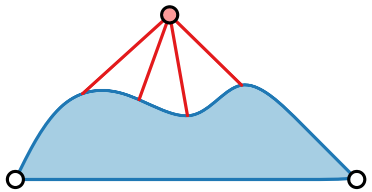
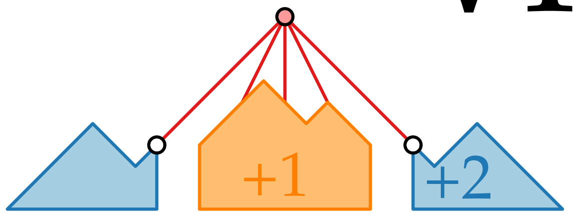
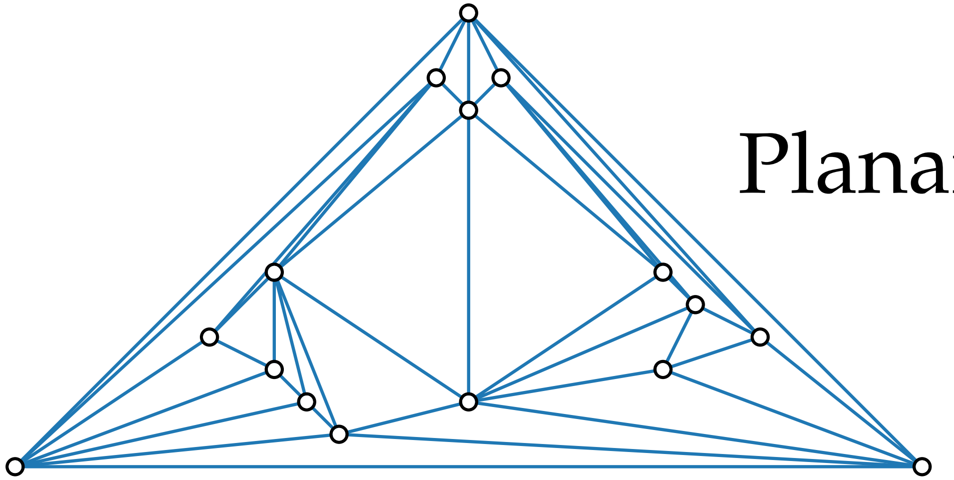


Visualization of Graphs



Lecture 4:
Straight-Line Drawings of Planar Graphs I: Canonical Ordering and Shift Method



Part I:
Planar Straight-Line Drawings

Philipp Kindermann

Motivation

Why planar and straight-line?

Motivation

Why planar and straight-line?

[Bennett, Ryall, Spaltzeholz and Gooch '07]

The Aesthetics of Graph Visualization

3.2. Edge Placement Heuristics

By far the most agreed-upon edge placement heuristic is to *minimize the number of edge crossings* in a graph [BMRW98, Har98, DH96, Pur02, TR05, TBB88]. The importance of avoiding edge crossings has also been extensively validated in terms of user preference and performance (see Section 4). Similarly, based on perceptual principles, it is beneficial to *minimize the number of edge bends* within a graph [Pur02, TR05, TBB88]. Edge bends make edges more difficult to follow because an edge with a sharp bend is more likely to be perceived as two separate objects. This leads to the heuristic of *keeping edge bends uniform* with respect to the bend's position on the edge and its angle [TR05]. If an edge must be bent to satisfy other aesthetic criteria, the angle of the bend should be as little as possible, and the bend placement should evenly divide the edge.

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Drawing conventions

- No crossings $\Rightarrow$ planar
- No bends $\Rightarrow$ straight-line

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Drawing conventions

- No crossings ⇒ planar
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Drawing aestethics

- Area

Planar Graphs

Planar Graphs

Characterization

Planar Graphs

Characterization

Recognition

Planar Graphs

Characterization

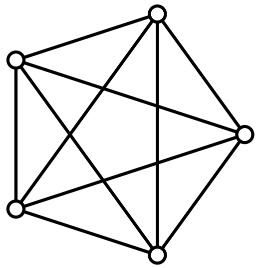
Recognition

Drawing

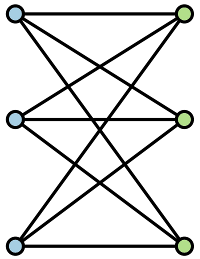
Planar Graphs

Theorem. [Kuratowski 1930]
G planar $\Leftrightarrow$
neither K_5 nor $K_{3,3}$ minor of G

K_5



$K_{3,3}$



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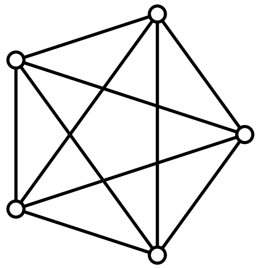

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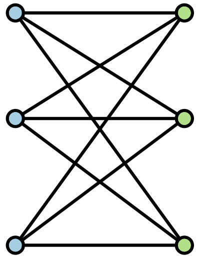

Characterization

Theorem. [Hopcroft & Tarjan 1974]
For a graph G with n vertices, there is an $\mathcal{O}(n)$ time algorithm to test whether G is planar.

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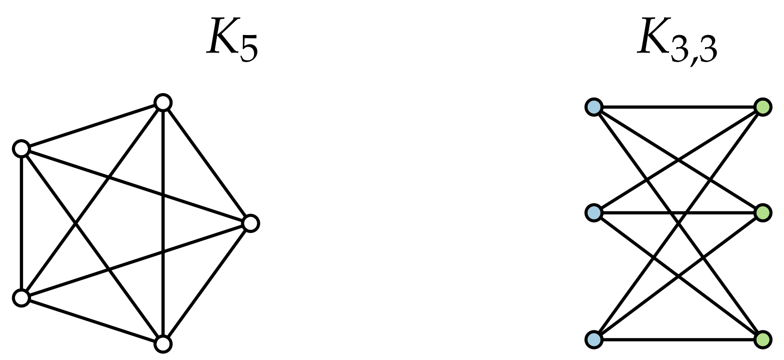

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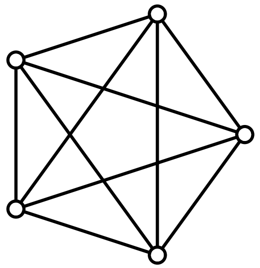

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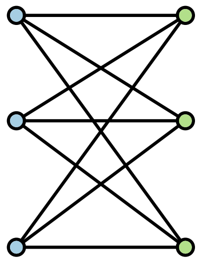

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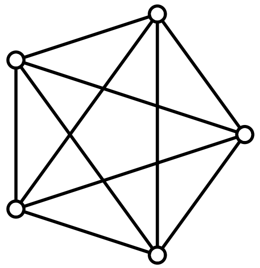

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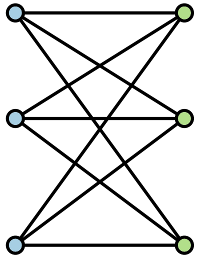

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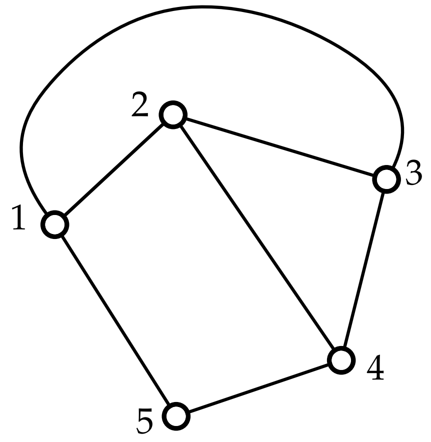
Drawing

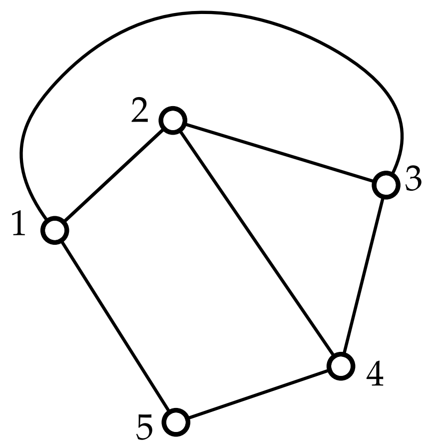
The algorithms implied by this theory produce drawings with area **not** bounded by any polynomial on n.

Triangulations



Triangulations

A **plane triangulation** is a plane graph where every face is a triangle.



Triangulations

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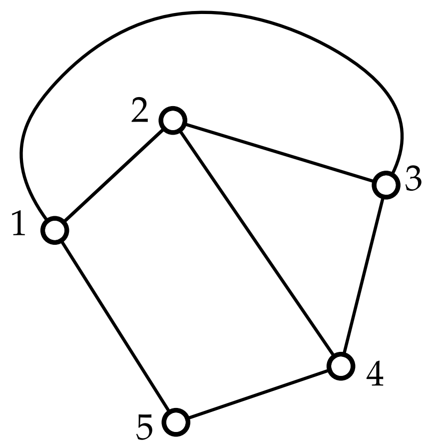

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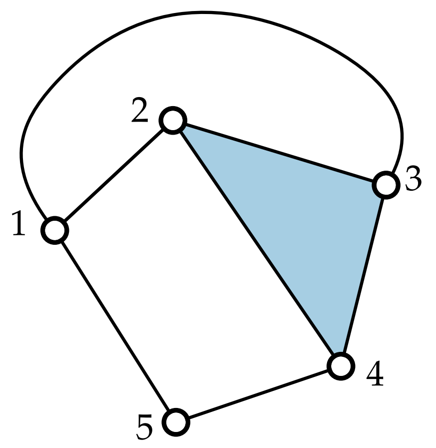

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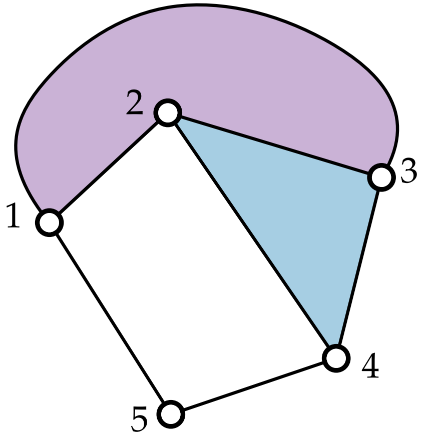

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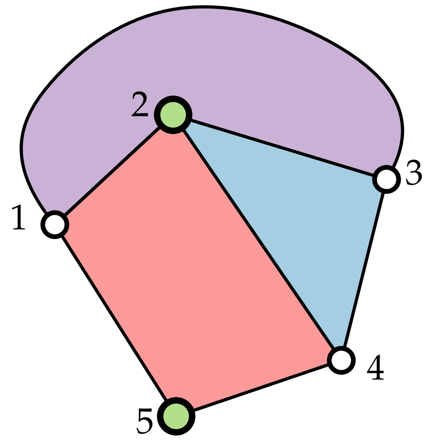

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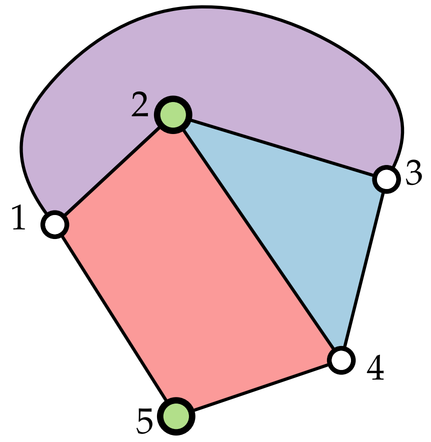

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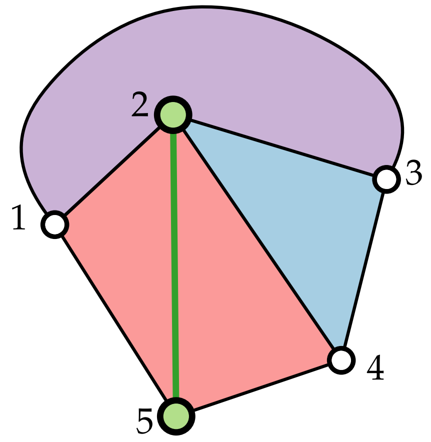

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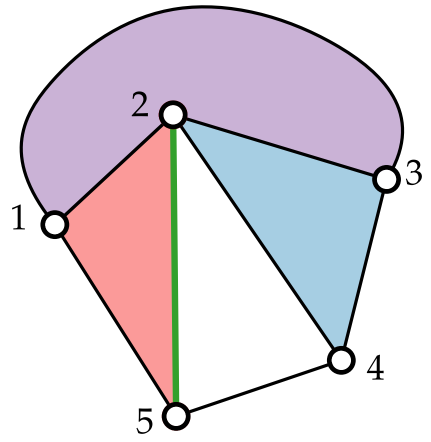

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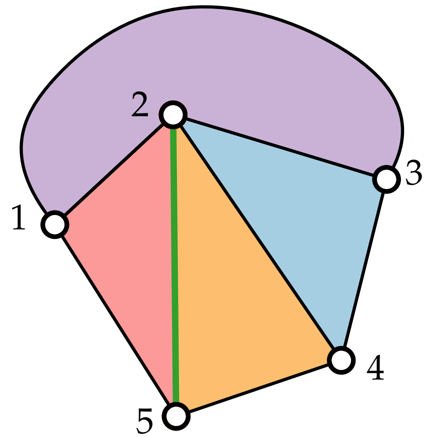

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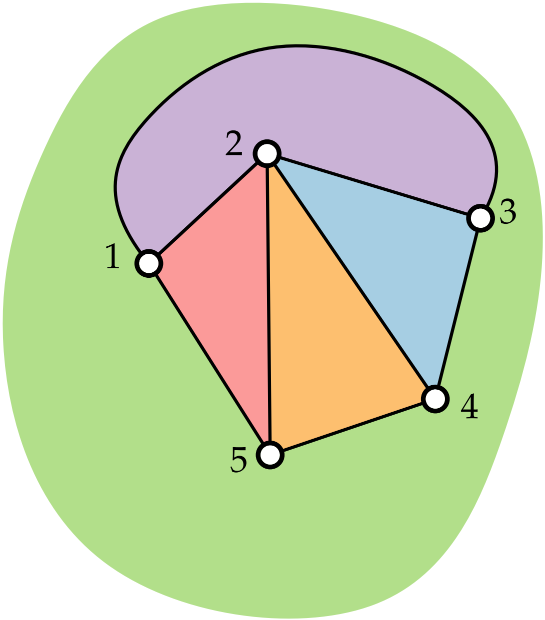

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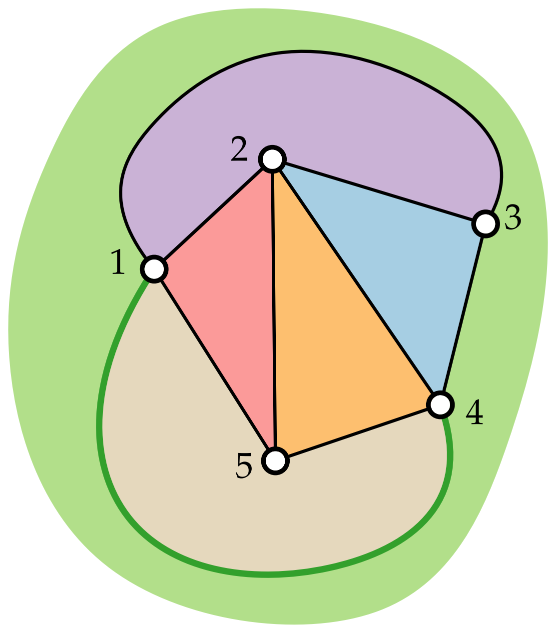

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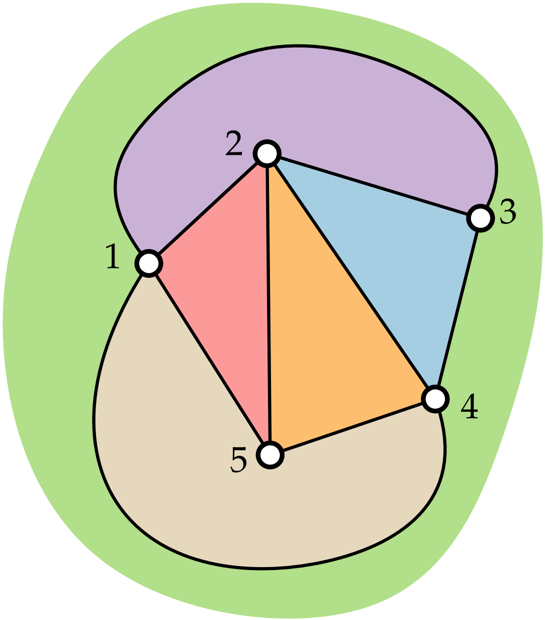

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A **plane (inner) triangulation** is a plane graph where every (inner) face is a triangle.

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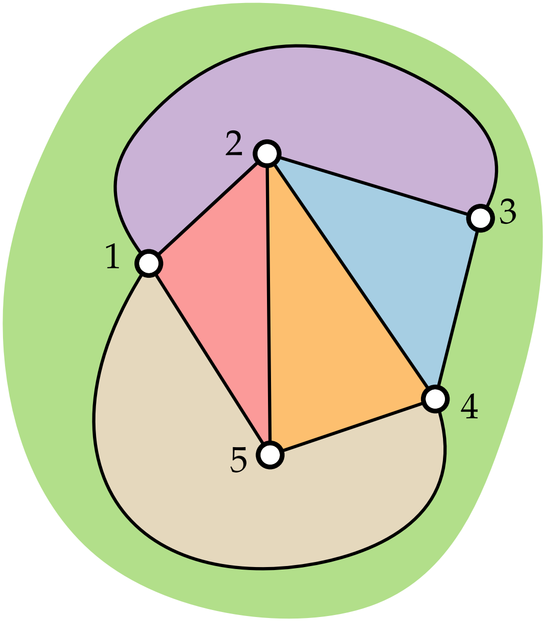

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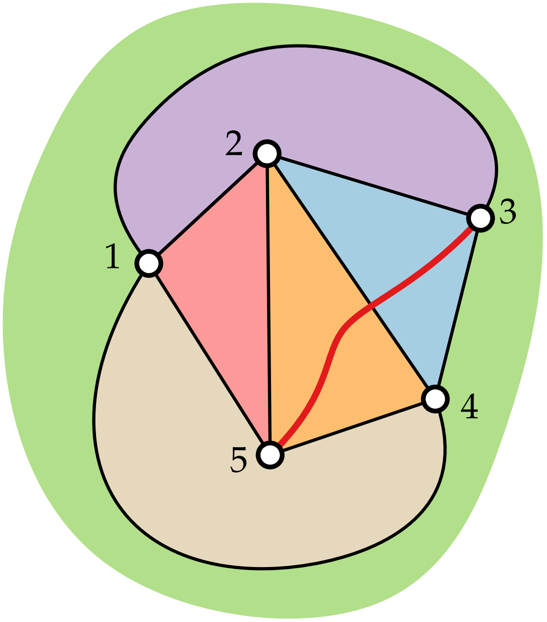

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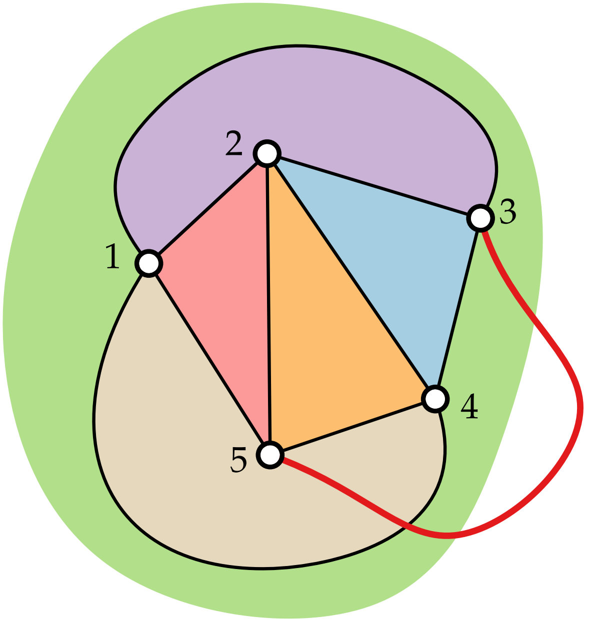

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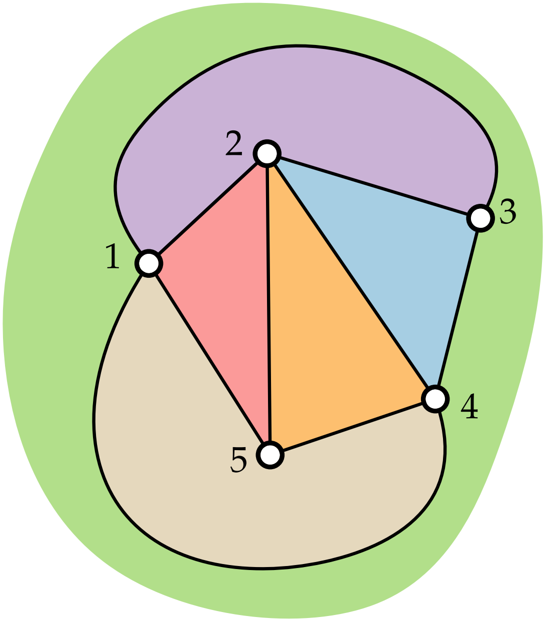

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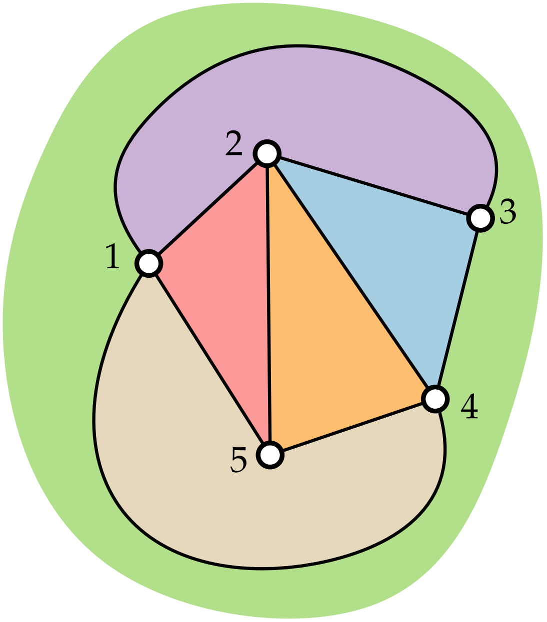

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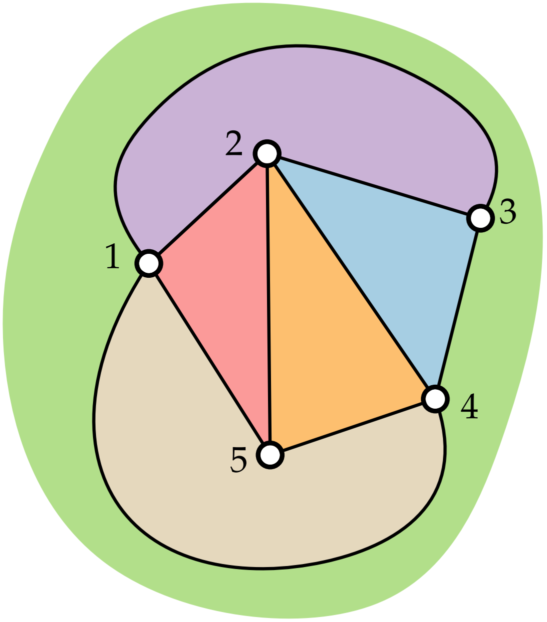

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A maximal plane graph is a plane triangulation.

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A plane triangulation is at least 3-connected and thus has a unique planar embedding.

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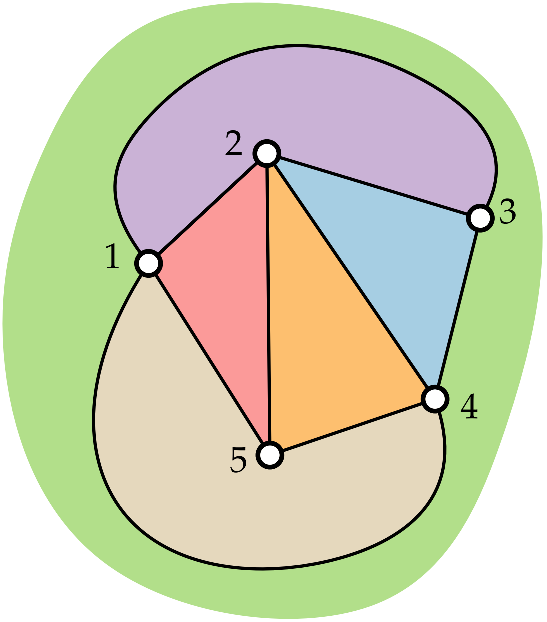

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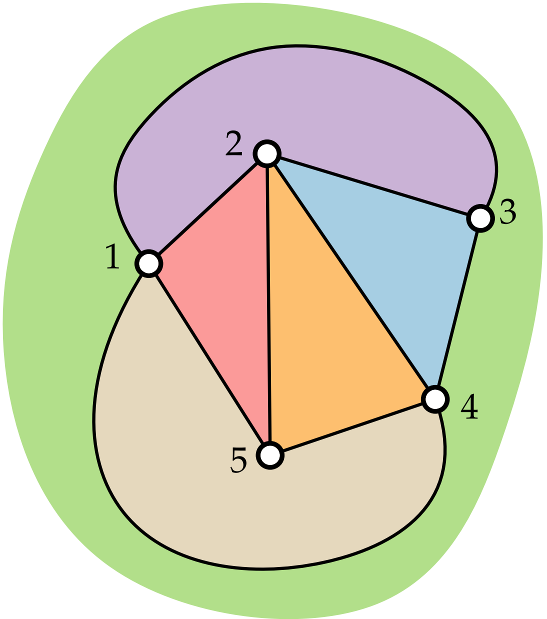

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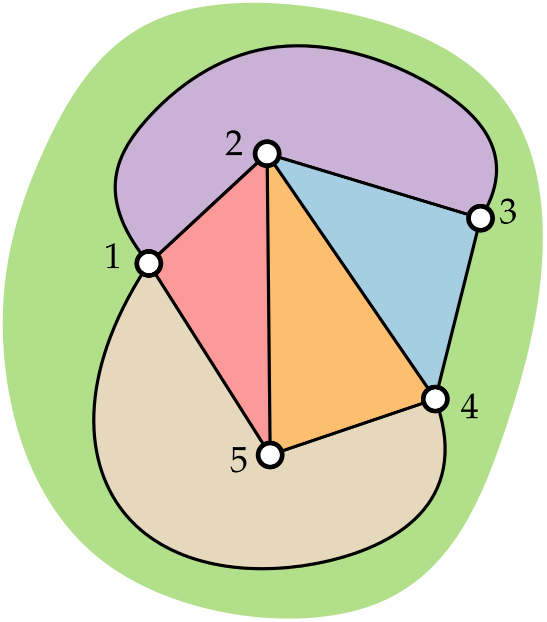

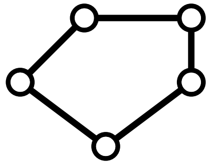

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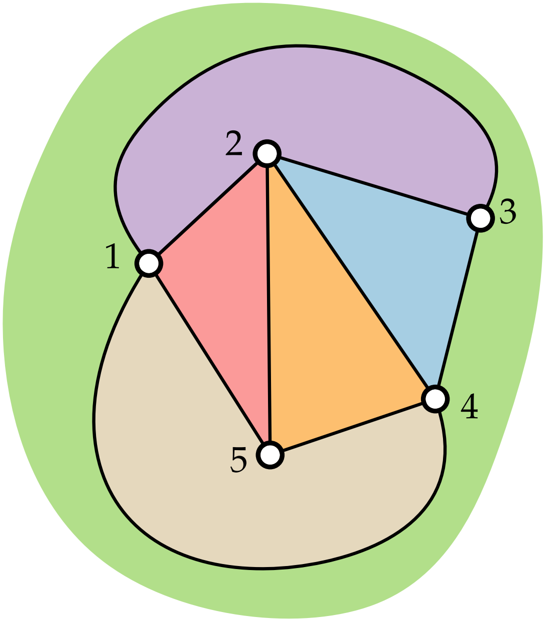

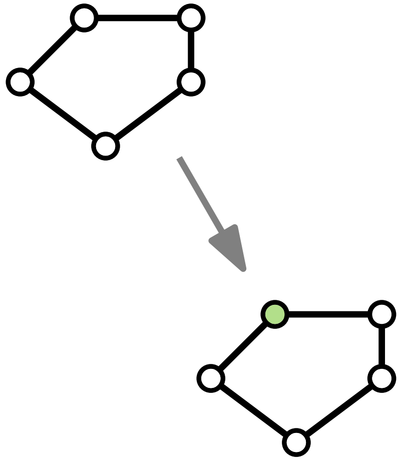

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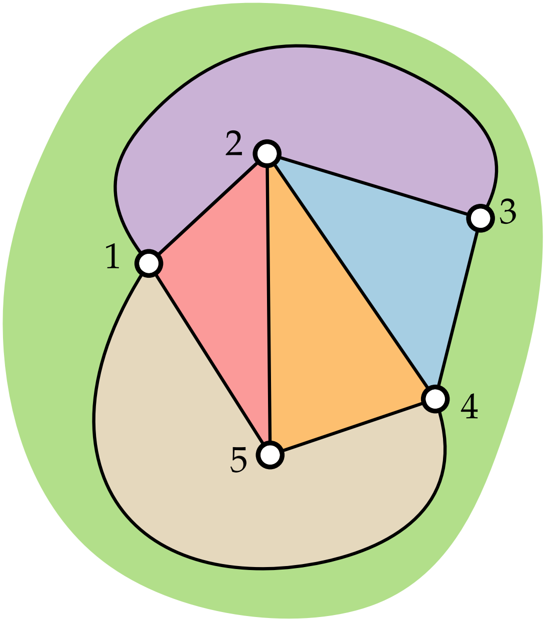

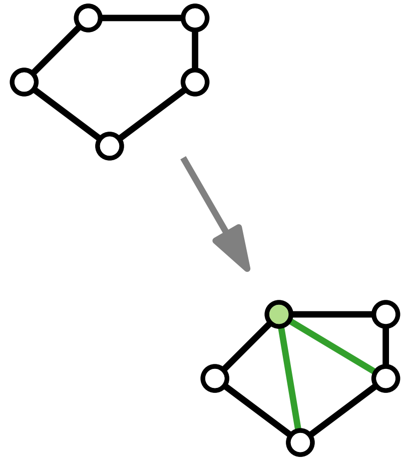

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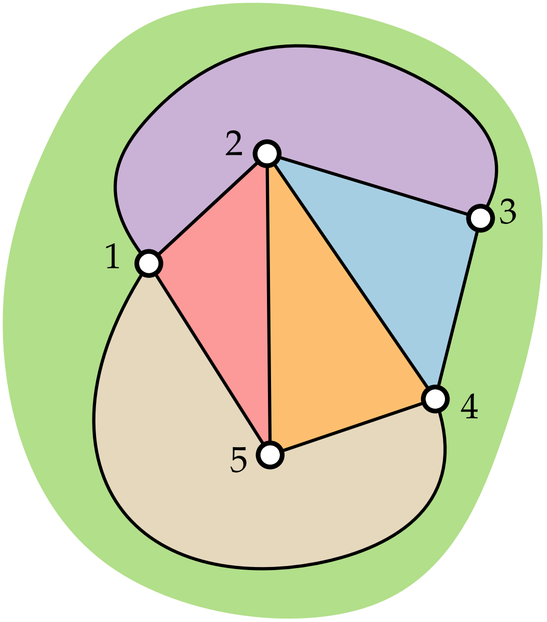

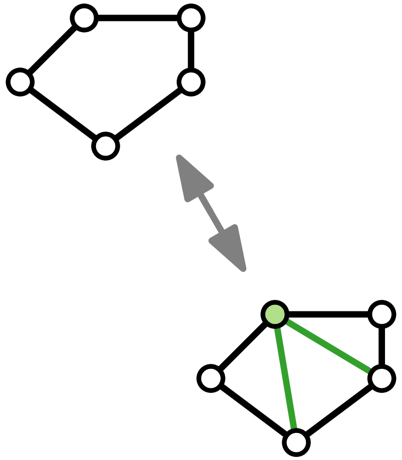

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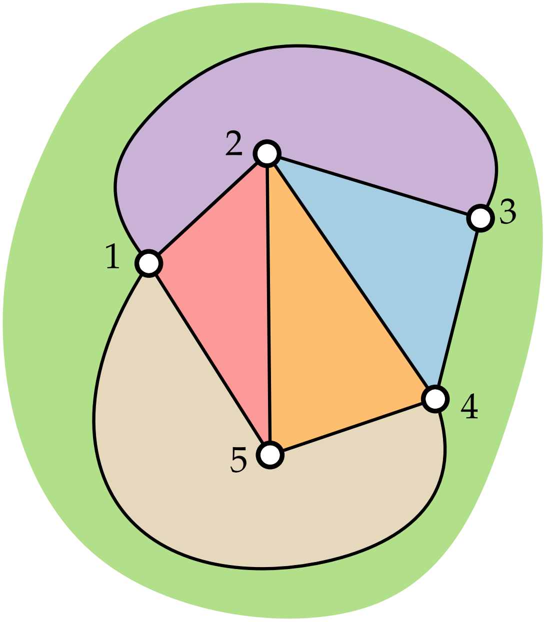

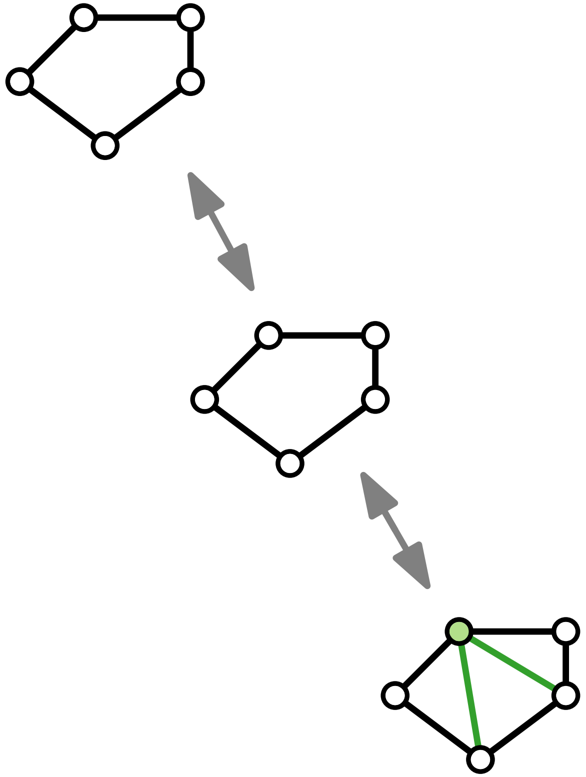

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Lemma.
Every plane graph is subgraph of a plane triangulation.

