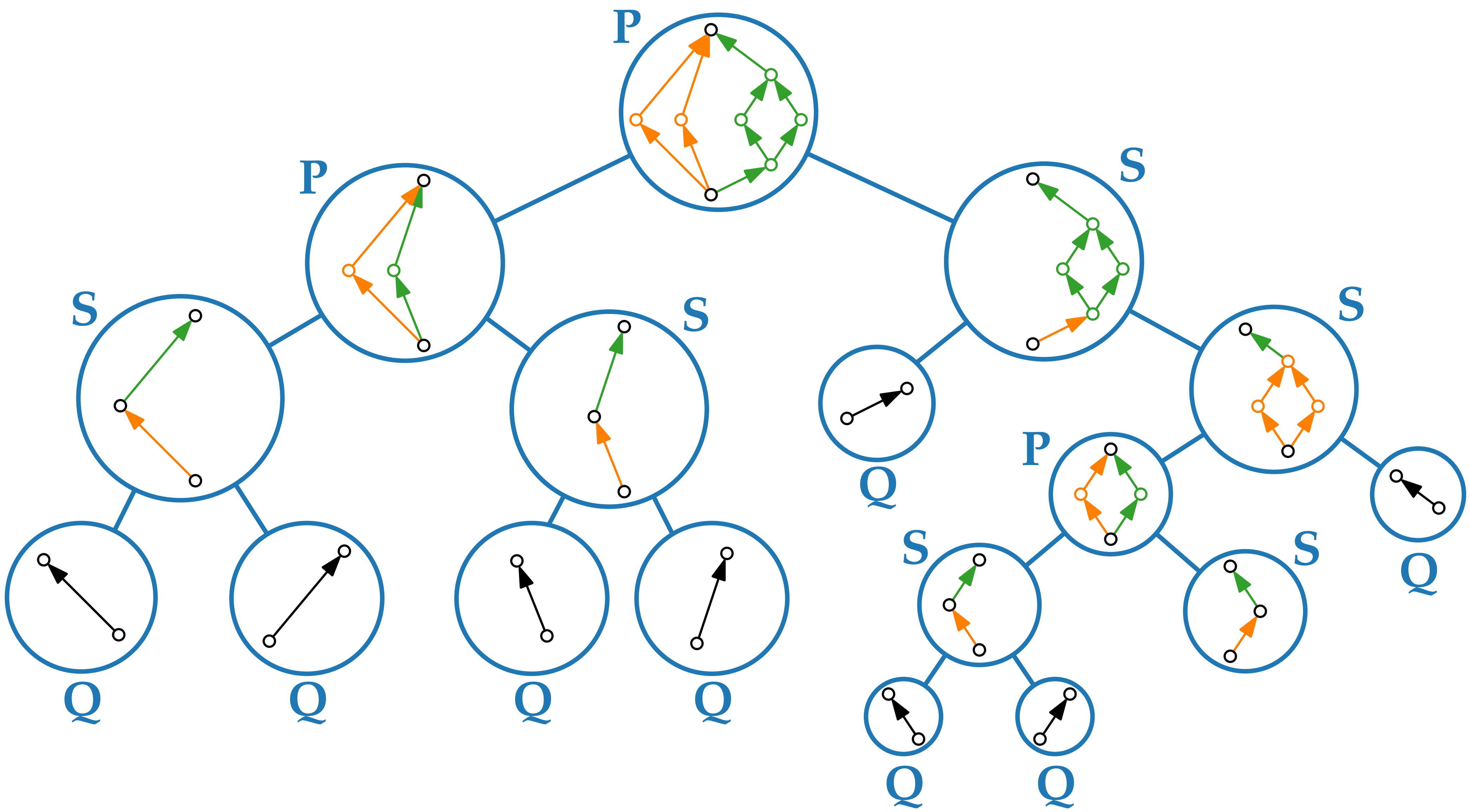

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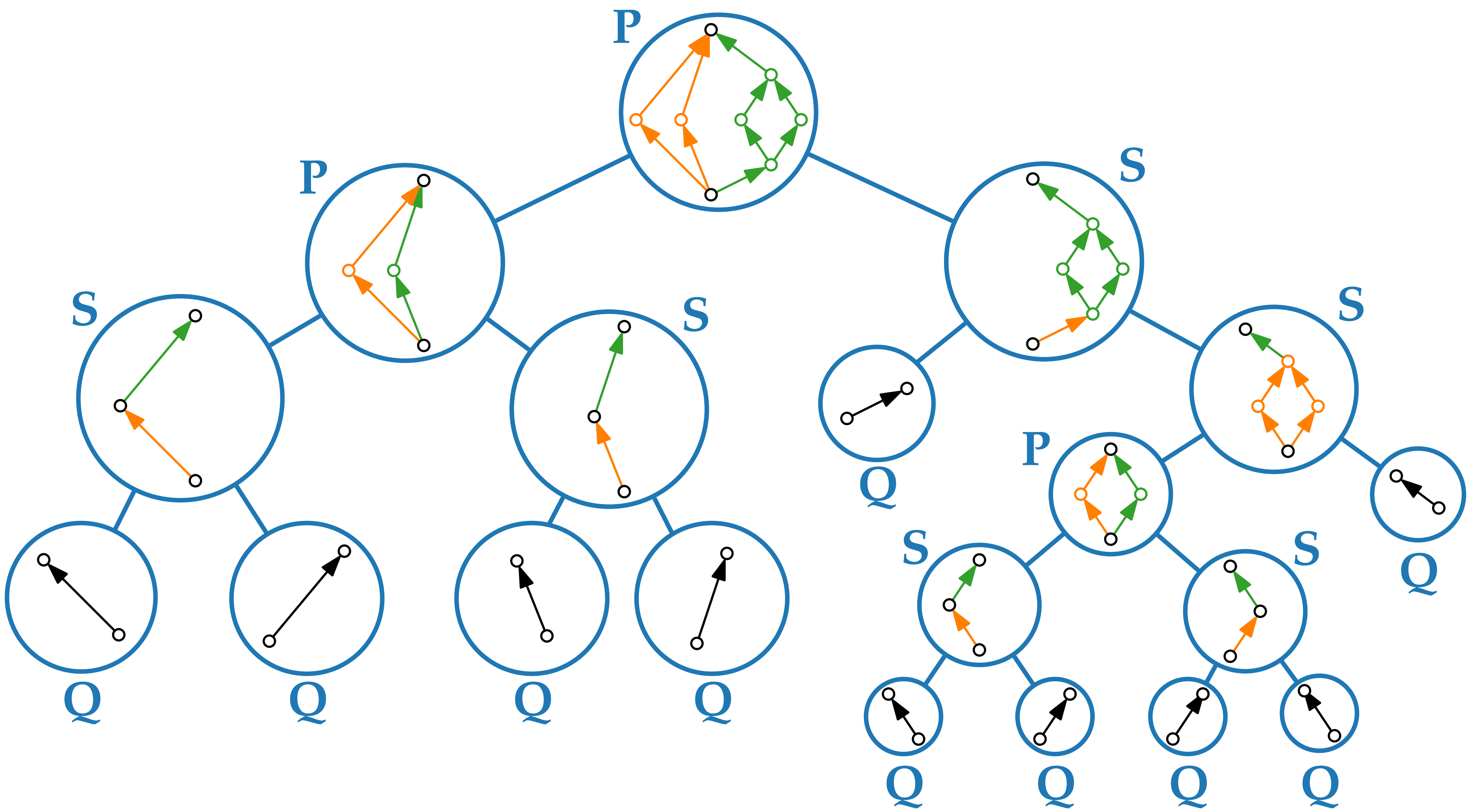
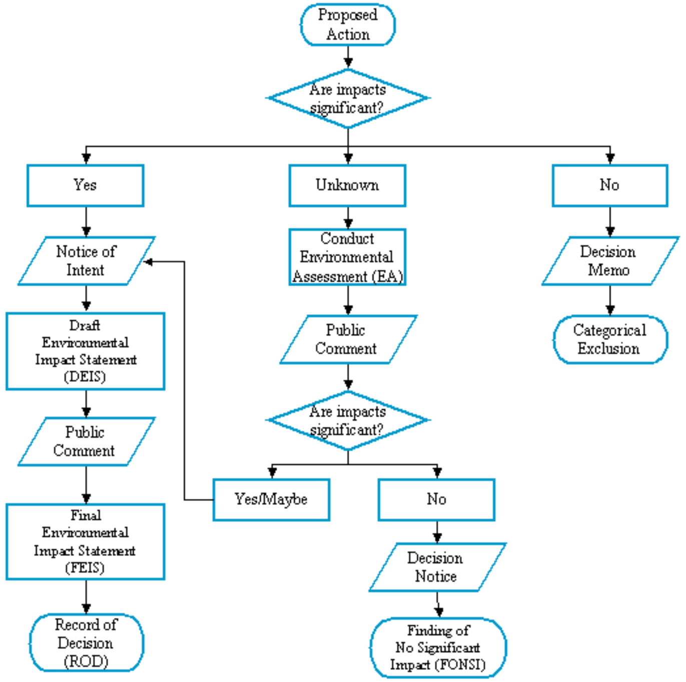
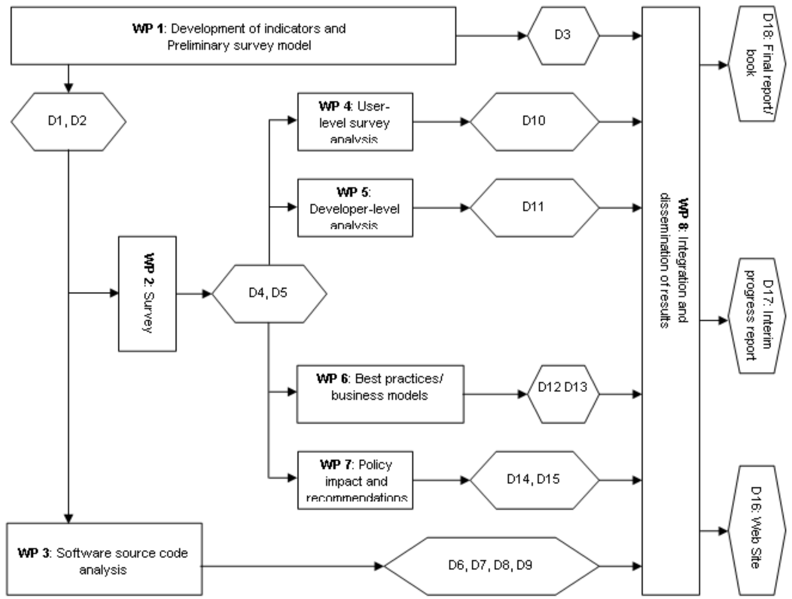


Series-Parallel Graphs – Applications



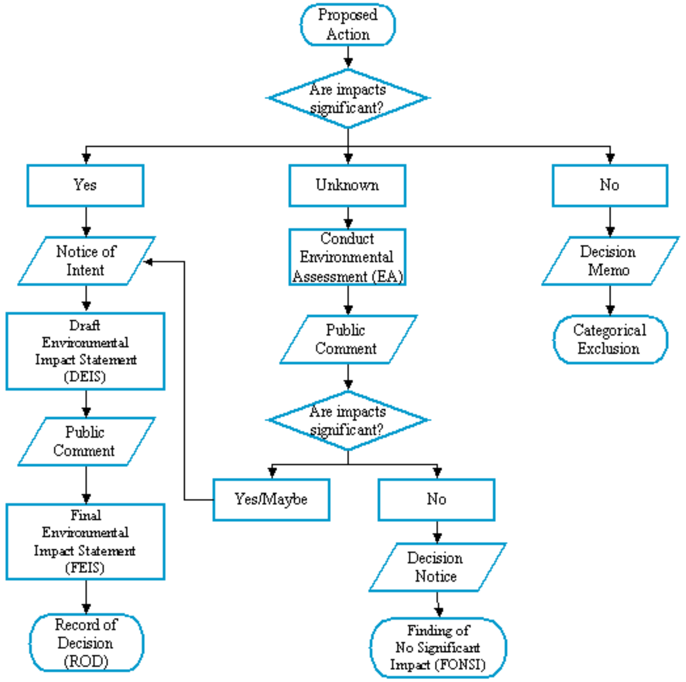
Flowcharts



PERT-Diagrams

(Program Evaluation and Review Technique)

Series-Parallel Graphs – Applications



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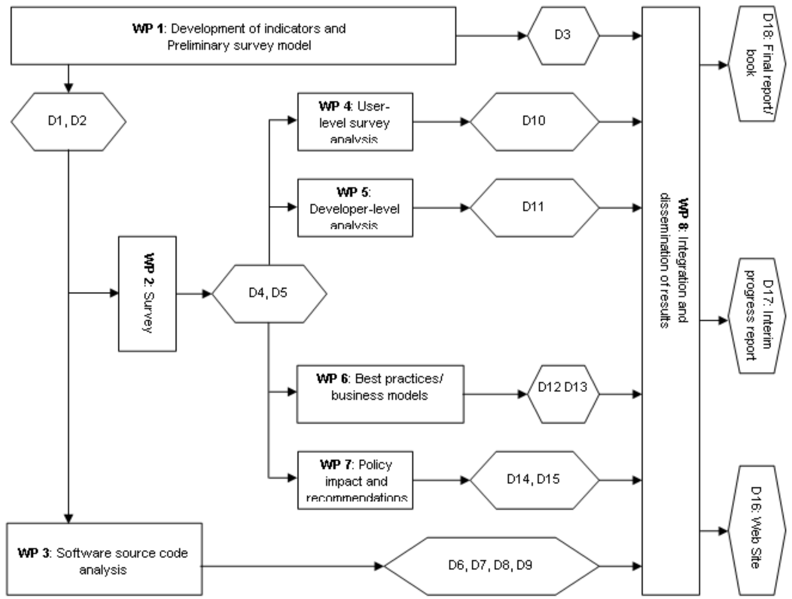
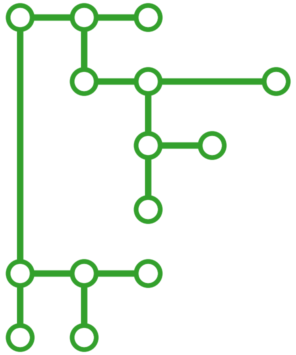


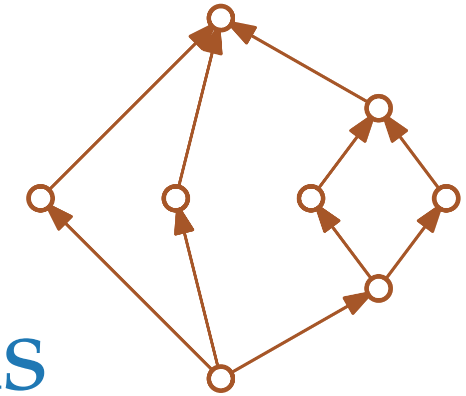
PERT-Diagrams
(Program Evaluation and Review Technique)

Computational complexity:
Linear time algorithms for $\mathcal{NP}$-hard problems
(e.g. Maximum Matching, MIS, Hamiltonian Completion)

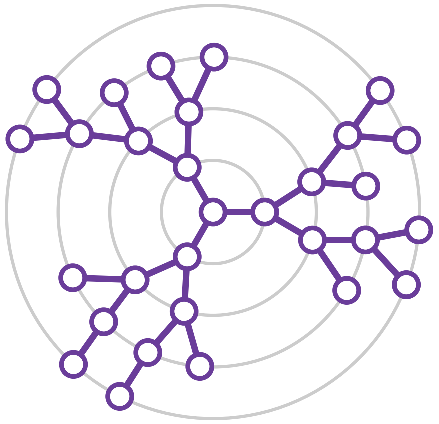
Universität Trier



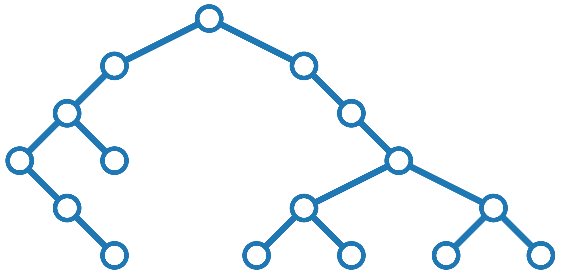
Visualization of Graphs



Lecture 2: Drawing Trees and Series-Parallel Graphs



Part VI: Drawings of Series-Parallel Graphs

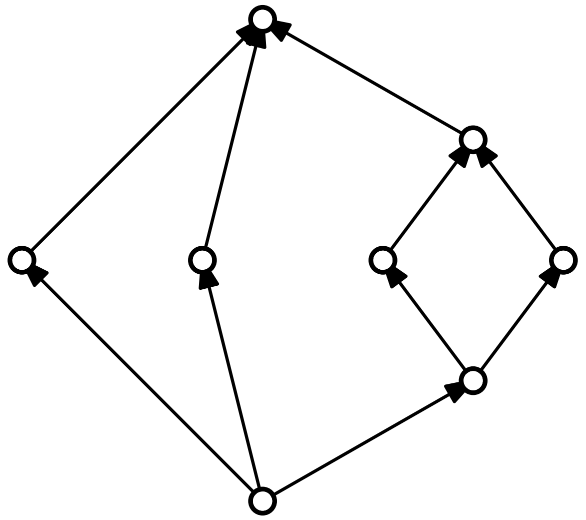


Philipp Kindermann

Series-Parallel Graphs – Drawing Style

Drawing conventions

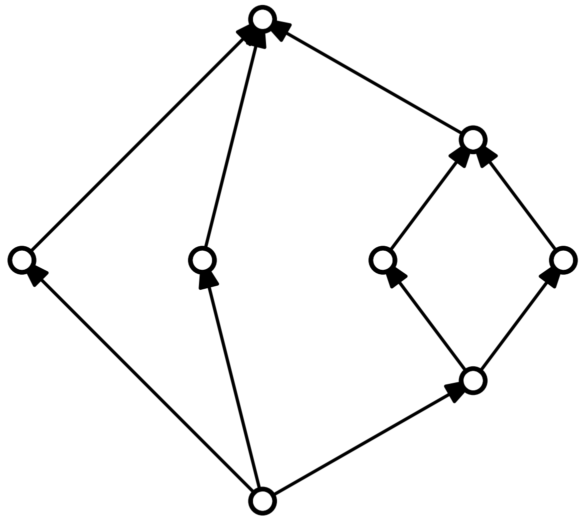
Drawing aesthetics



Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity

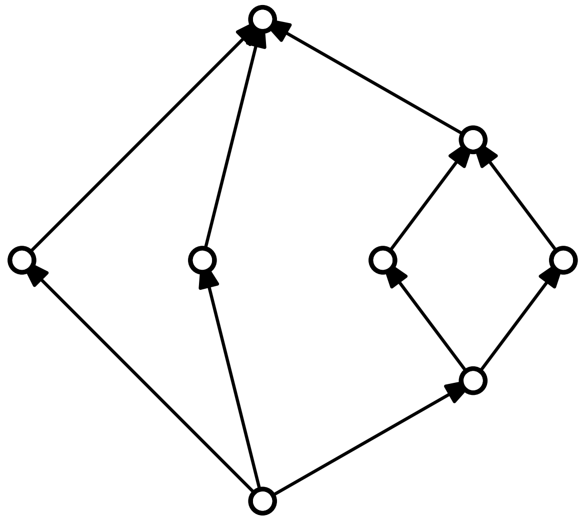


Drawing aesthetics

Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity
- Straight-line edges



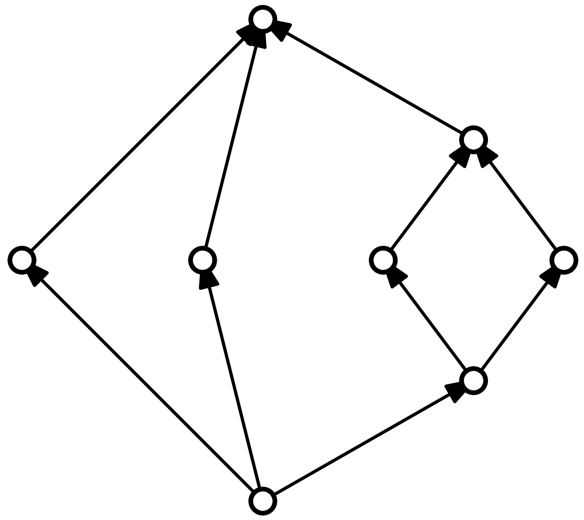
Drawing aesthetics

Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity
- Straight-line edges
- Upward

Drawing aesthetics



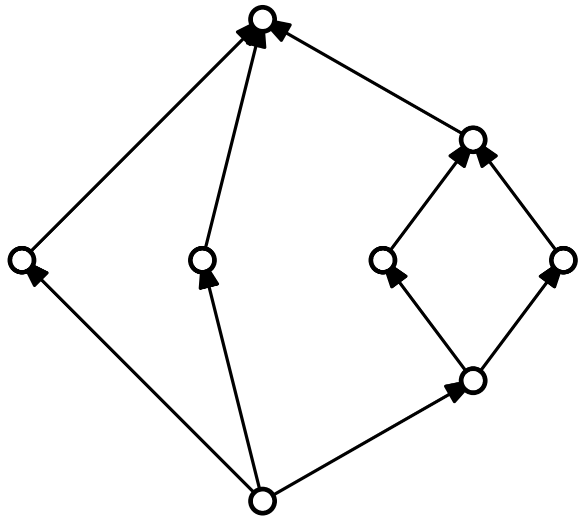
Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity
- Straight-line edges
- Upward

Drawing aesthetics

- Area



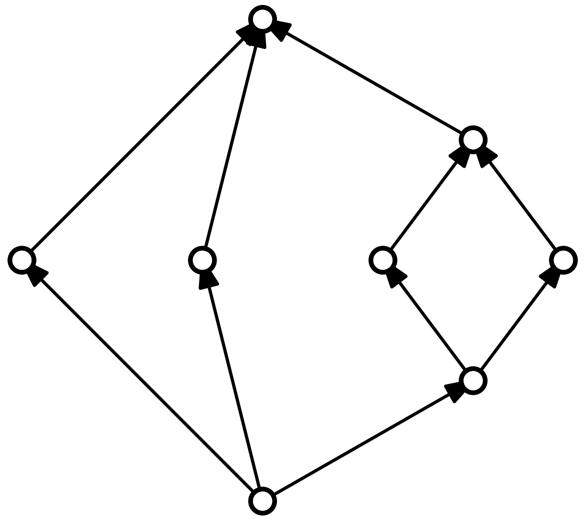
Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity
- Straight-line edges
- Upward

Drawing aesthetics

- Area
- Symmetry



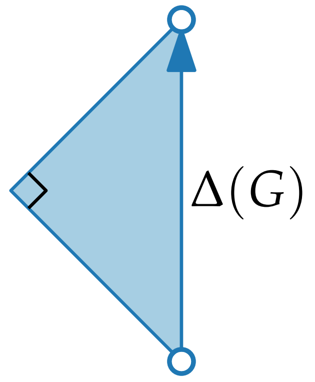
Series-Parallel Graphs – Straight-Line Drawings

Divide & conquer algorithm using the decomposition tree

Series-Parallel Graphs – Straight-Line Drawings

Divide & conquer algorithm using the decomposition tree

- Draw G inside a right-angled isosceles bounding triangle $\Delta(G)$



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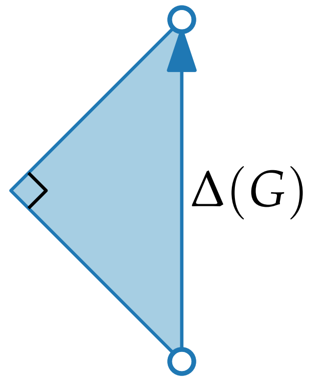
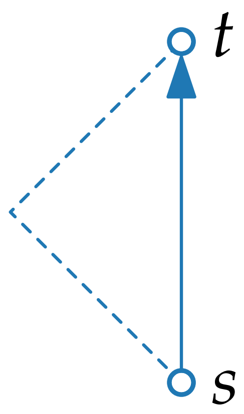


Base case: Q-nodes



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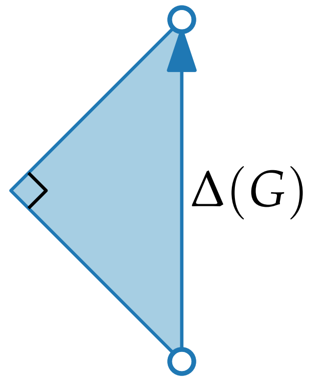
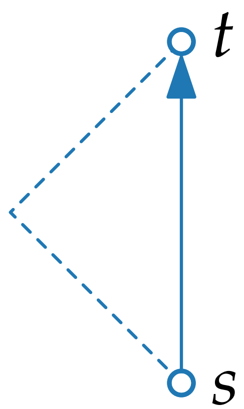


Base case: Q-nodes

Divide: Draw G_1 and G_2 first



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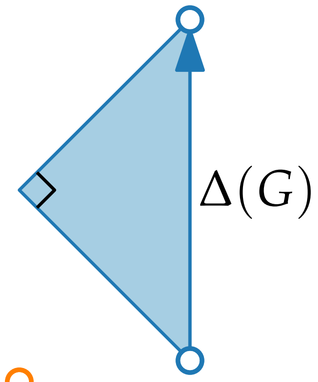

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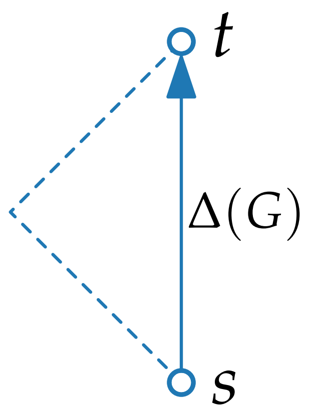

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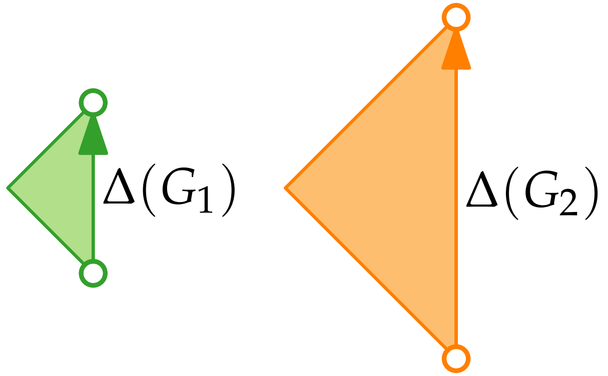
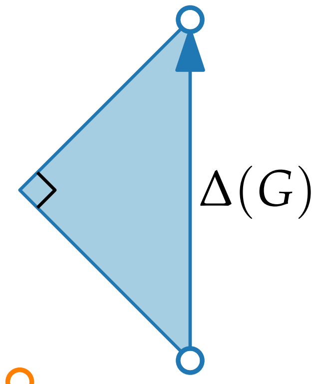


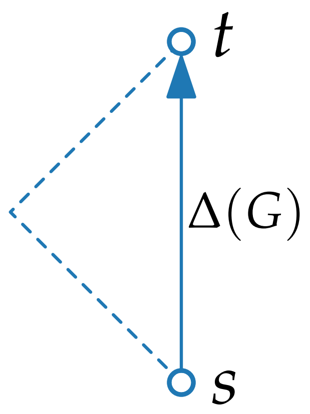
Series-Parallel Graphs – Straight-Line Drawings



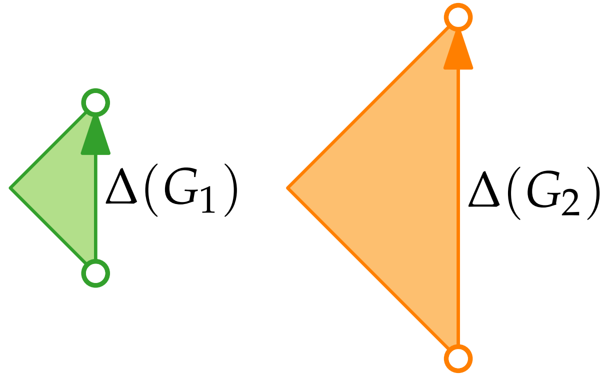
Divide & conquer algorithm using the decomposition tree

- Draw G inside a right-angled isosceles bounding triangle $\Delta(G)$

Base case: Q-nodes



Divide: Draw G_1 and G_2 first

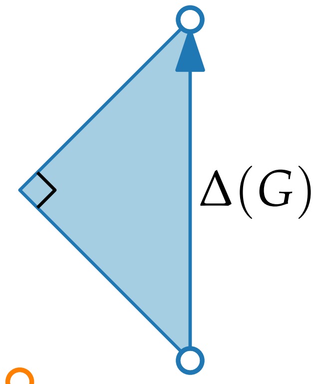


Conquer:

Series-Parallel Graphs – Straight-Line Drawings

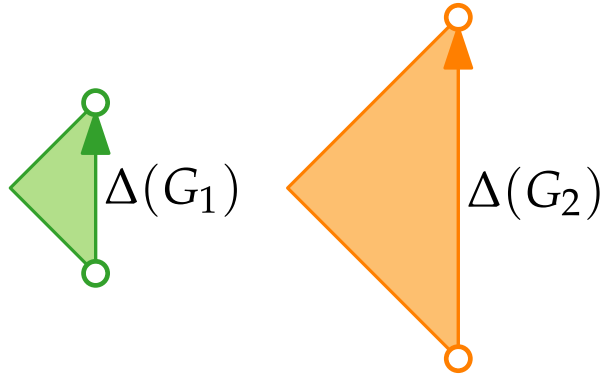
Divide & conquer algorithm using the decomposition tree

- Draw G inside a right-angled isosceles bounding triangle $\Delta(G)$



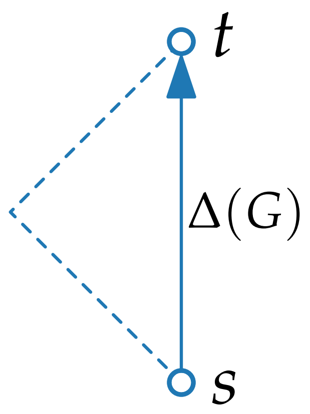
Base case: Q-nodes

Divide: Draw G_1 and G_2 first



Conquer:

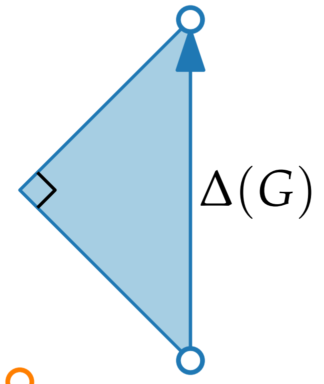
- S-nodes / series composition



Series-Parallel Graphs – Straight-Line Drawings

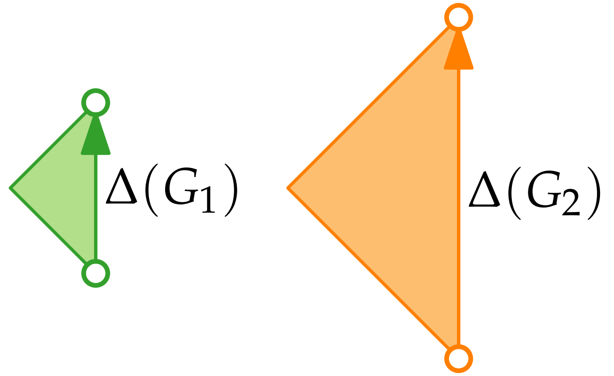
Divide & conquer algorithm using the decomposition tree

- Draw G inside a right-angled isosceles bounding triangle $\Delta(G)$



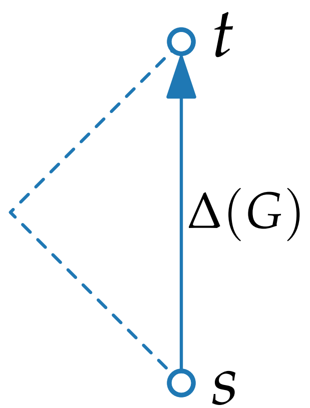
Base case: Q-nodes

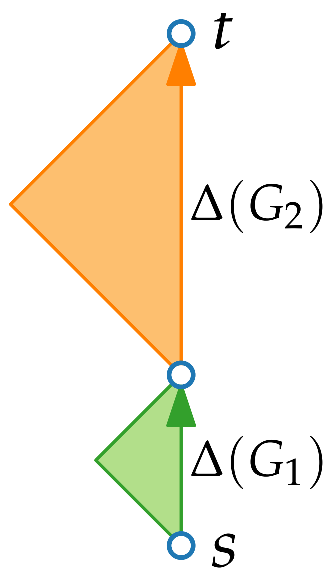
Divide: Draw G_1 and G_2 first



Conquer:

- S-nodes / series composition





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Divide & conquer algorithm using the decomposition tree

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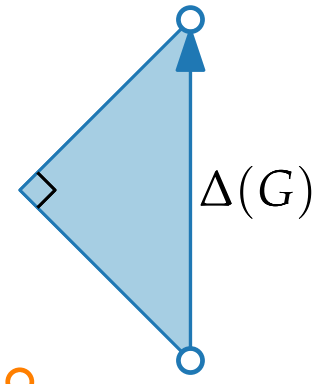

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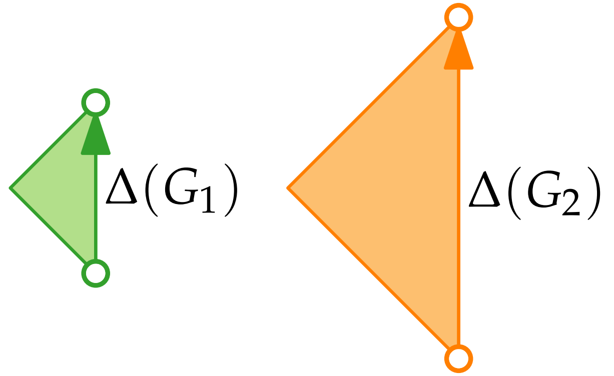

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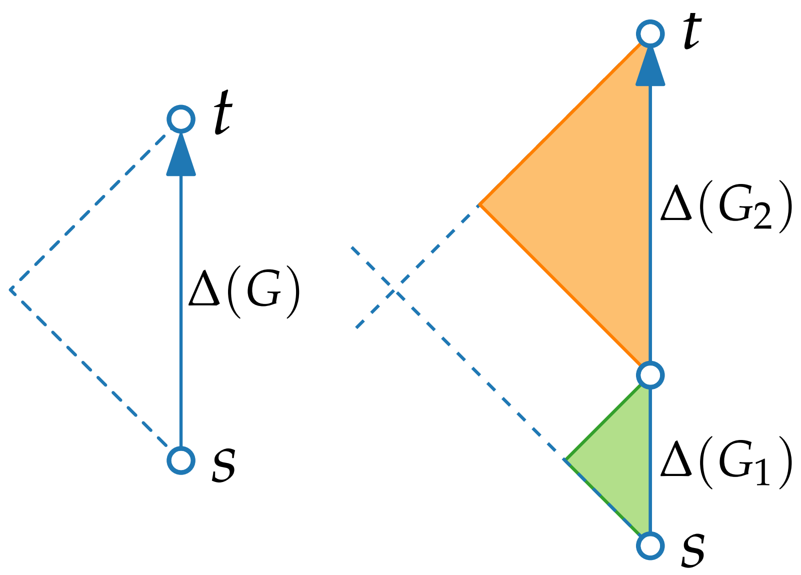

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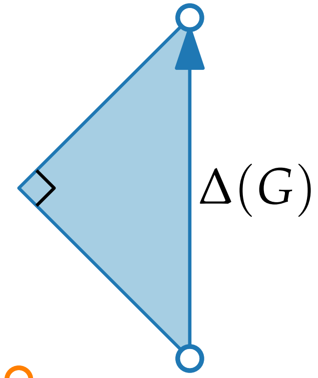

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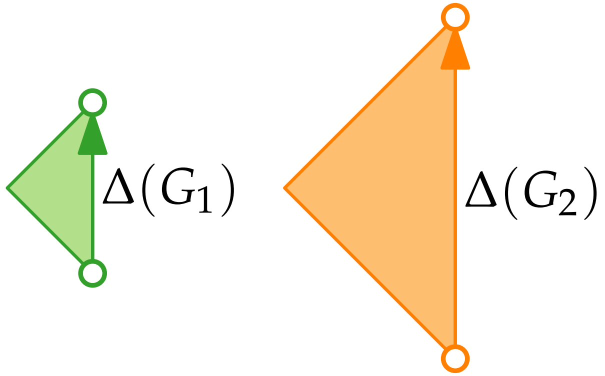

Conquer:

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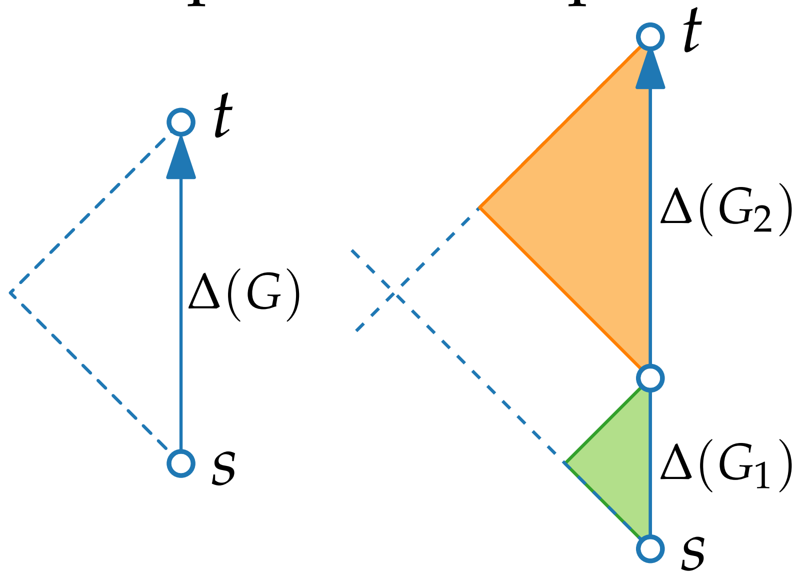