Triangulations

with planar embedding

A **plane (inner) triangulation** is a plane graph where every (inner) face is a triangle.

A **maximal planar graph** is a planar graph where adding any edge would destroy planarity.

Observation.
A maximal plane graph is a plane triangulation.

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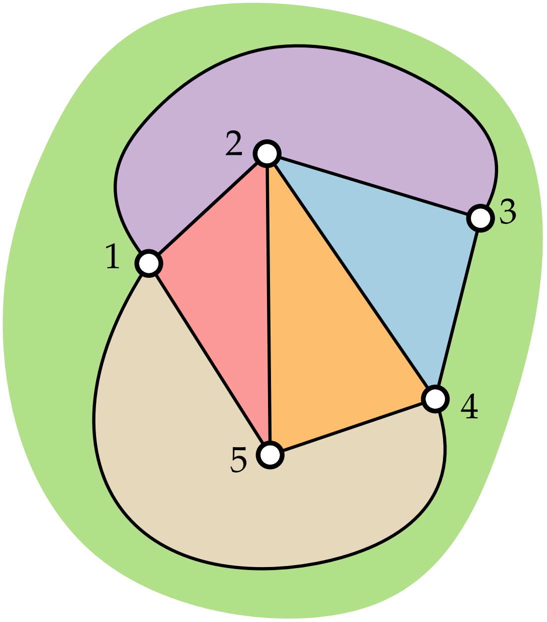

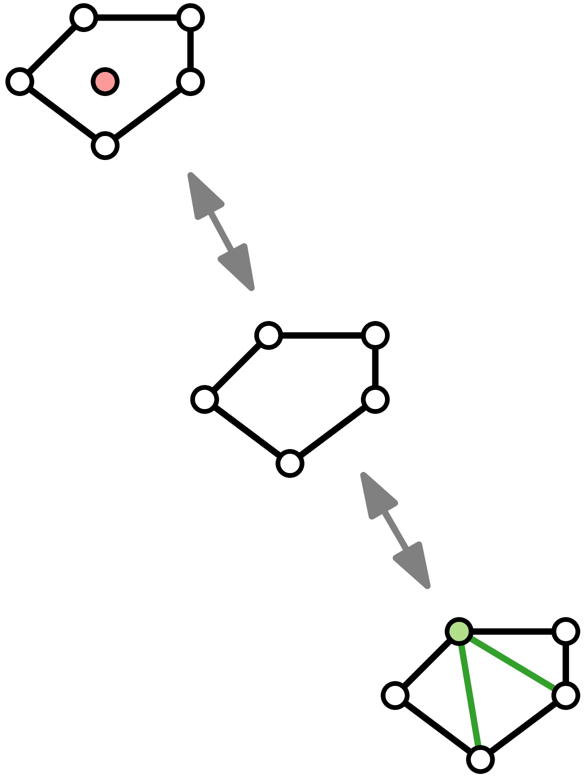

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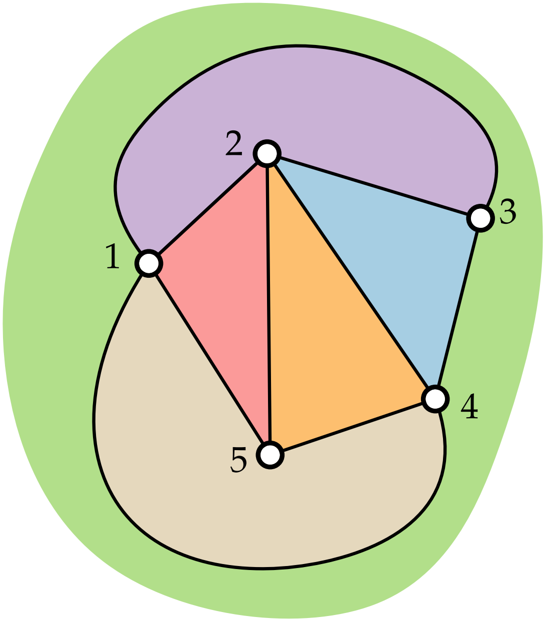

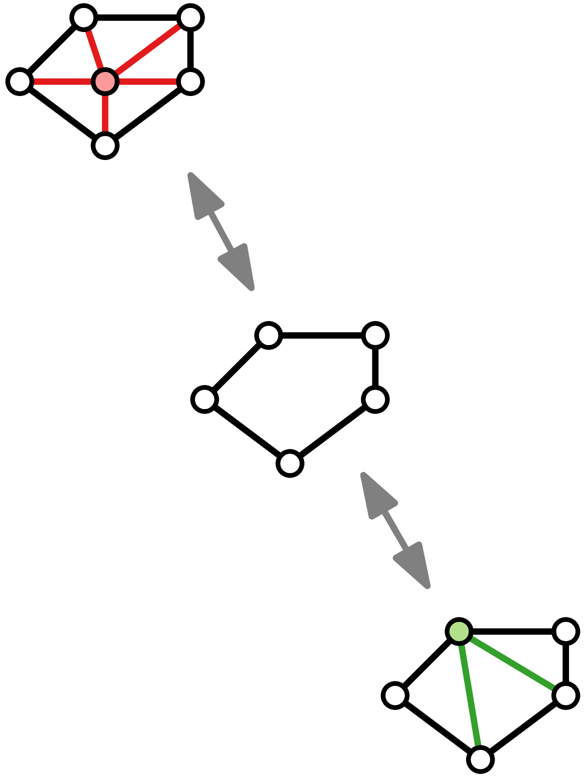

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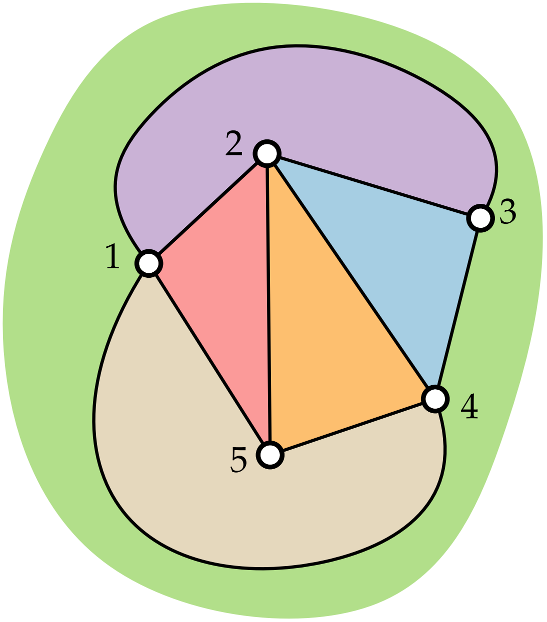

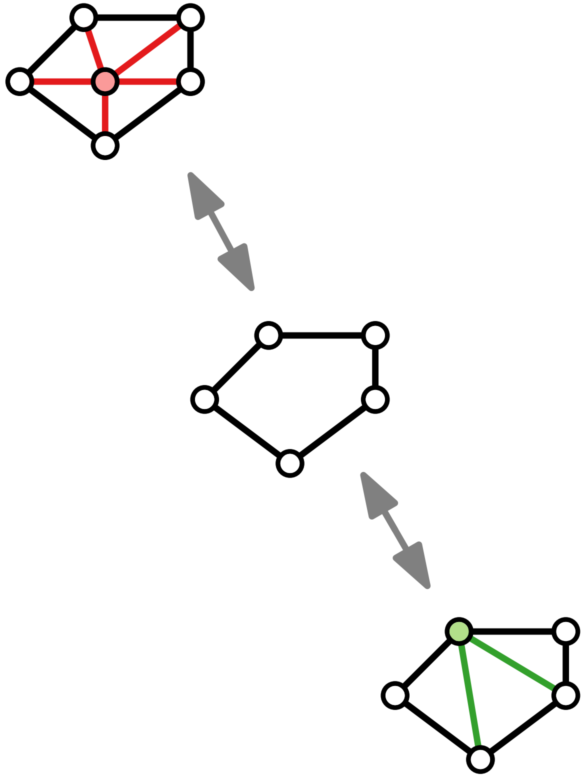

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Tutte's algorithm creates a planar straight-line drawing for every planar graph.

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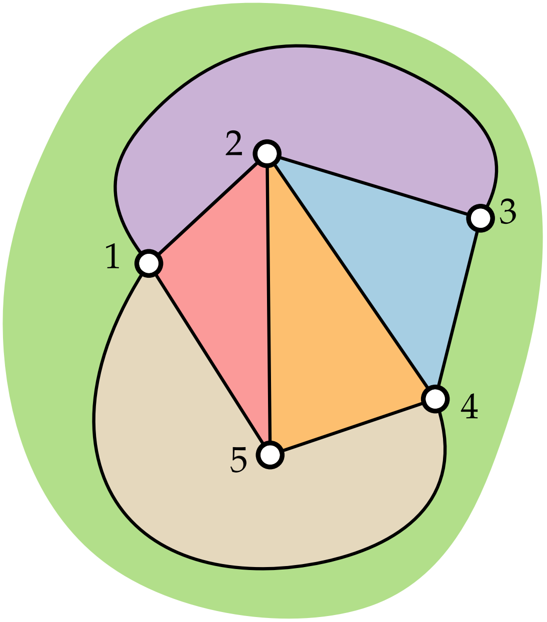

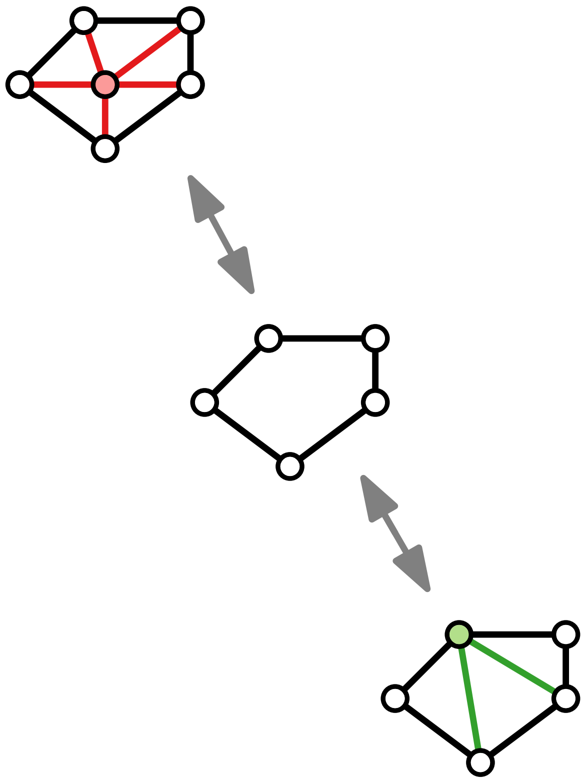

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Planar Straight-Line Drawings

Theorem. [De Fraysseix, Pach, Pollack '90]
Every n-vertex planar graph has a planar straight-line drawing of size

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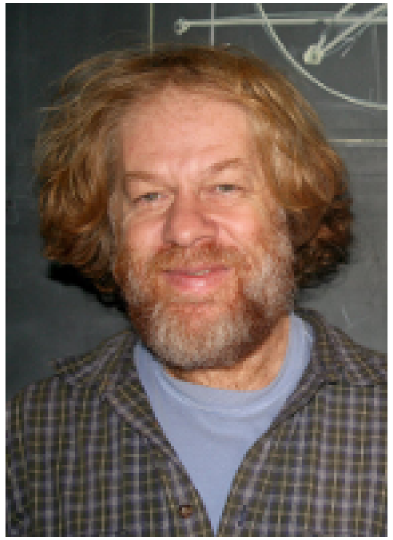

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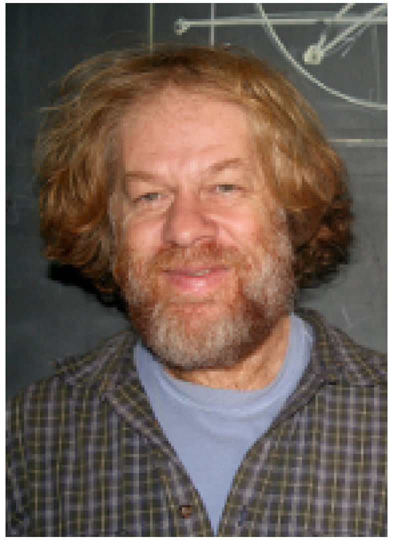

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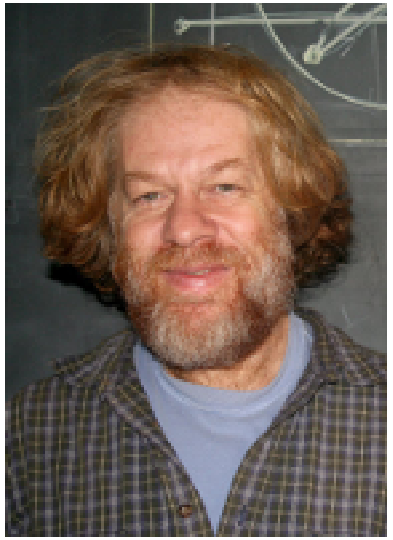

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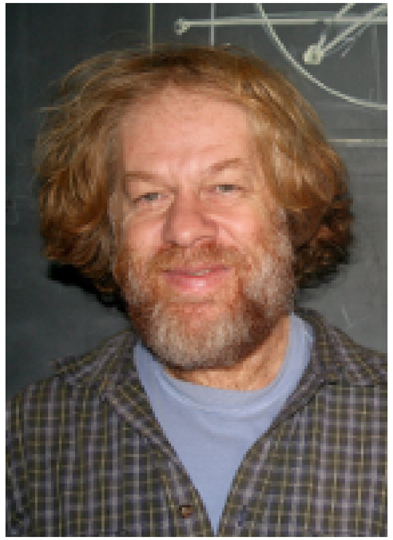

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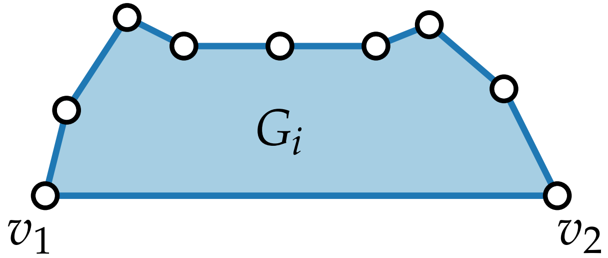

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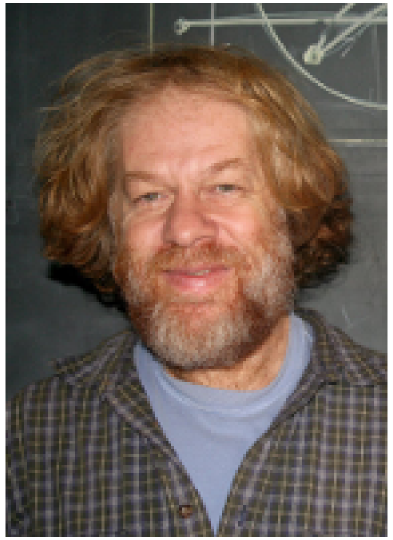

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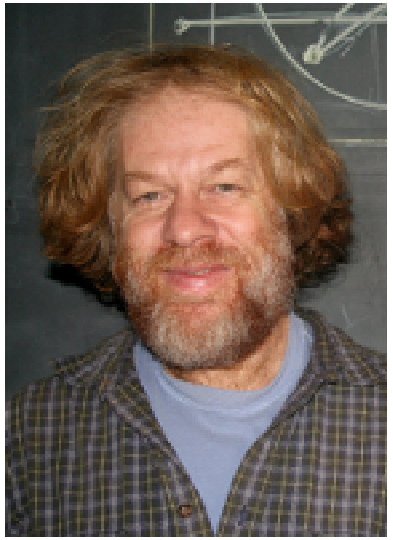

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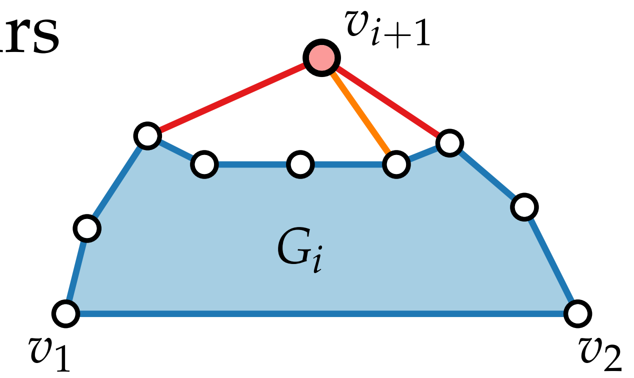

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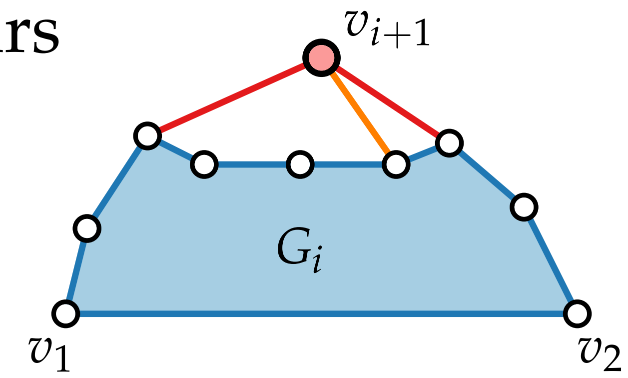

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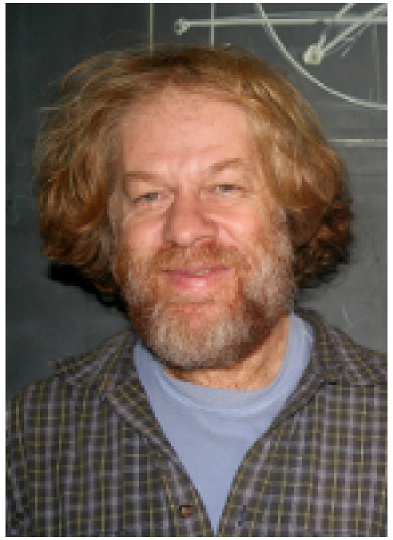

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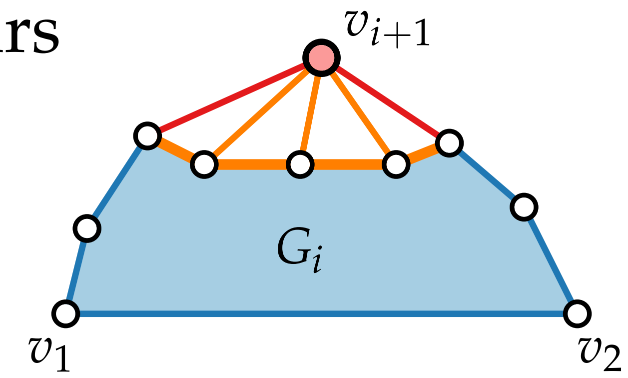

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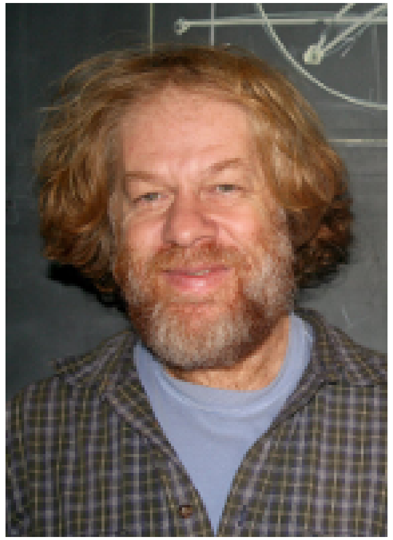

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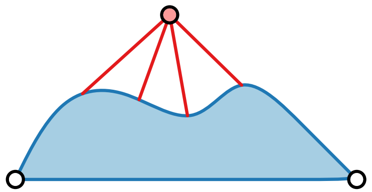
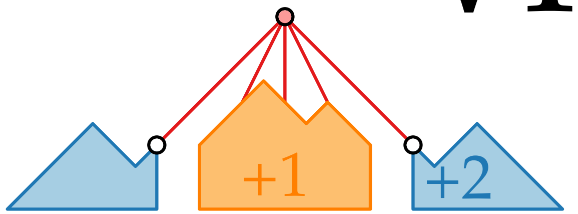
Universität Trier

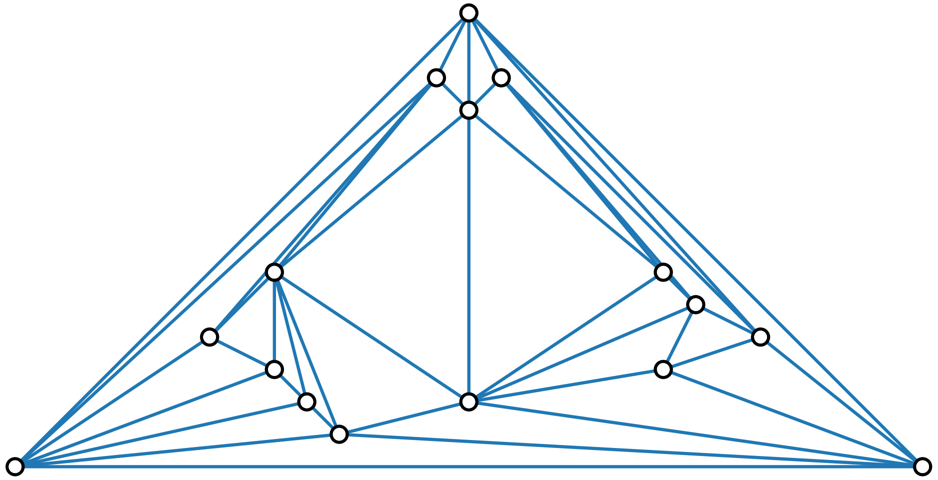


Visualization of Graphs



Lecture 4:
Straight-Line Drawings of Planar Graphs I: Canonical Ordering and Shift Method

Part II:
Canonical Order



Philipp Kindermann

Canonical Order – Definition

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Let $G = (V, E)$ be a triangulated plane graph on $n \geq 3$ vertices.

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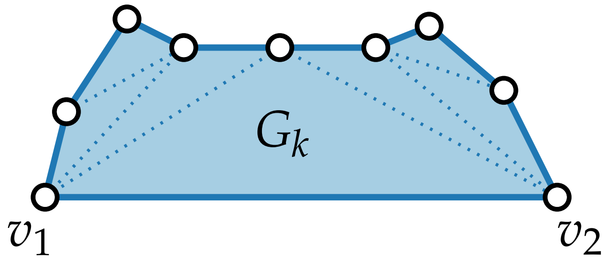

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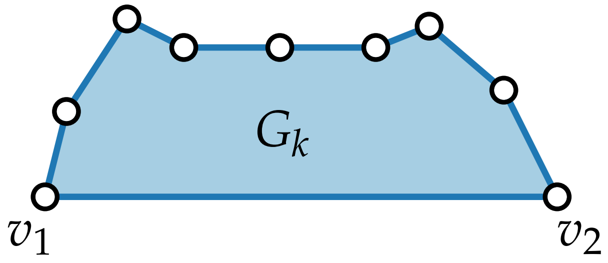

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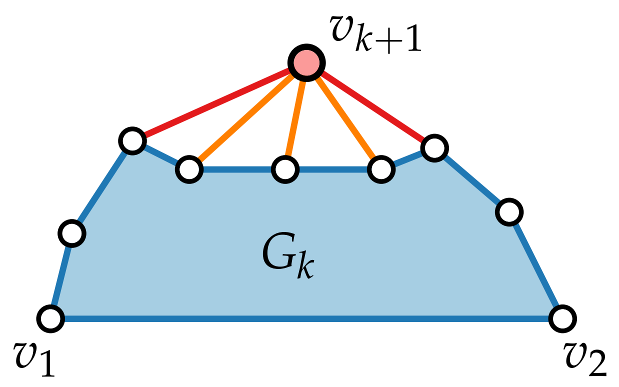

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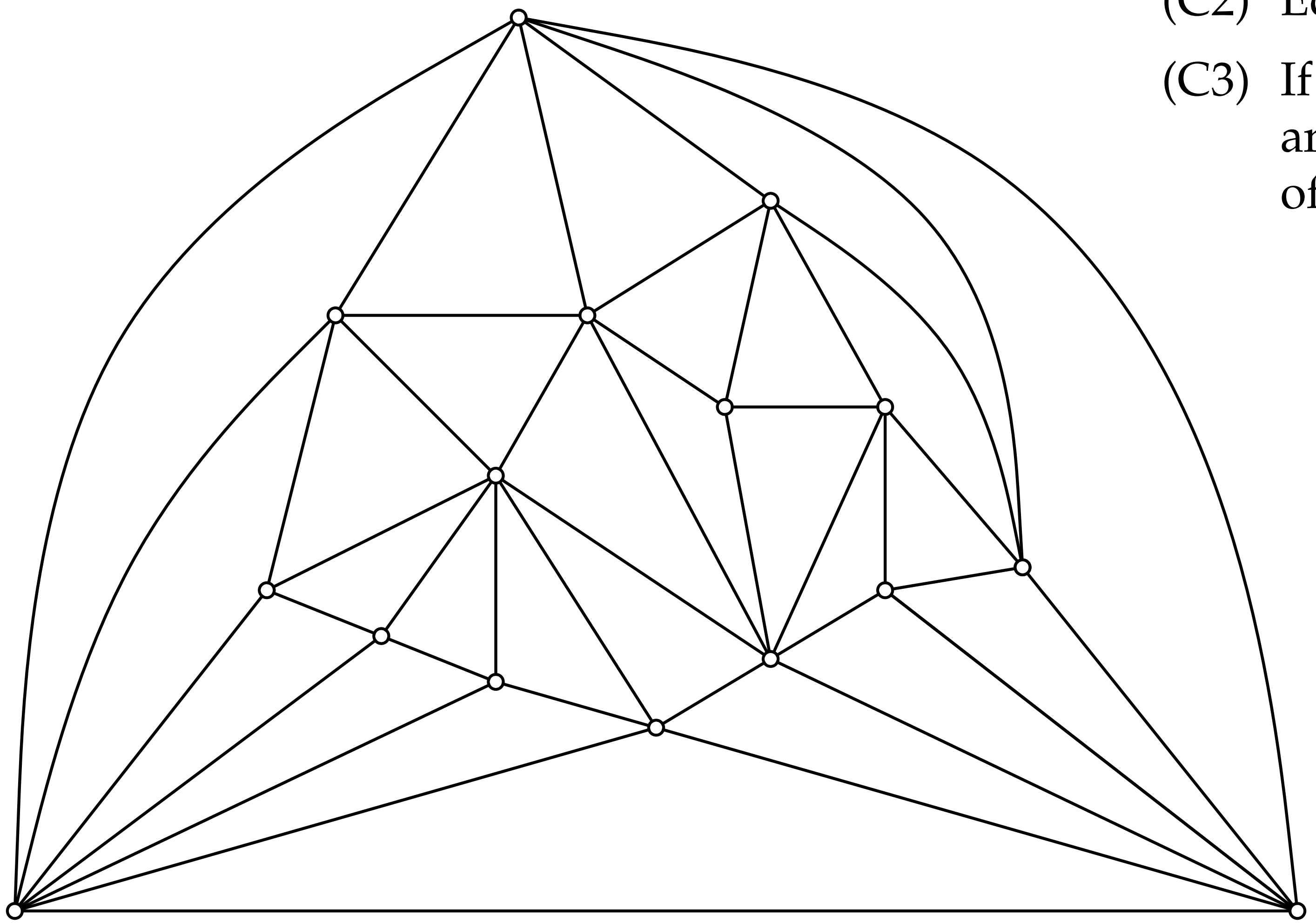

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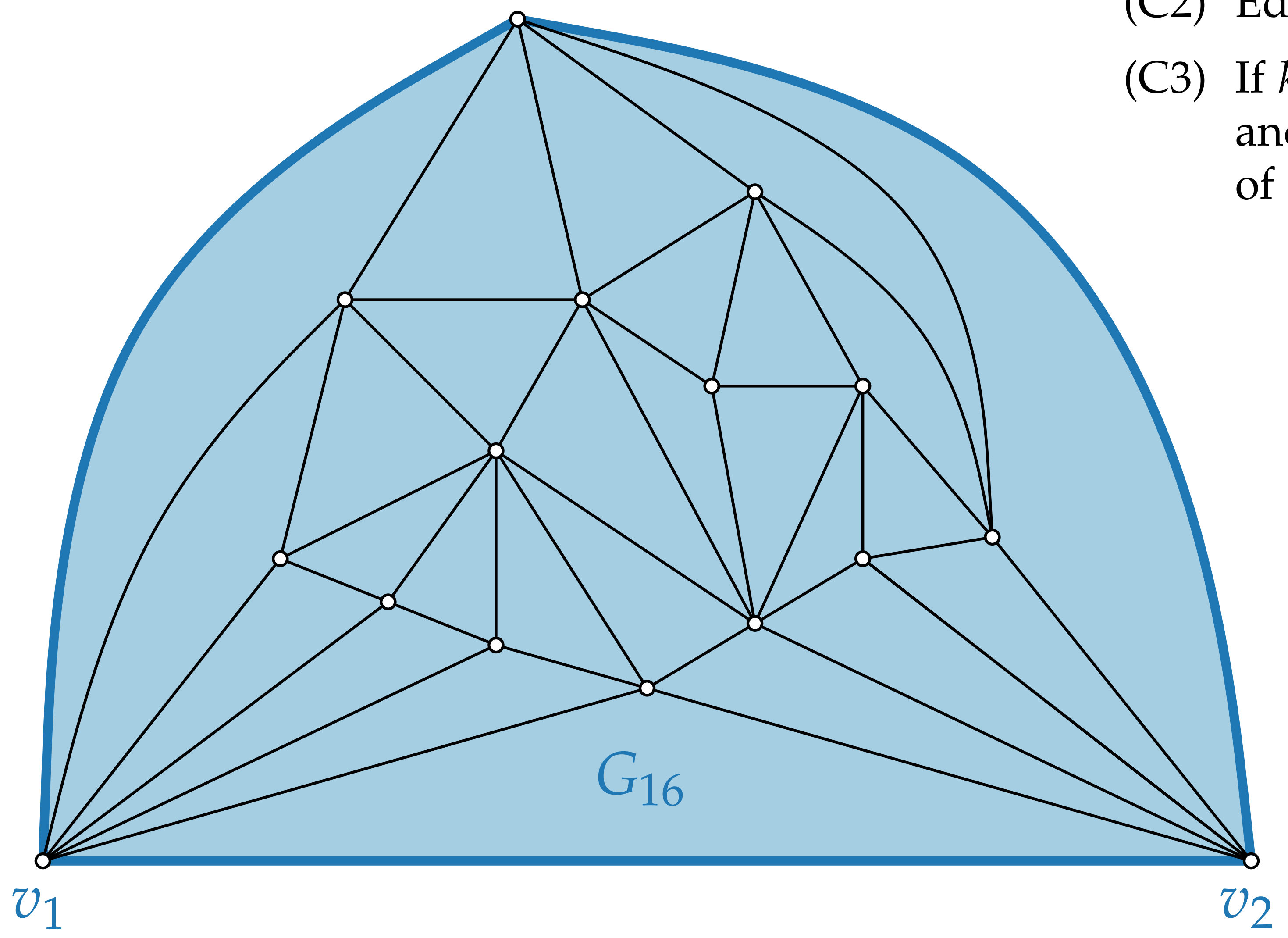

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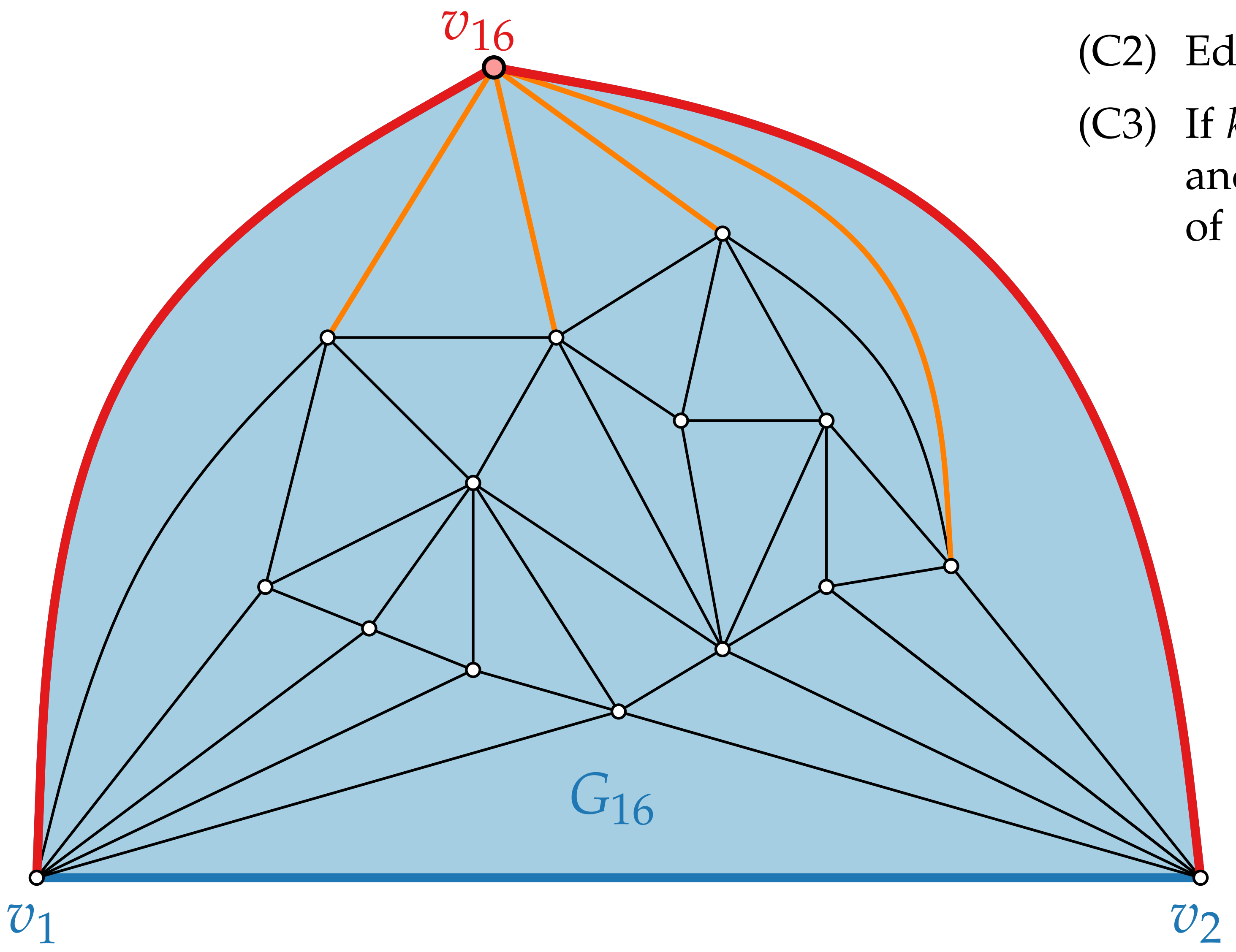