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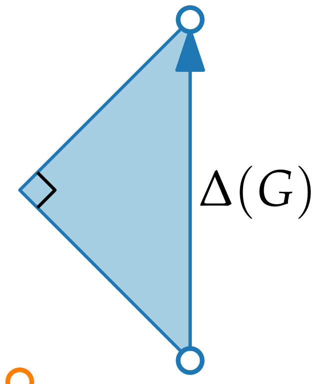

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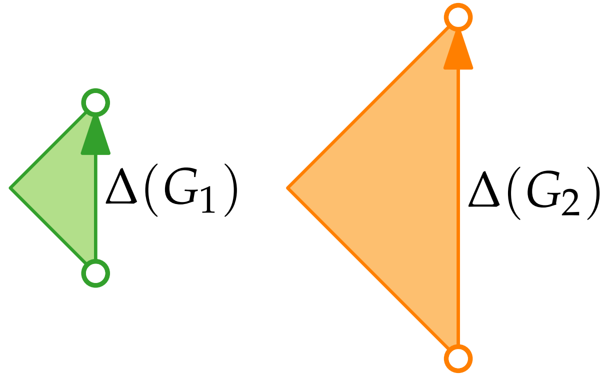

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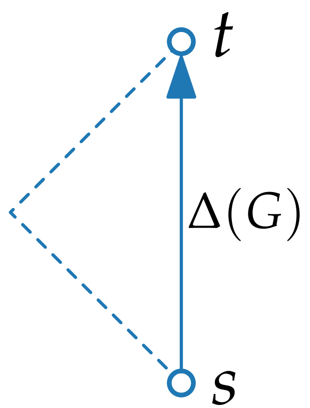

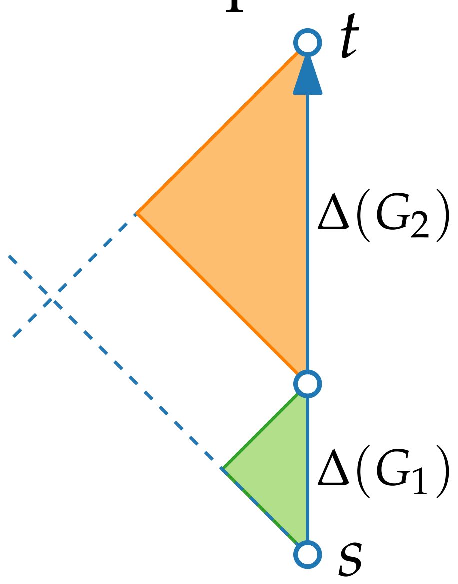

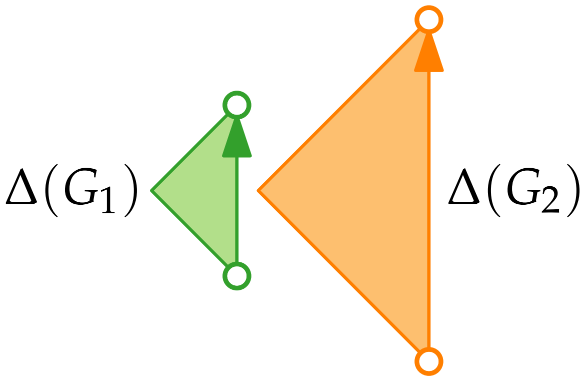

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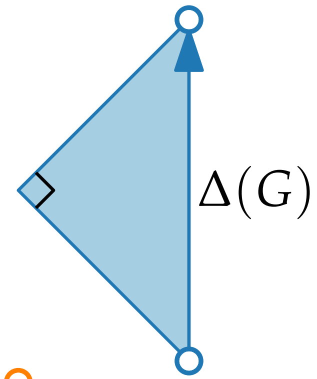

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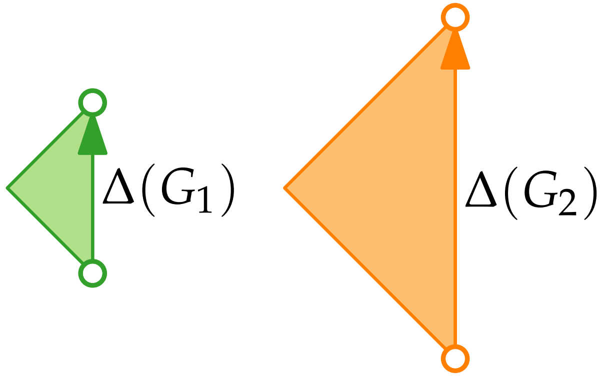

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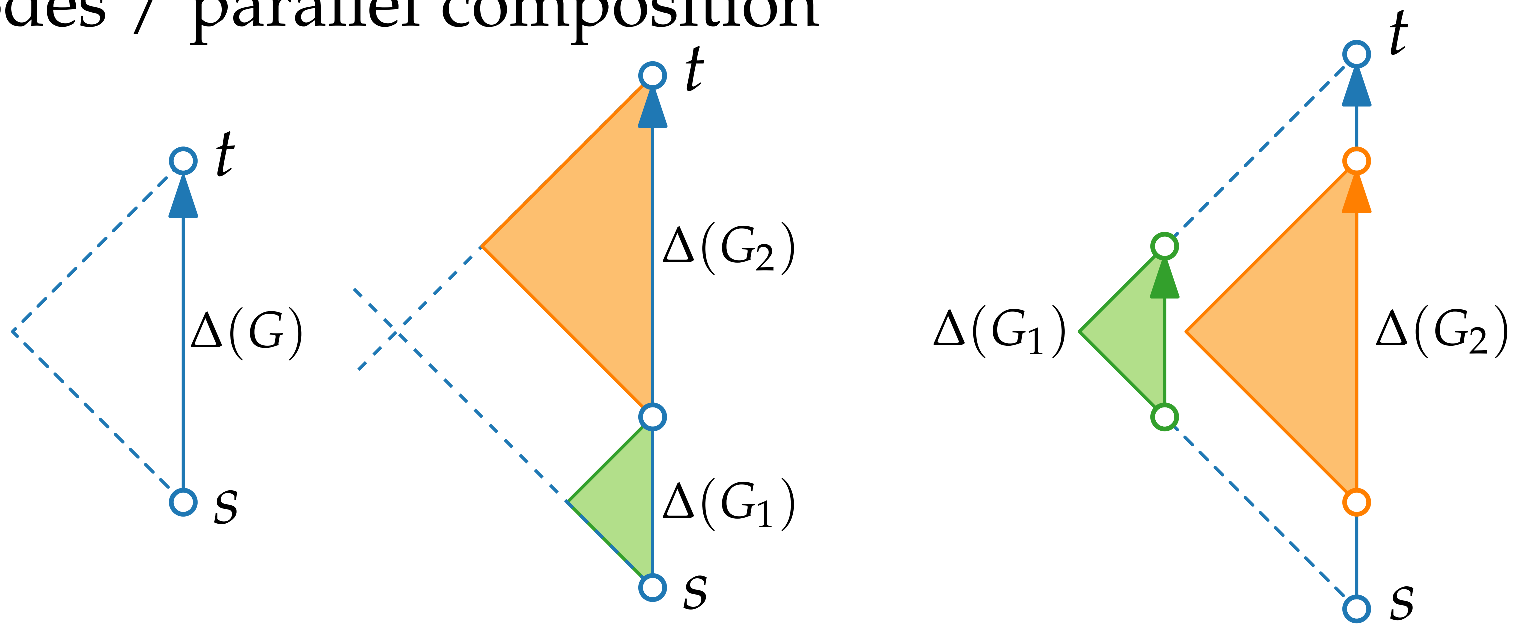

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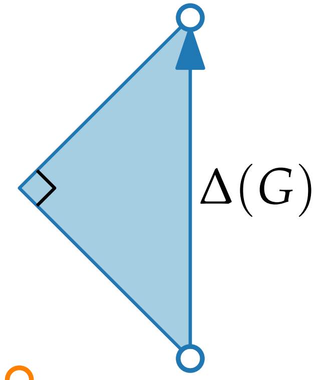

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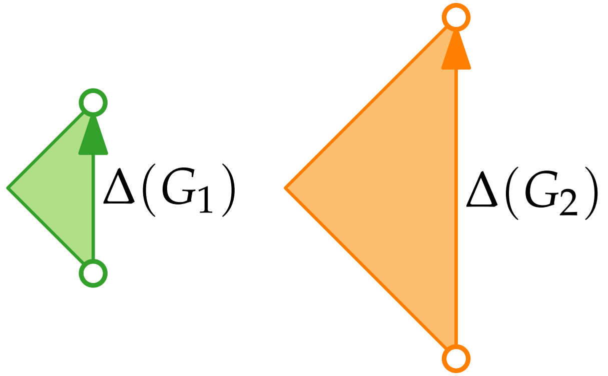

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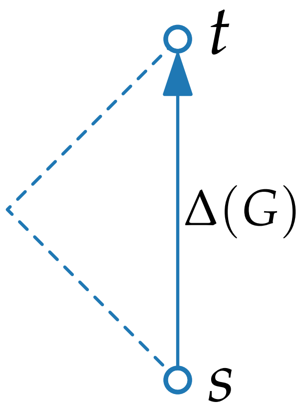

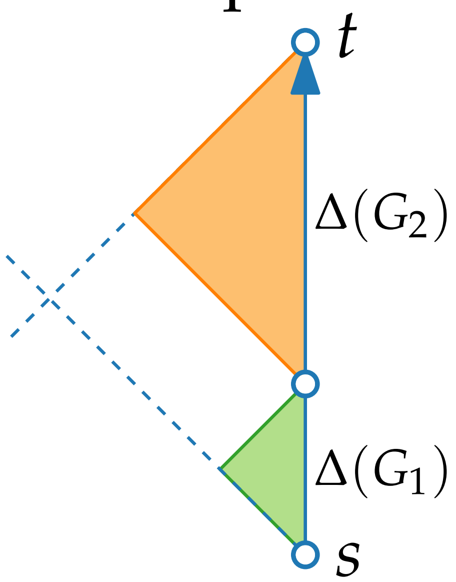

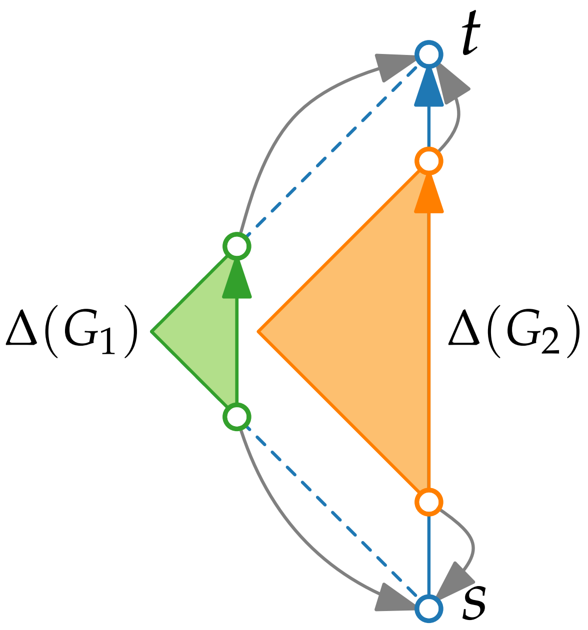

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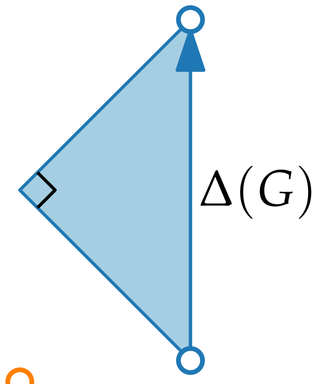

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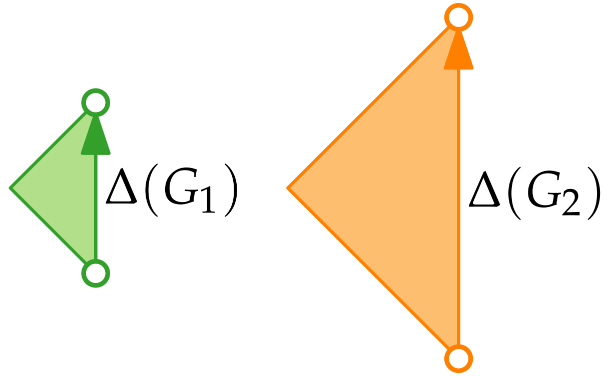

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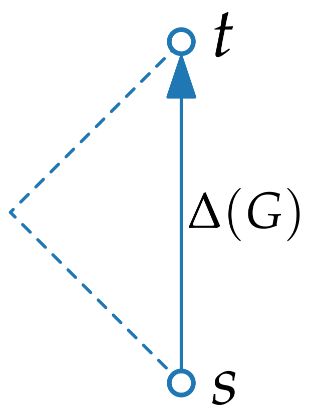
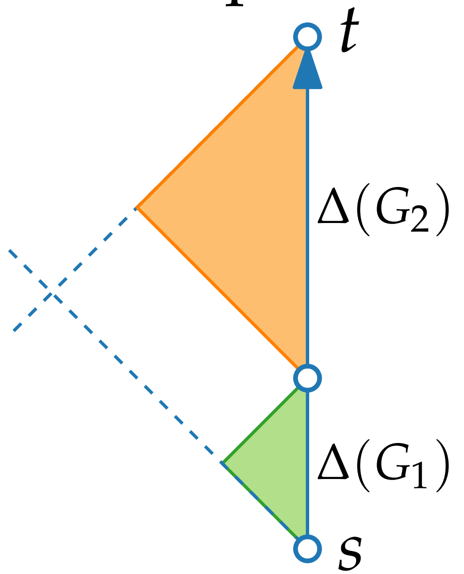
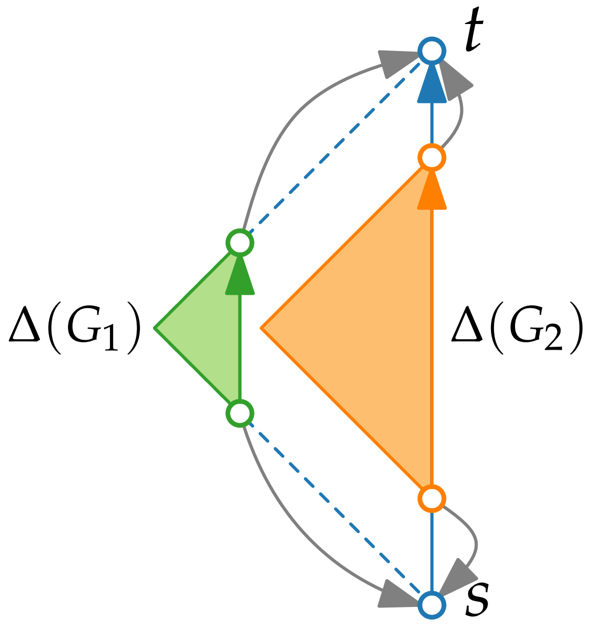
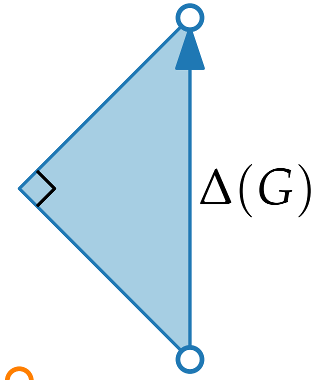






Do you see any problem?

Series-Parallel Graphs – Straight-Line Drawings



Divide & conquer algorithm using the decomposition tree

- Draw G inside a right-angled isosceles bounding triangle $\Delta(G)$

Base case: Q-nodes

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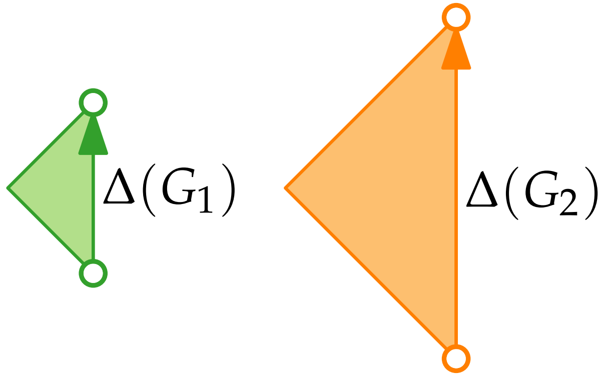

Conquer:

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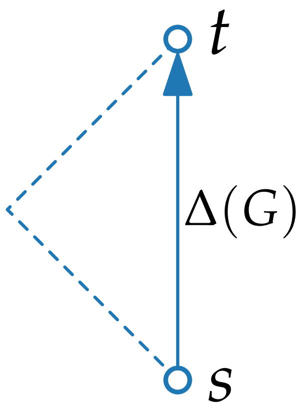

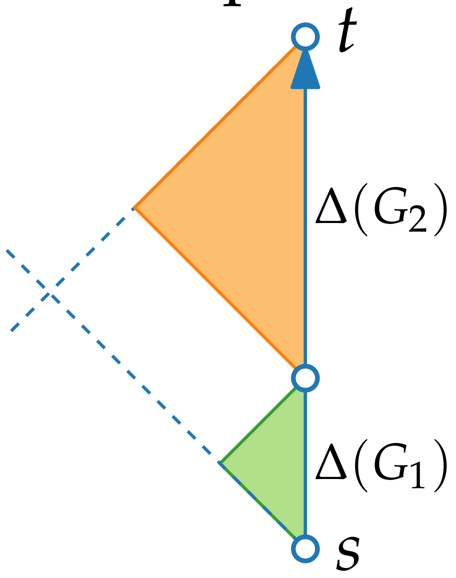

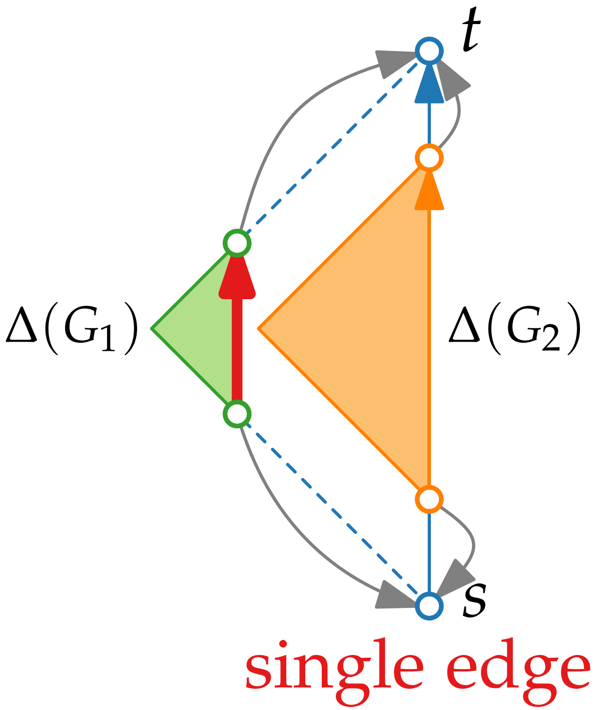

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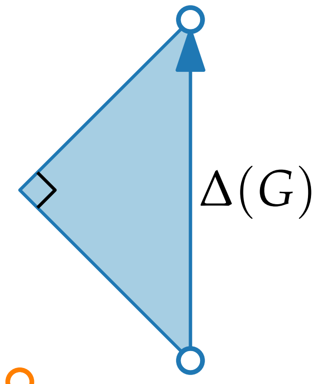

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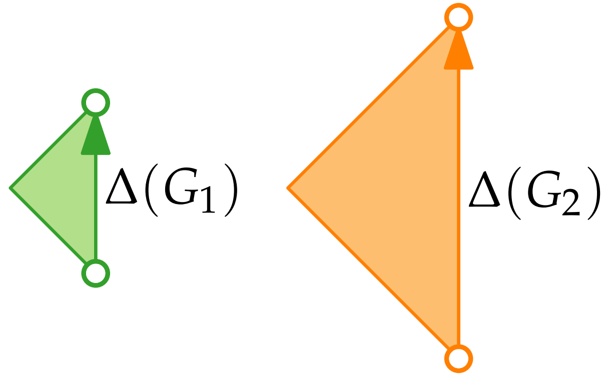

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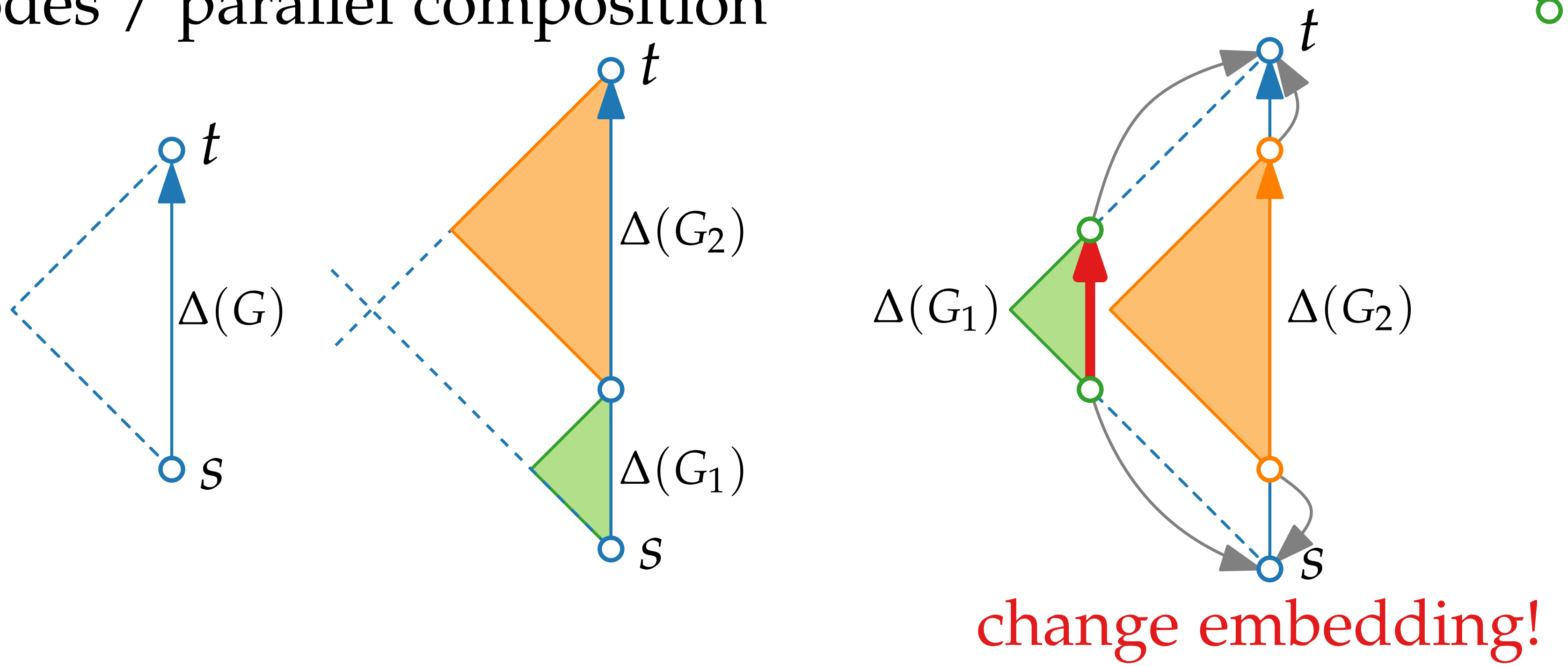

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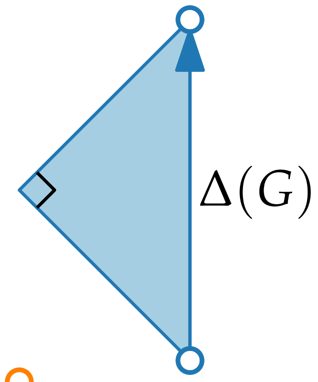

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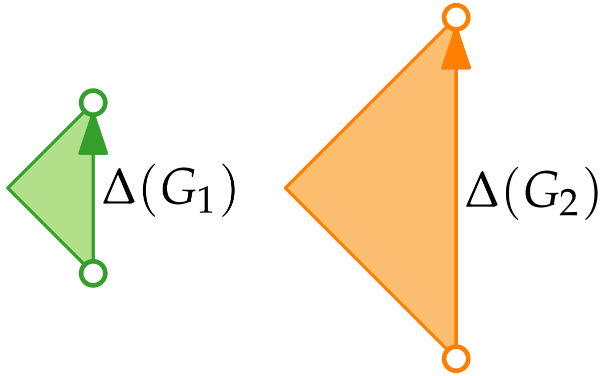

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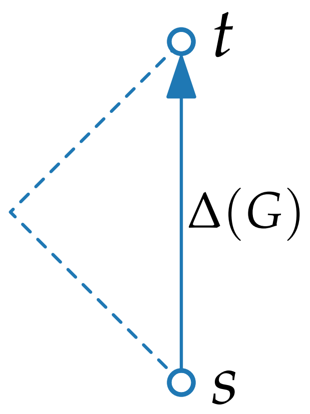

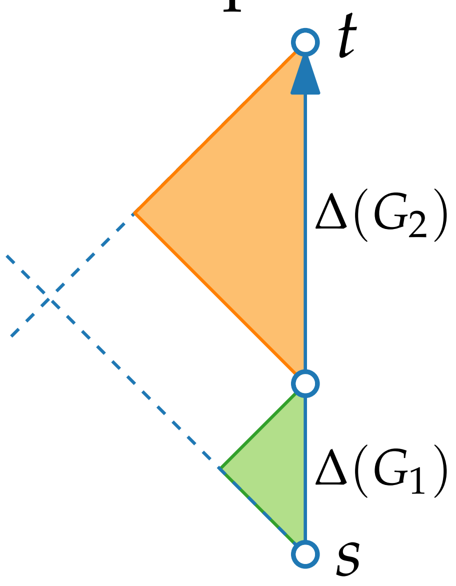

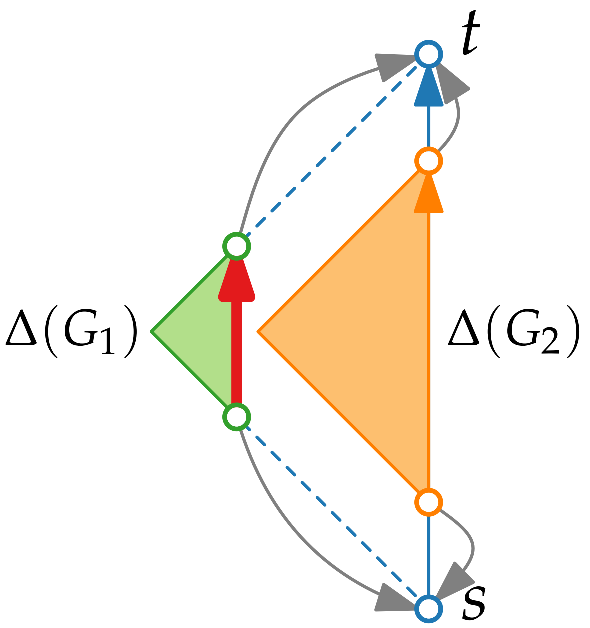

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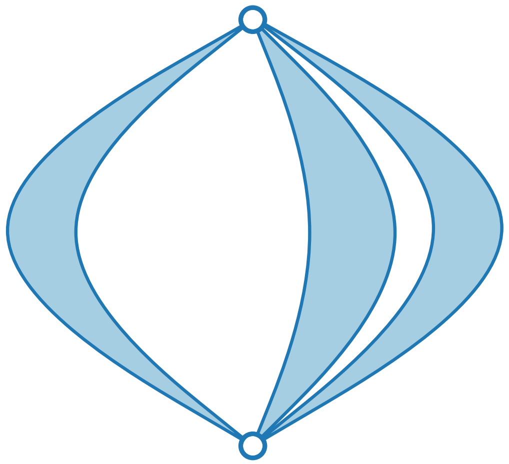

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Divide & conquer algorithm using the decomposition tree

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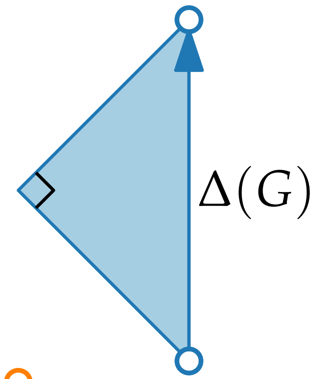

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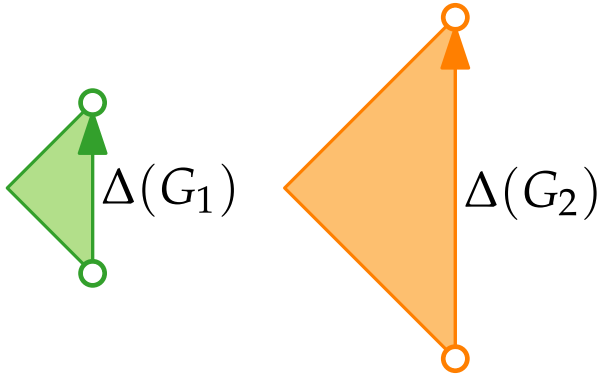

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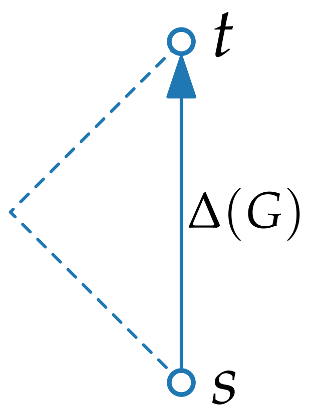

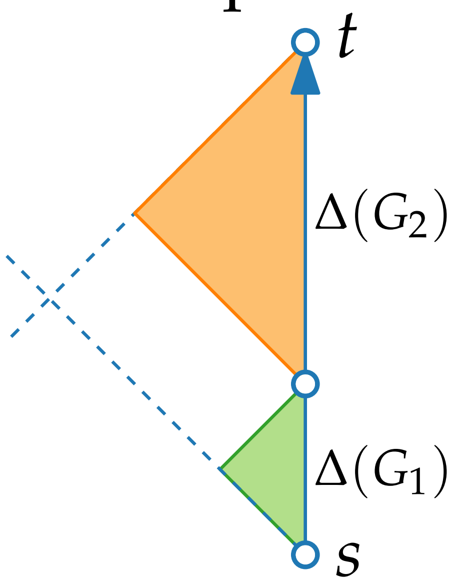

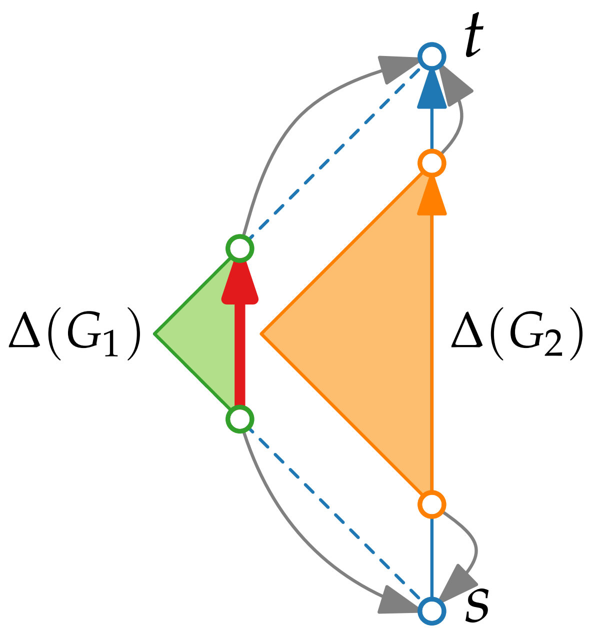

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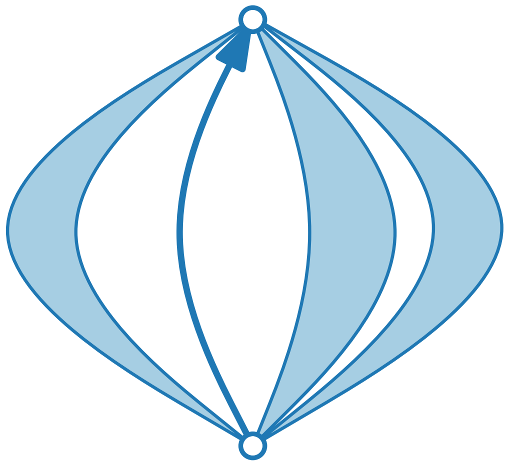

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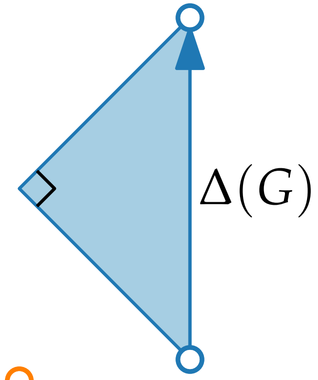

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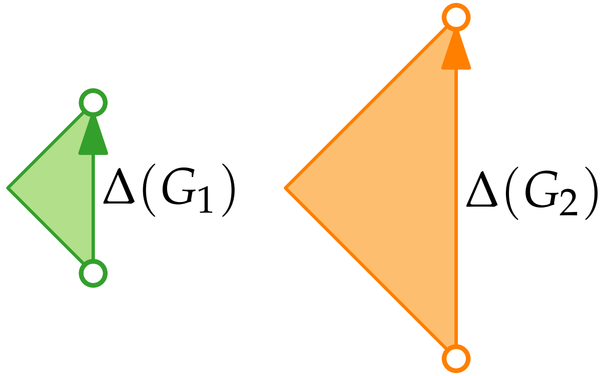

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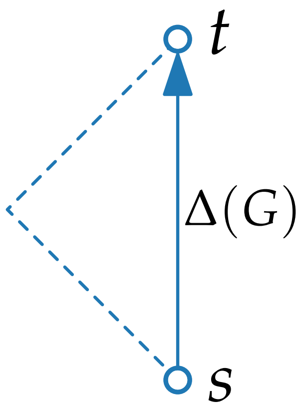

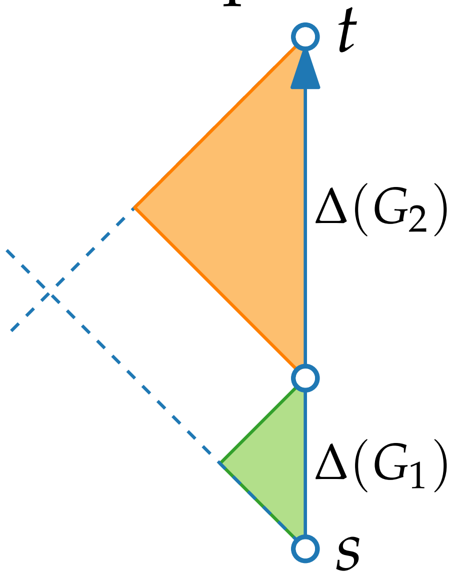

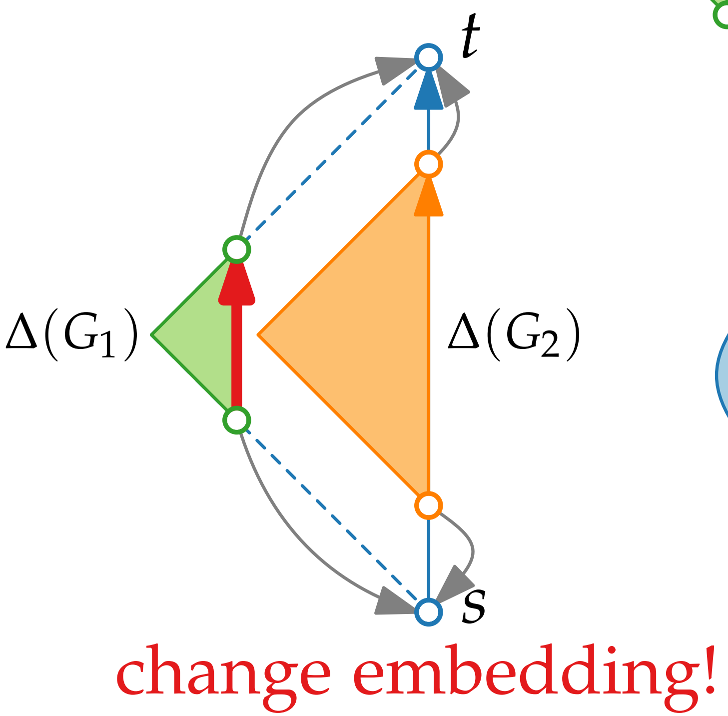

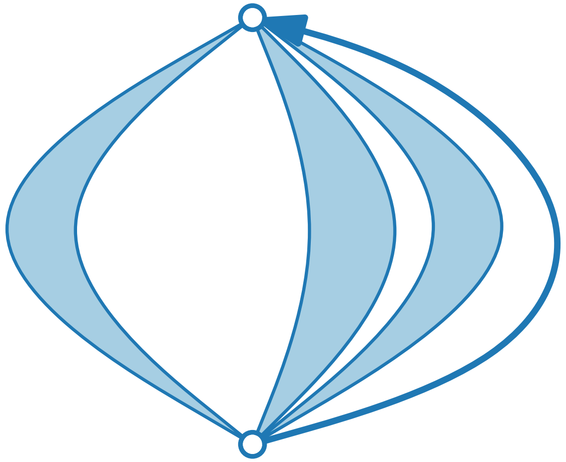

Series-Parallel Graphs – Straight-Line Drawings

- What makes parallel composition possible without creating crossings?