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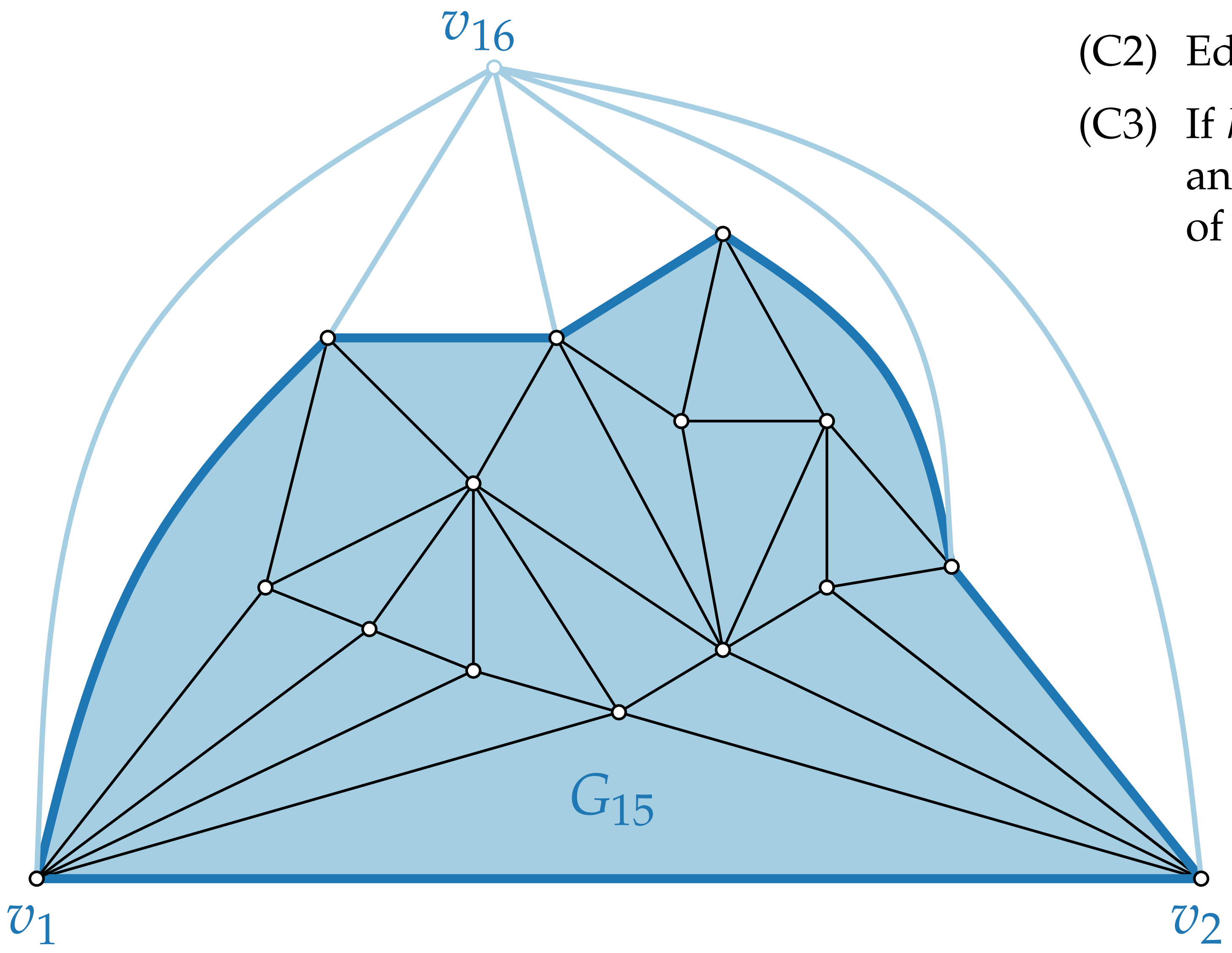

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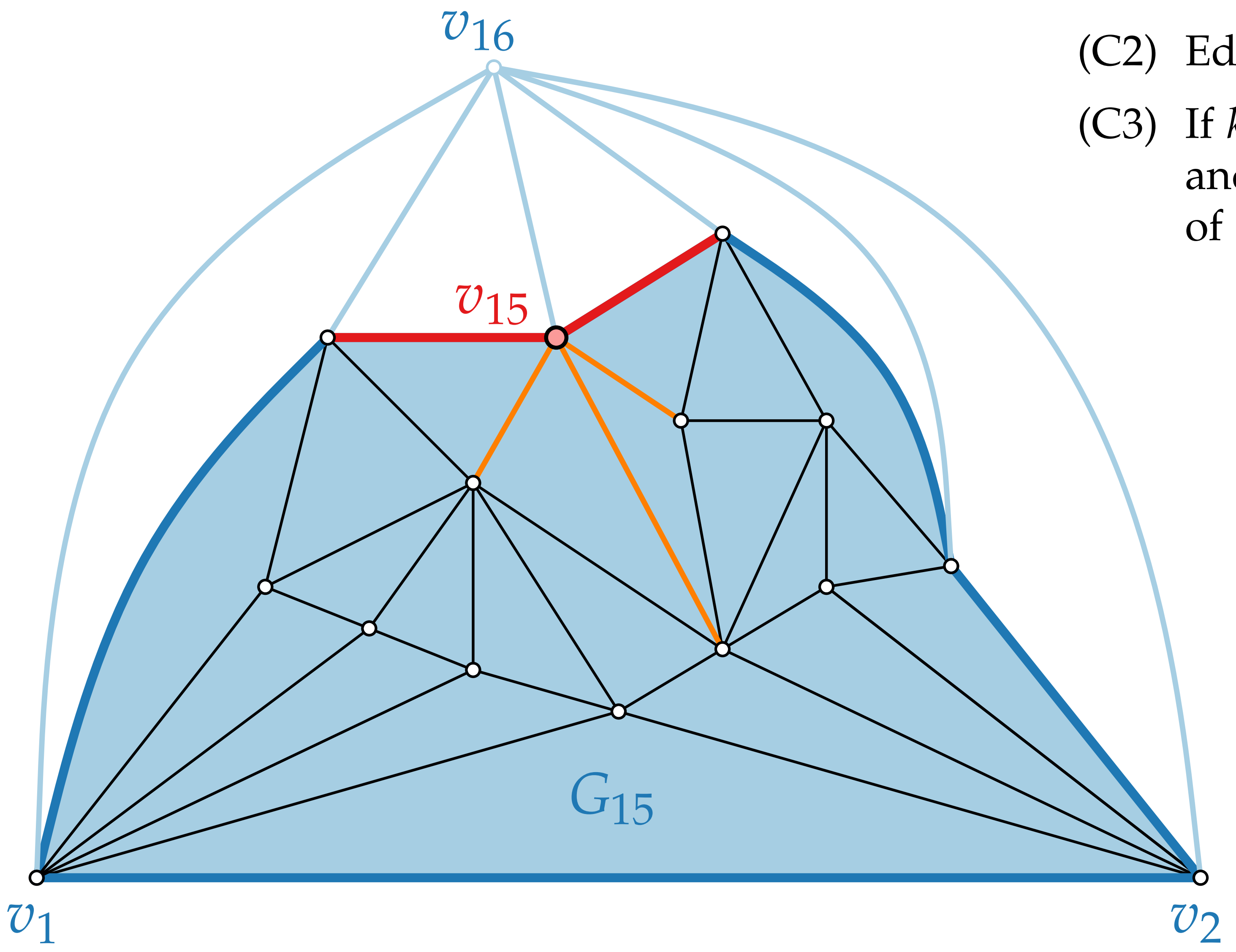

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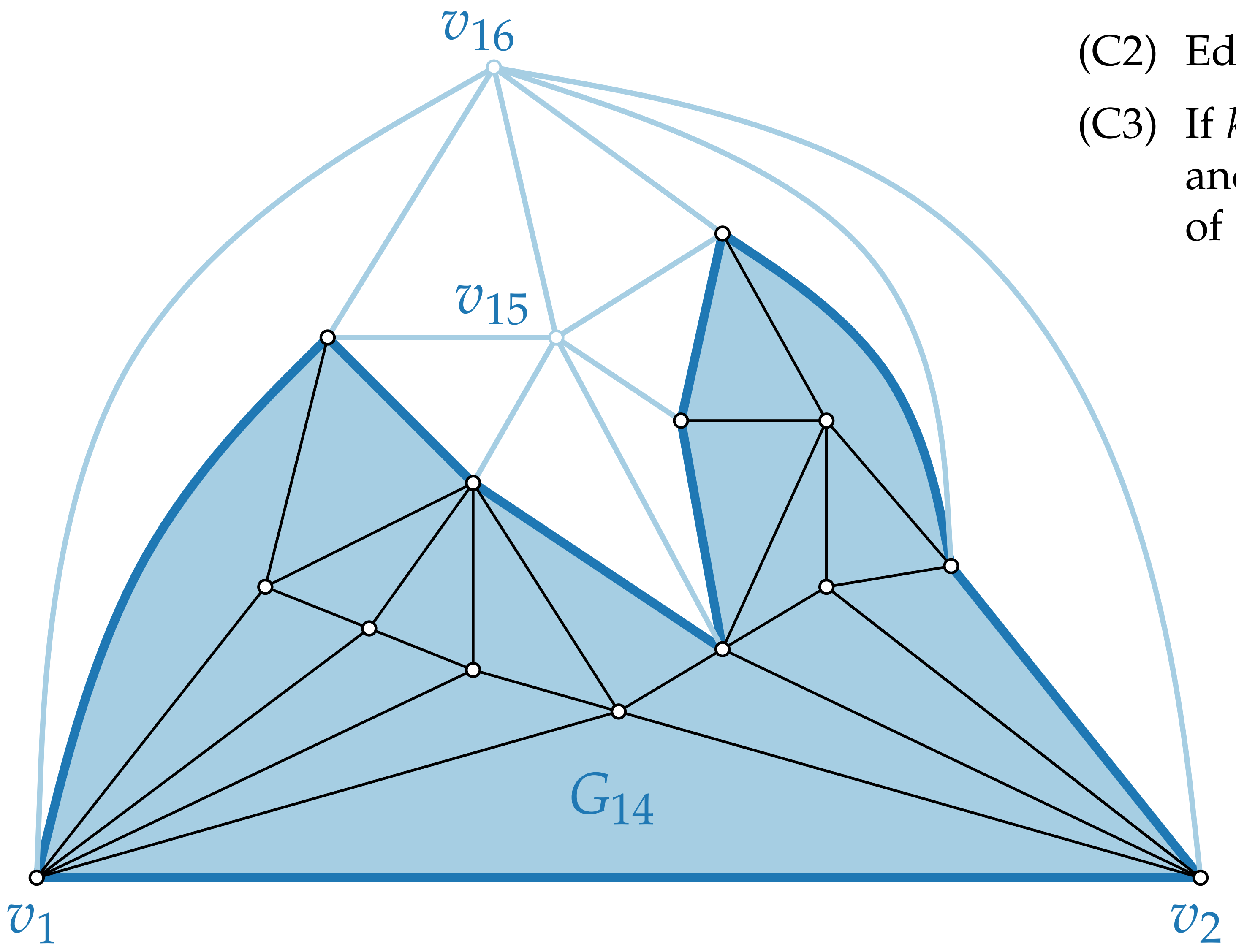

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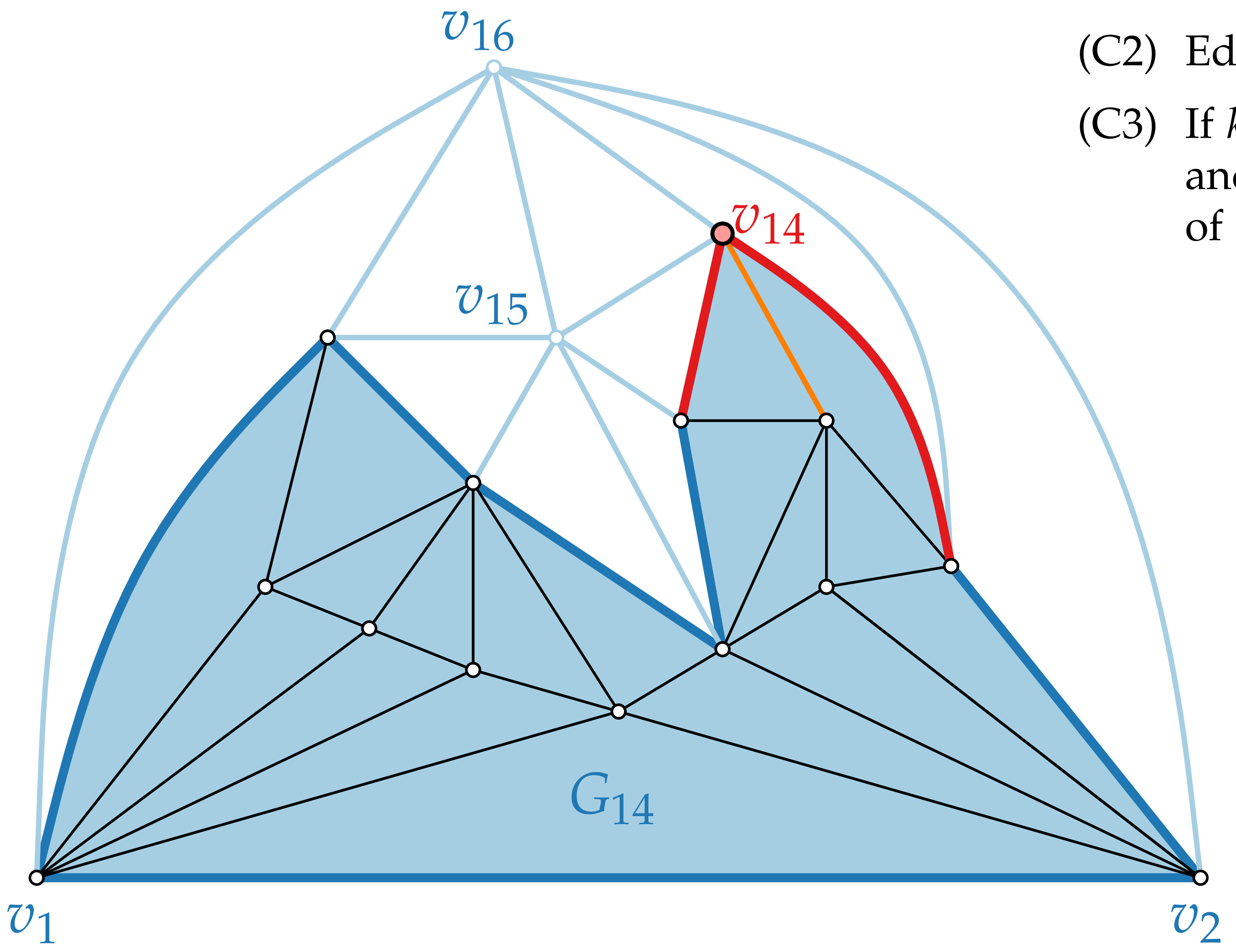

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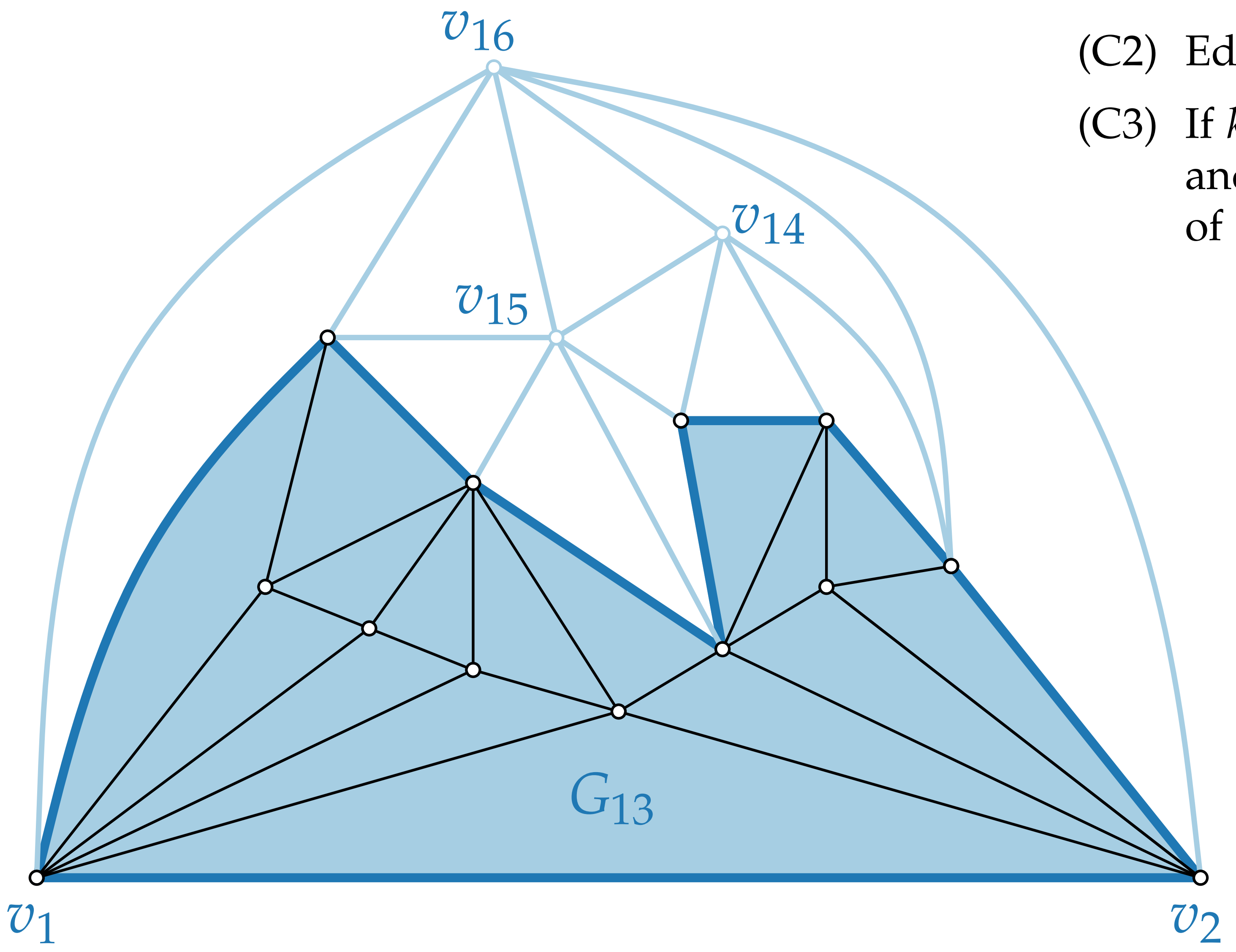

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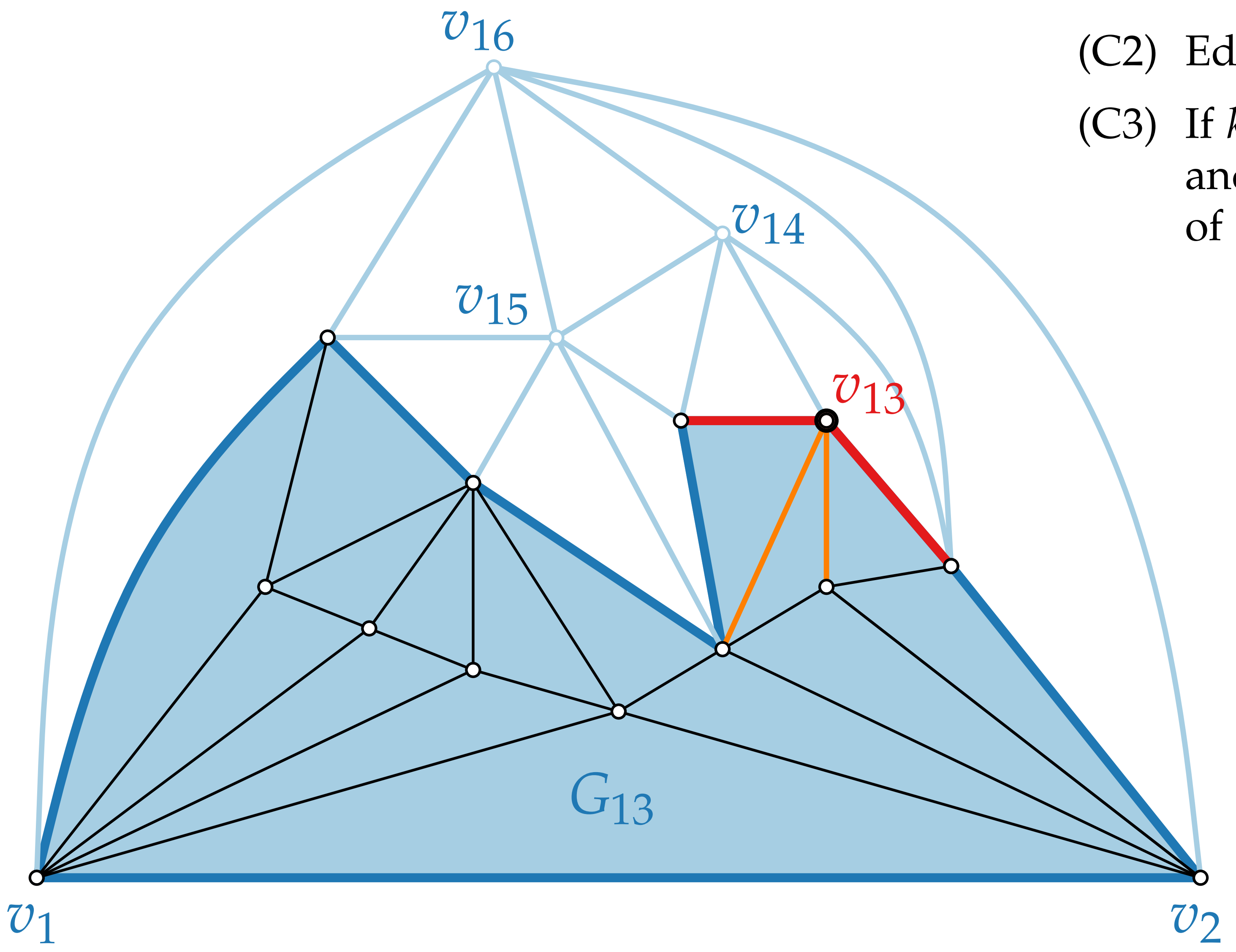

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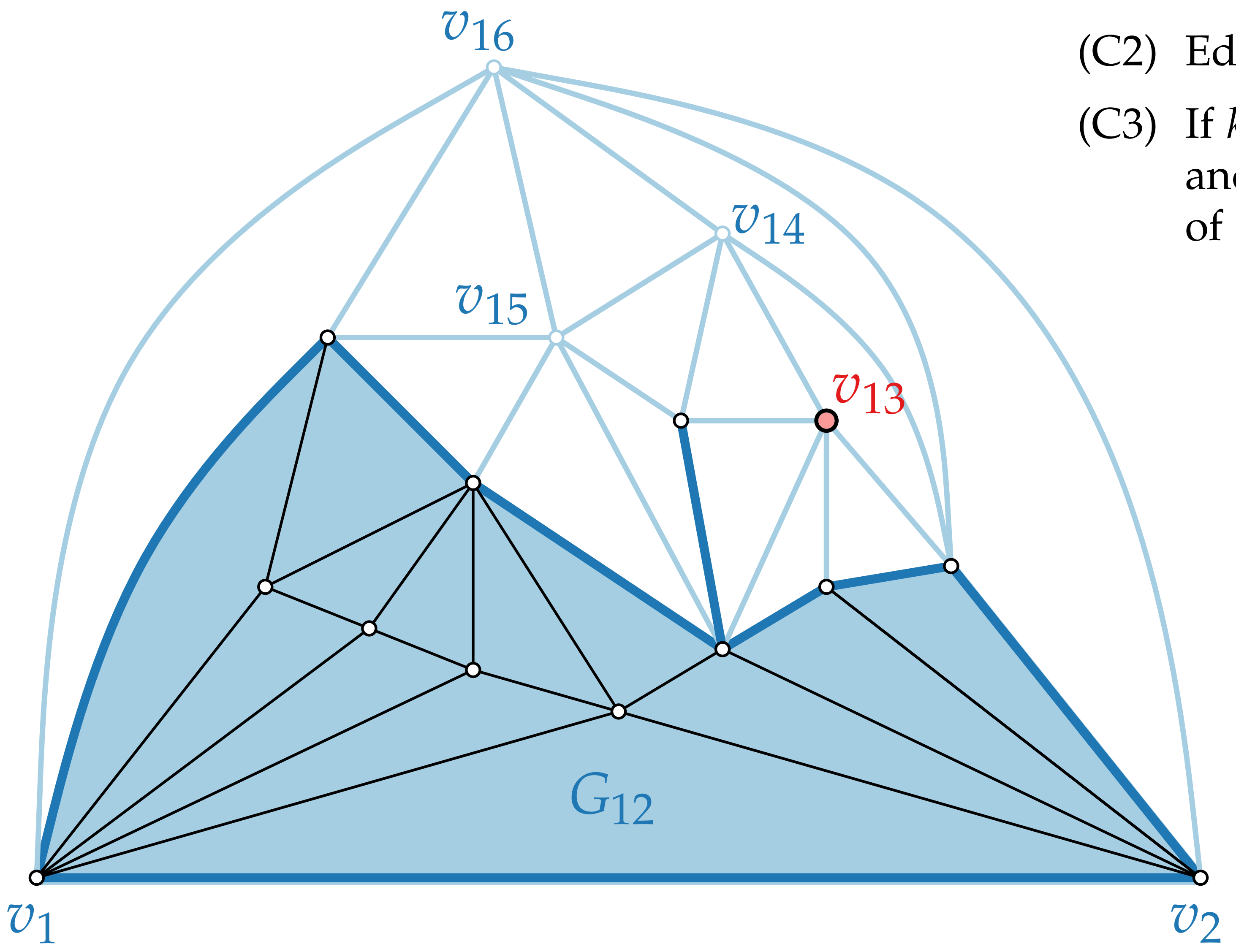

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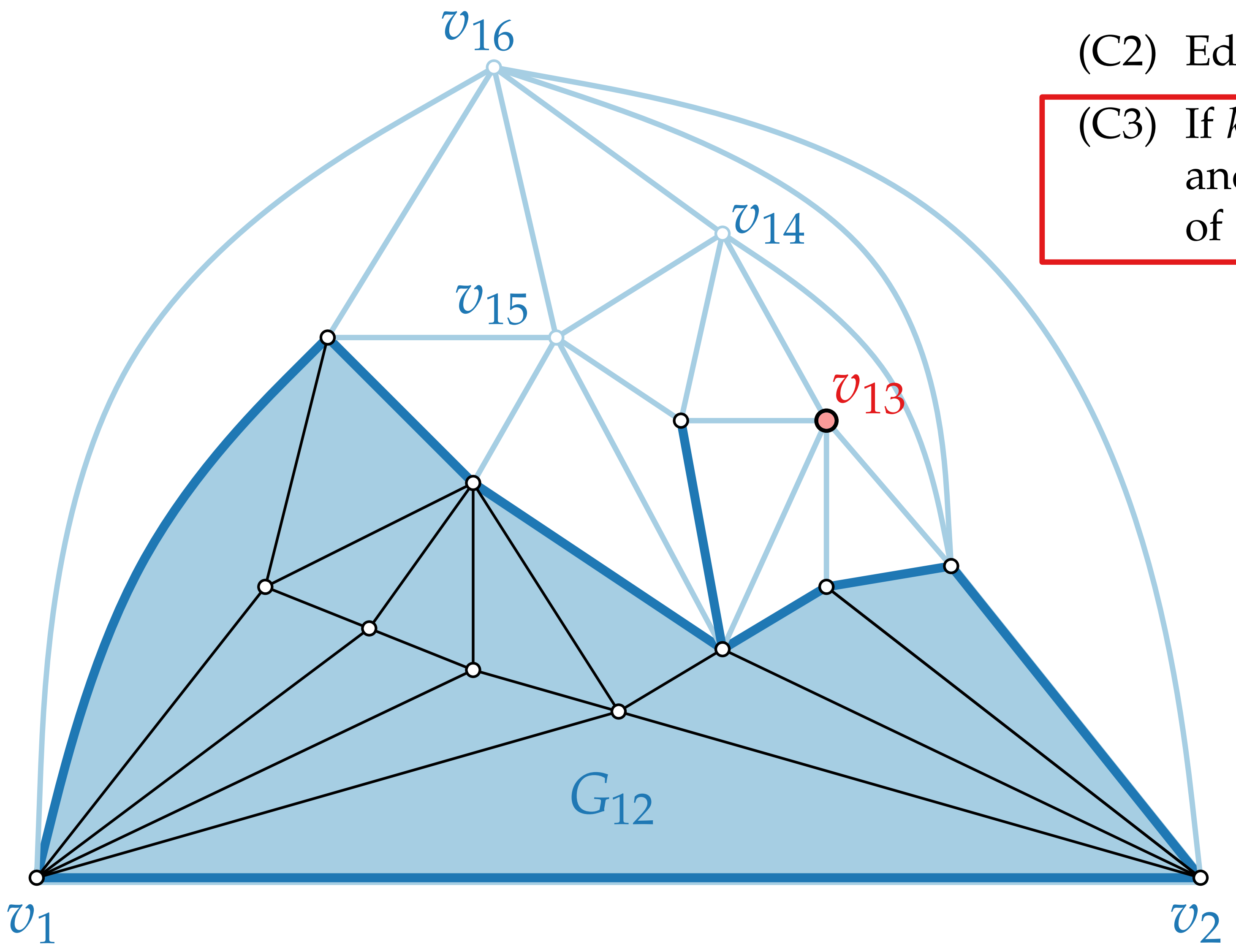

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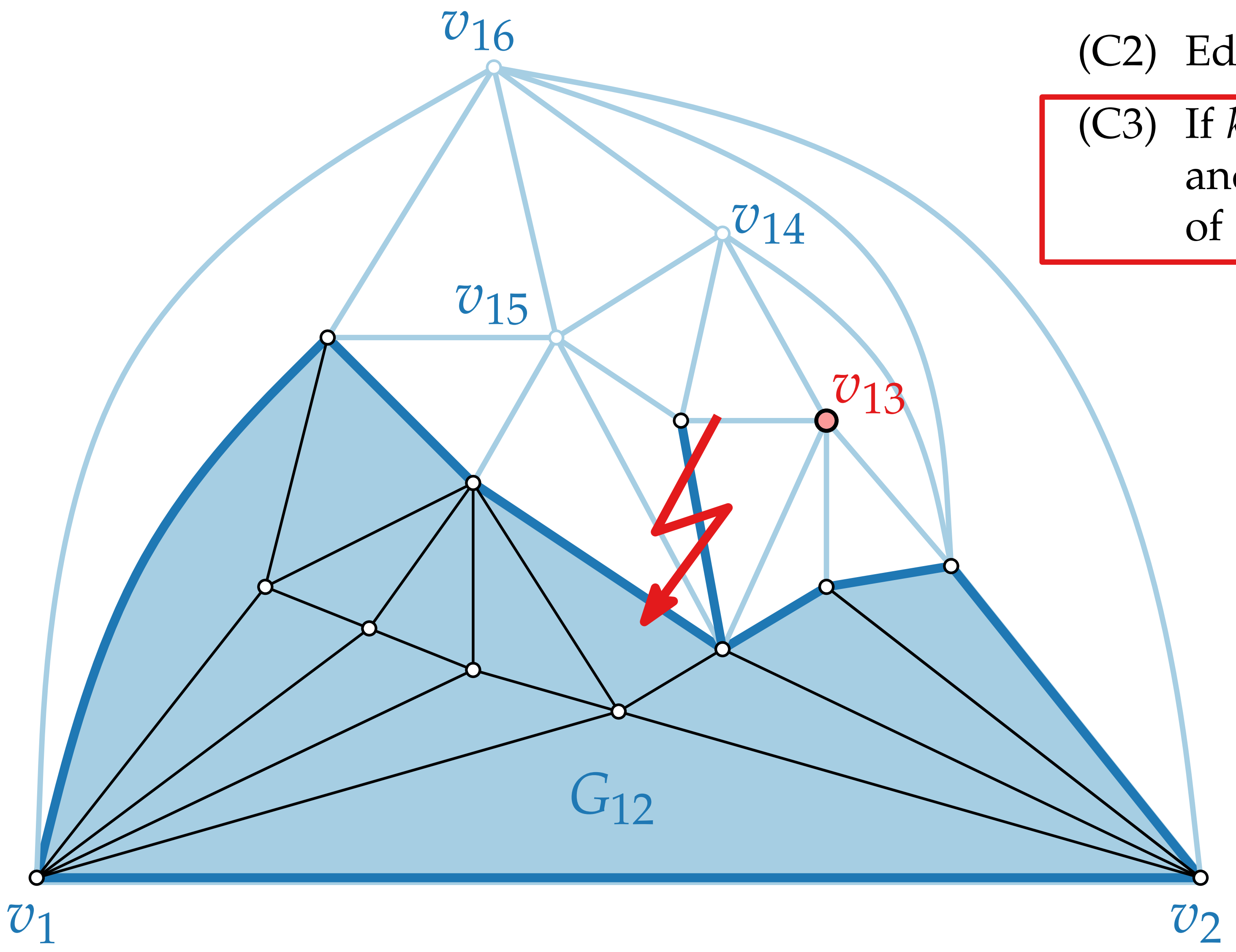

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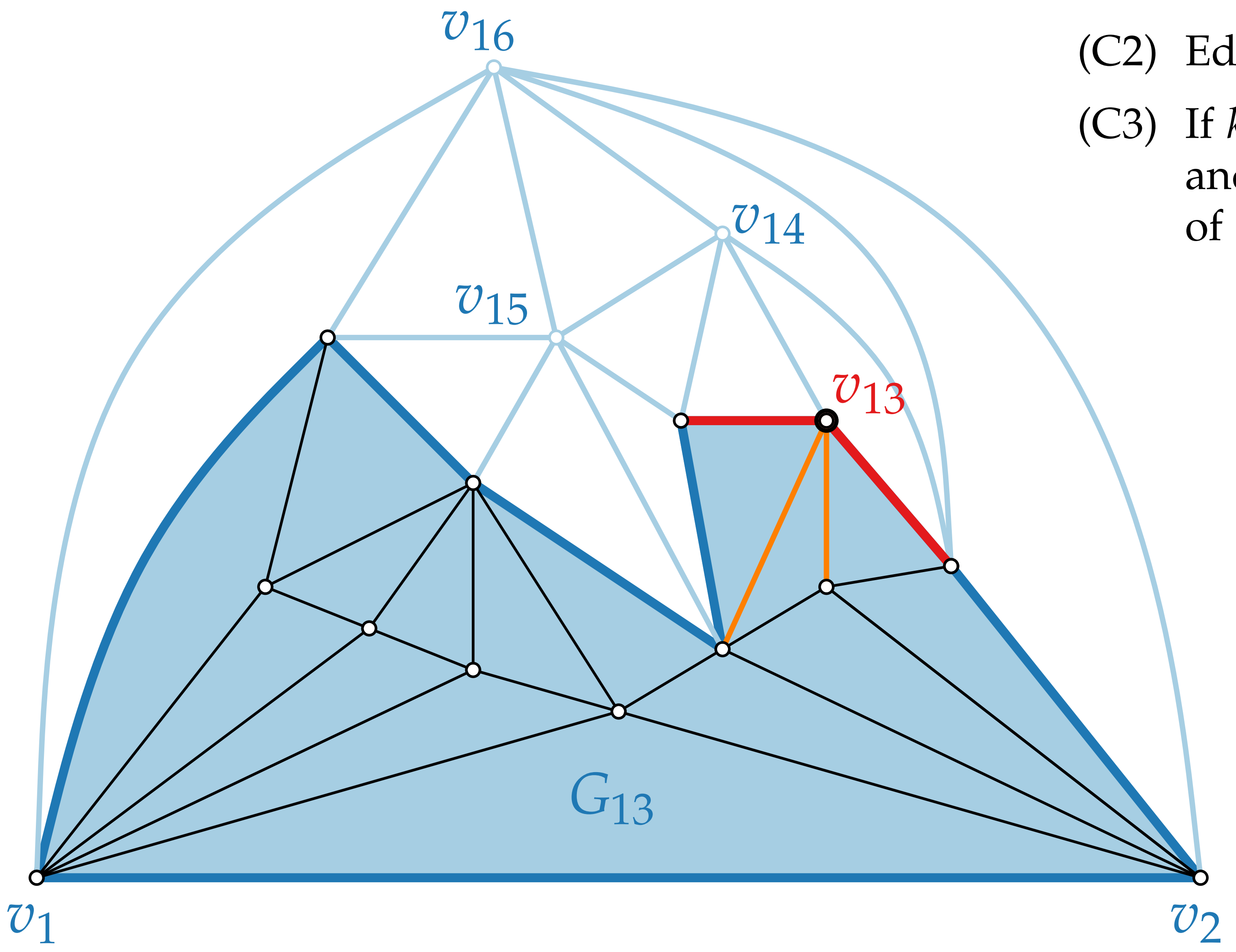

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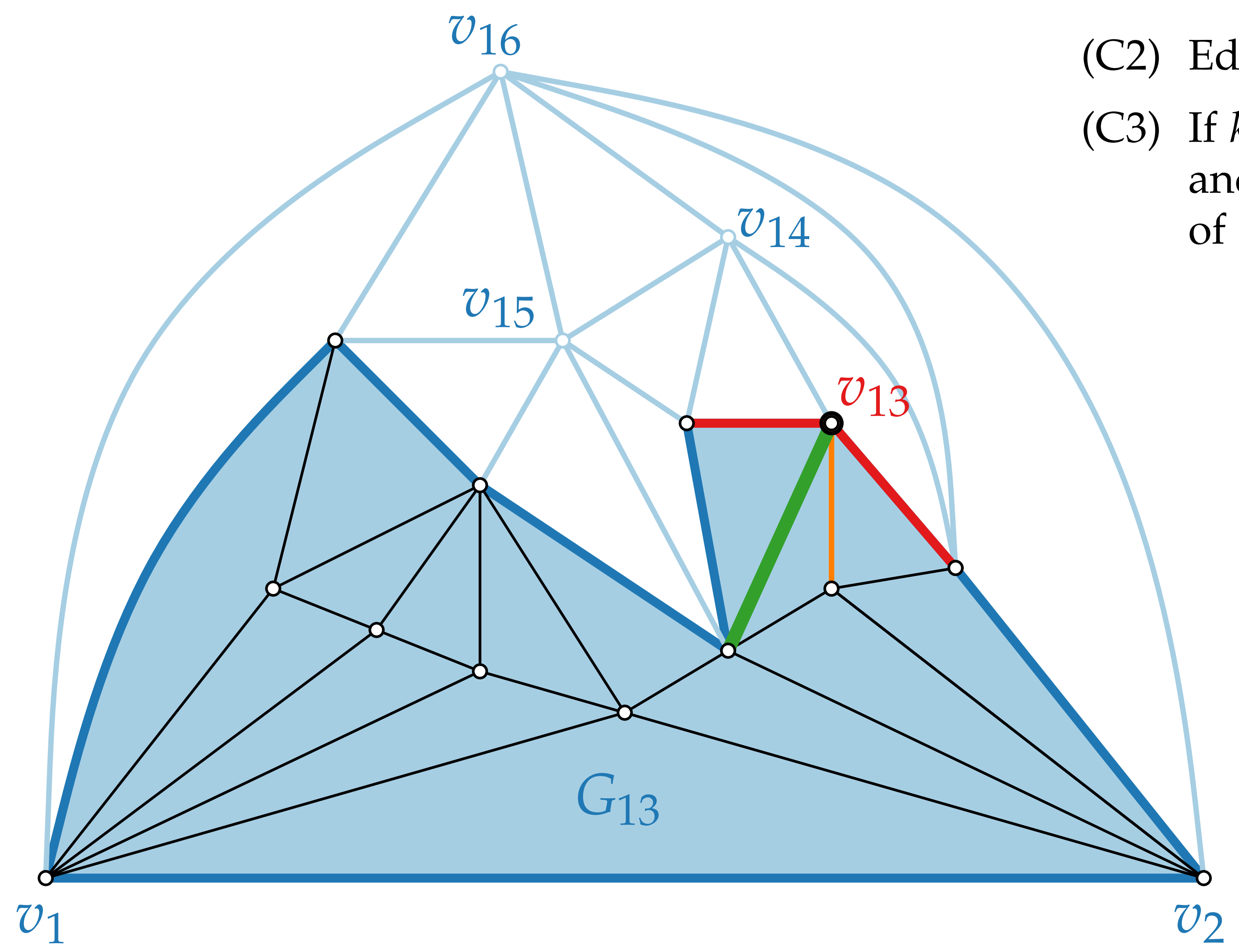

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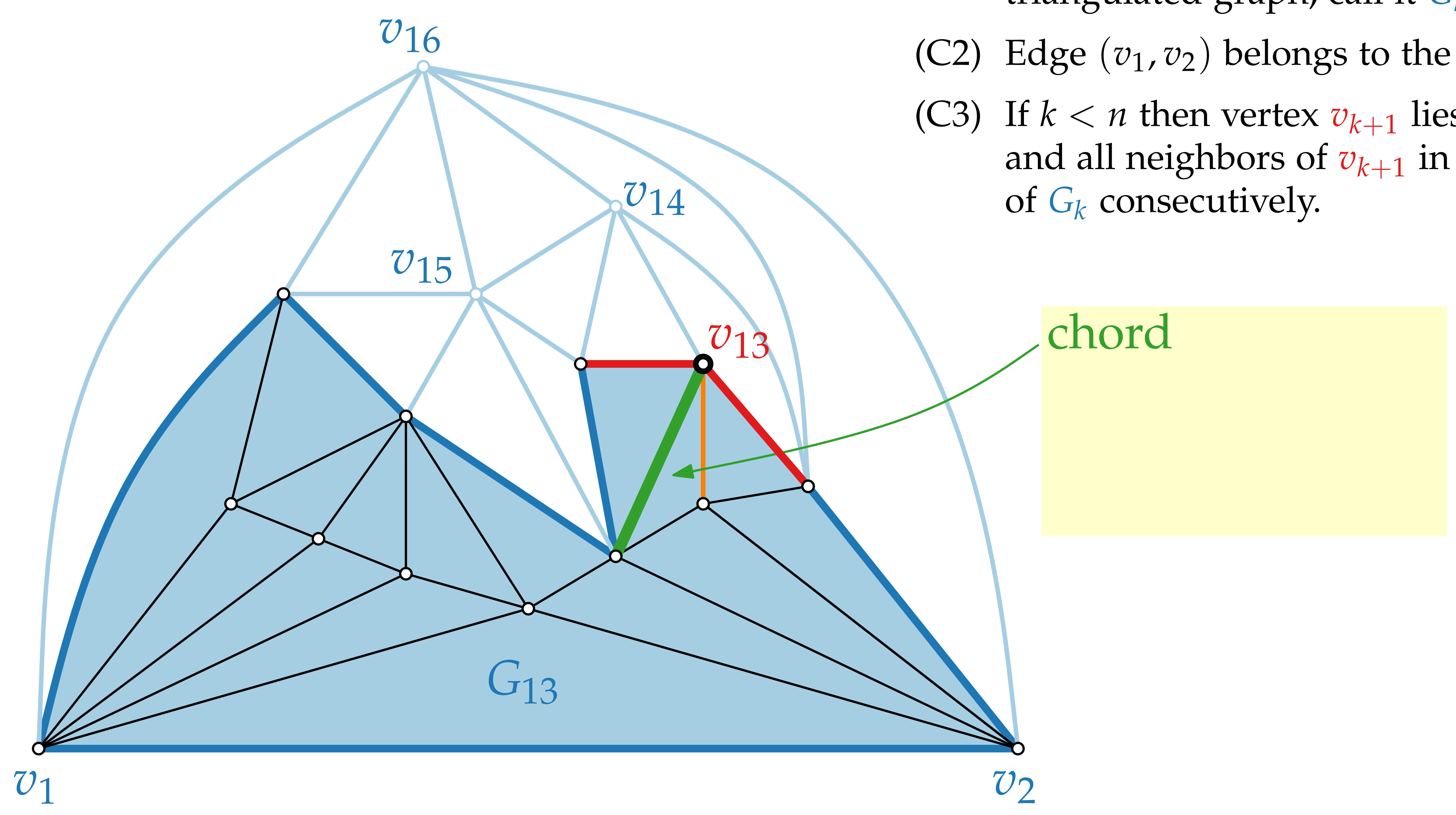

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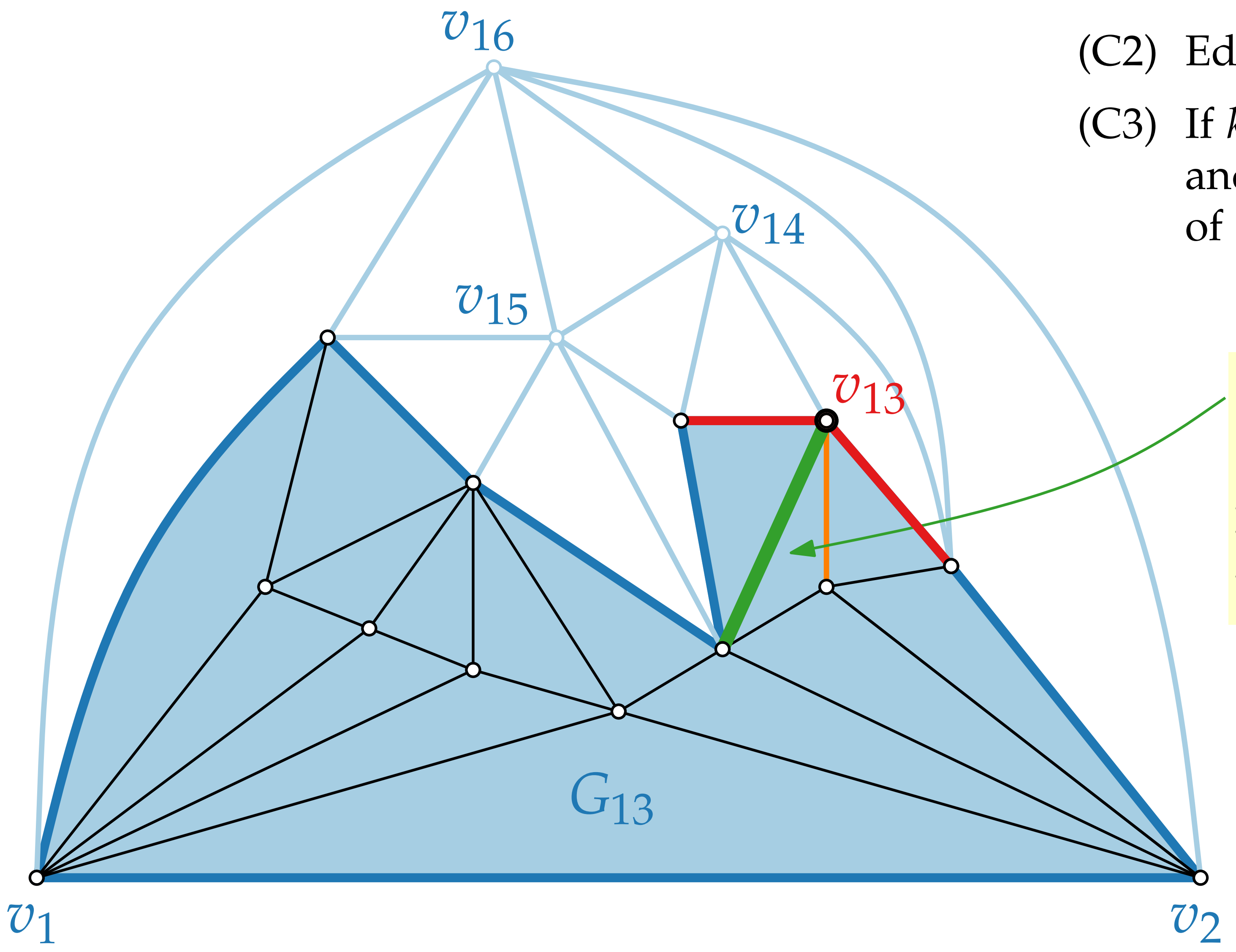

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chord
edge joining two nonadjacent vertices in a cycle

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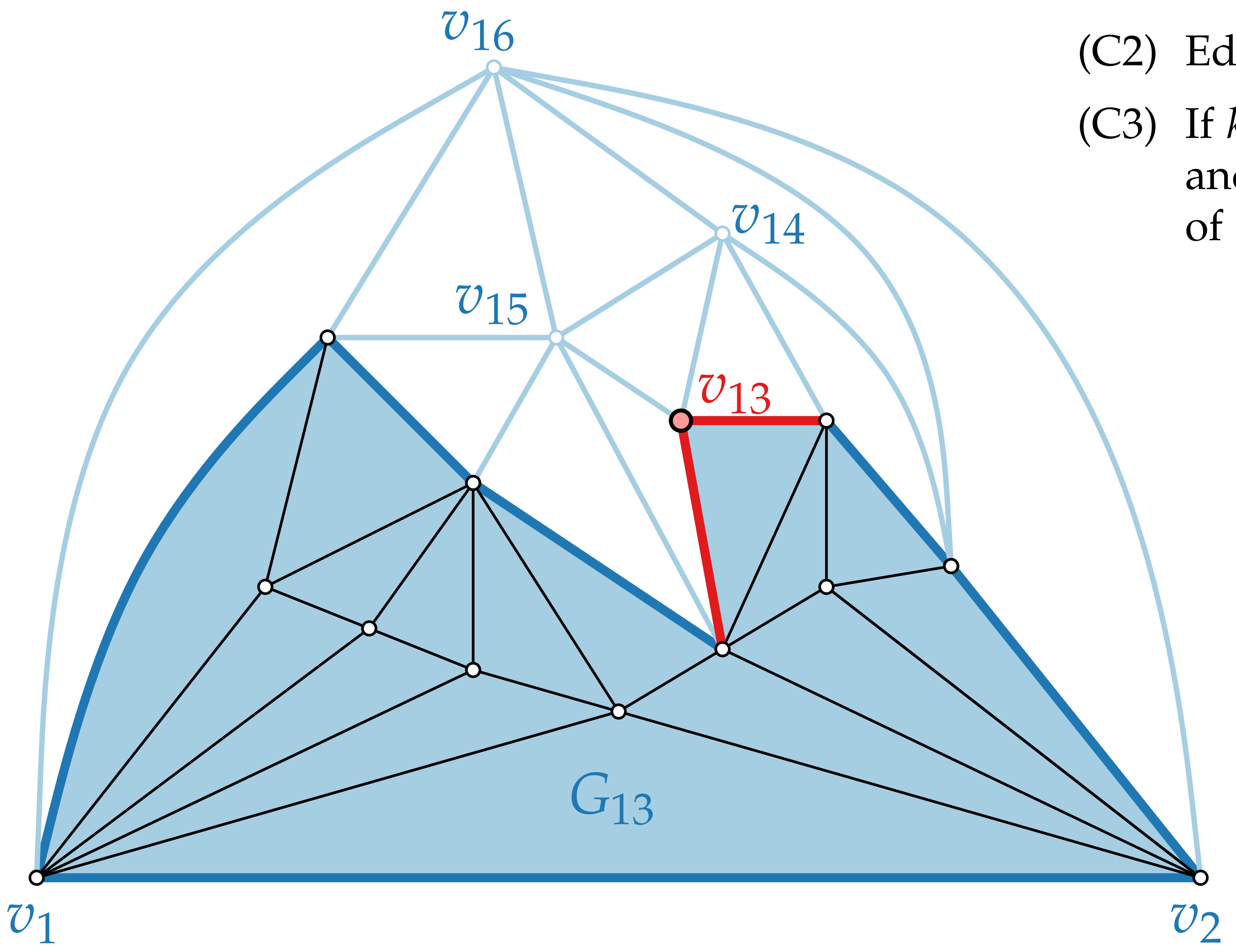

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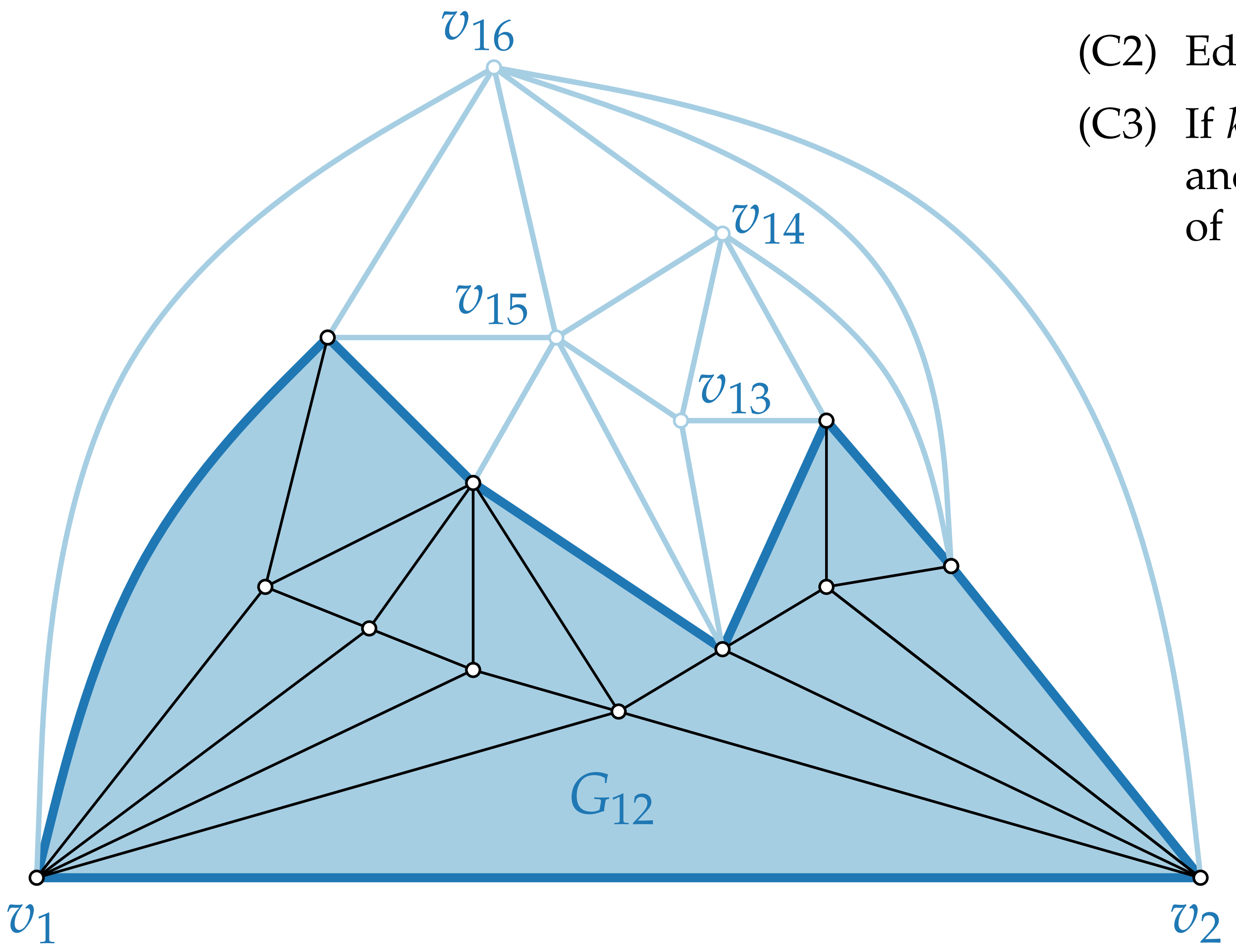

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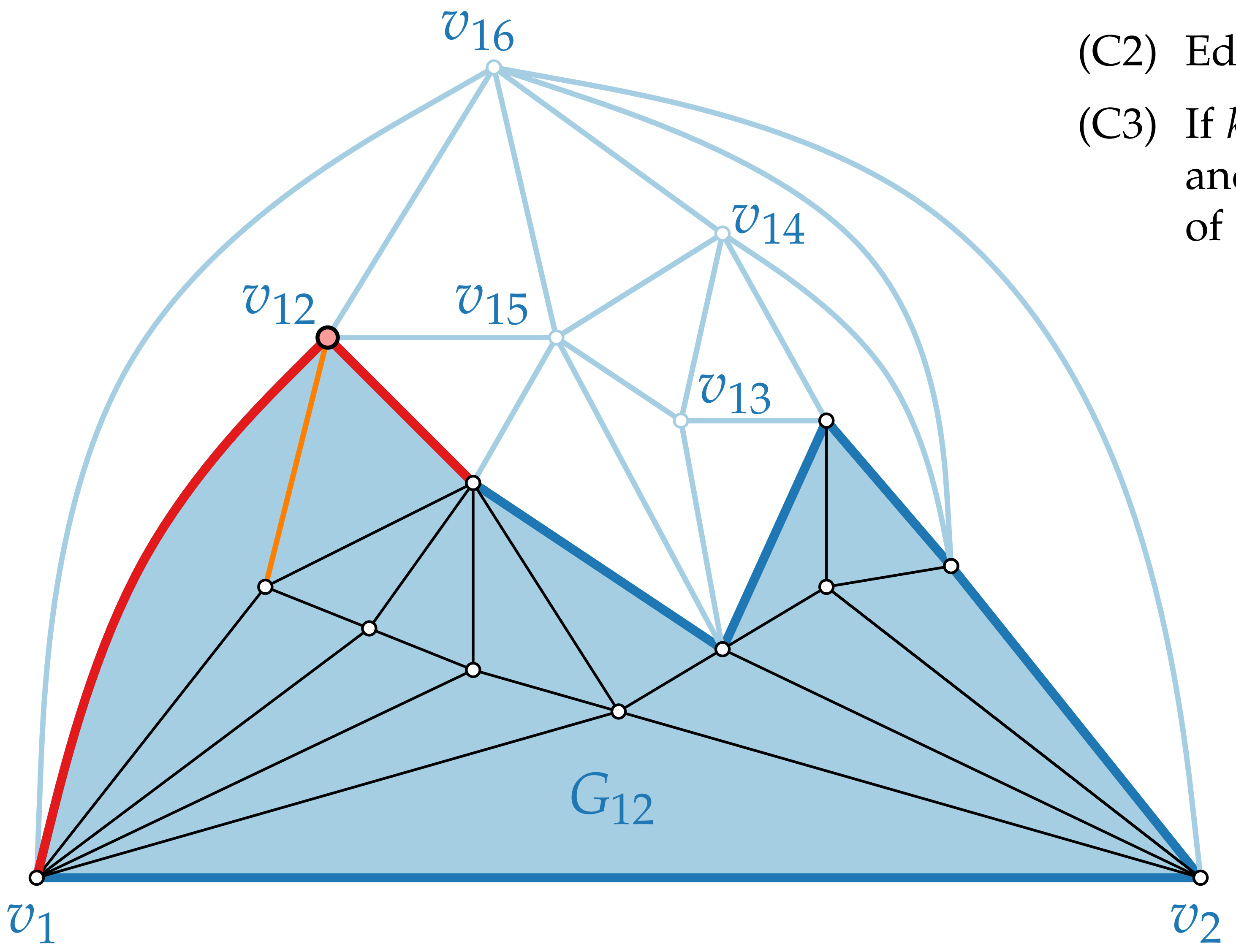

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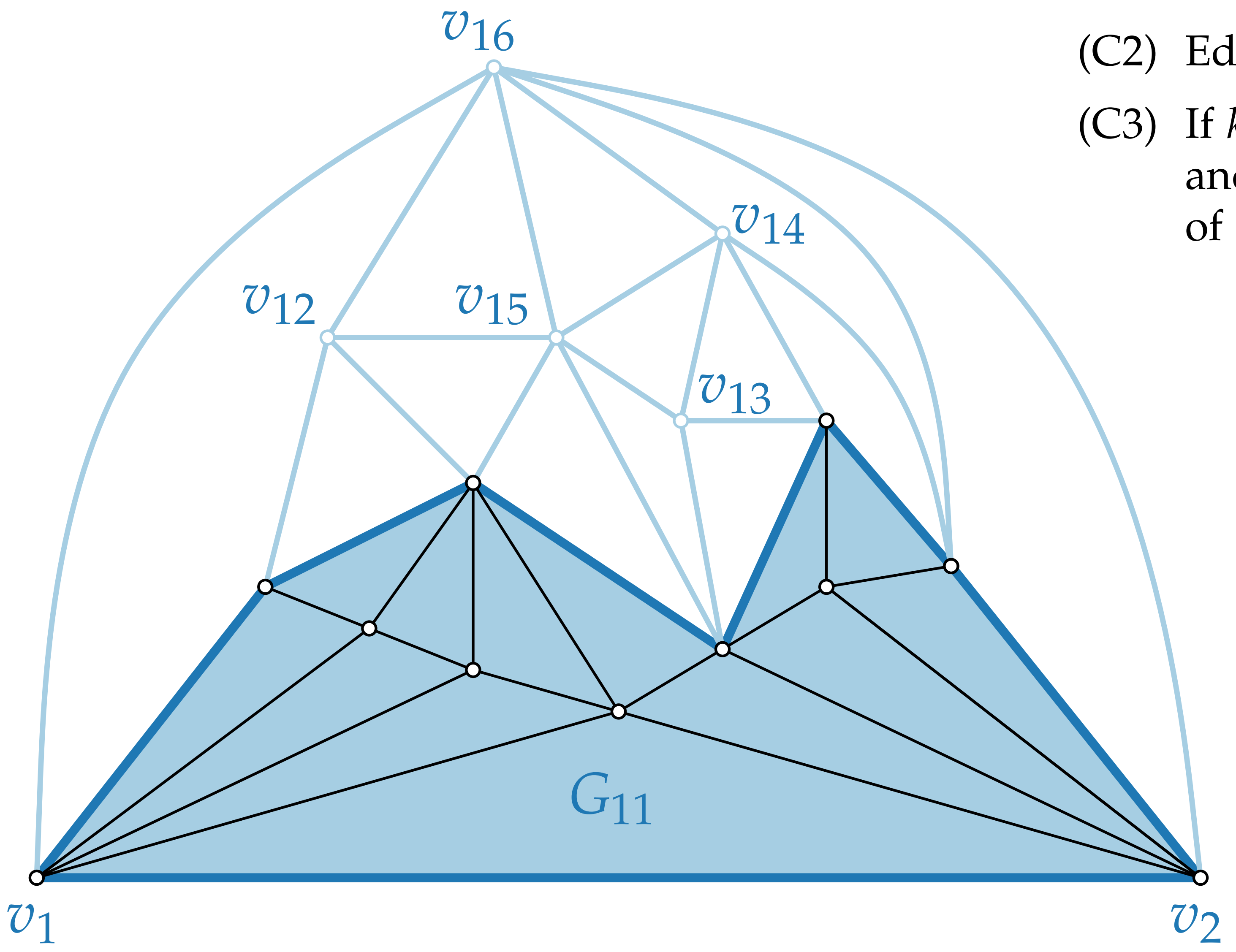

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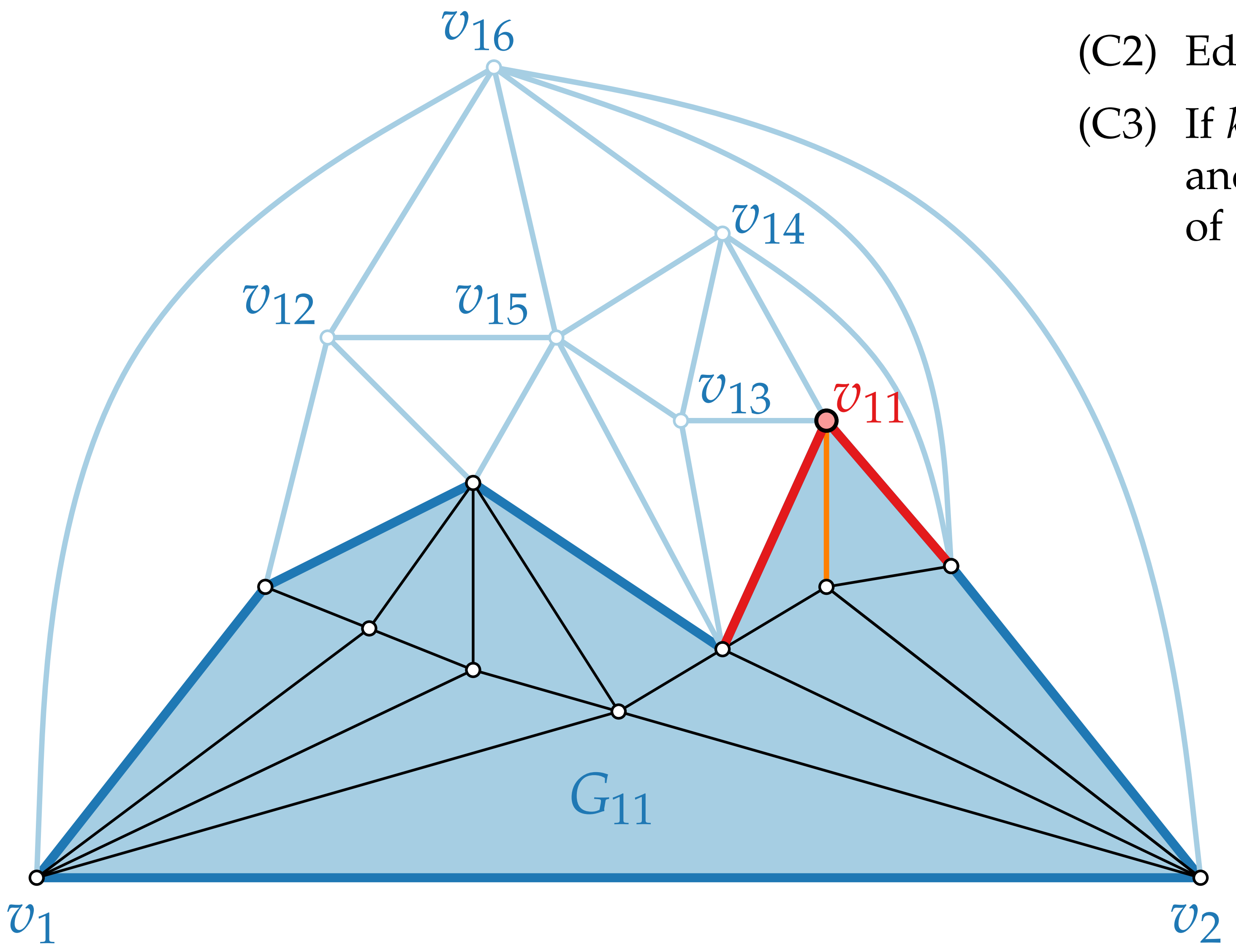