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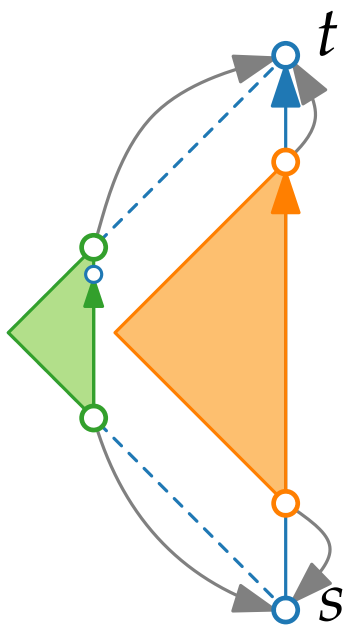

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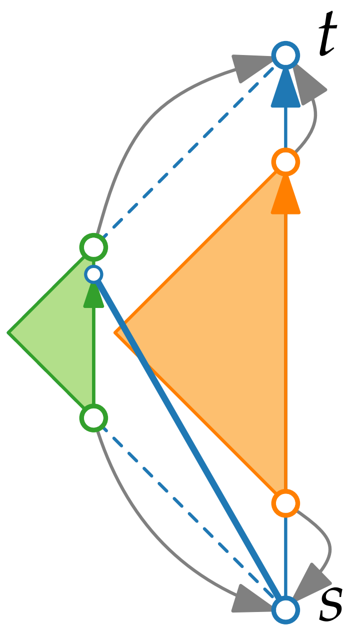

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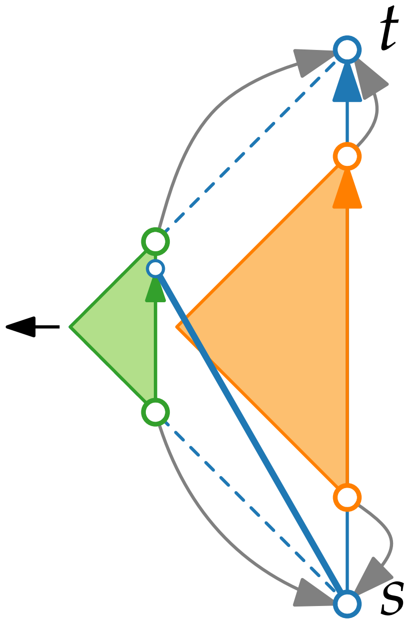

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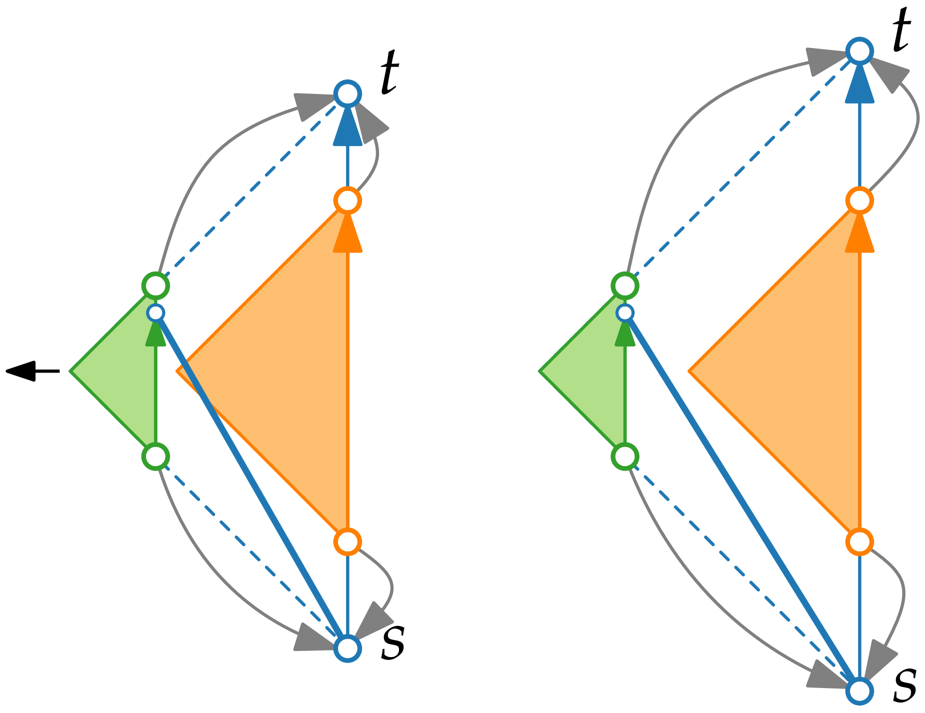

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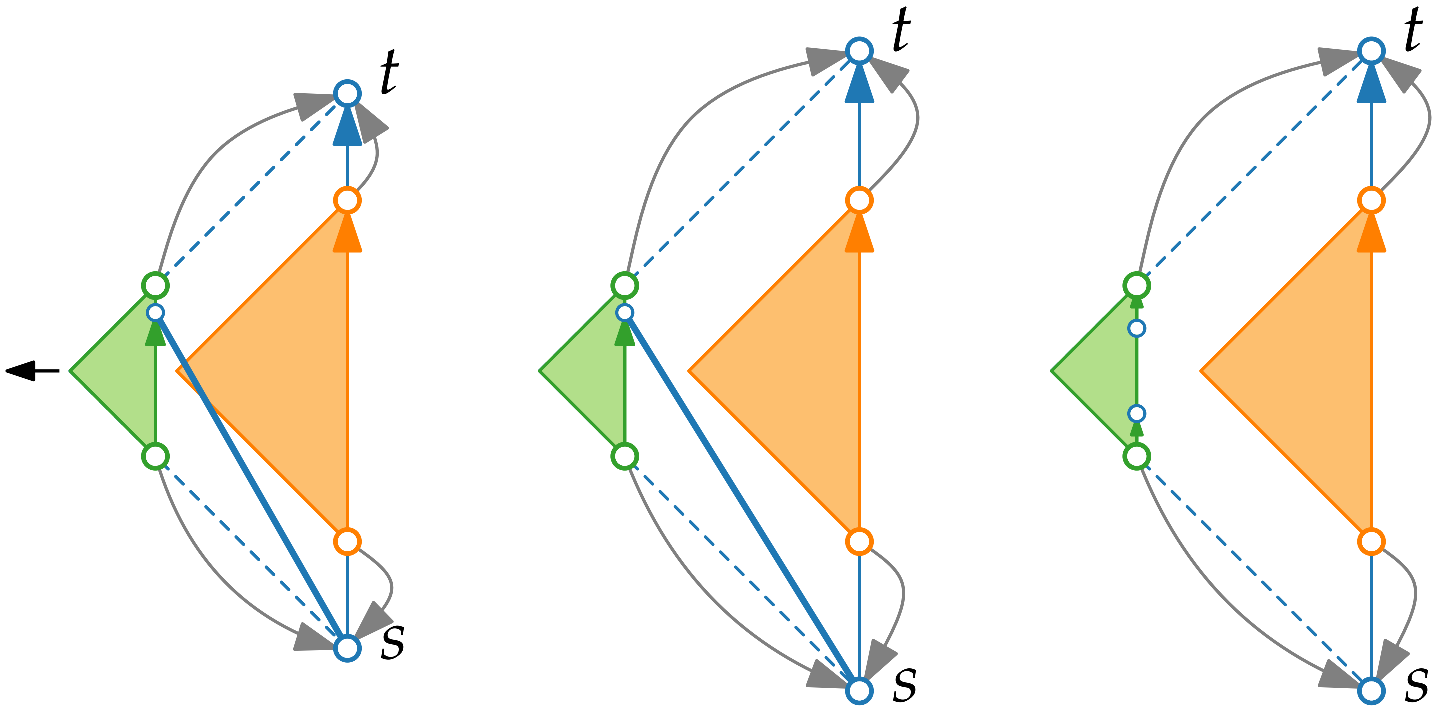

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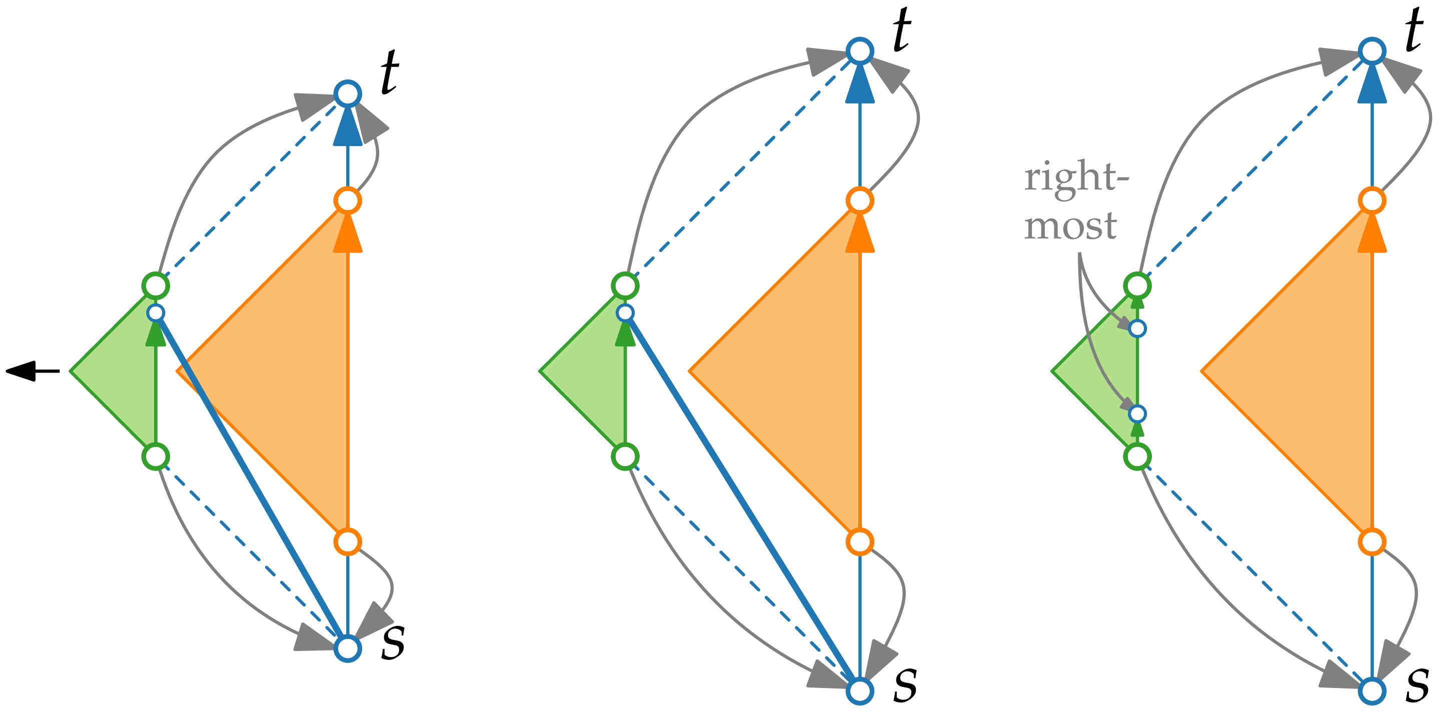

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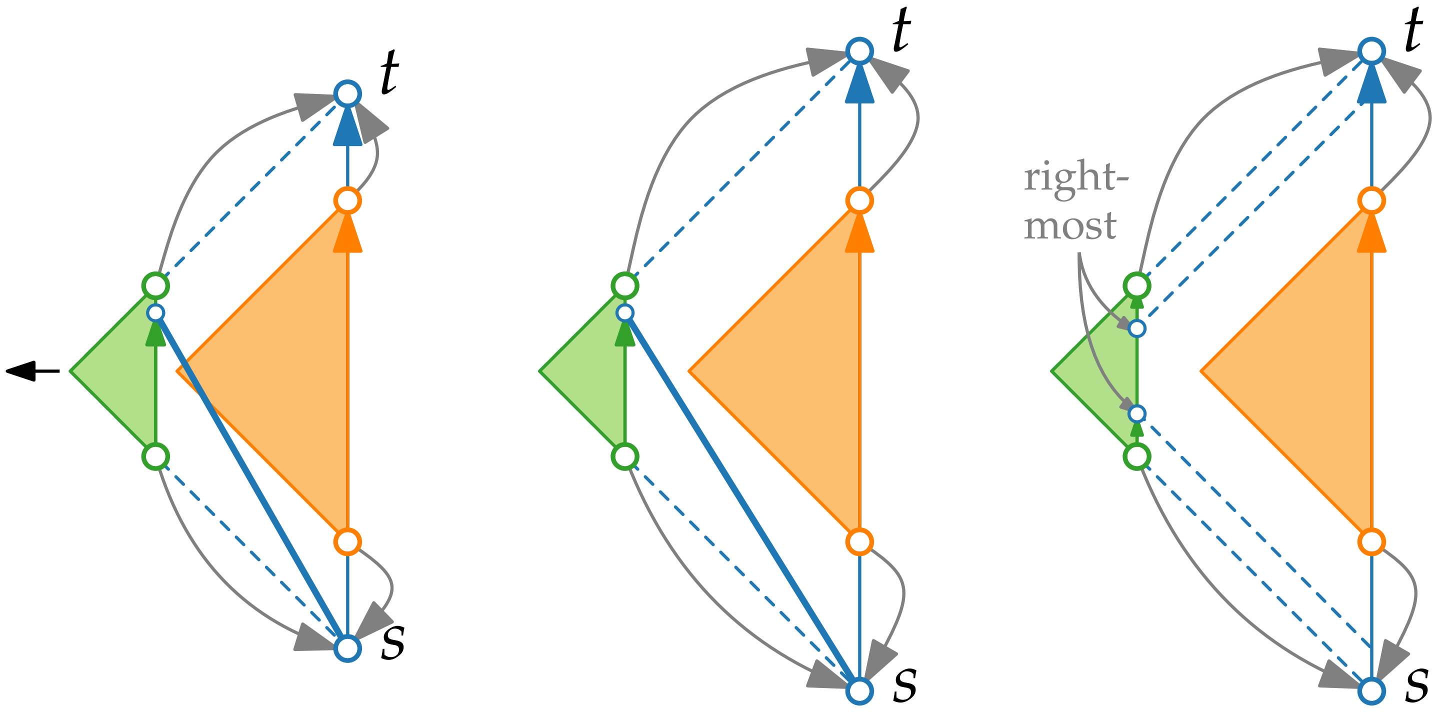

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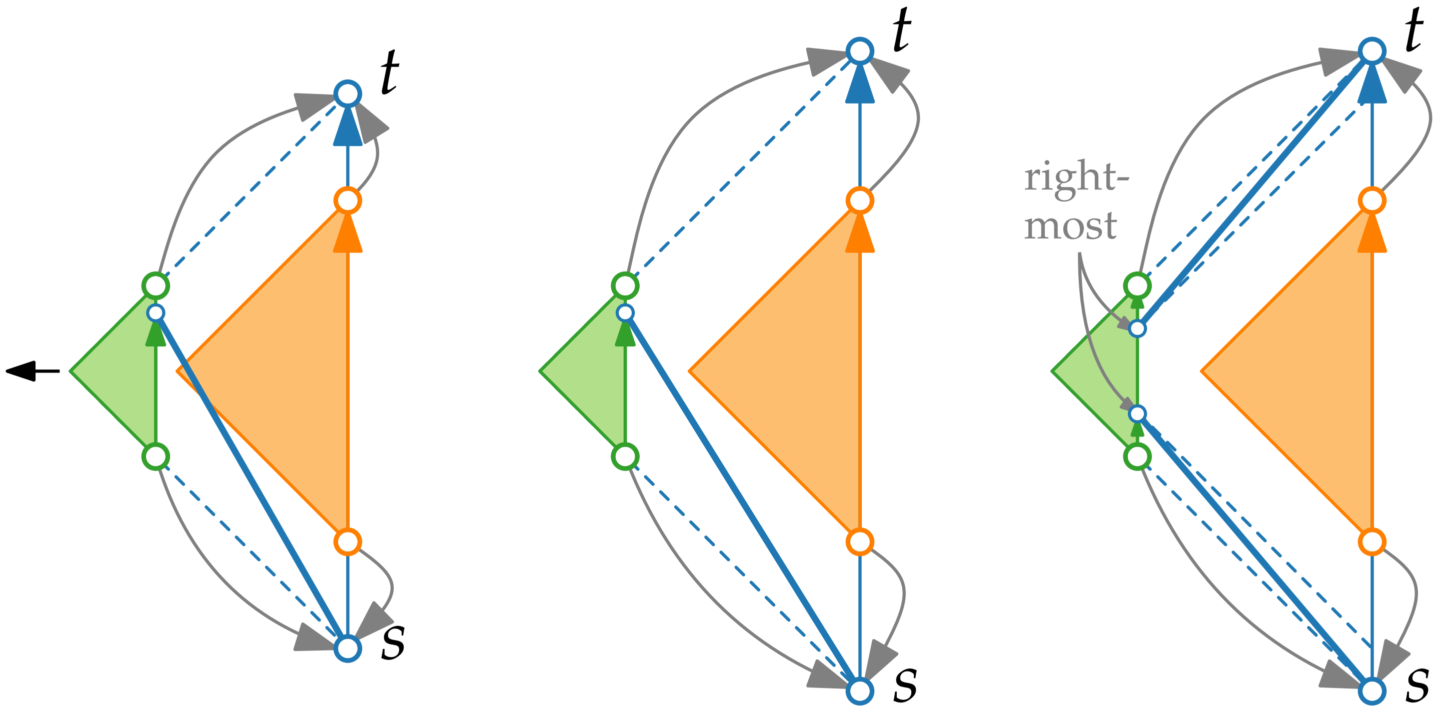

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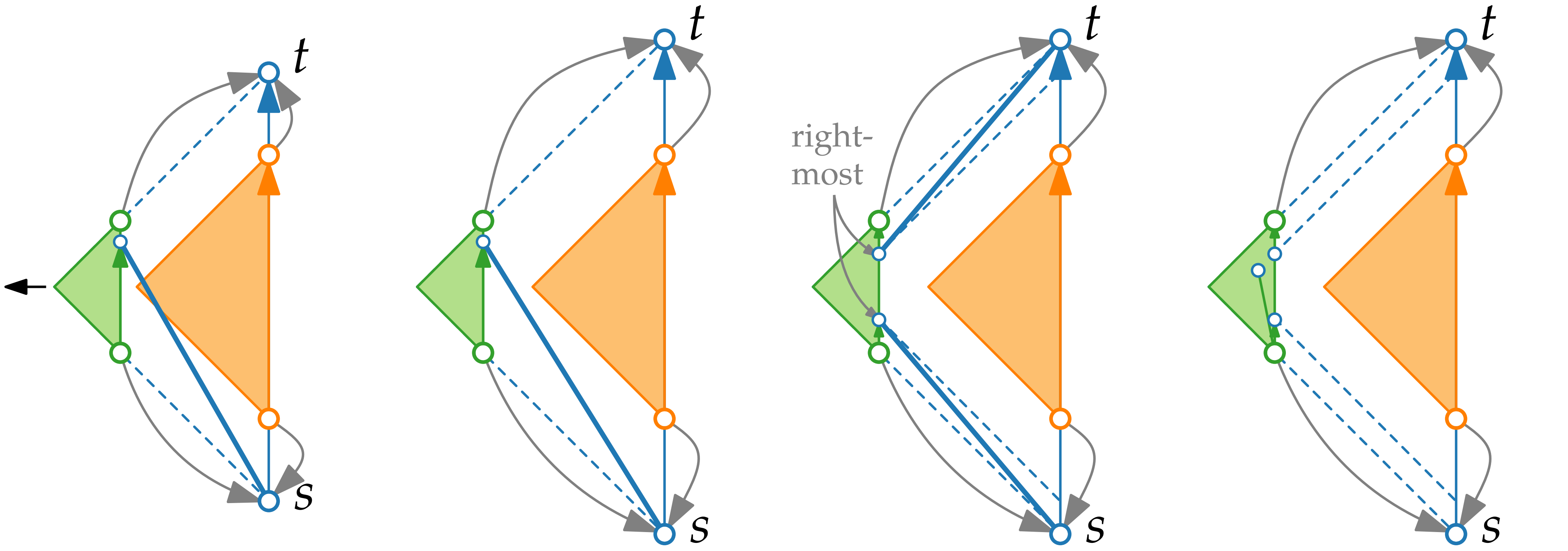