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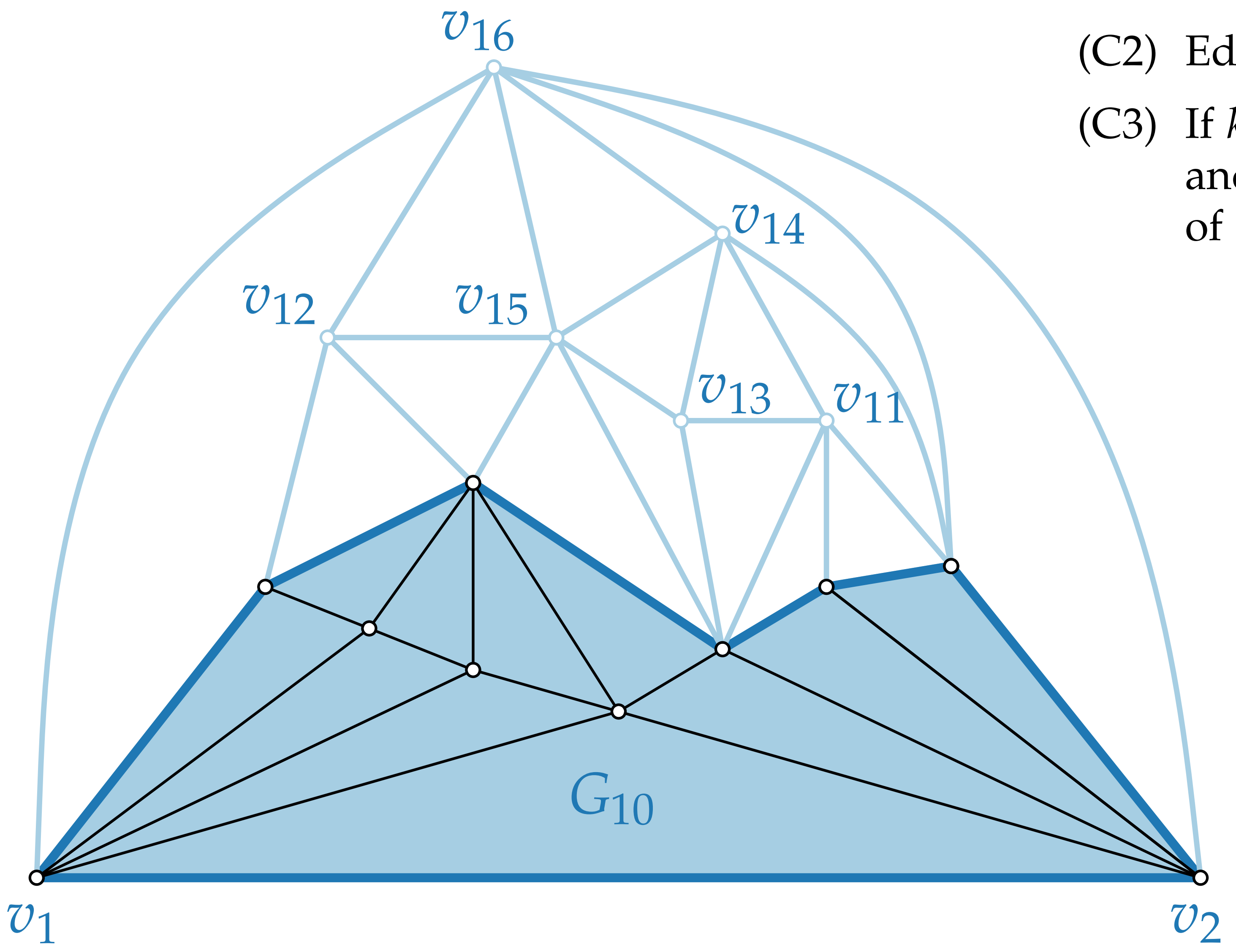

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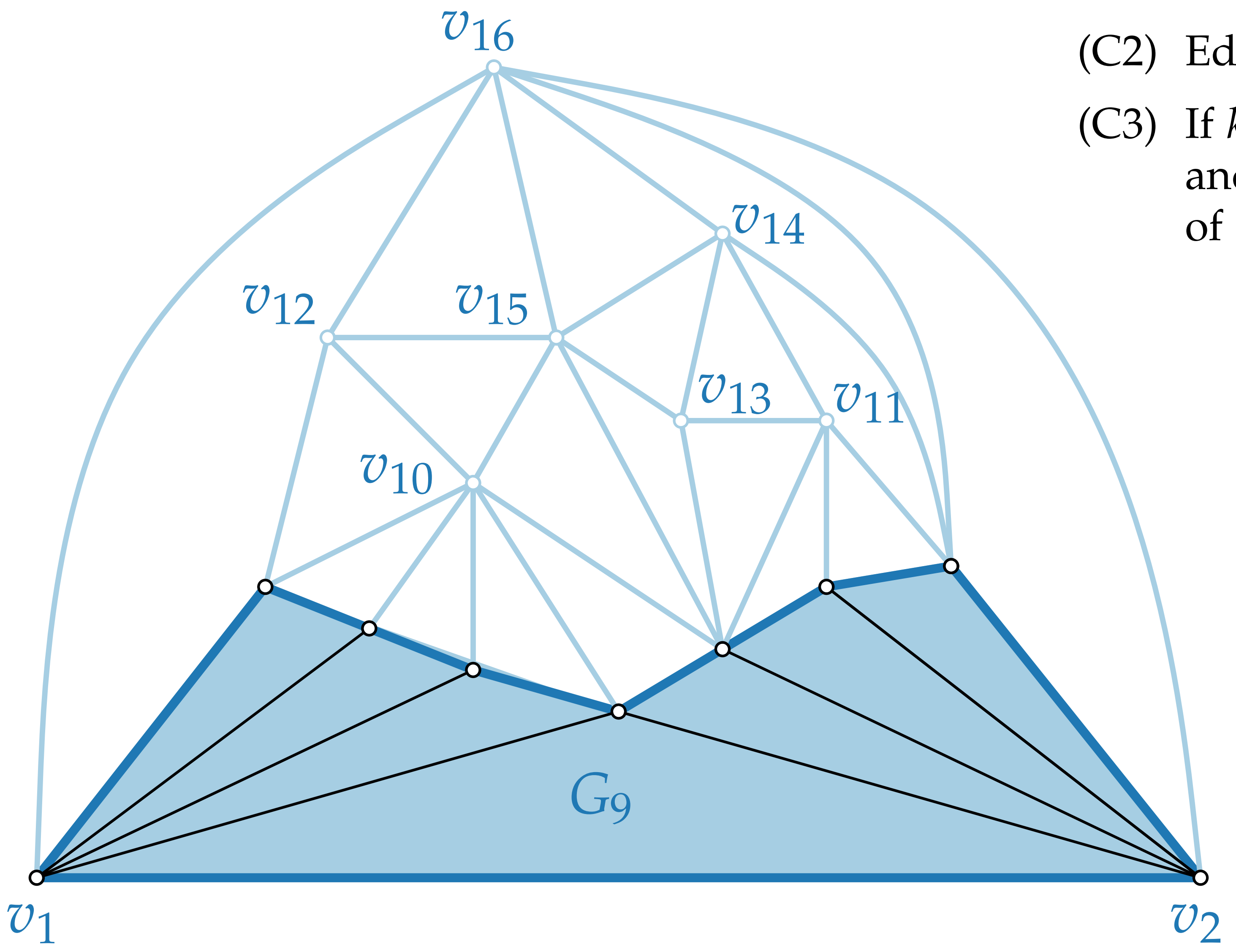

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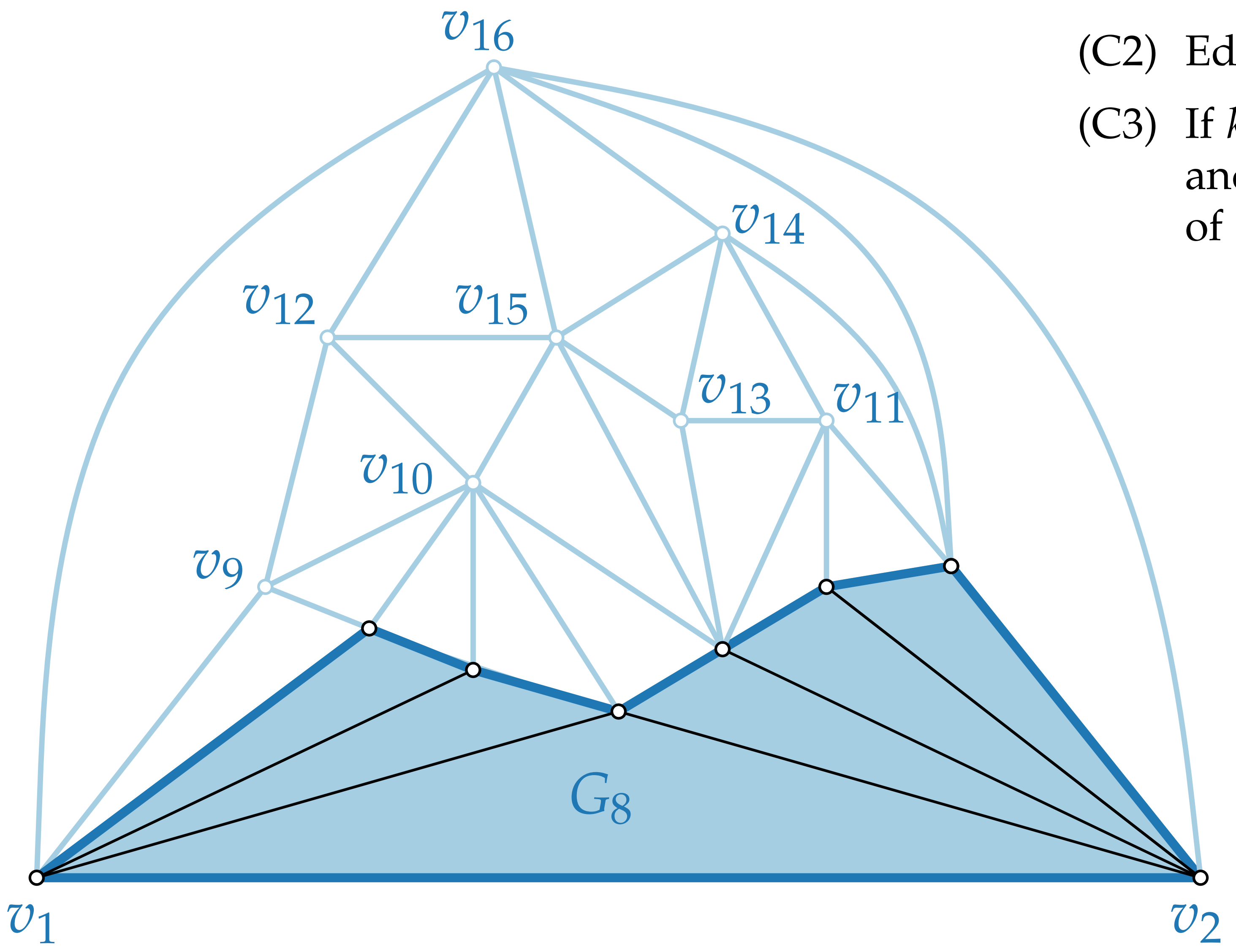

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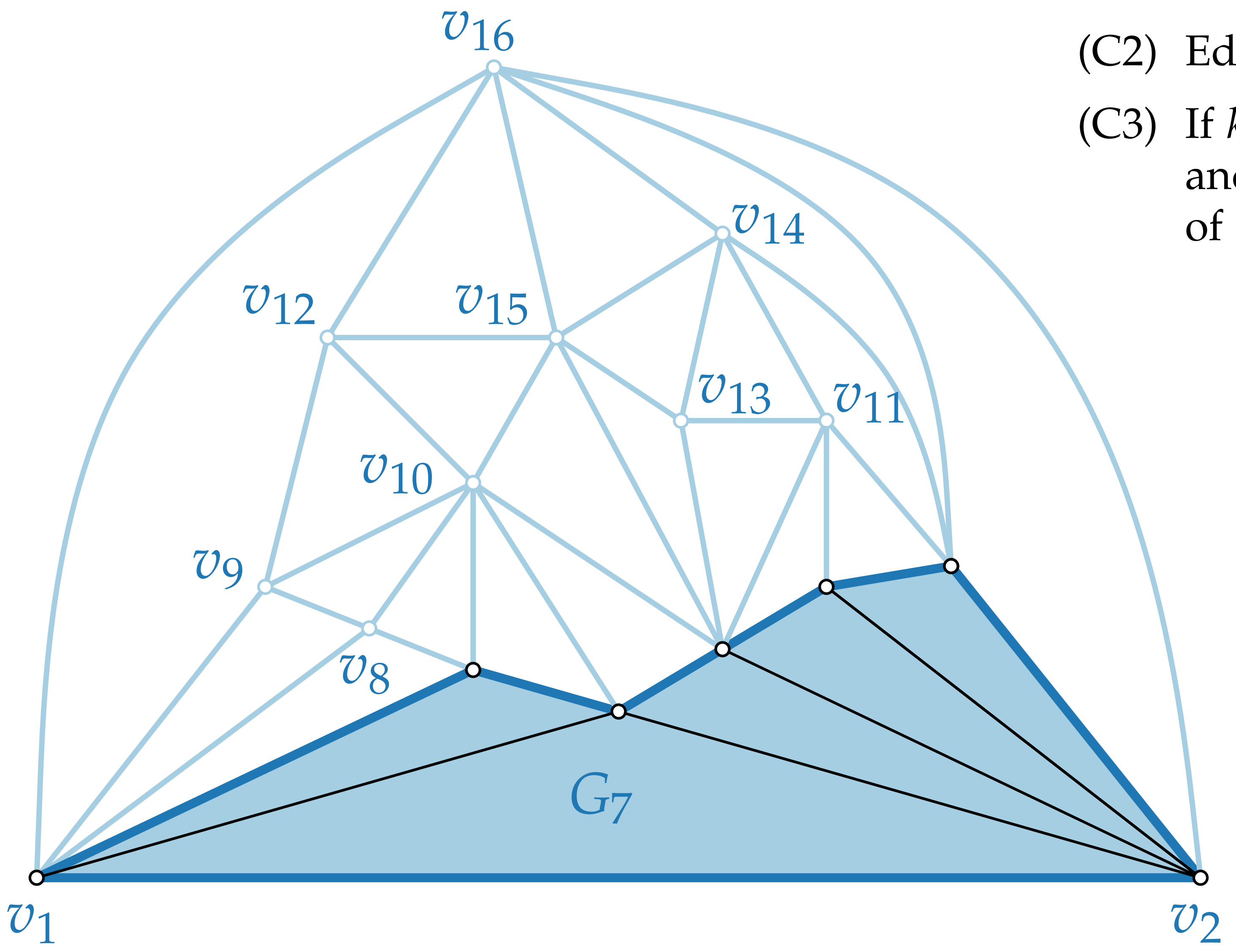

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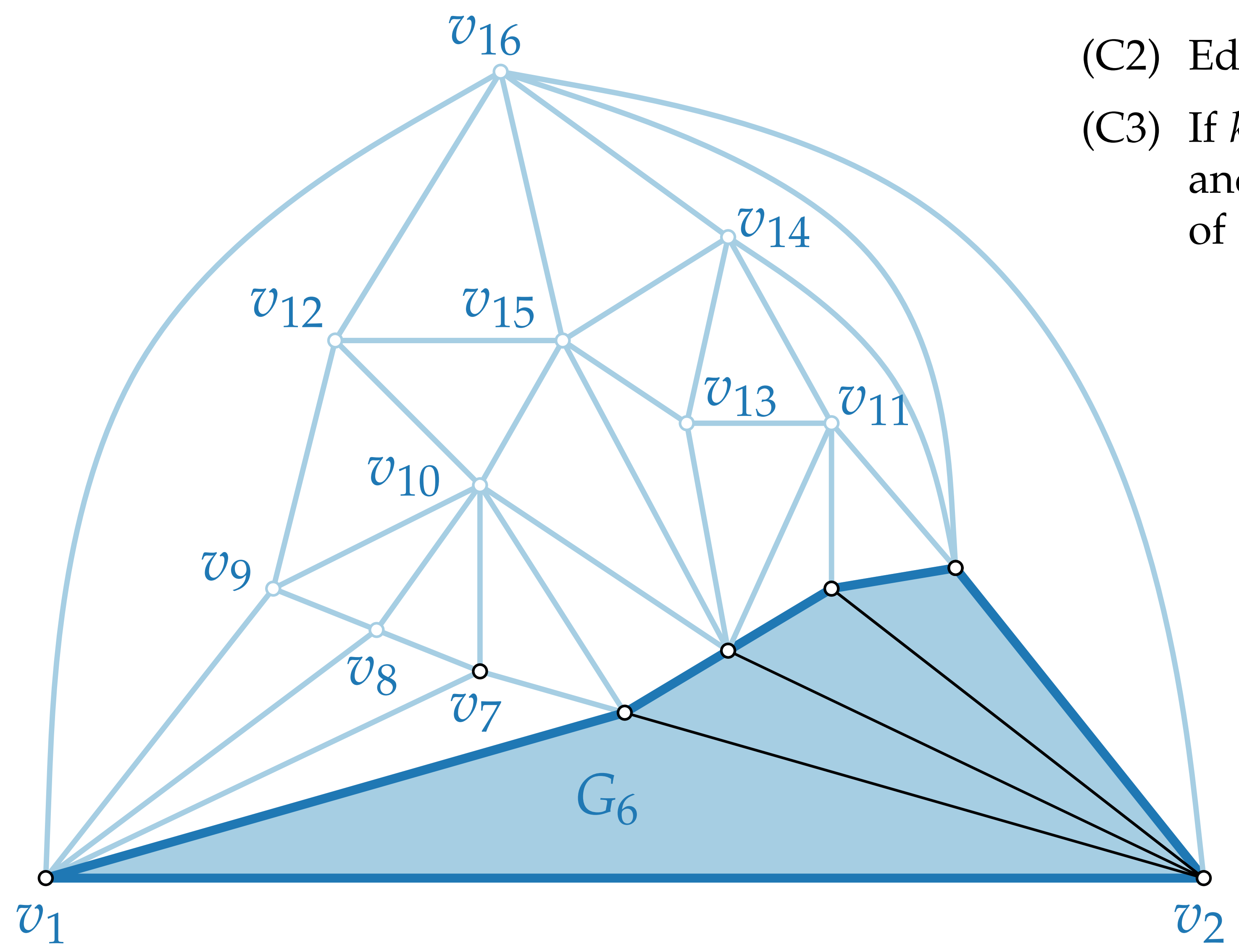

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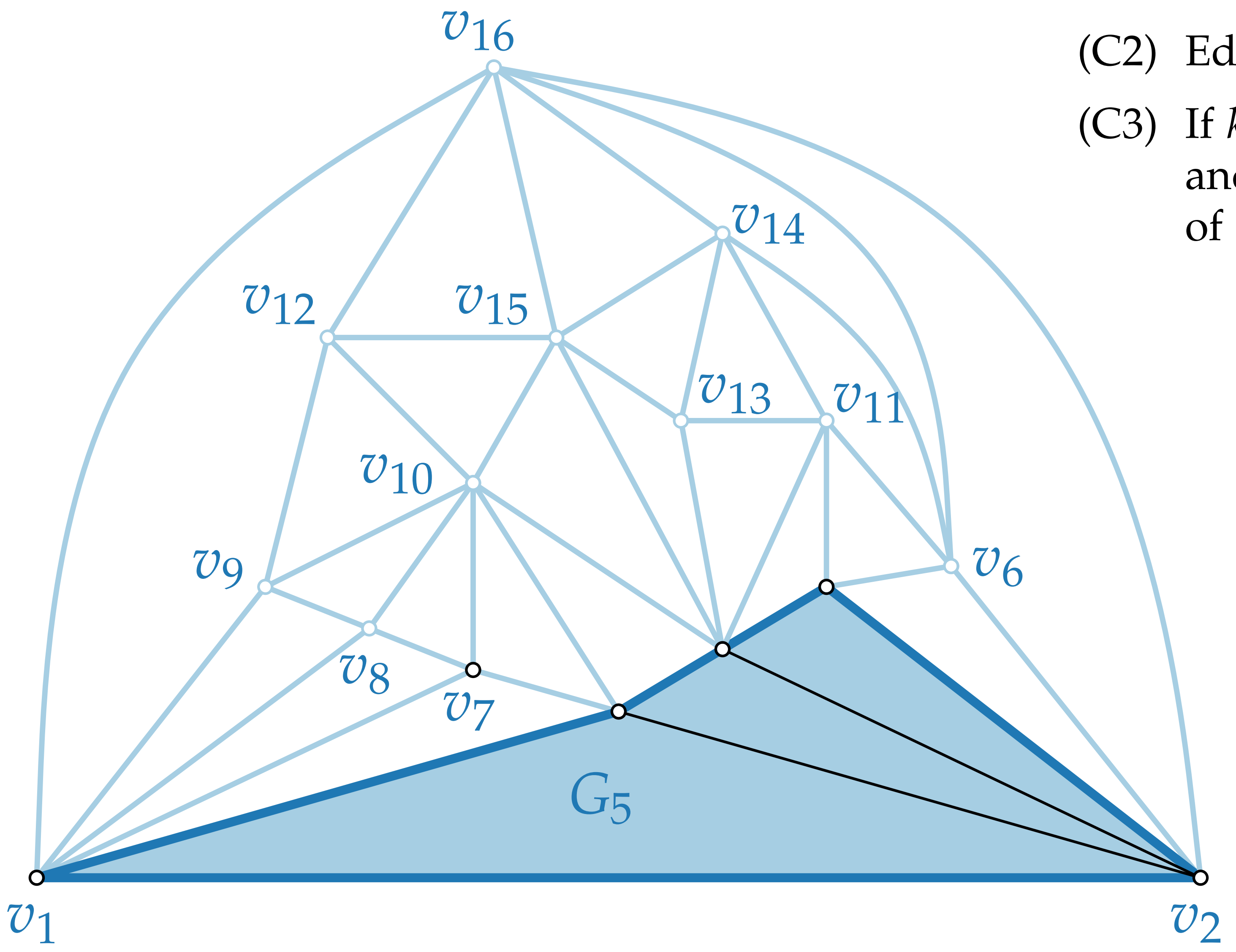

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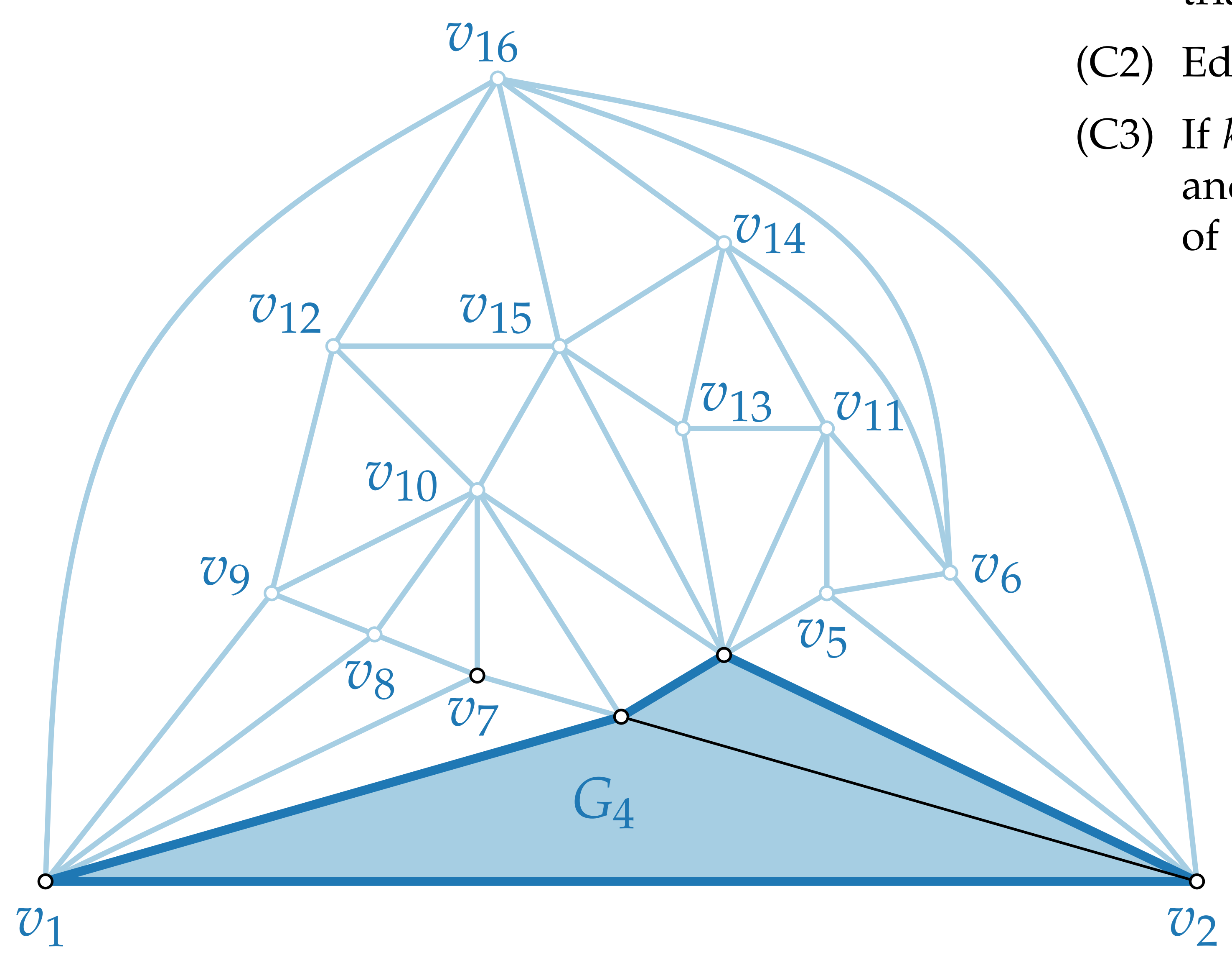

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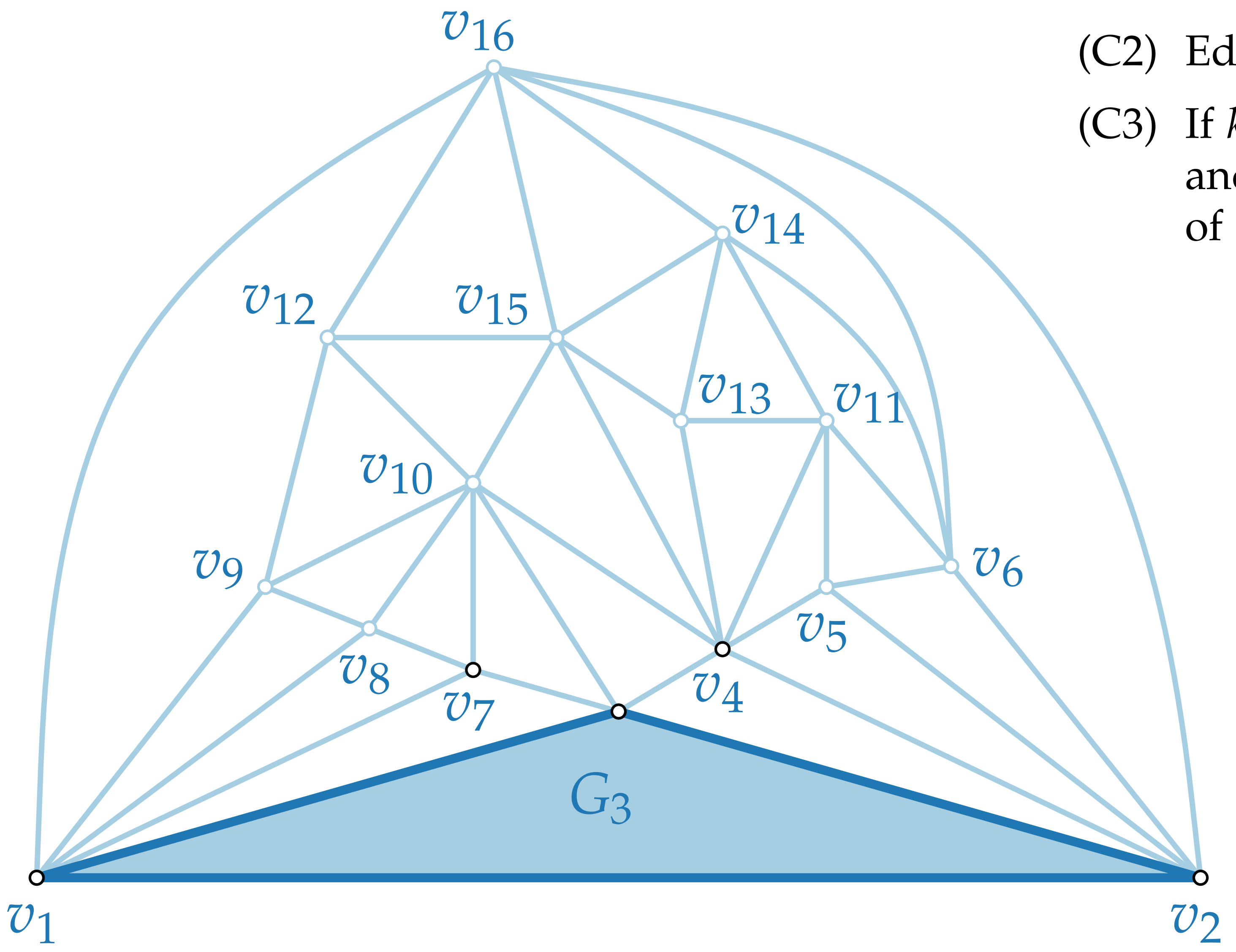

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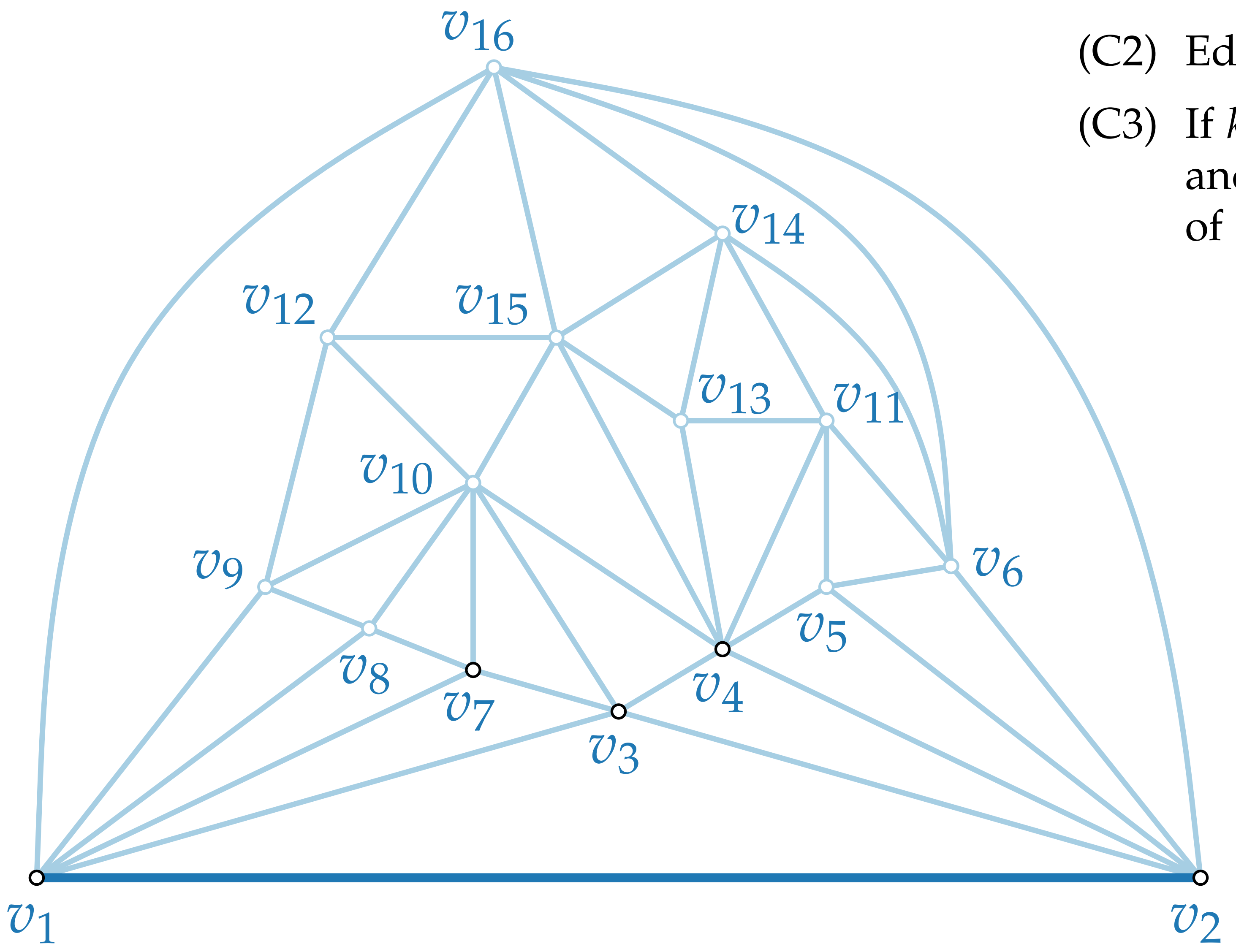

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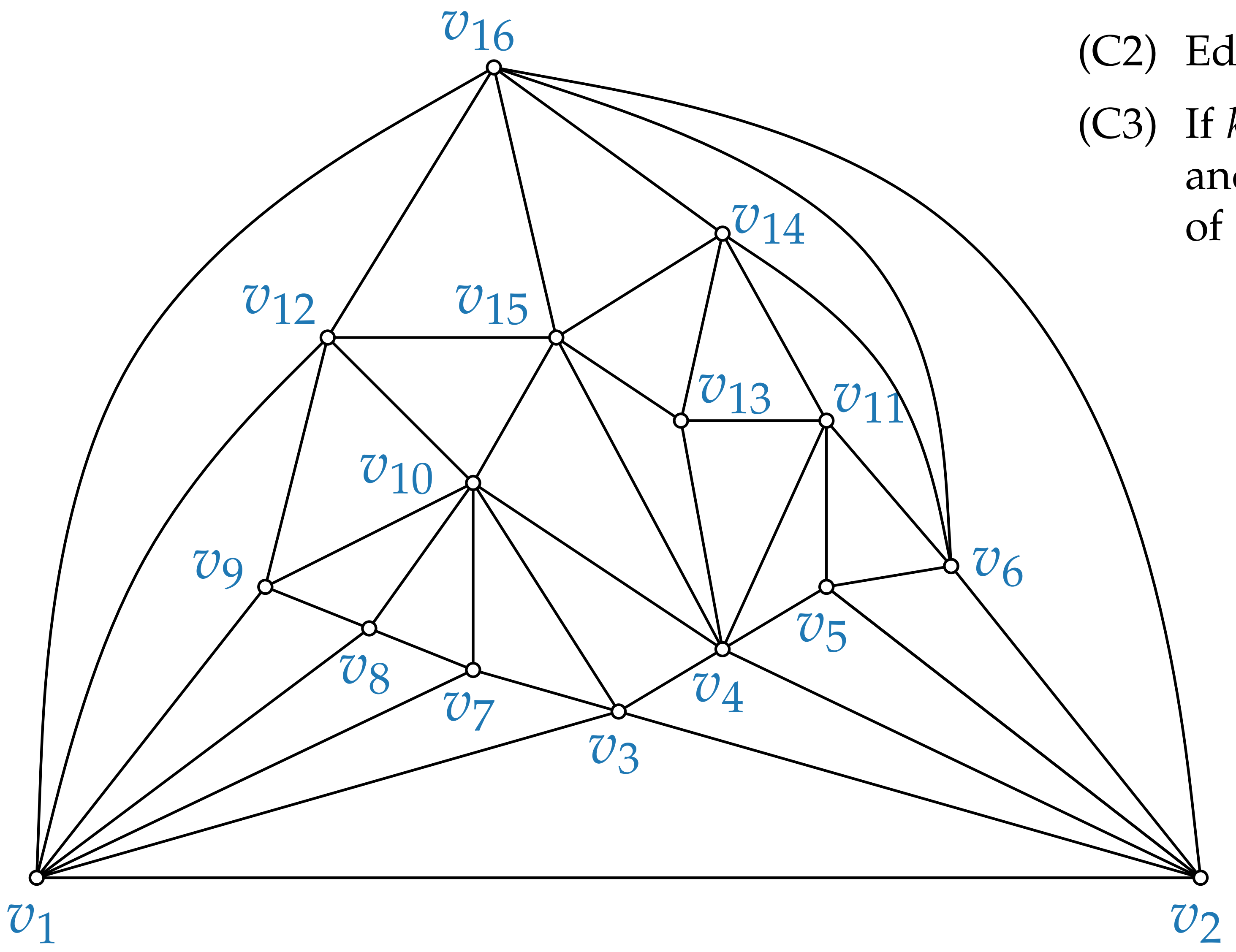

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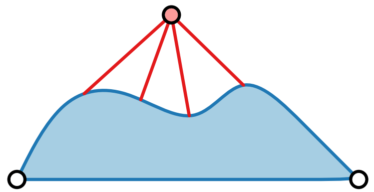
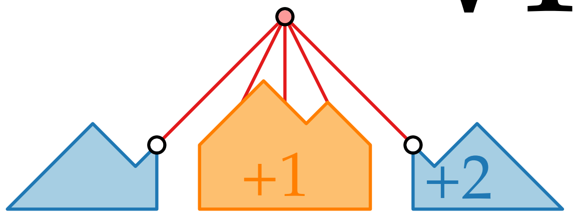
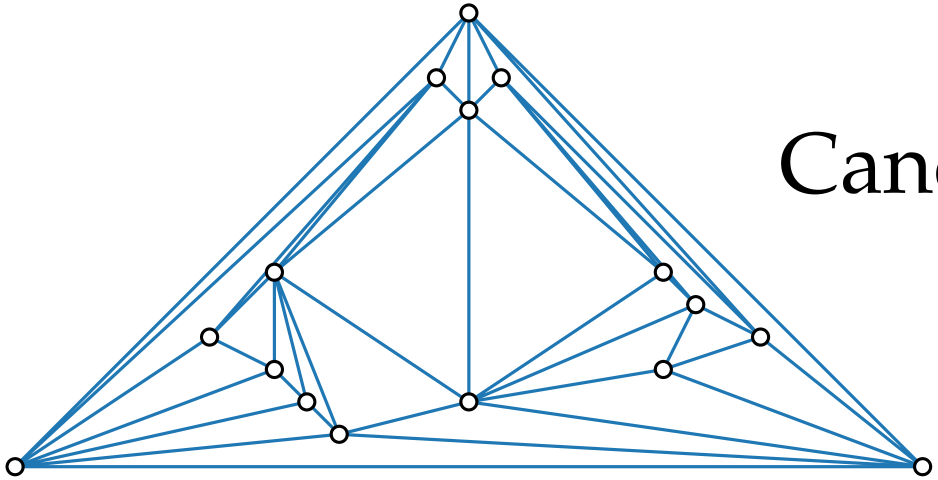
Universität Trier



Visualization of Graphs



Lecture 4:
Straight-Line Drawings of Planar Graphs I: Canonical Ordering and Shift Method



Part III:
Canonical Order – Existence

Philipp Kindermann

Canonical Order – Existence

Lemma.
Every triangulated plane graph has a canonical order.

(C1) G_k biconnected and internally triangulated

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Induction step:

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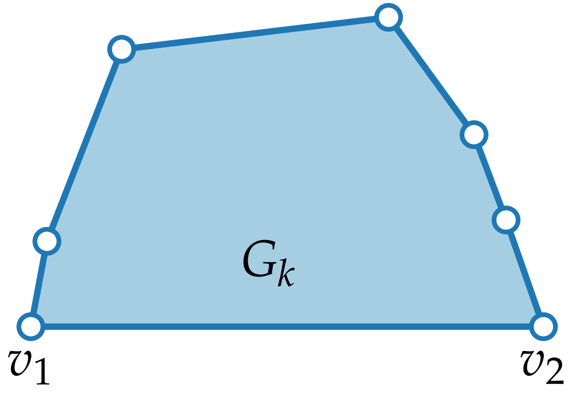

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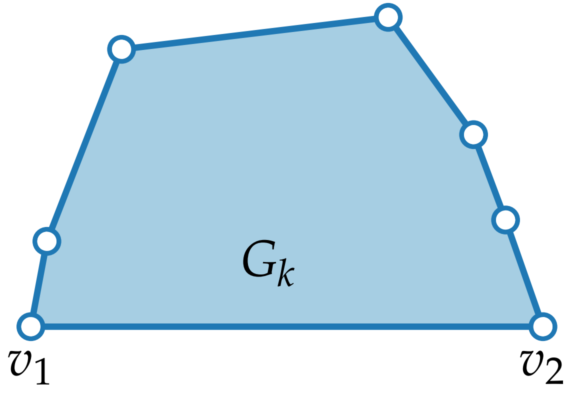

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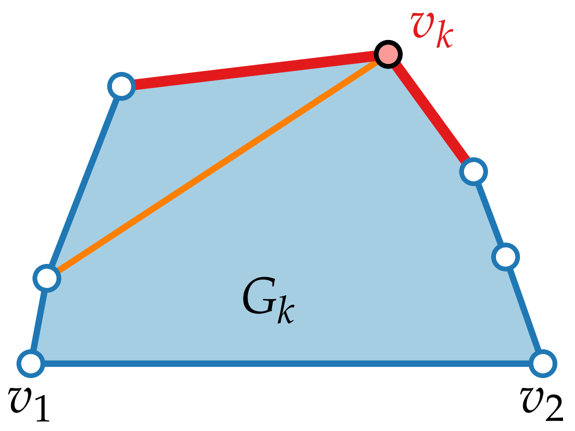

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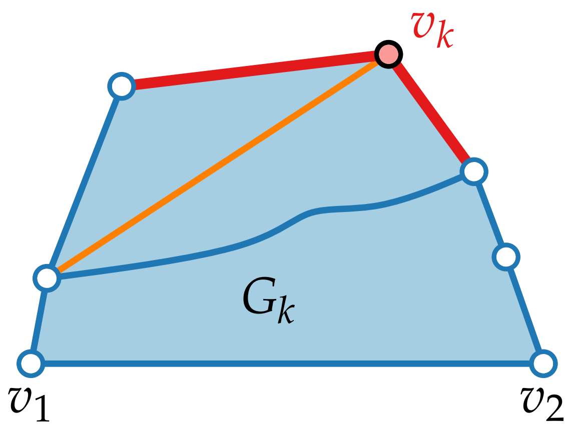

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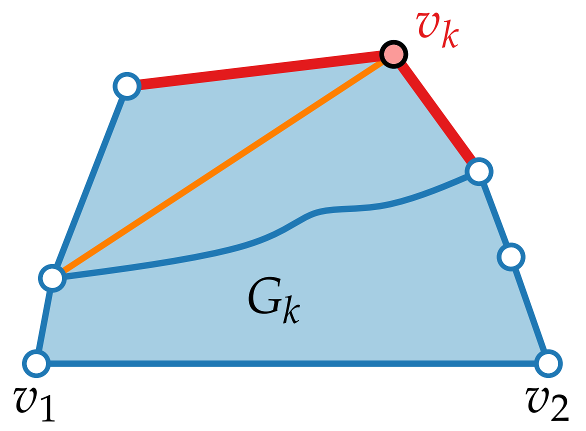

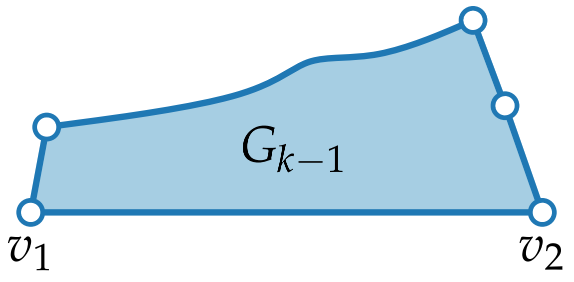

Canonical Order – Existence

(C1) G_k biconnected and internally triangulated

(C2) (v_1, v_2) on outer face of G_k

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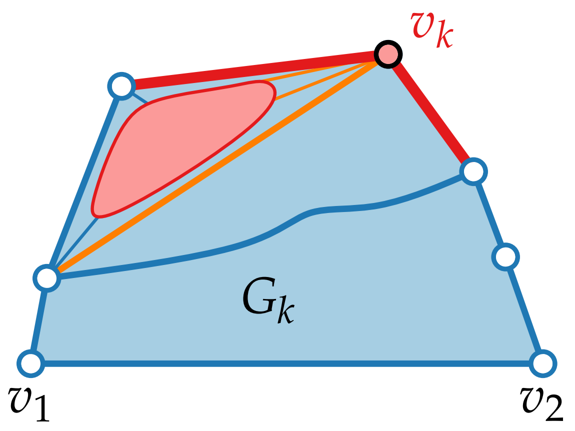

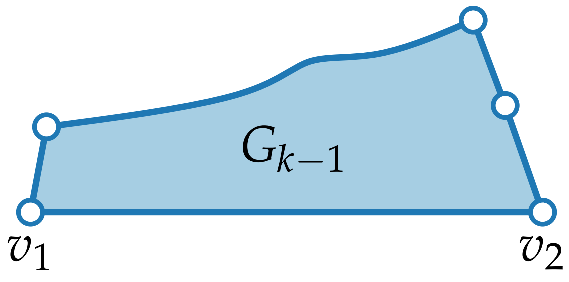

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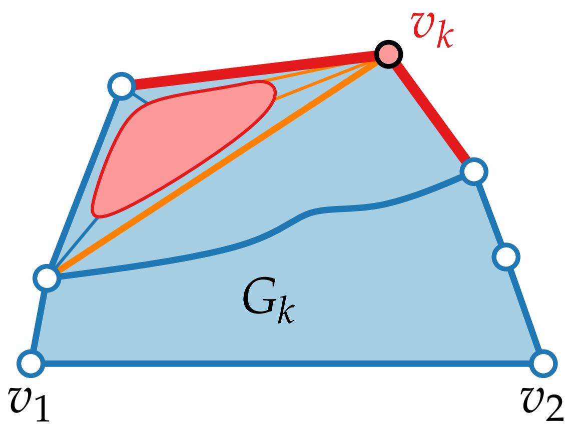

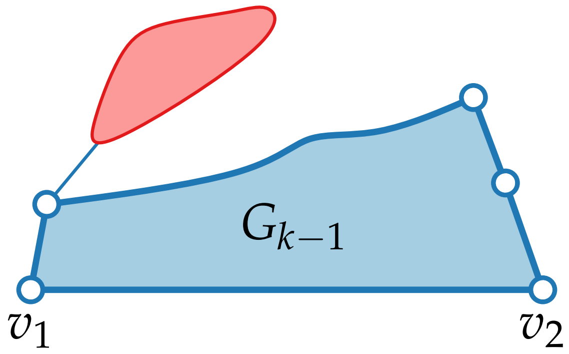

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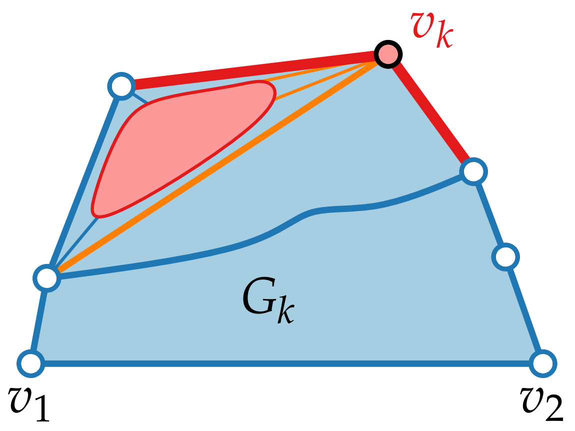

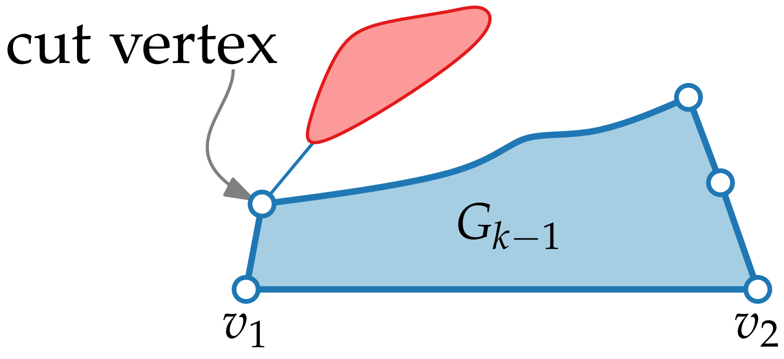

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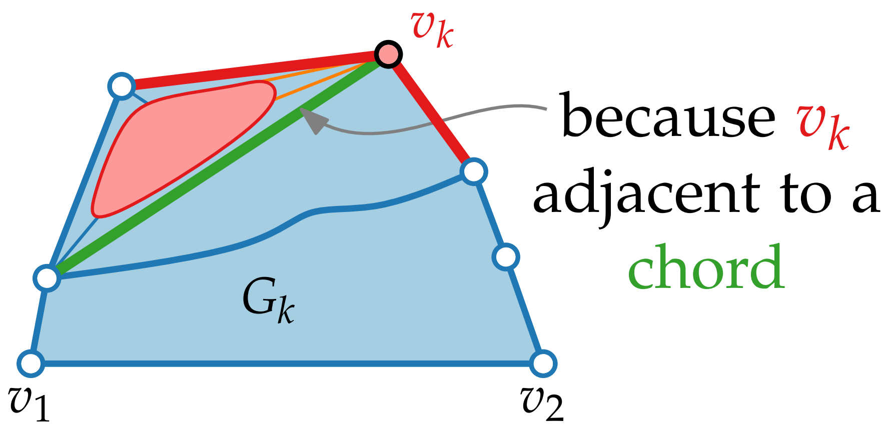

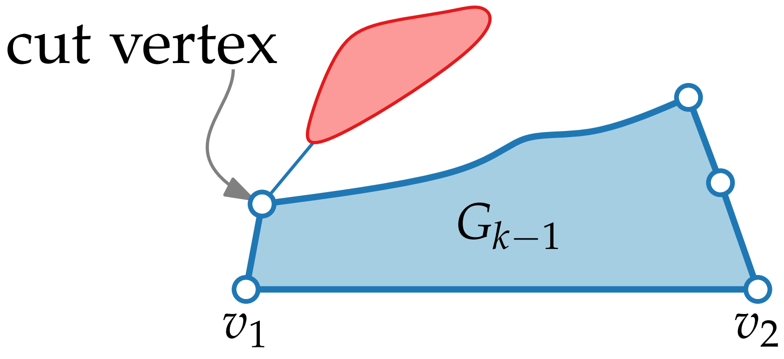

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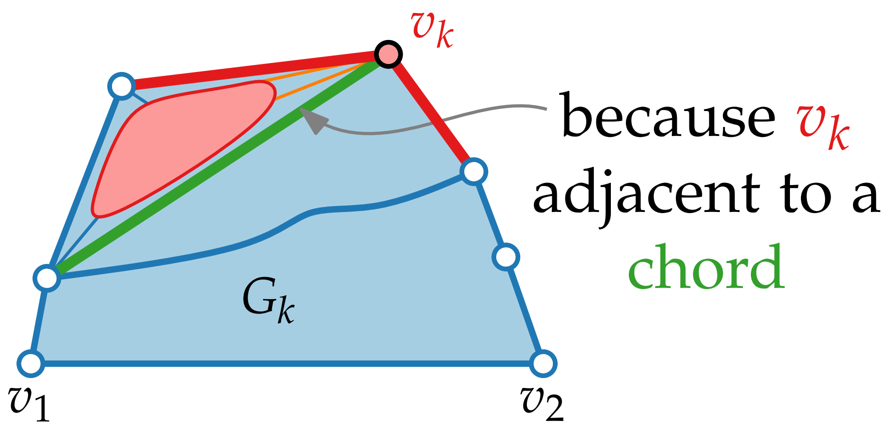

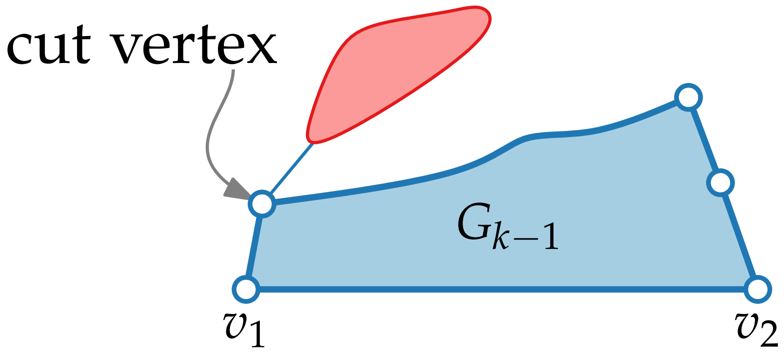

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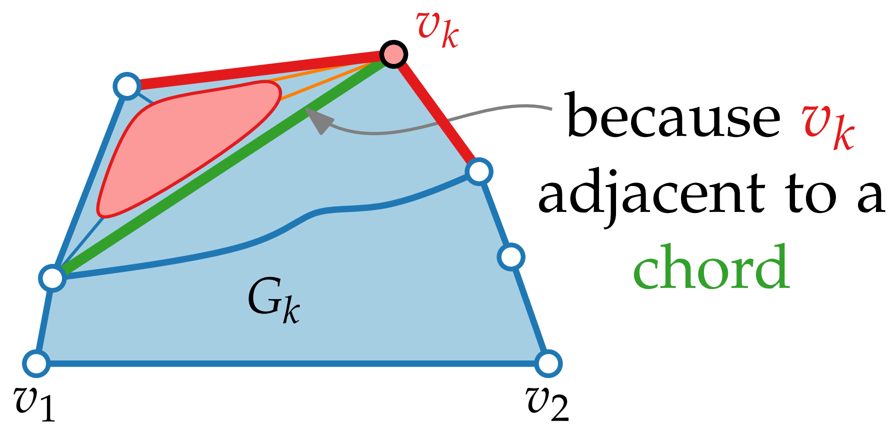

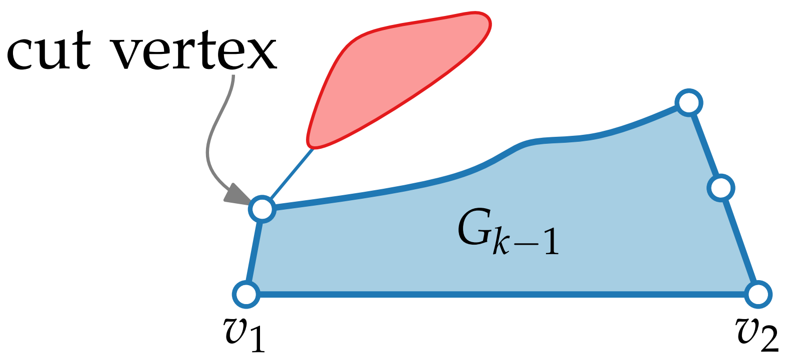

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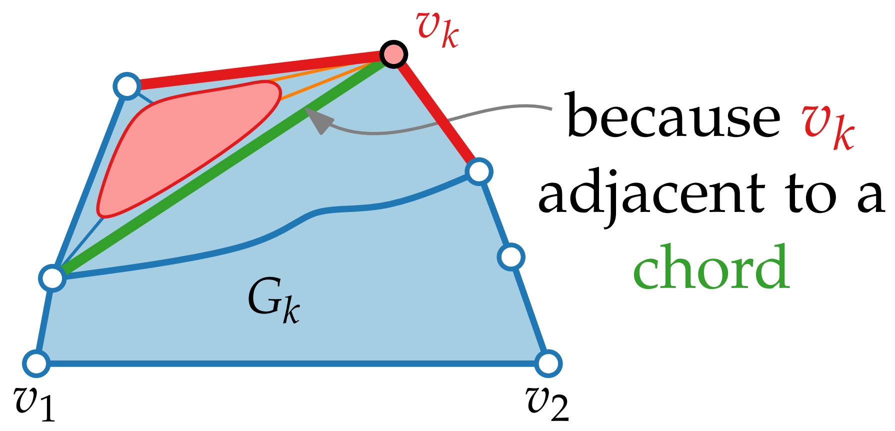
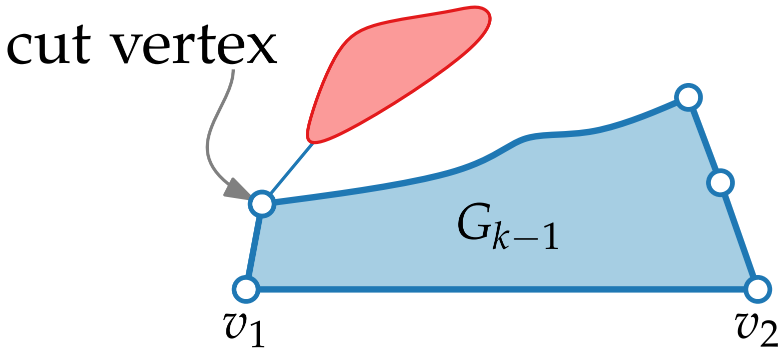
Have to show:

1. v_k not adjacent to chord is sufficient
2. Such v_k exists





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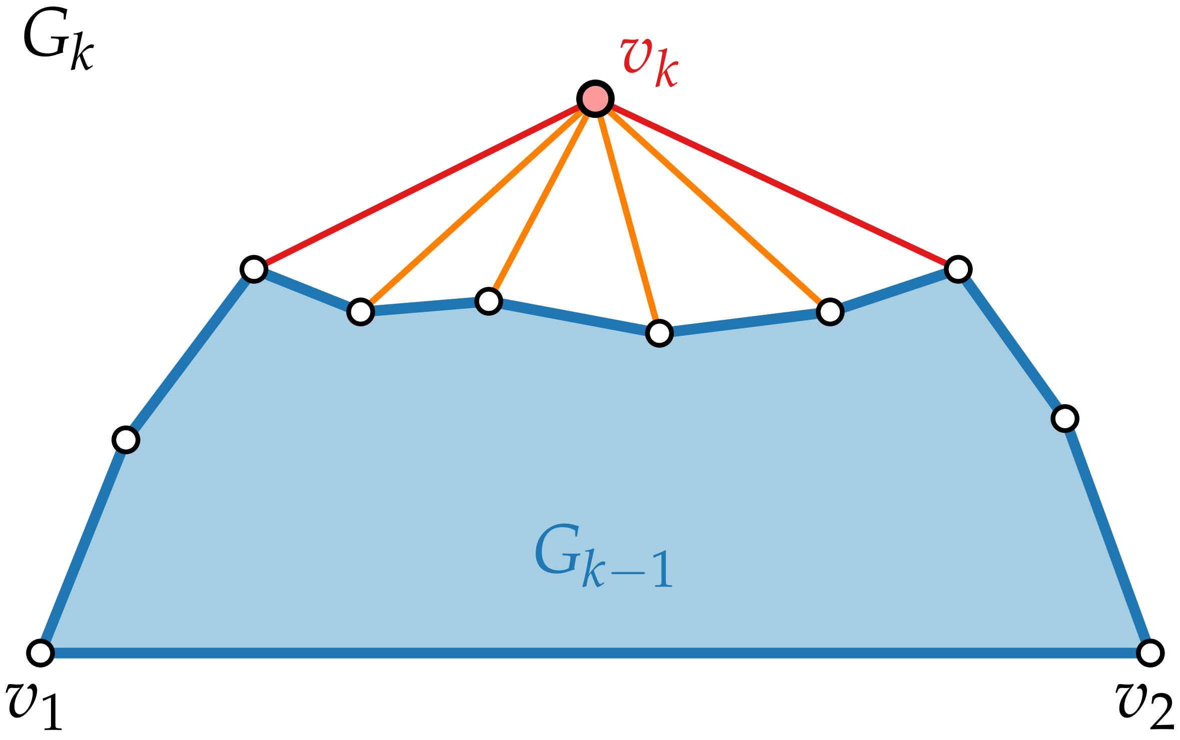

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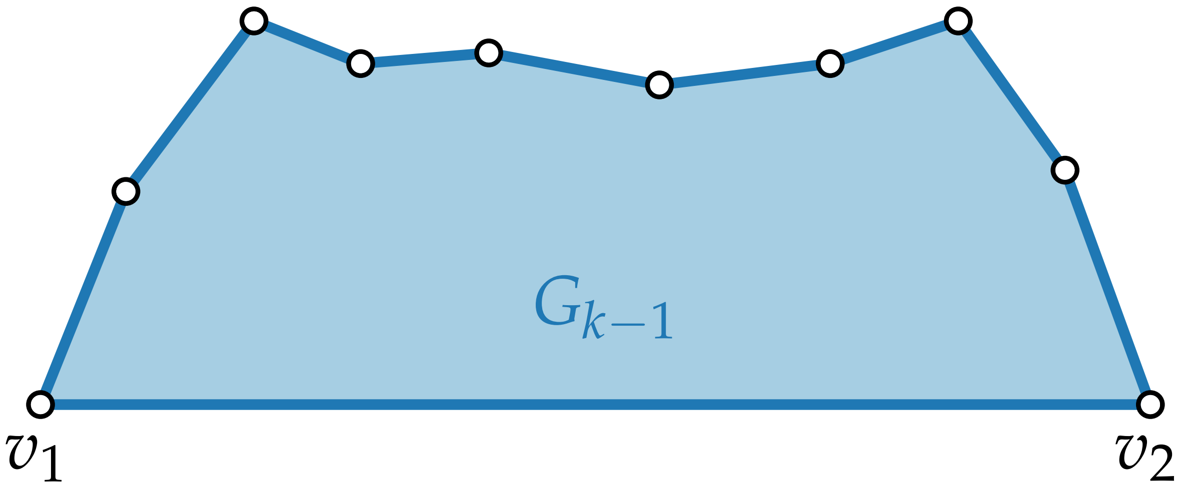

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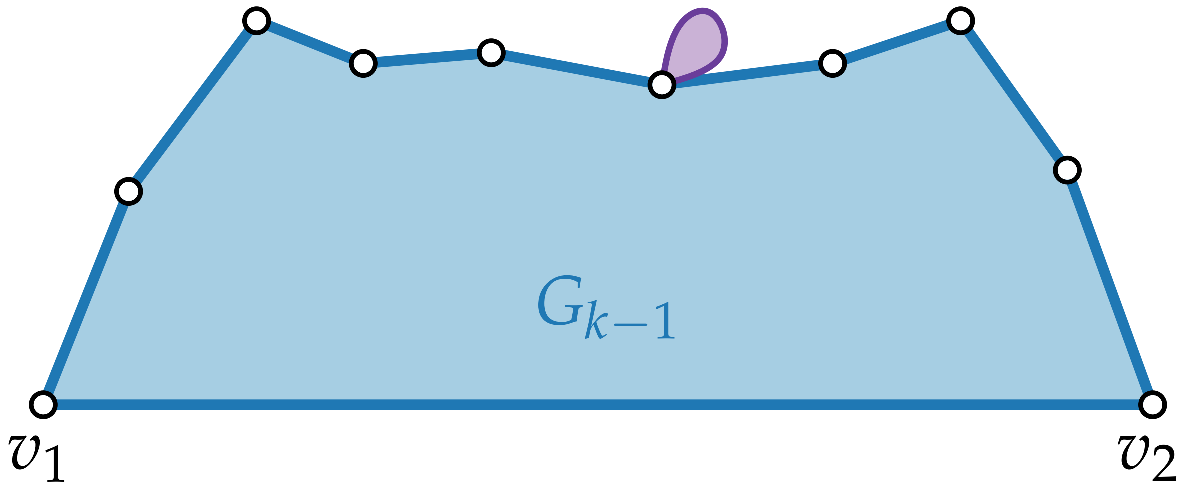

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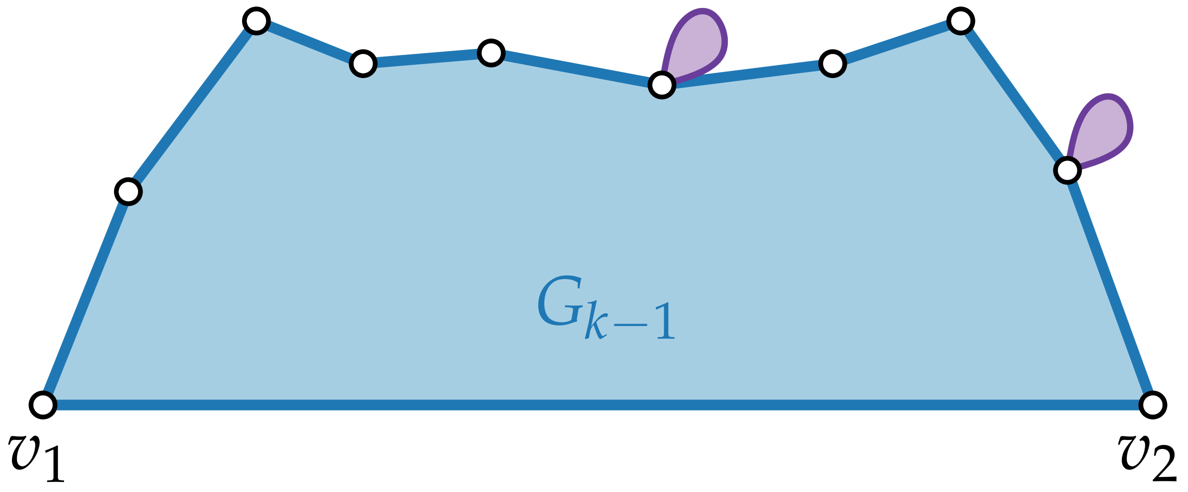

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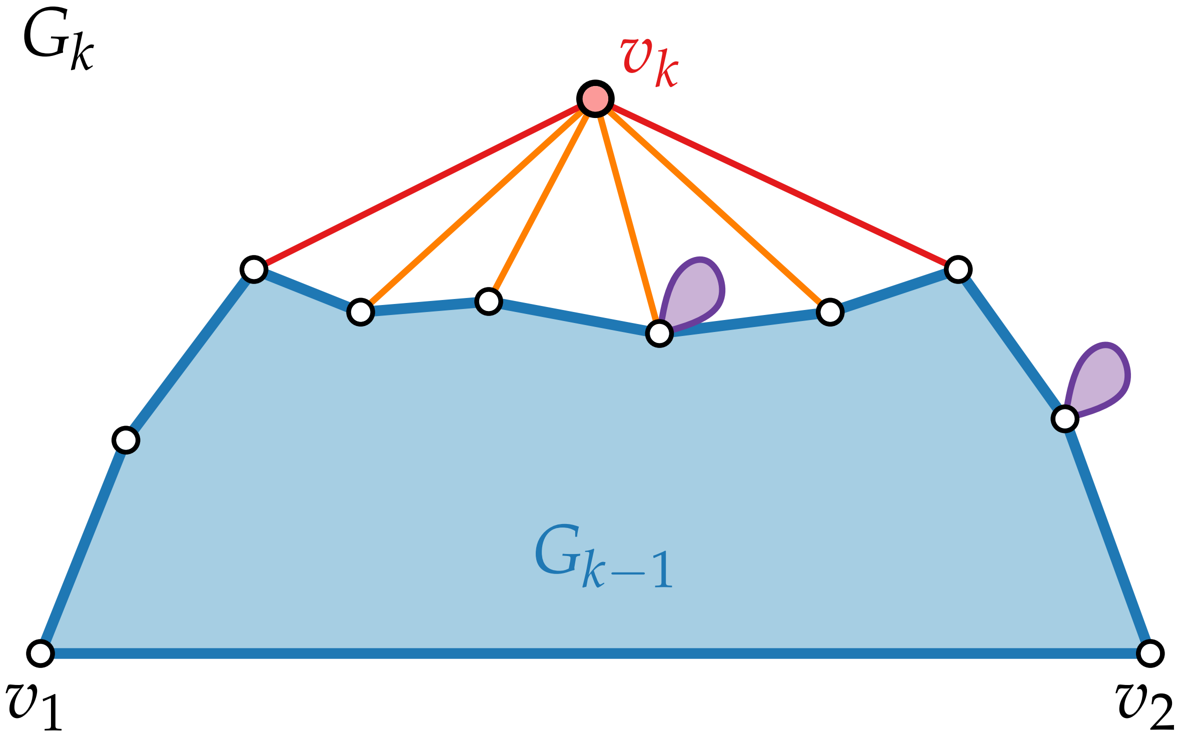

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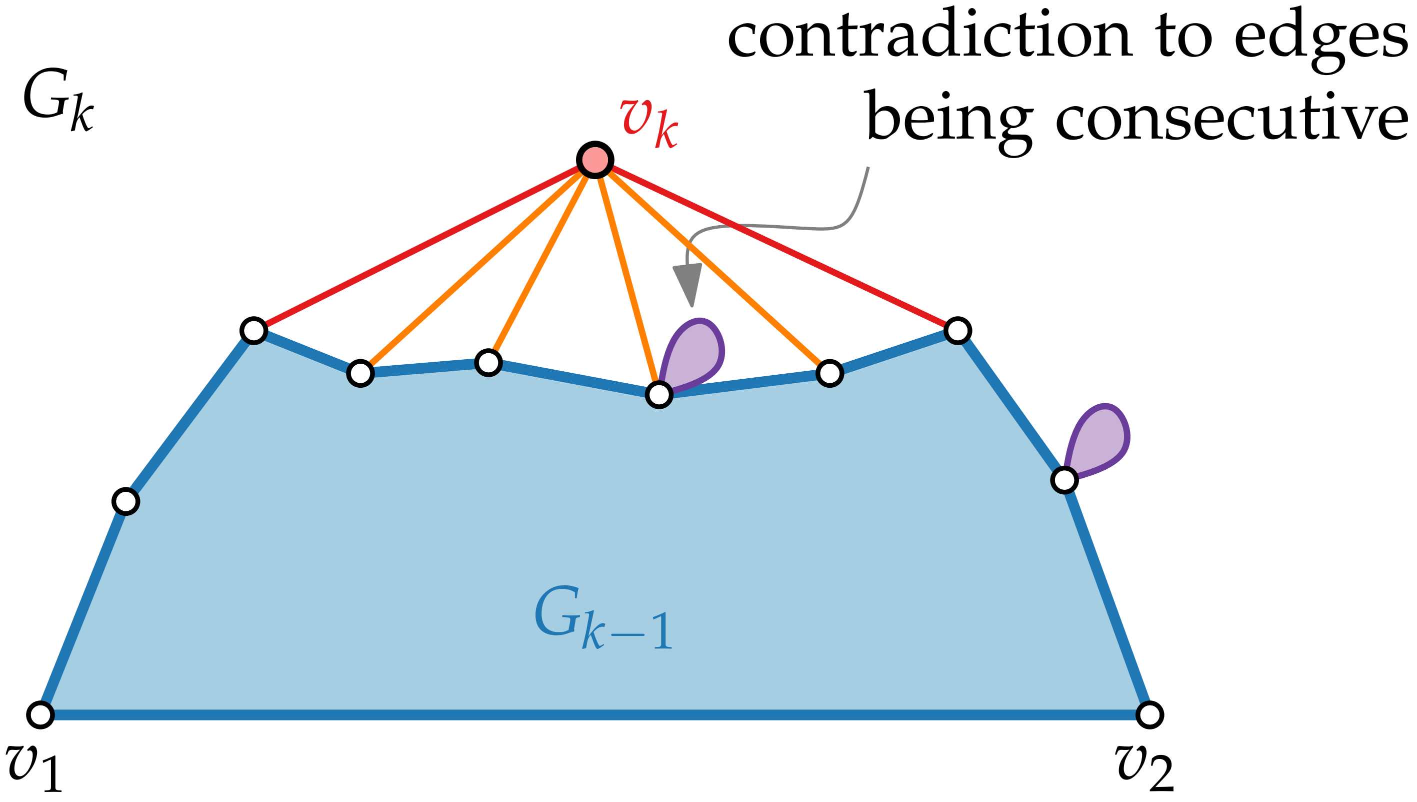