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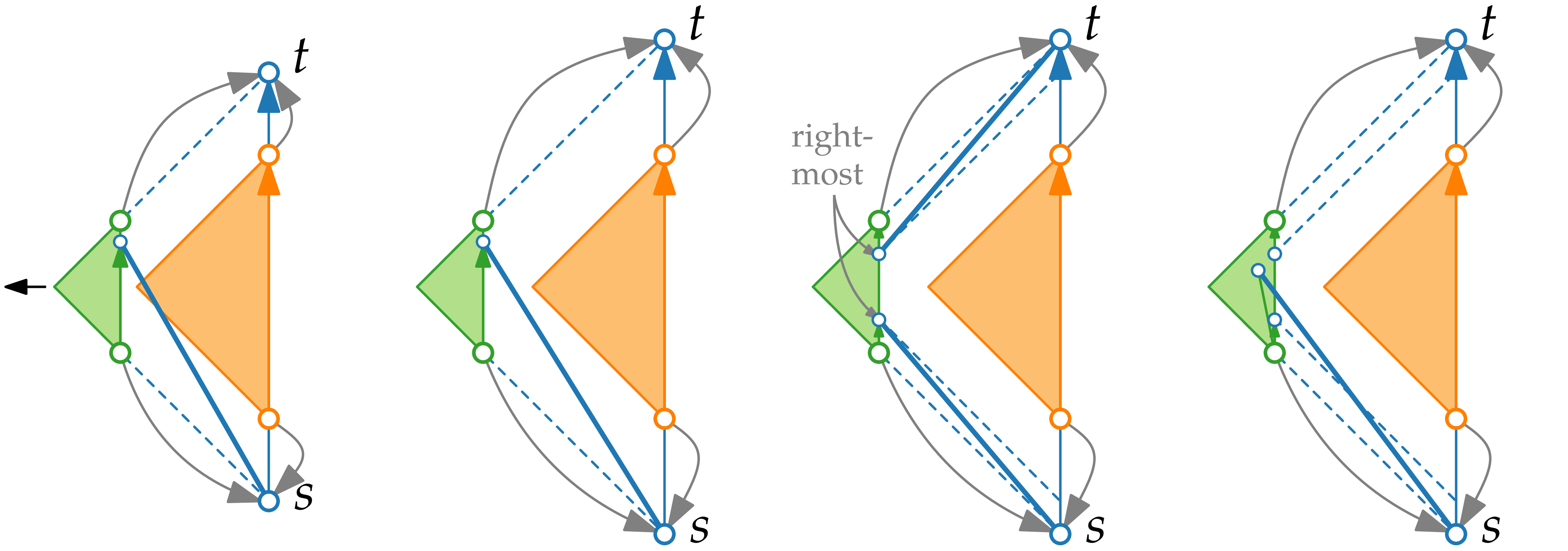

Series-Parallel Graphs – Straight-Line Drawings

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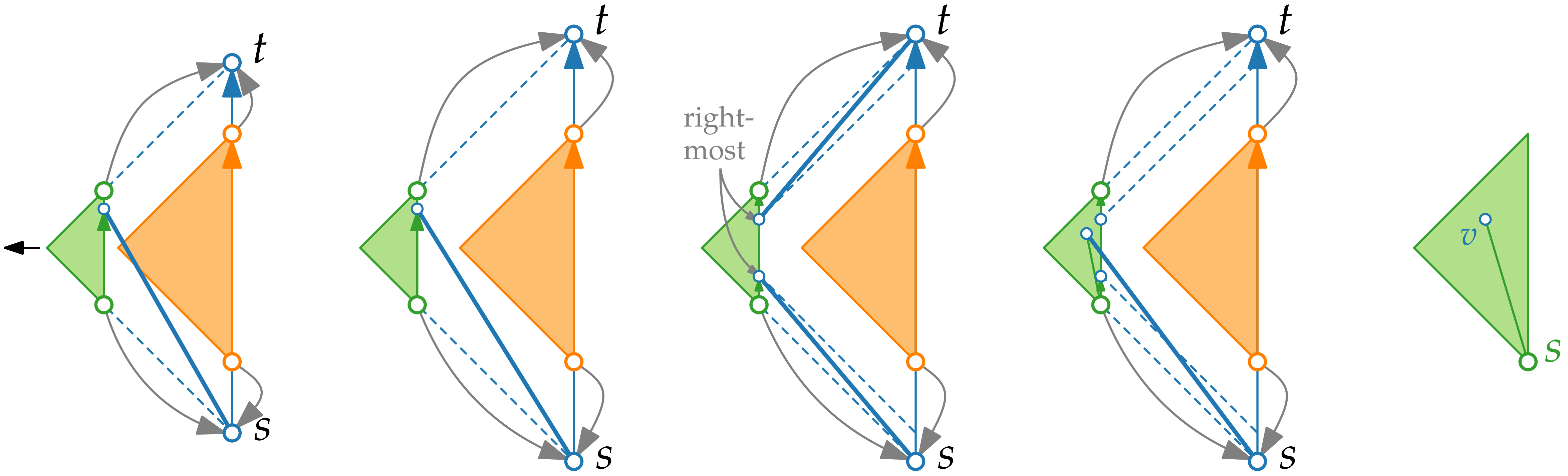

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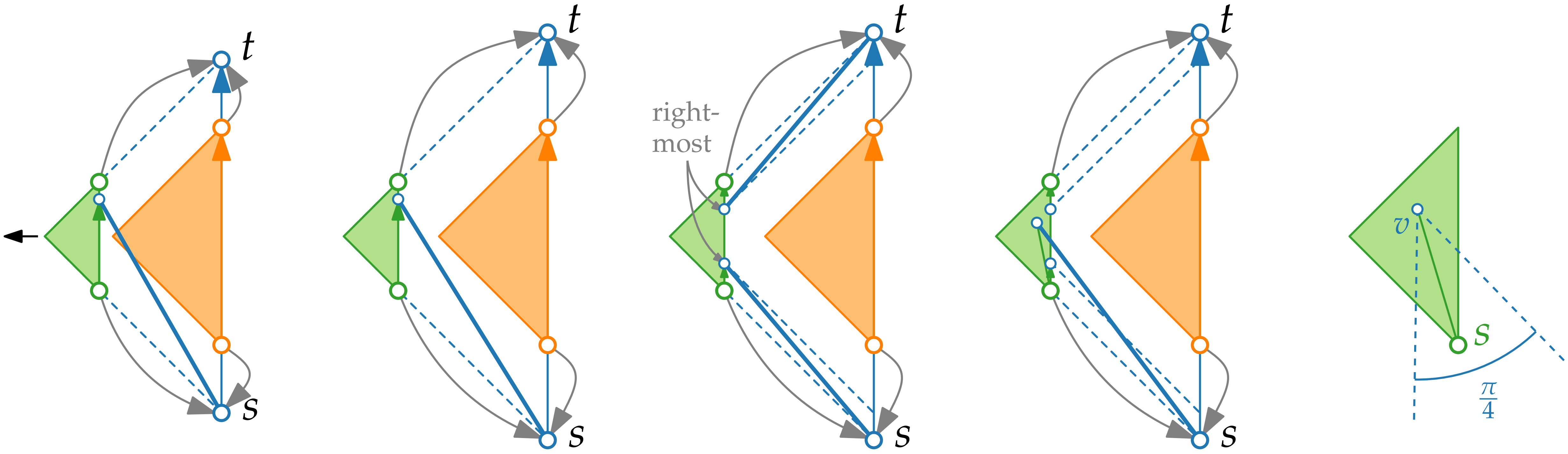

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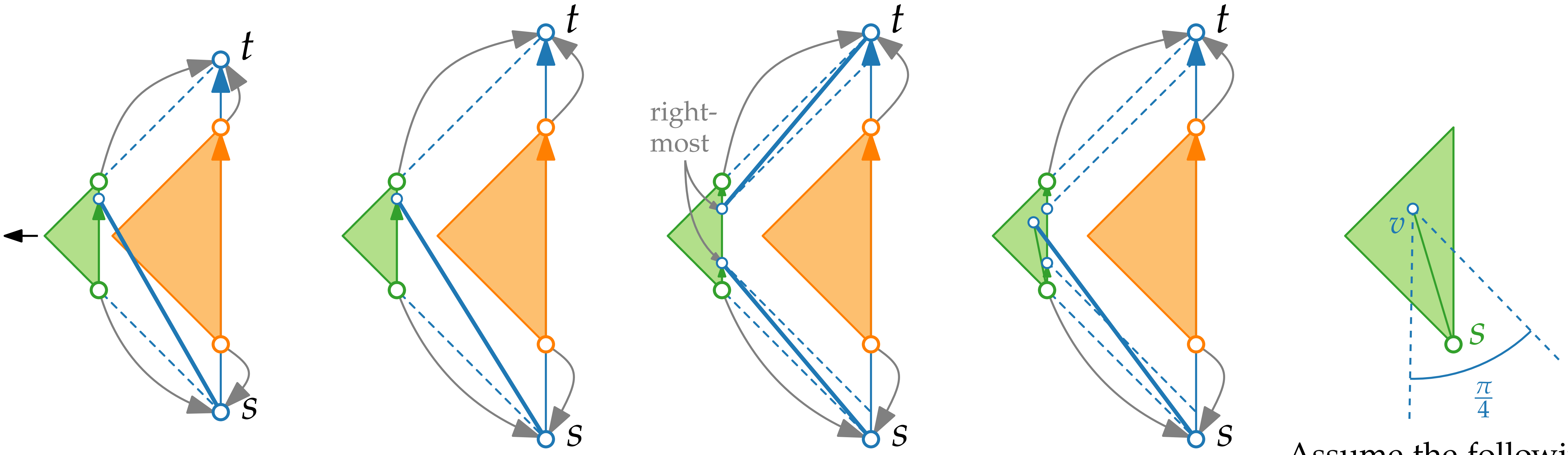

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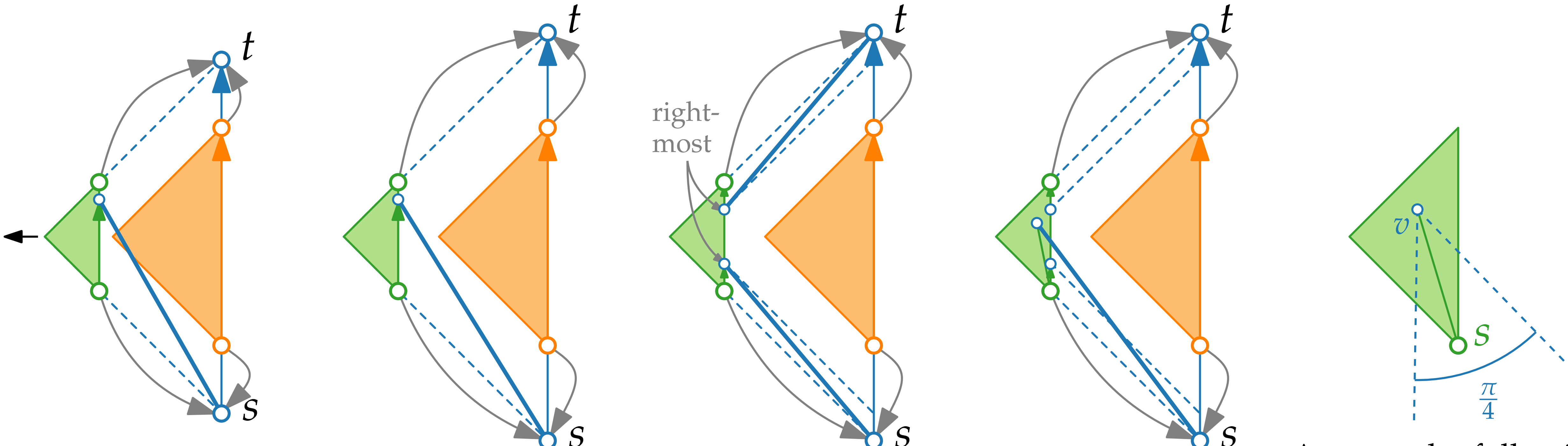

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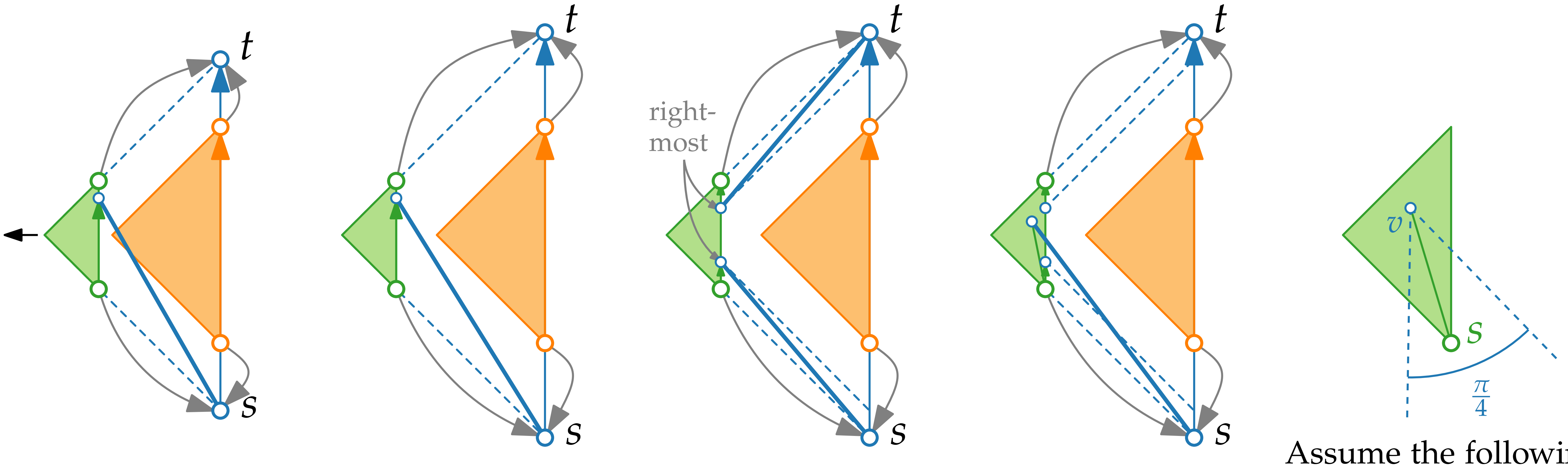

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> **Lemma.**
> The drawing produced by the algorithm is planar.

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Γ can be computed in $\mathcal{O}(n)$ time.

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Theorem. [Bertolazzi et al. 94]

There exists a $2n$-vertex series-parallel graph G_n such that any upward planar drawing of G_n that respects the embedding requires $\Omega(4^n)$ area.

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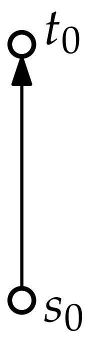

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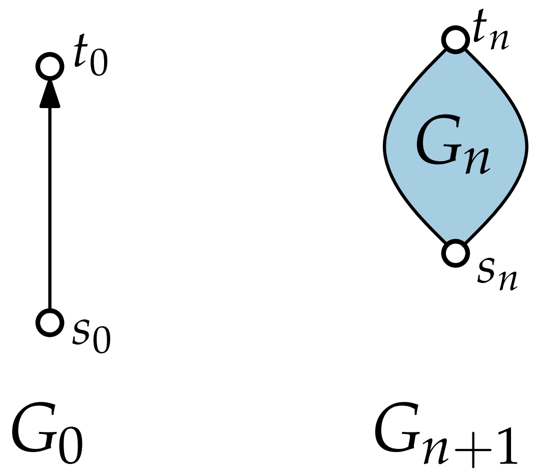

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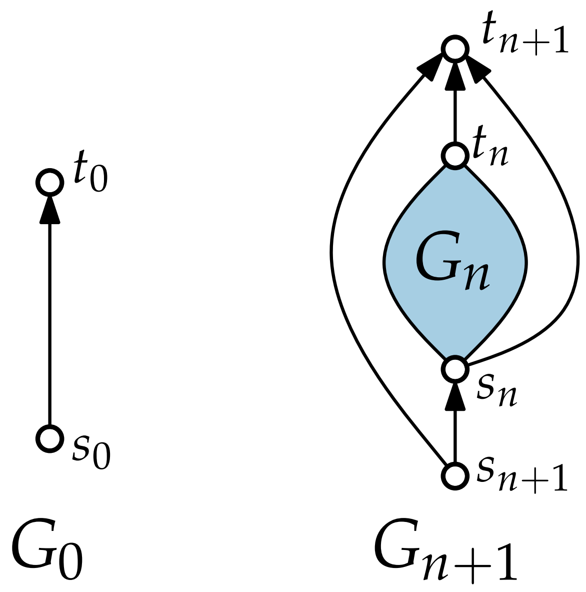

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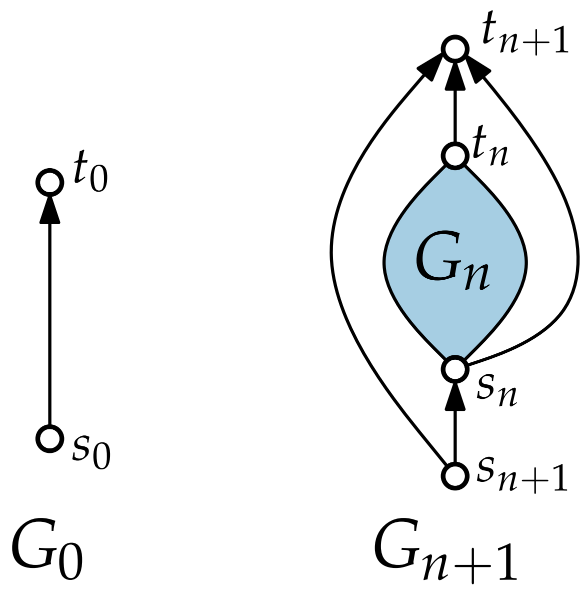