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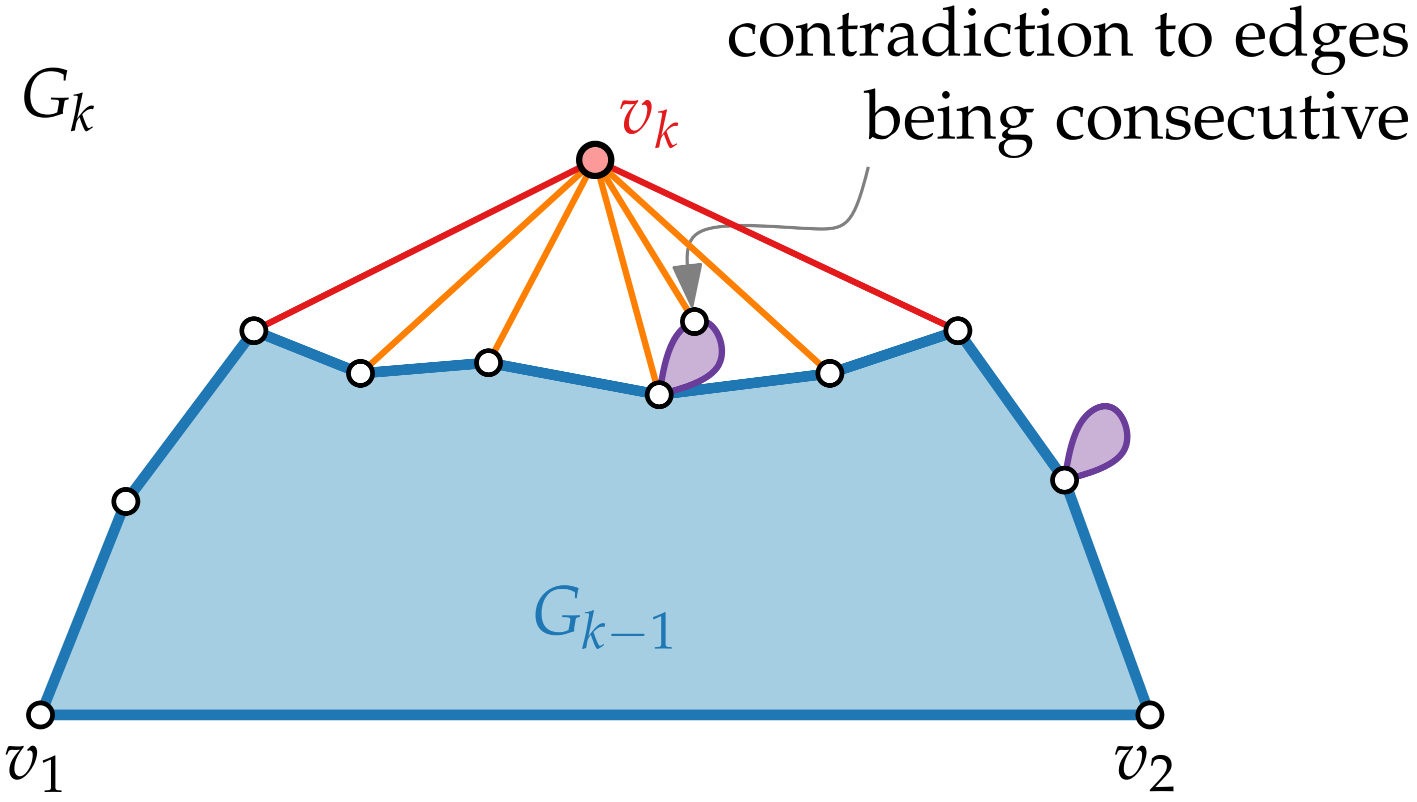

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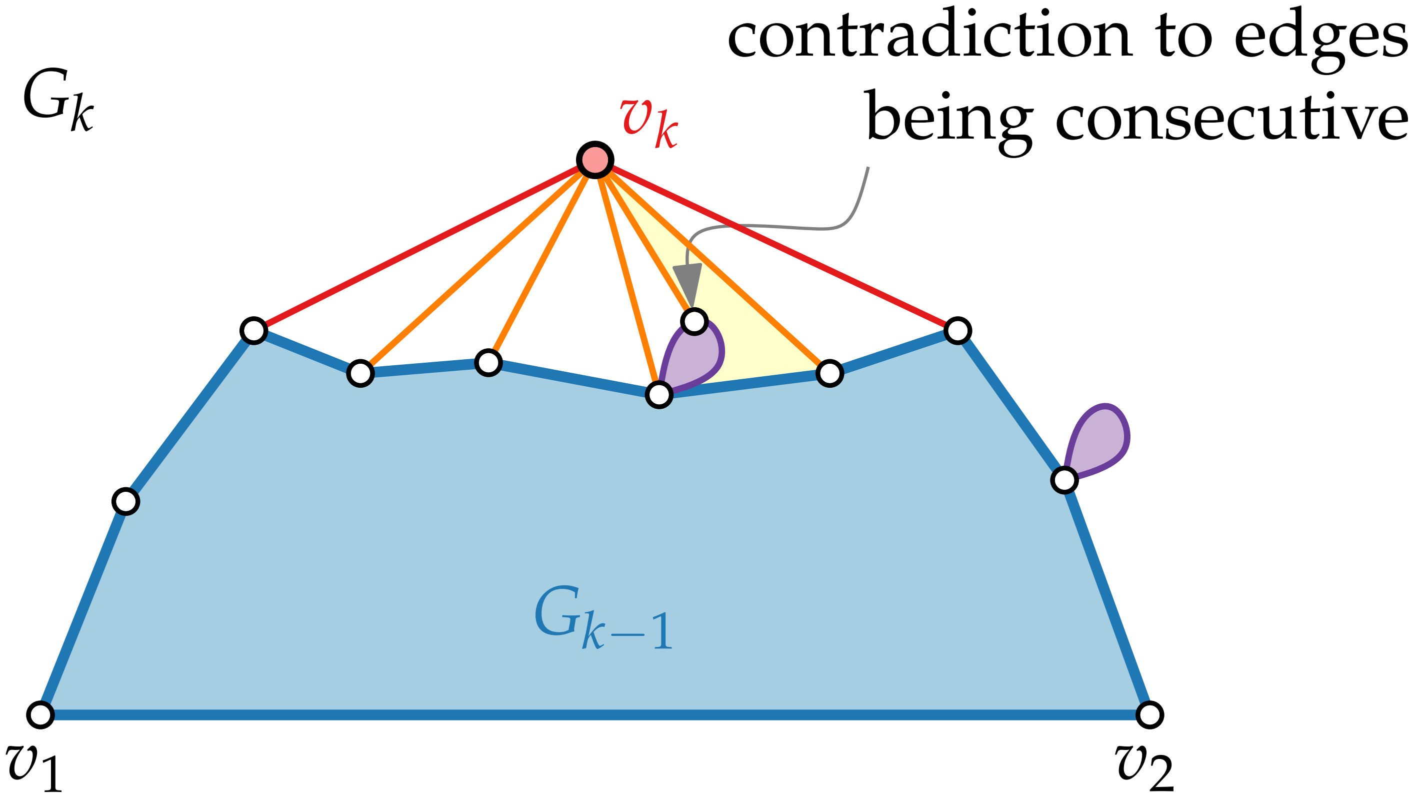

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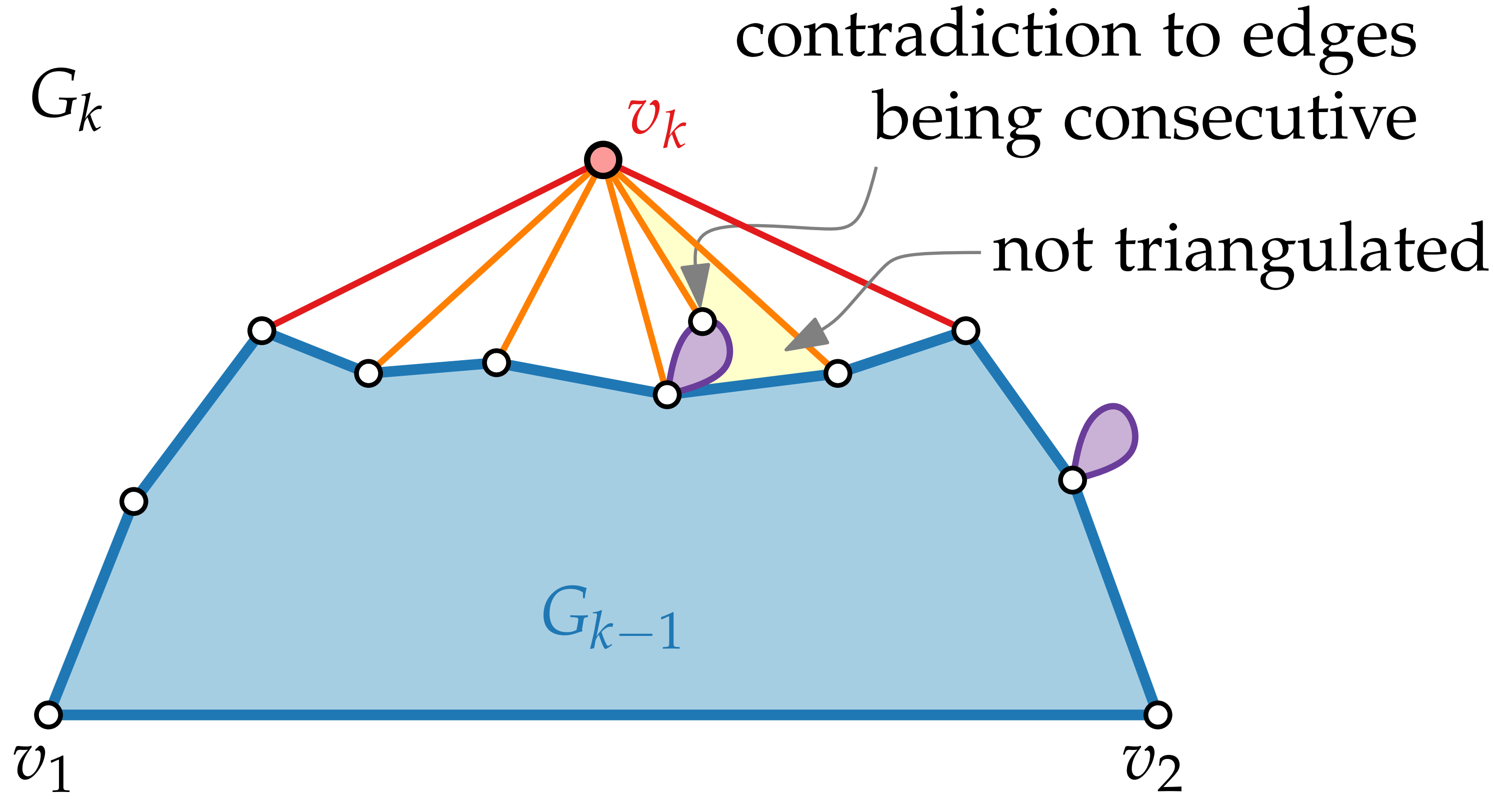

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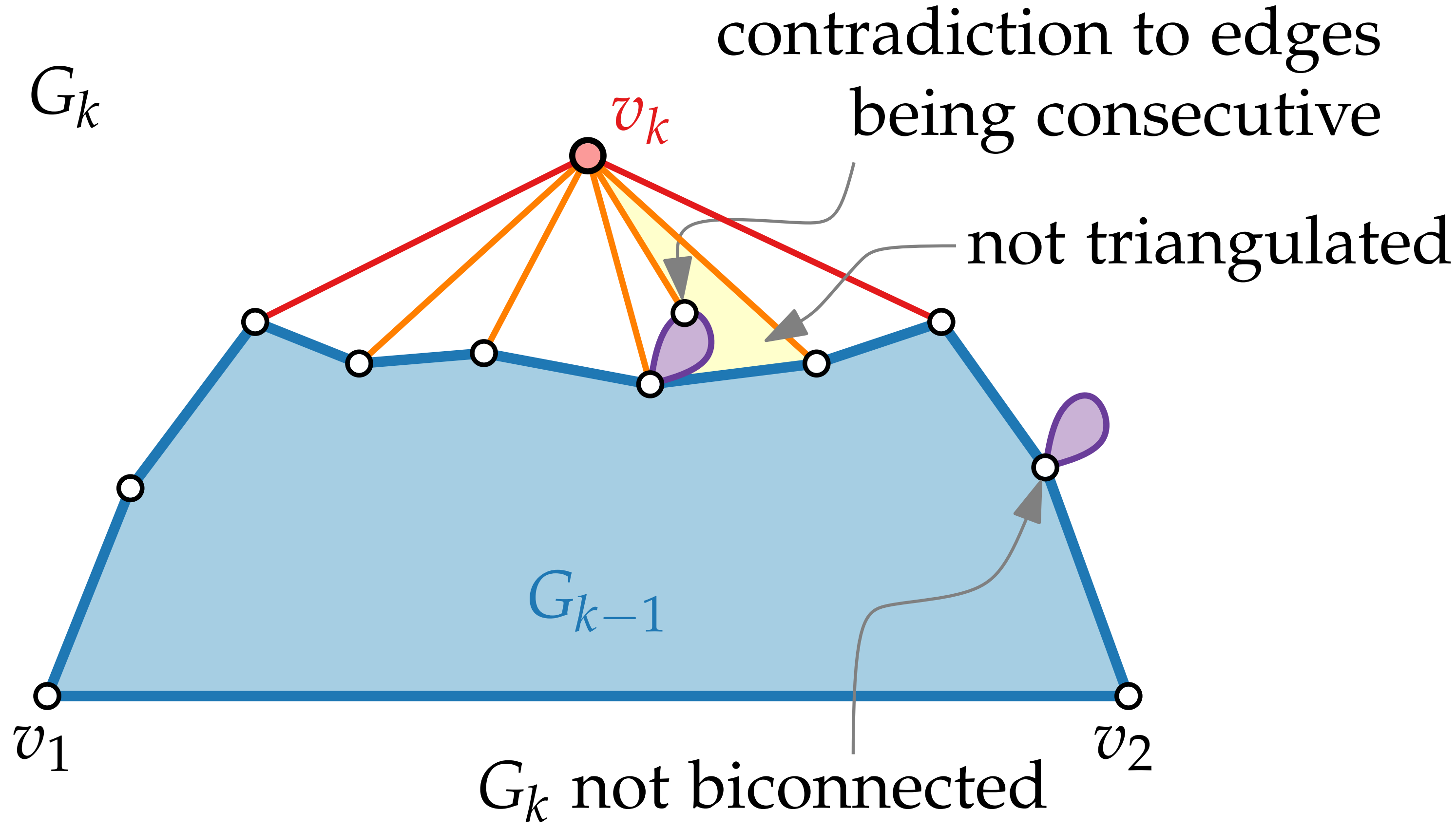

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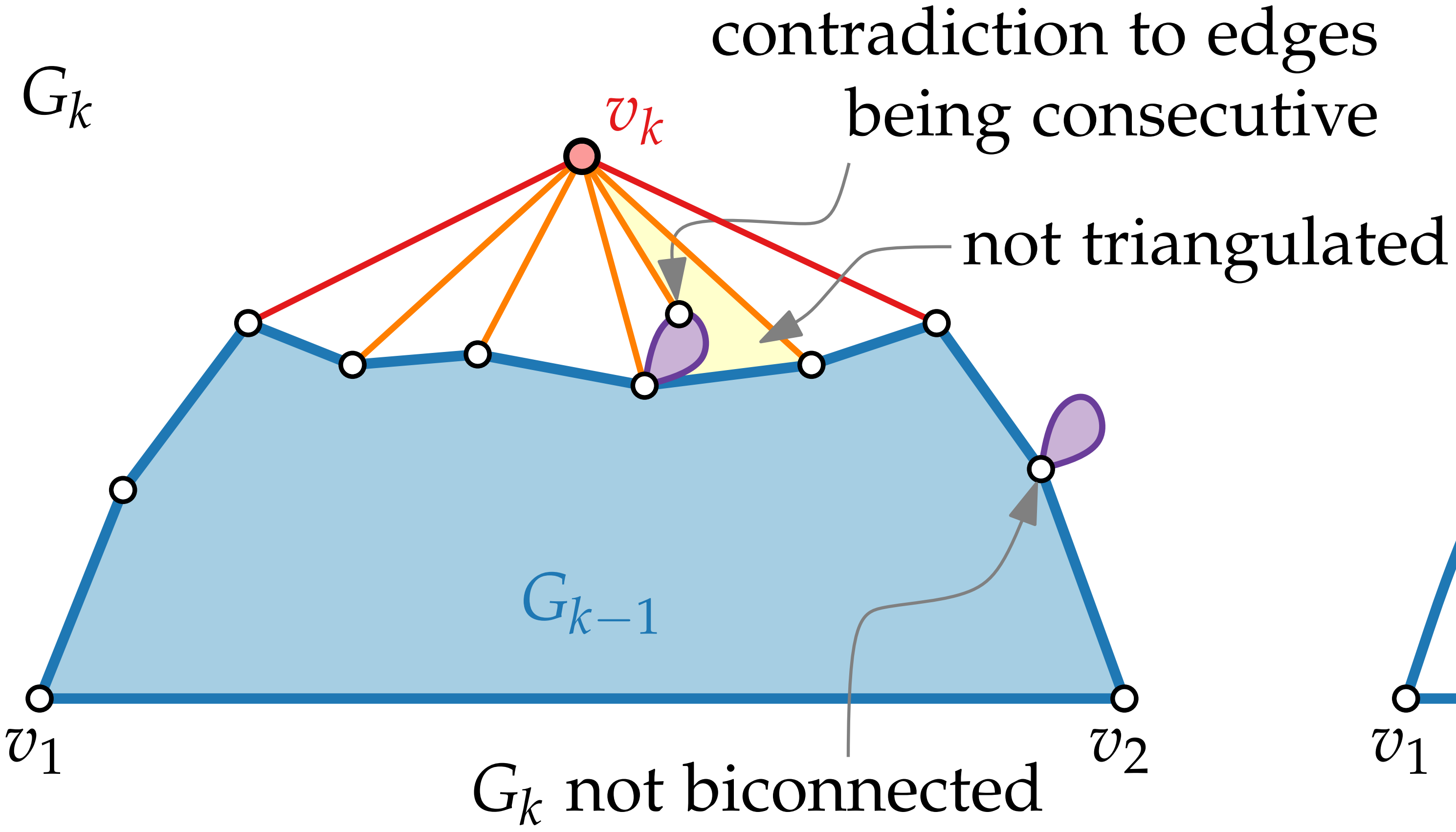

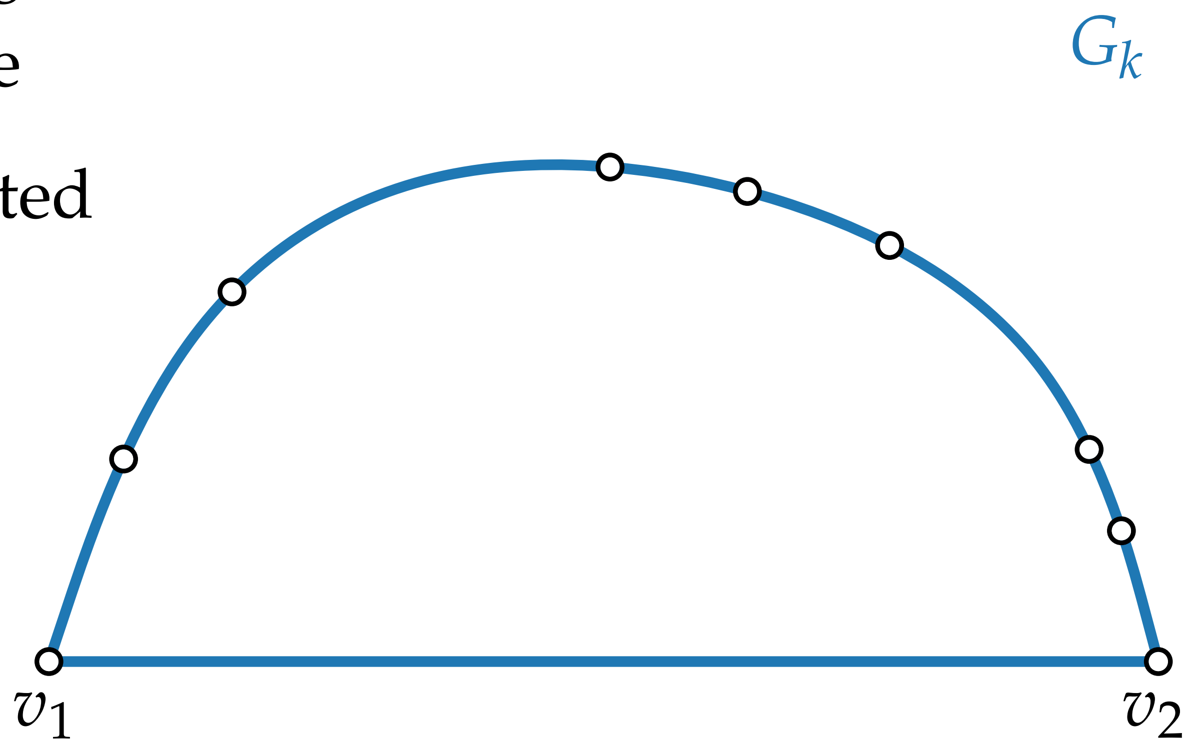

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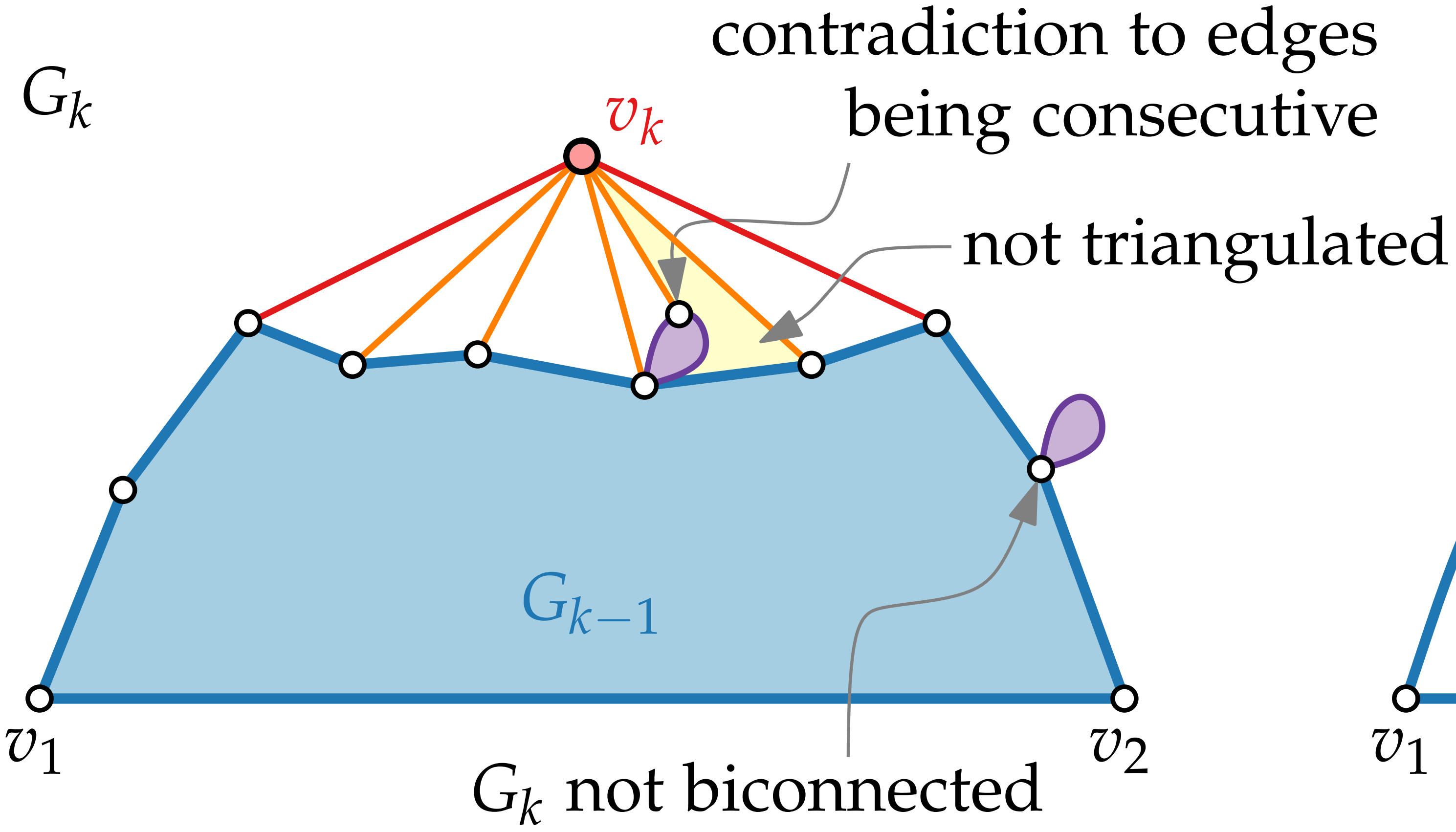

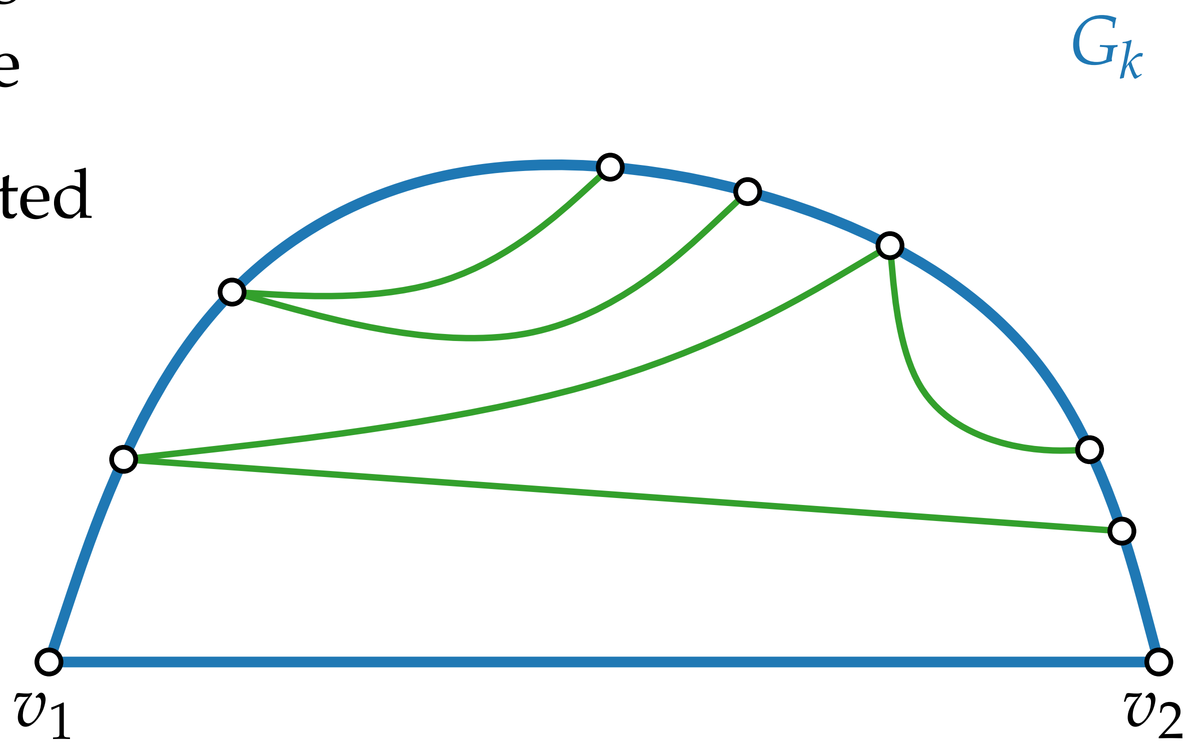

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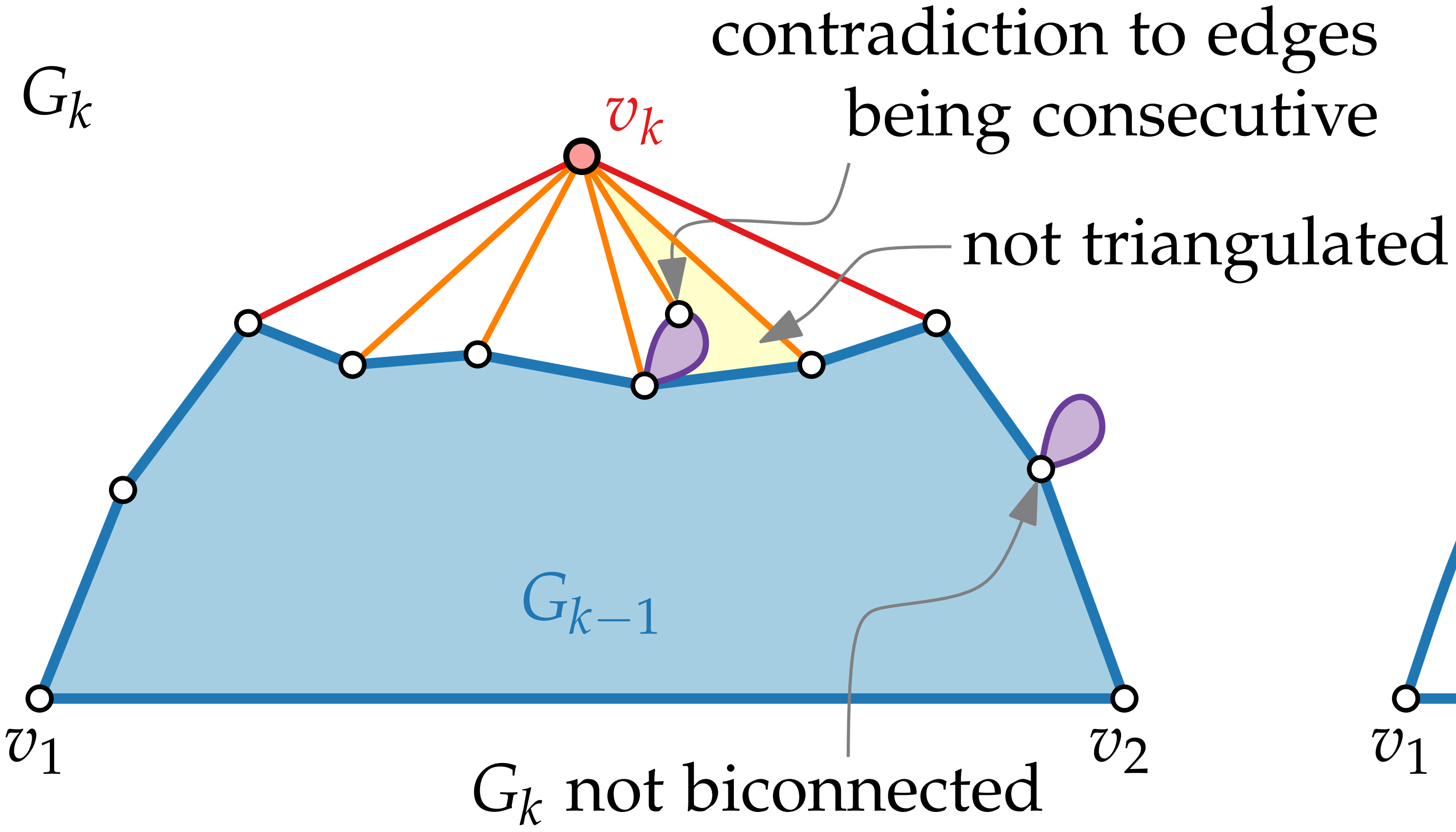