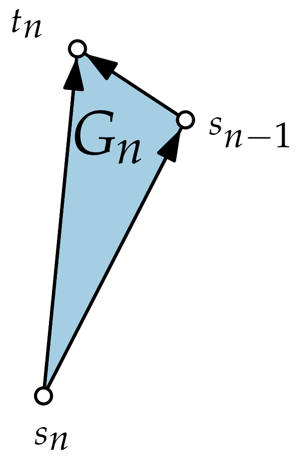

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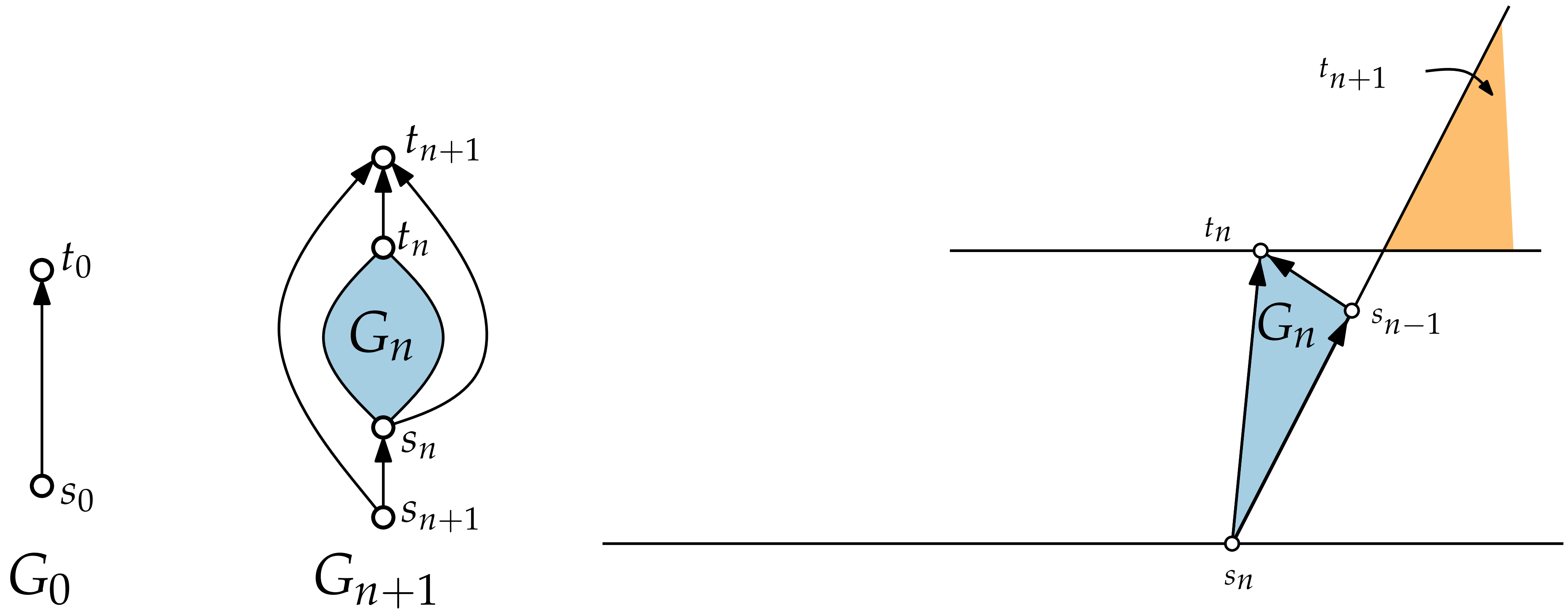

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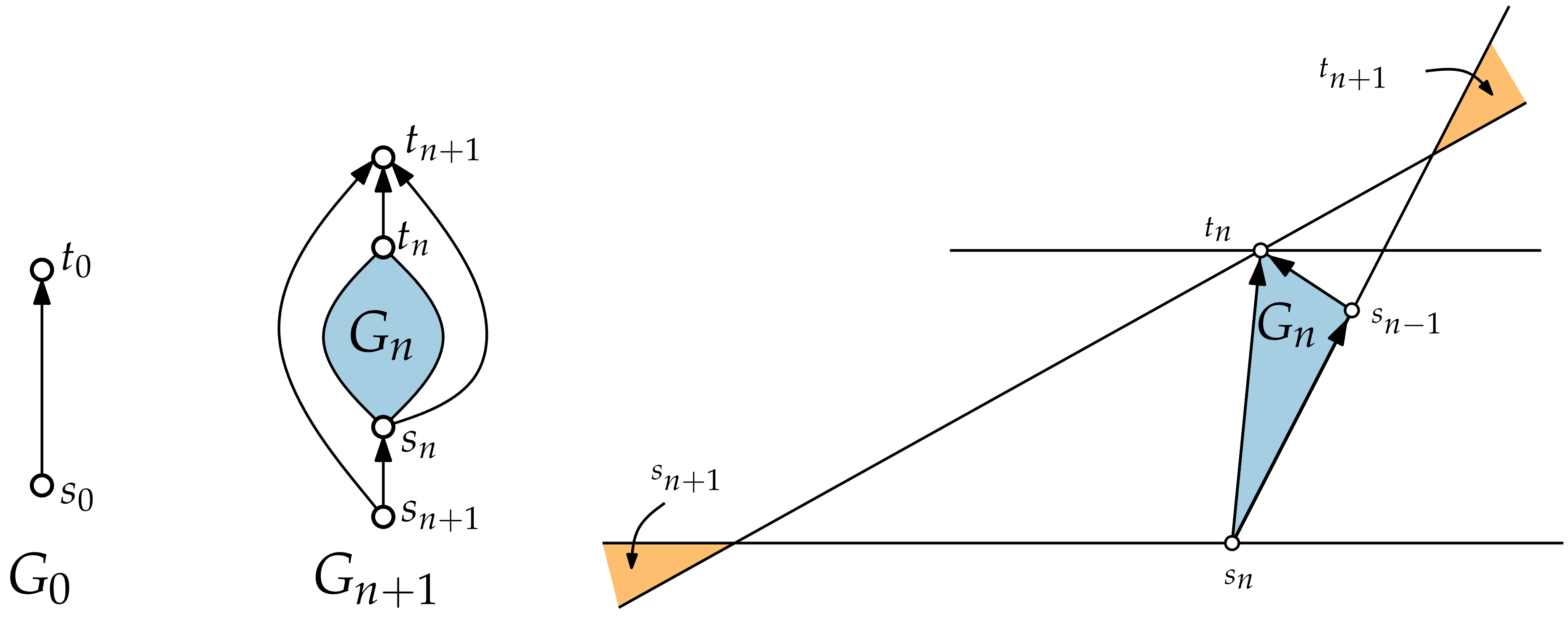

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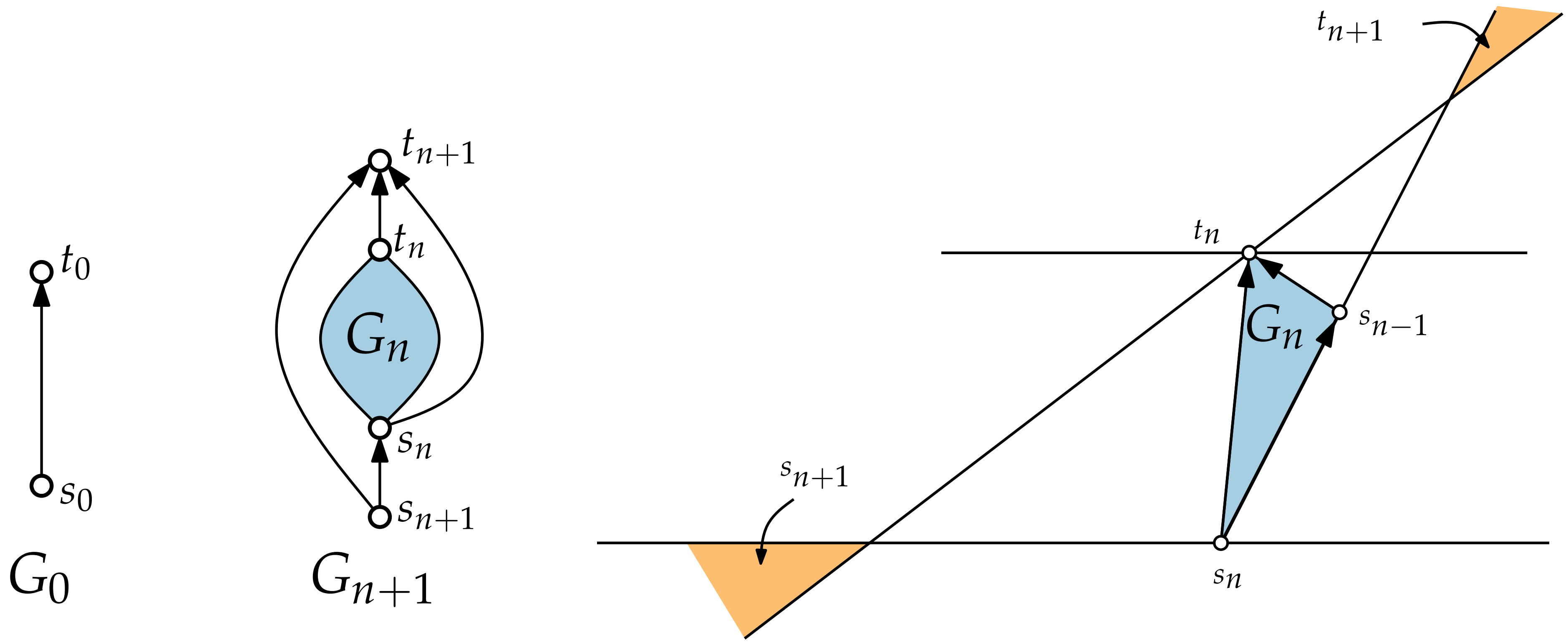

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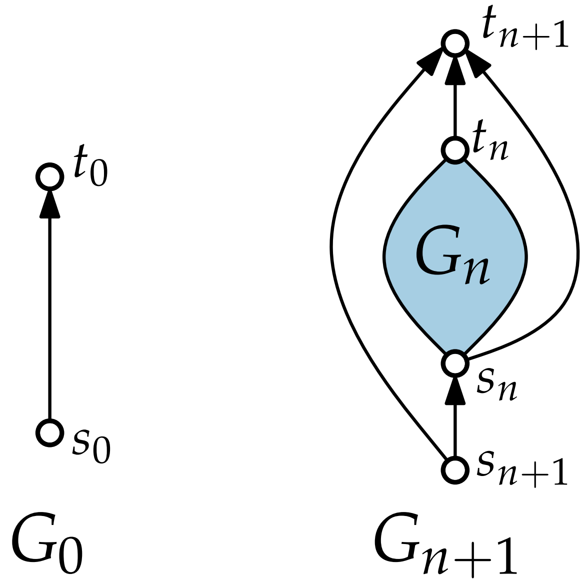

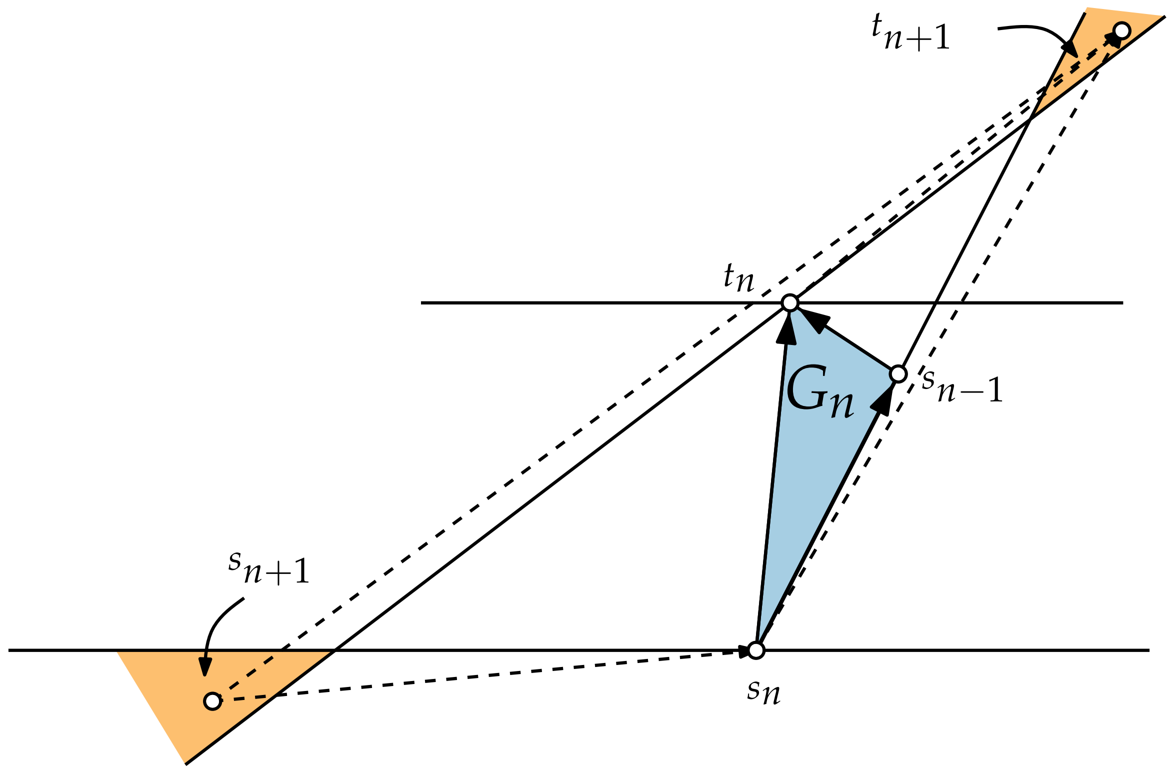

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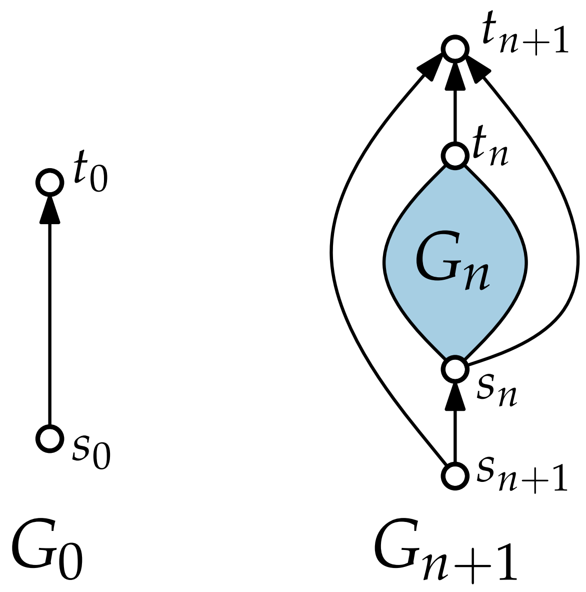

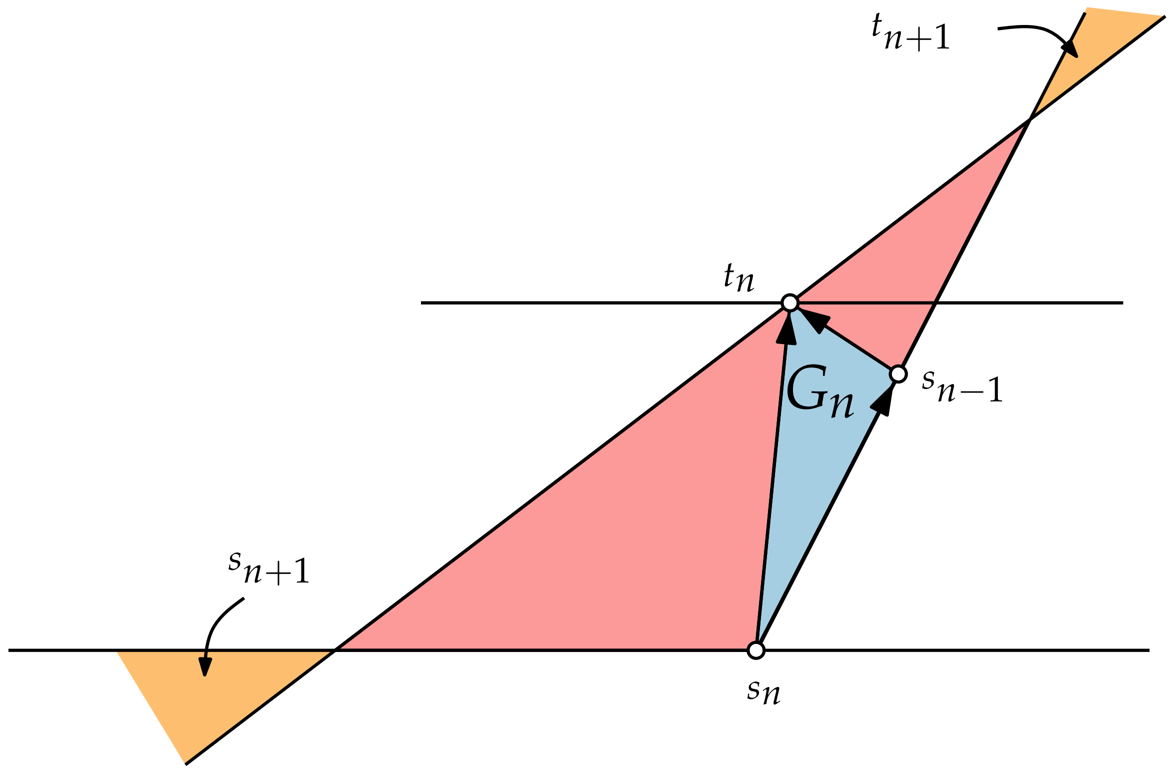

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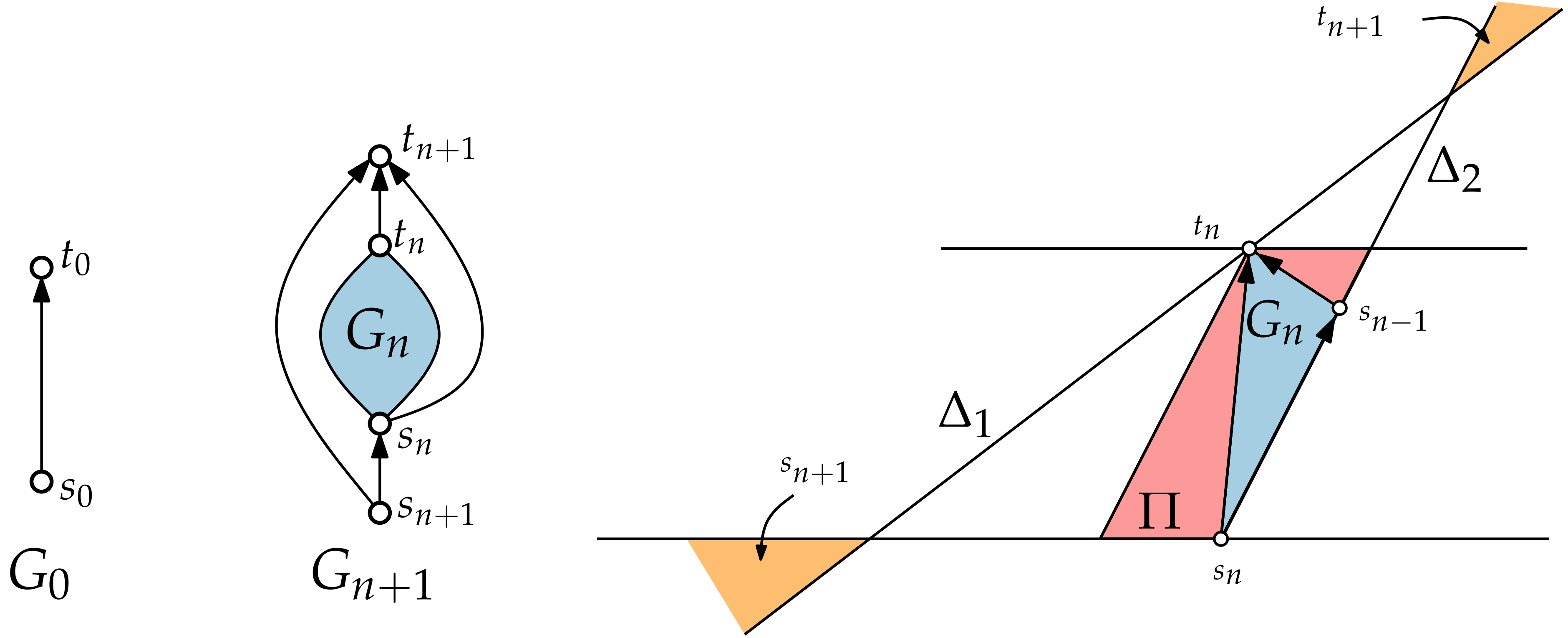

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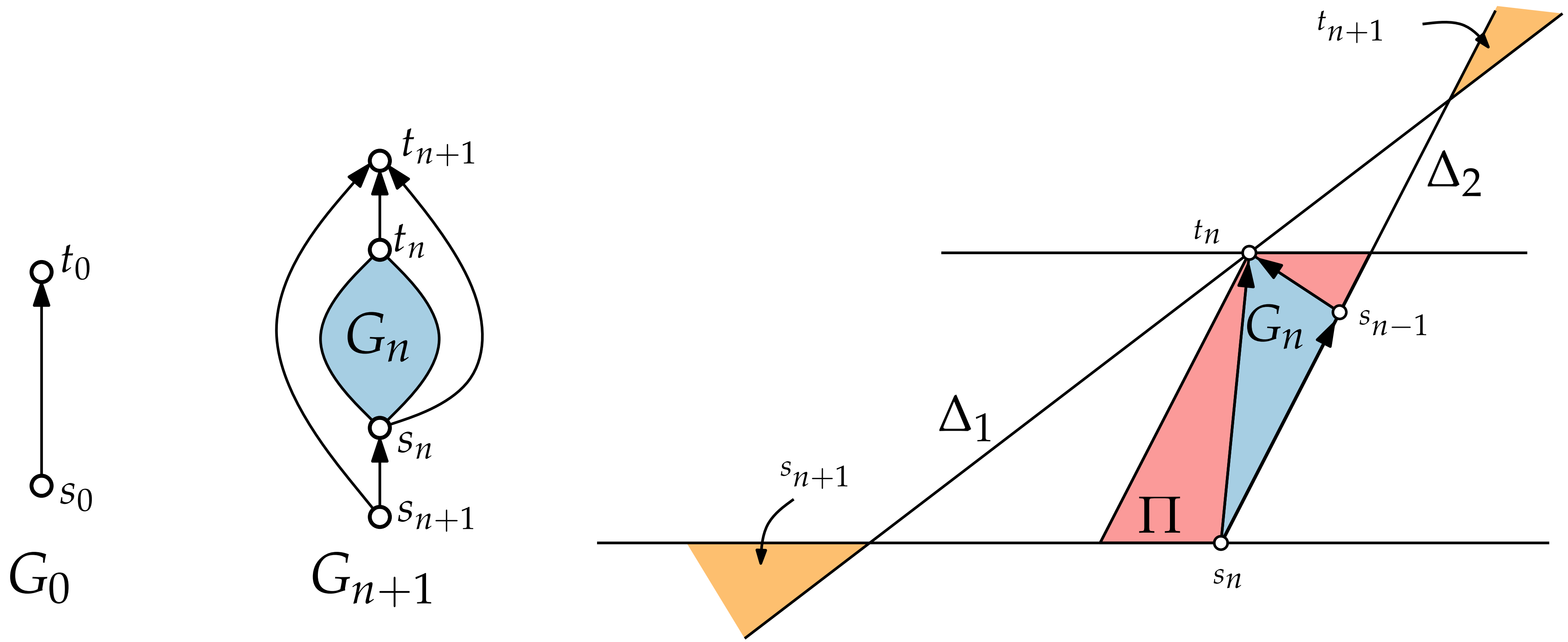

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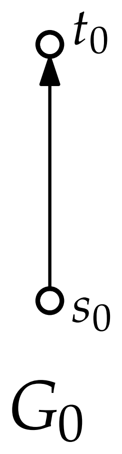

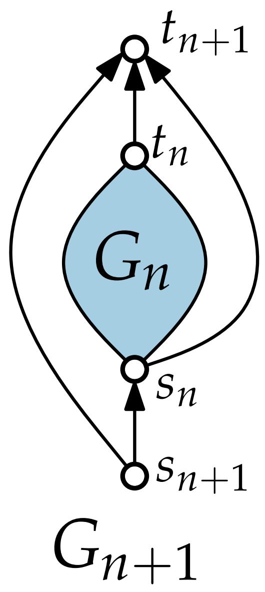

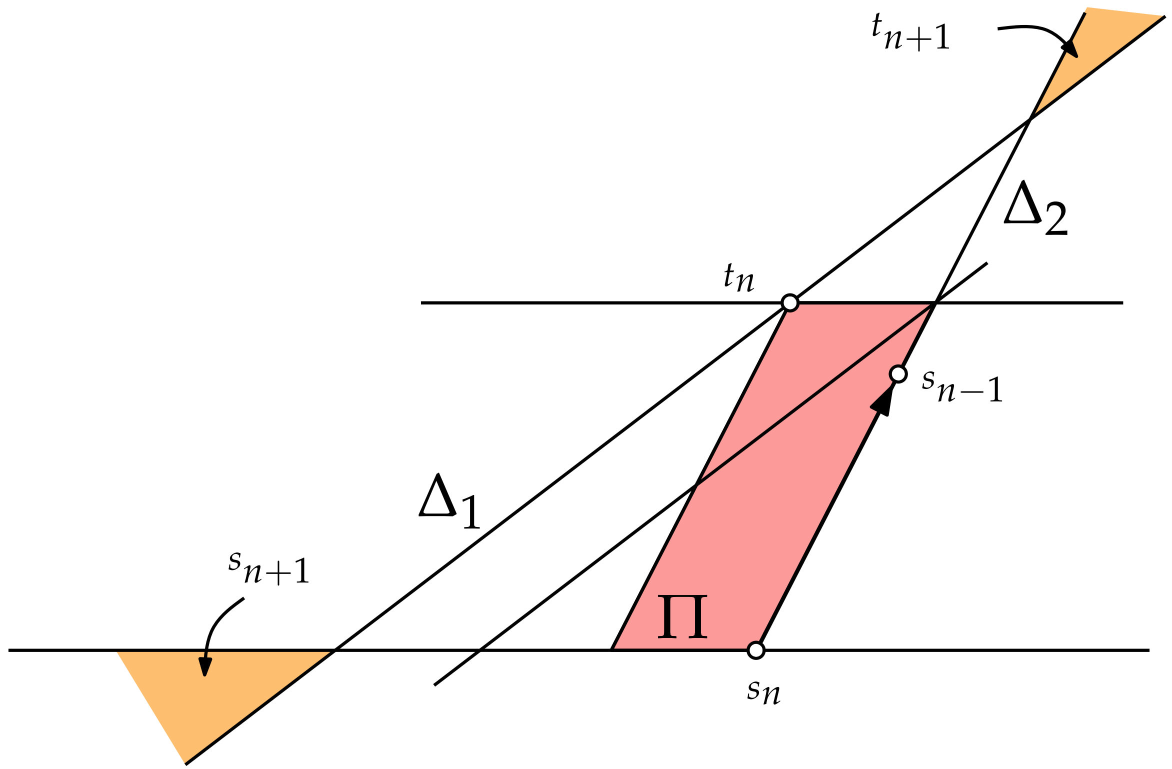

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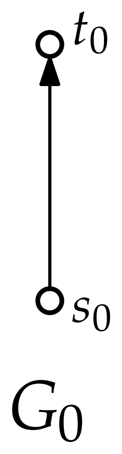

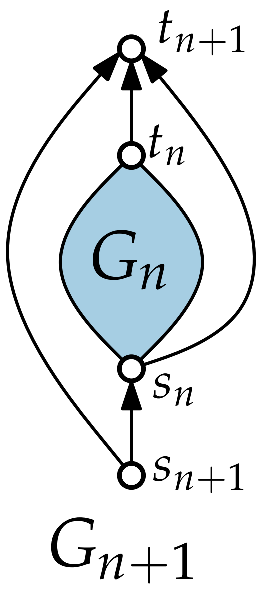

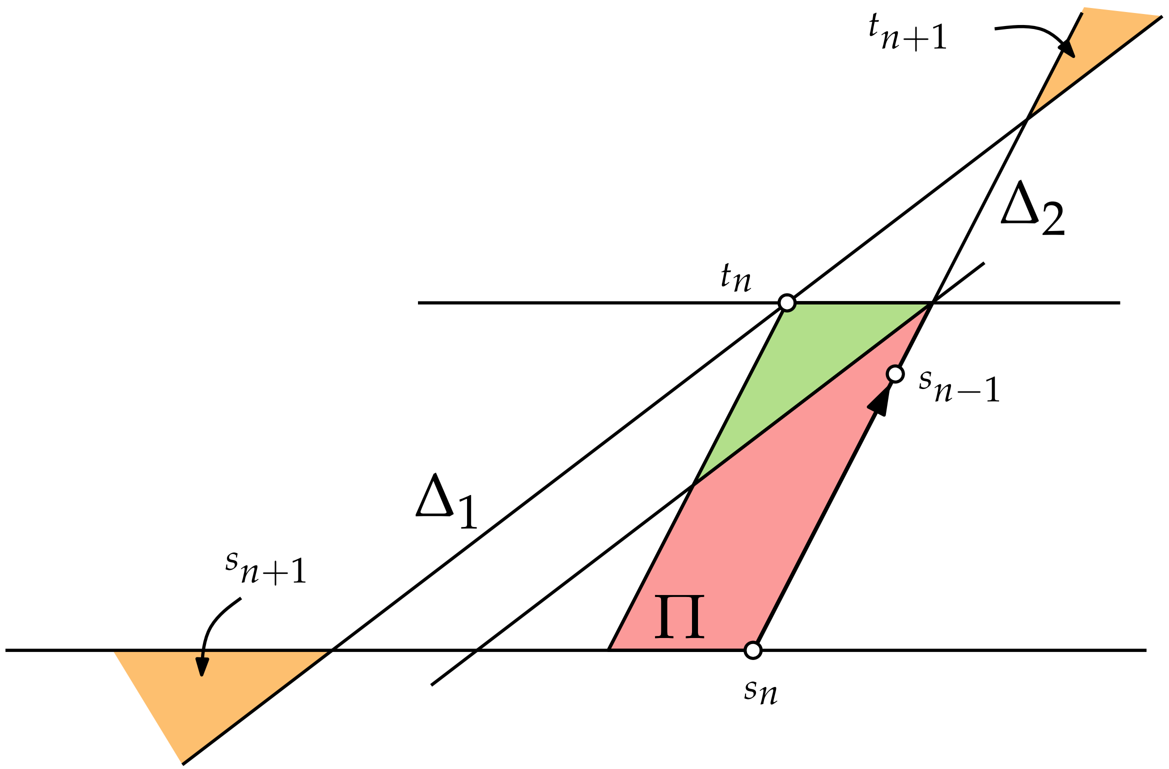

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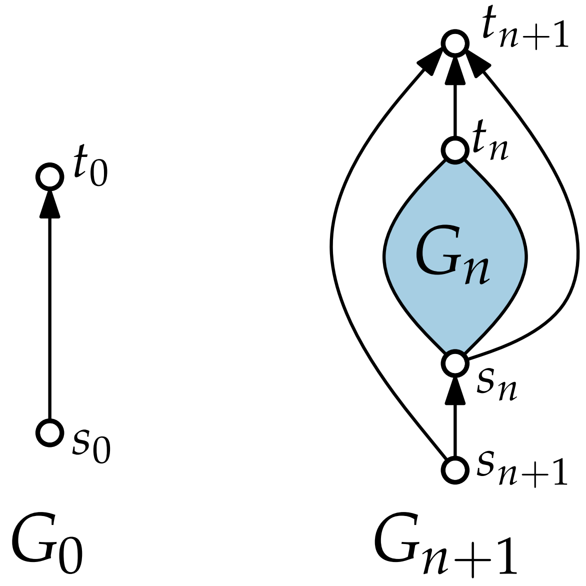

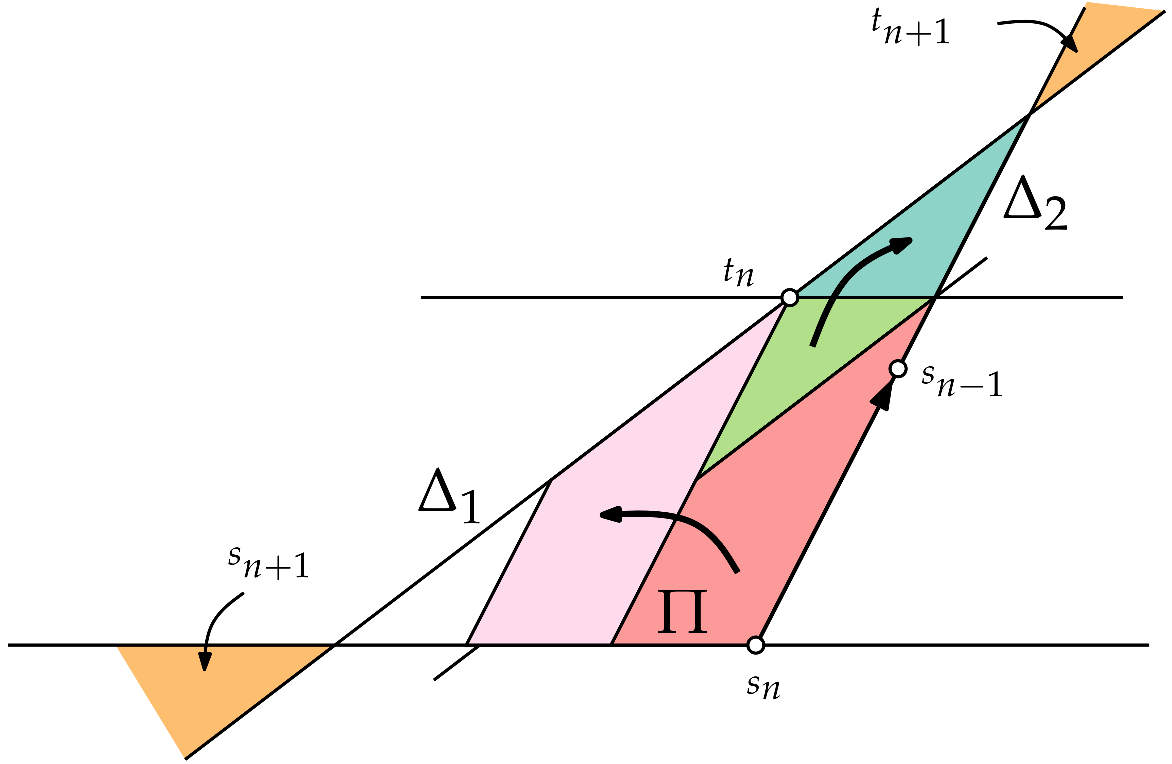

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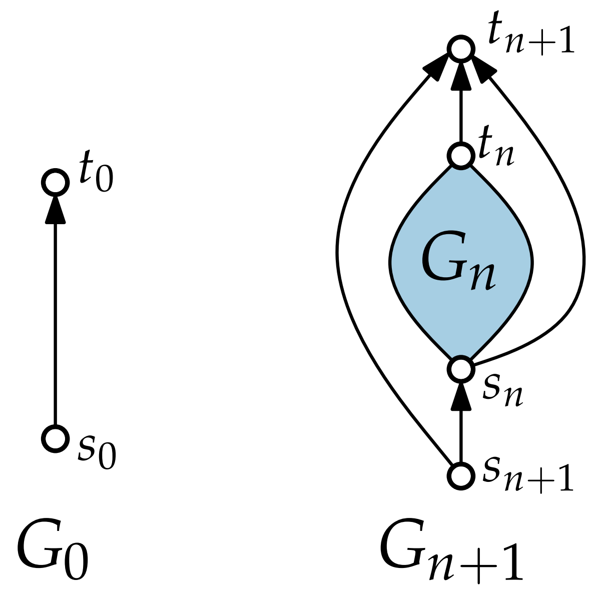

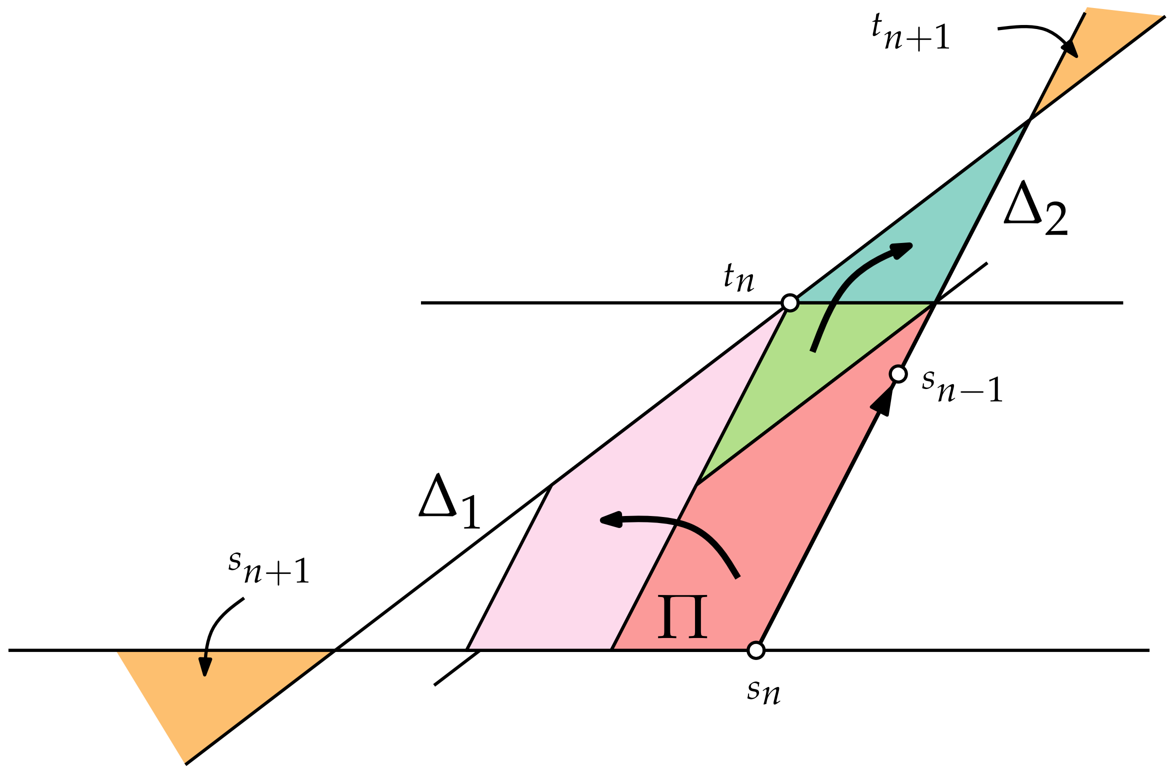

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