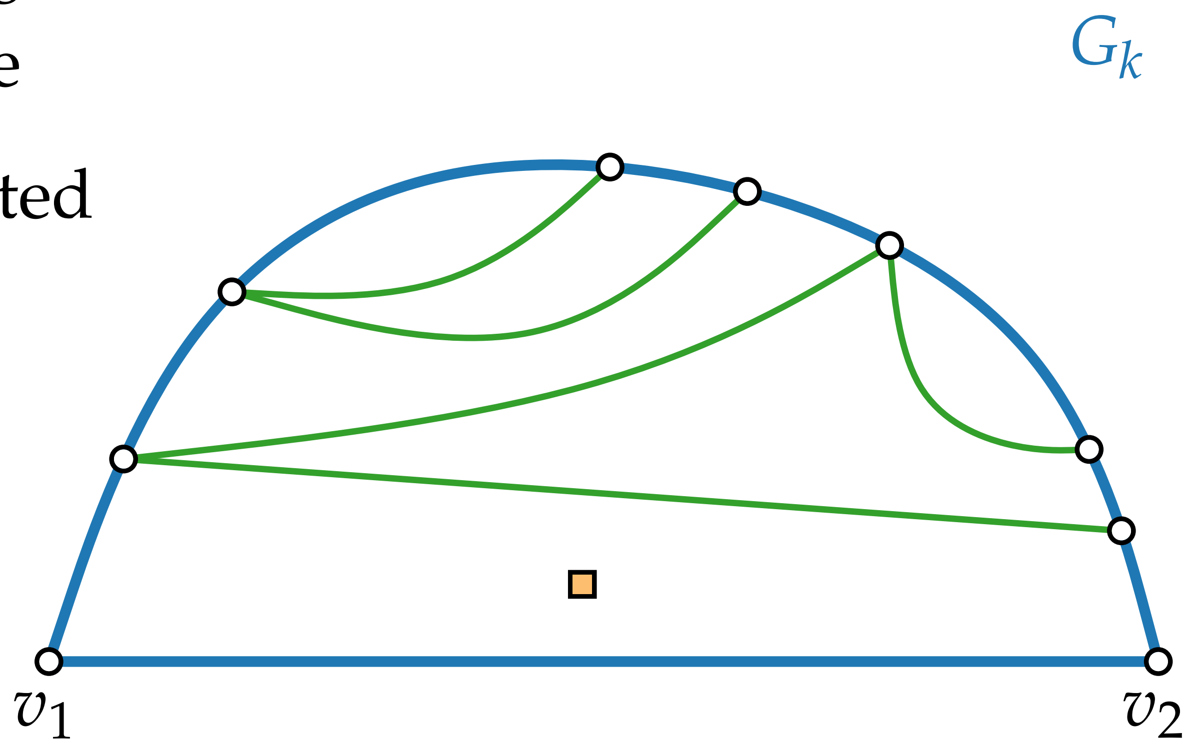

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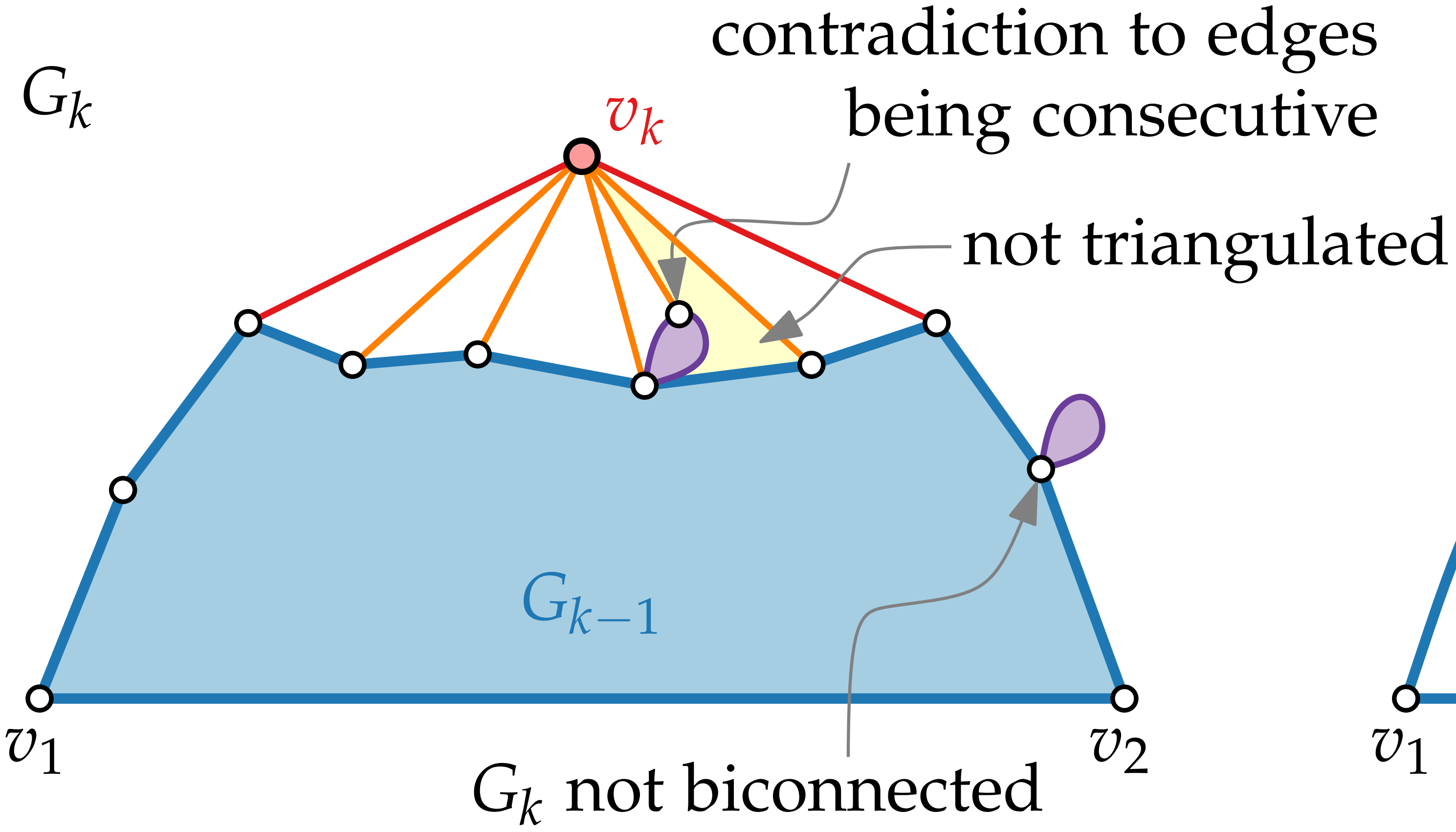

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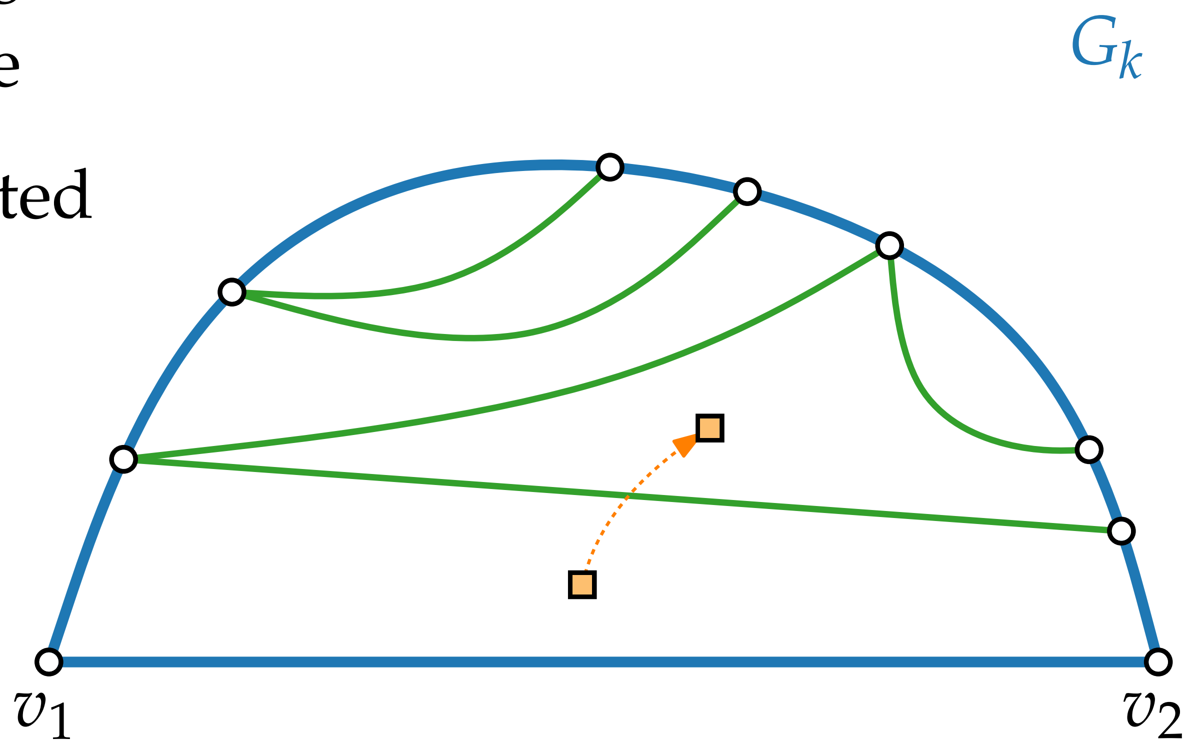

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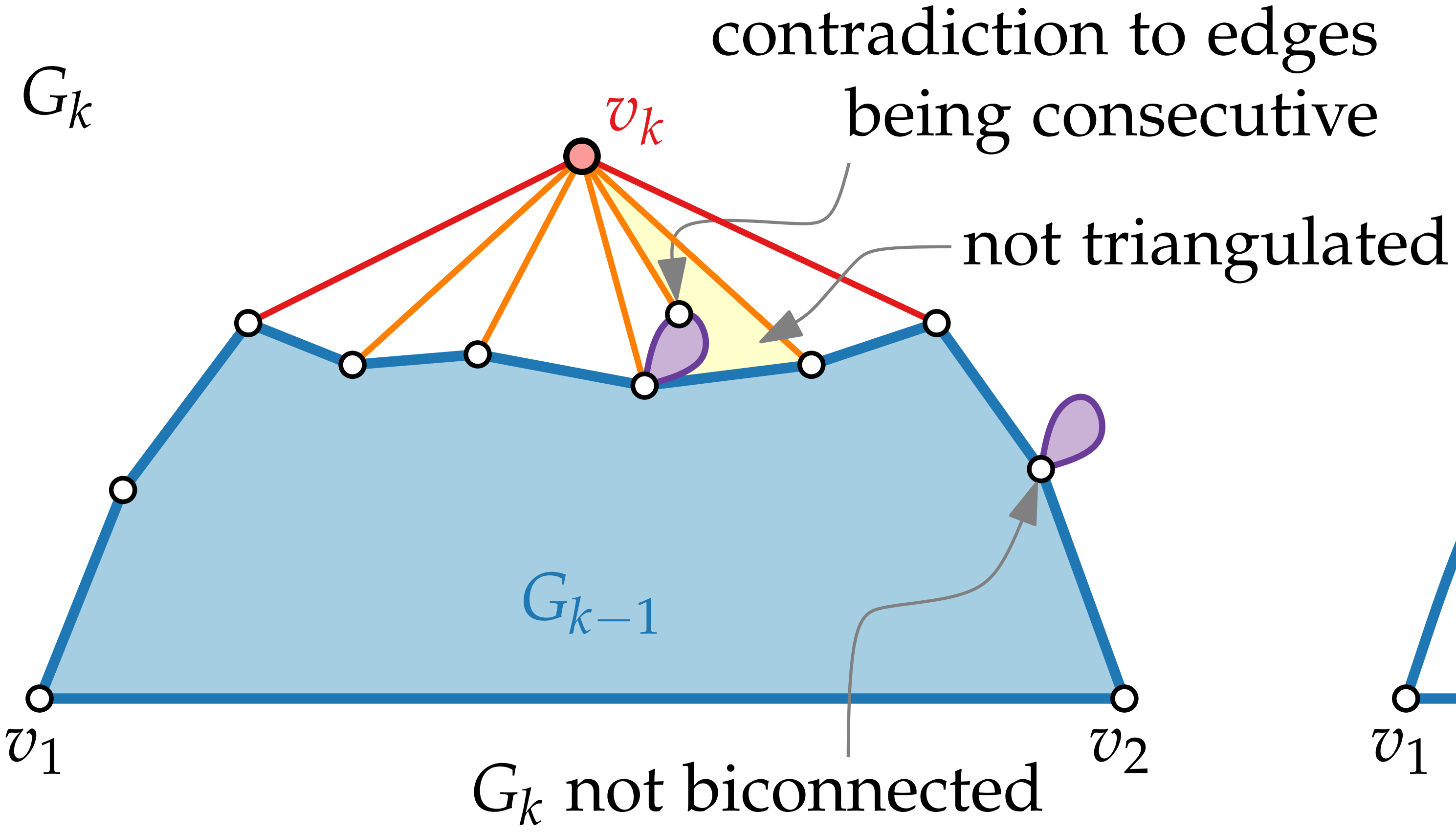

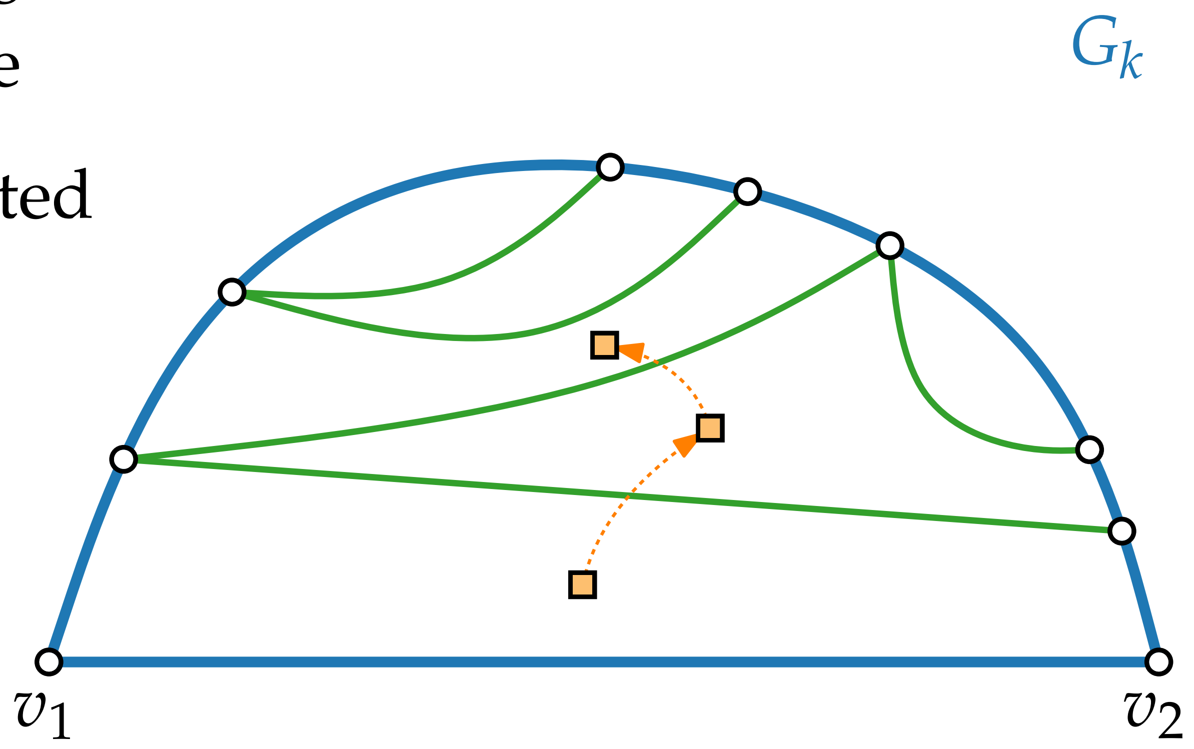

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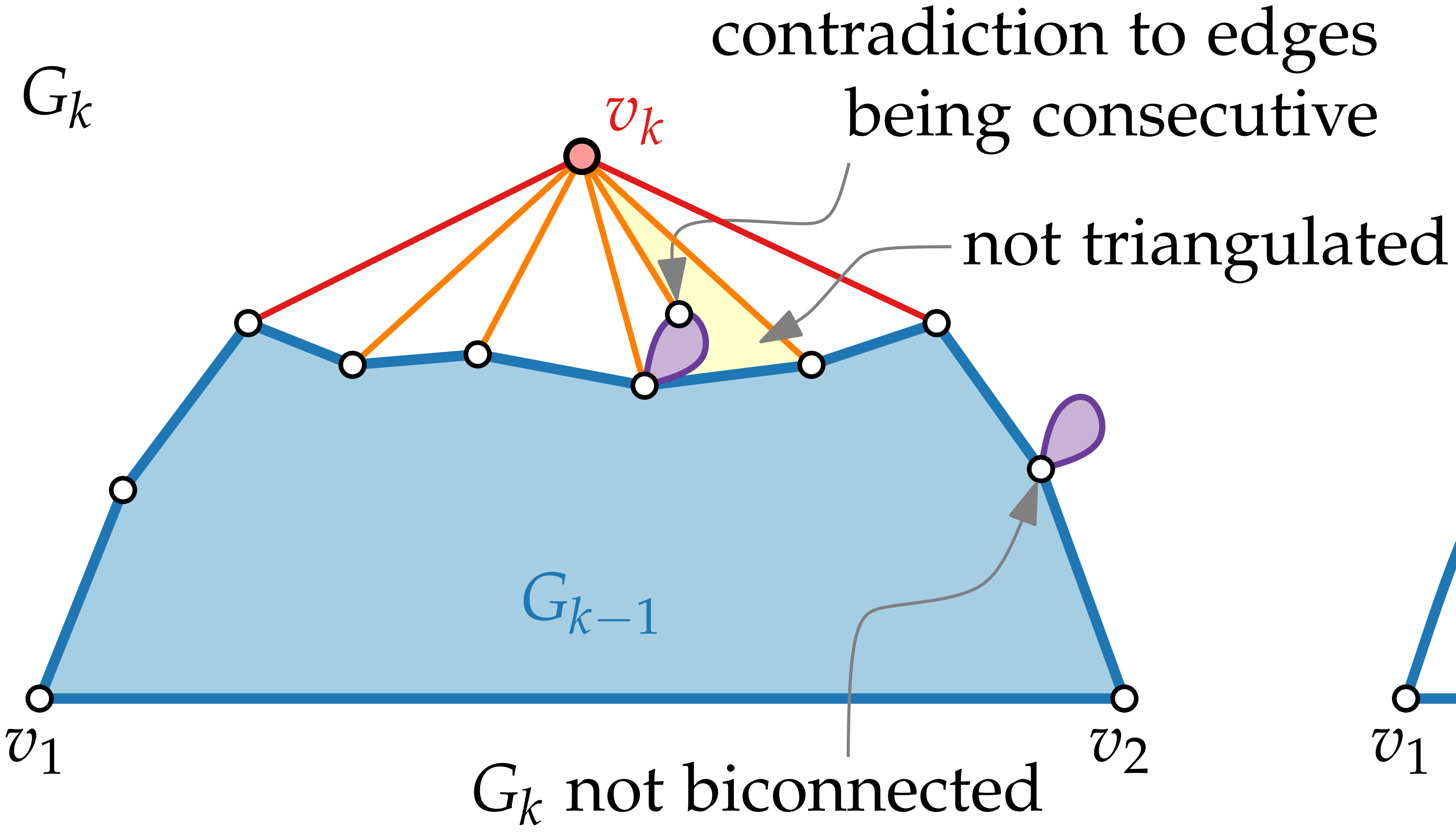

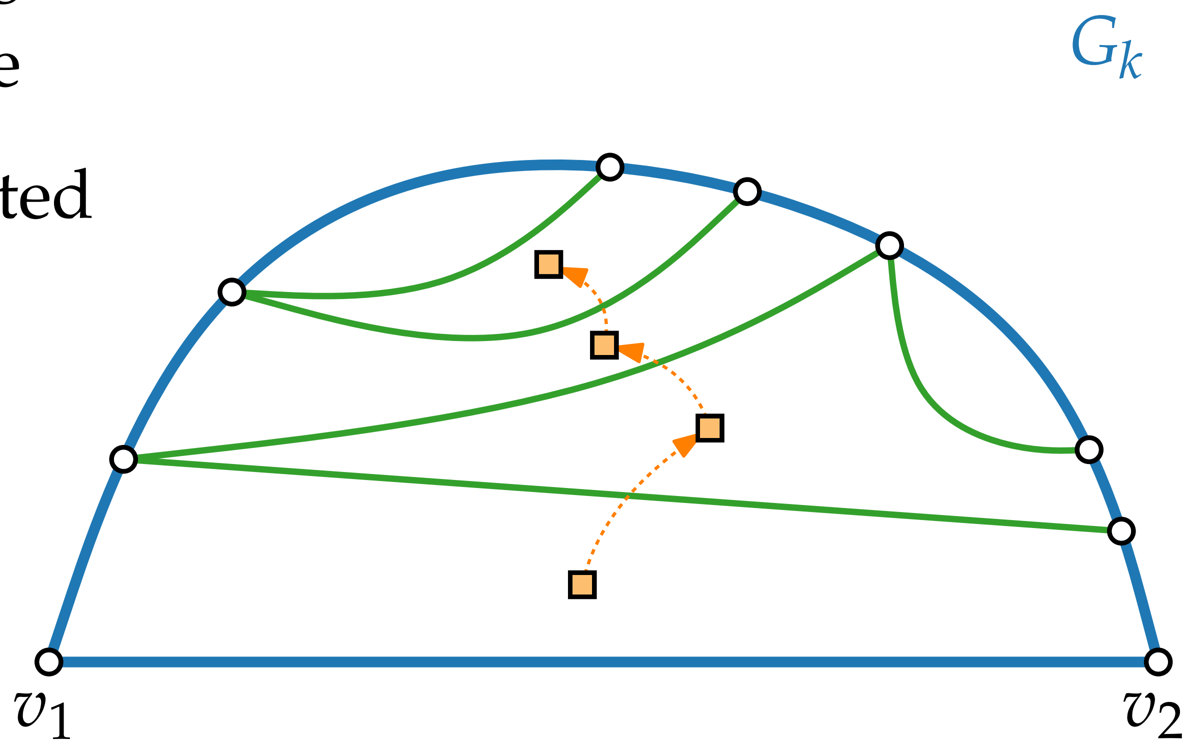

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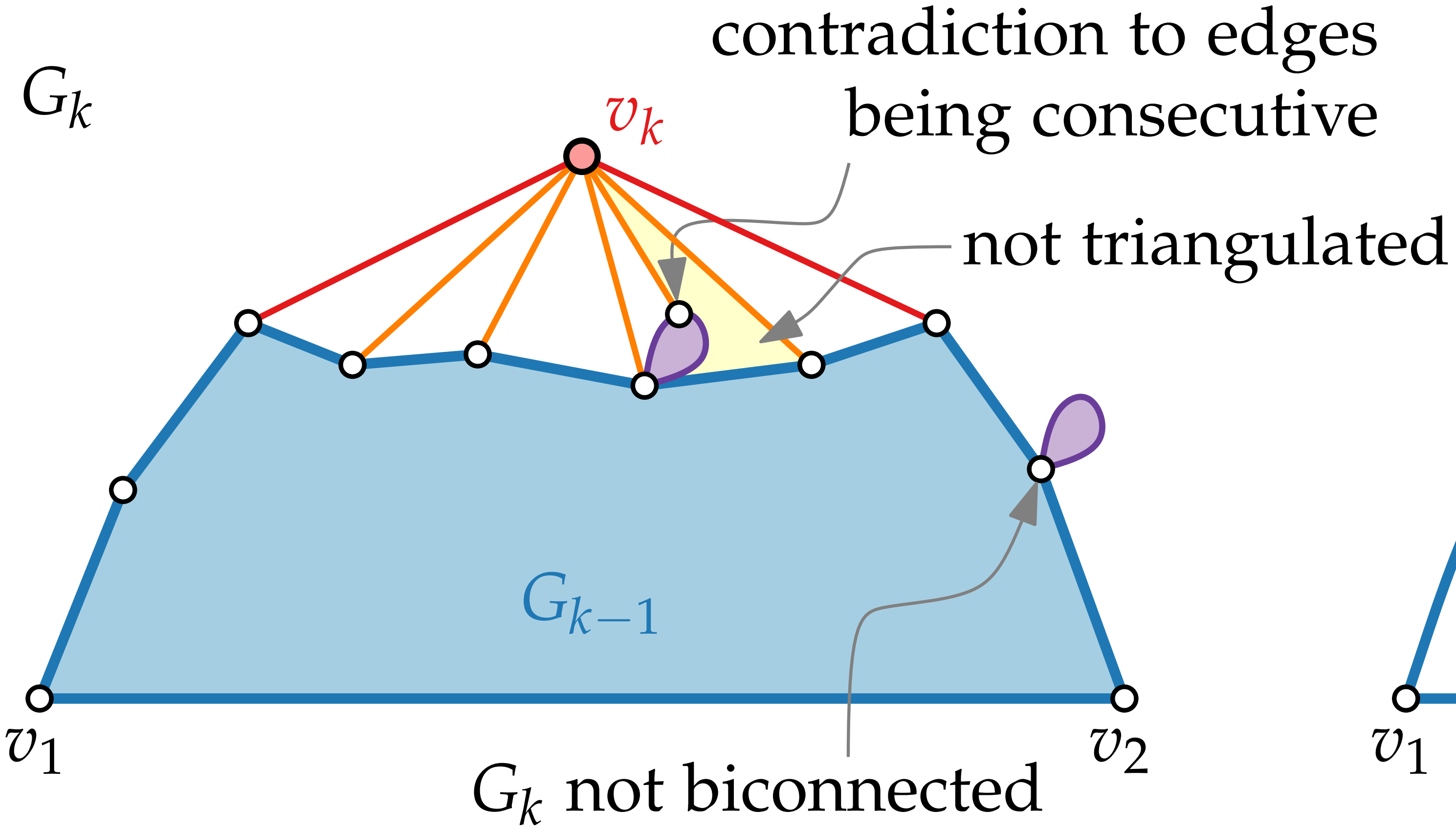

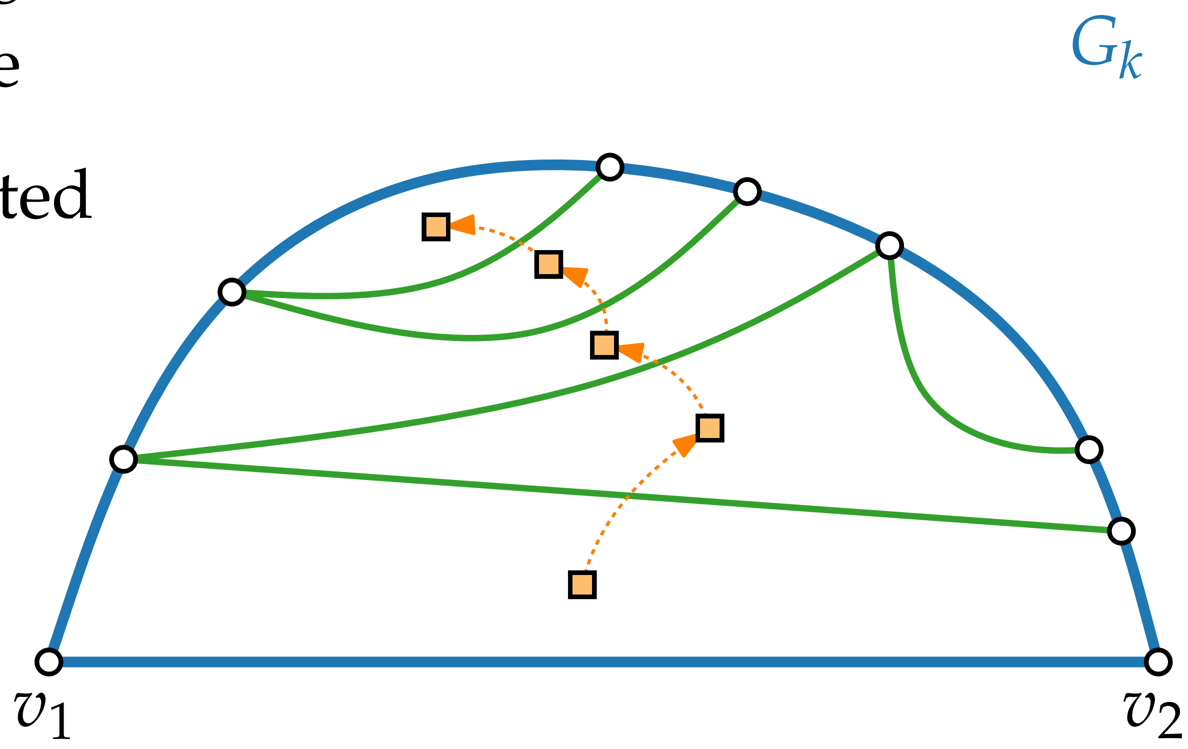

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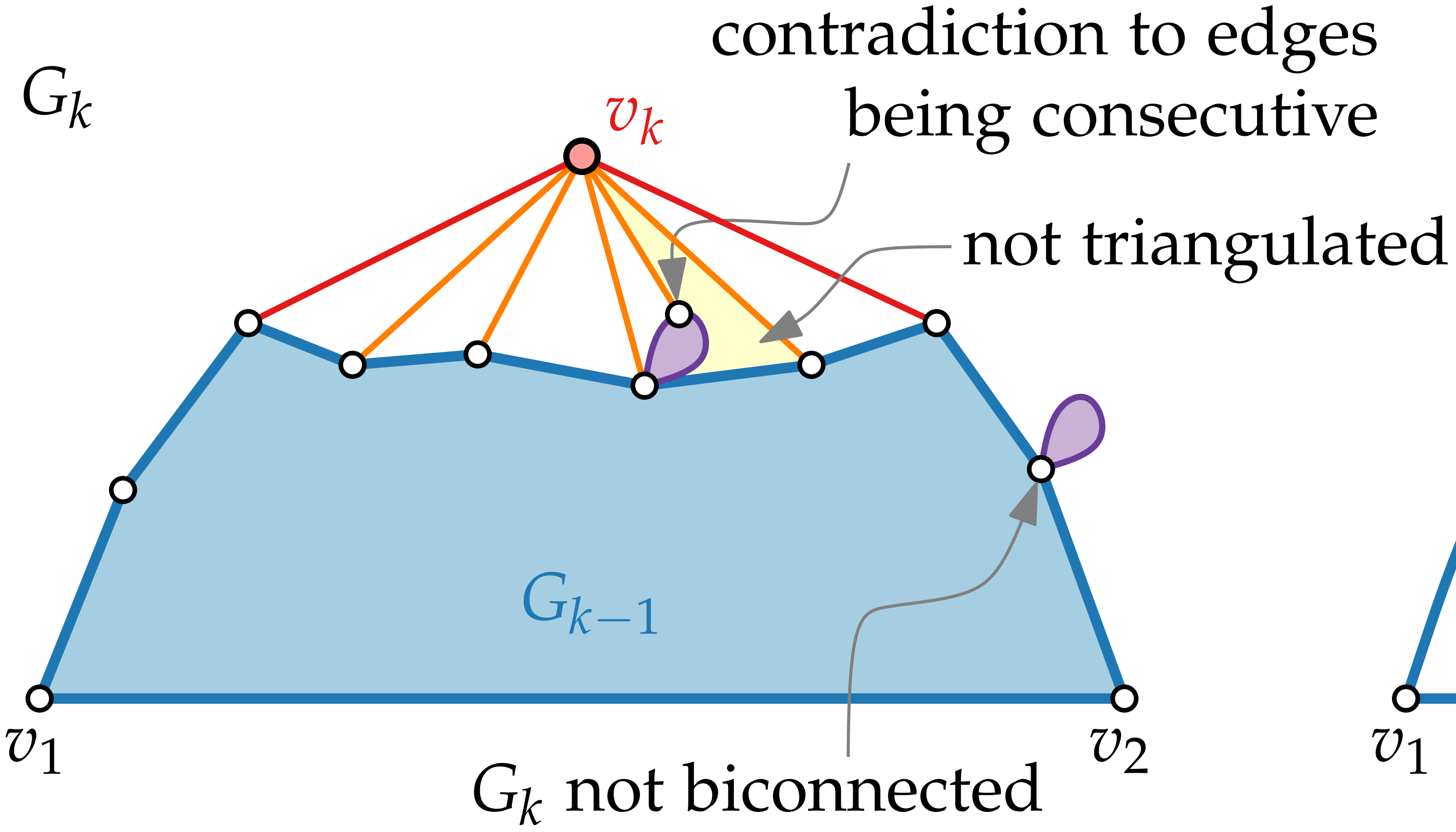

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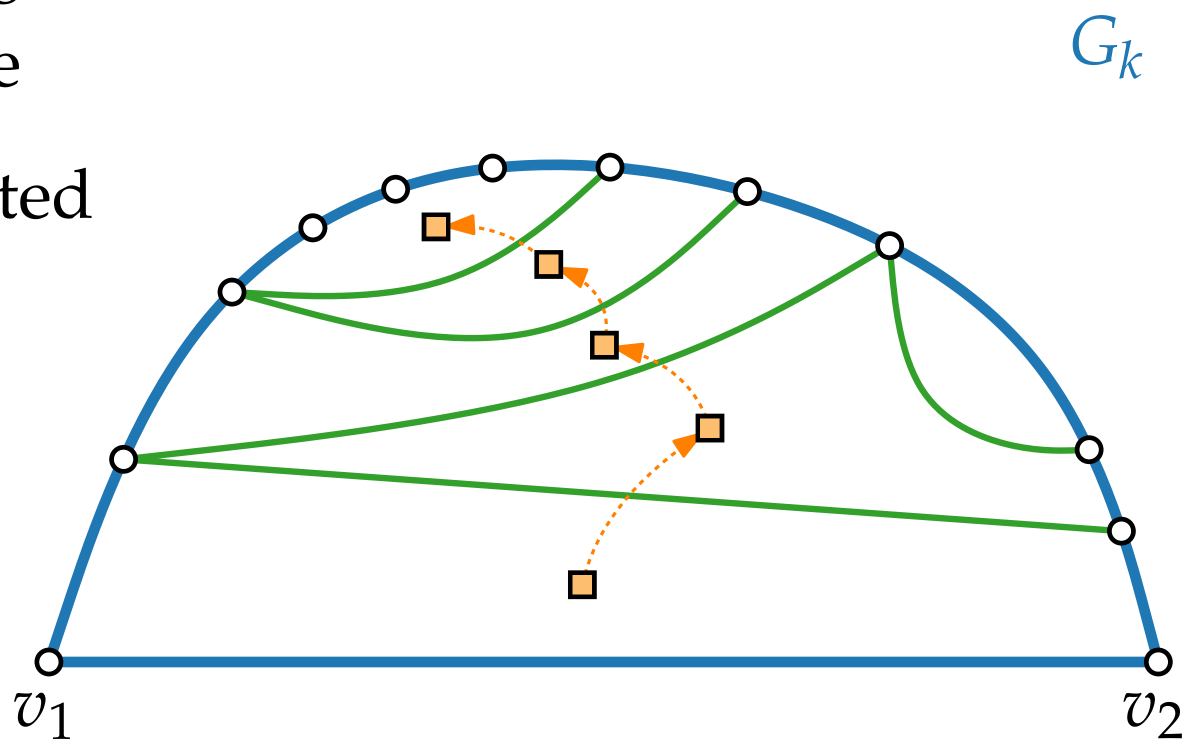

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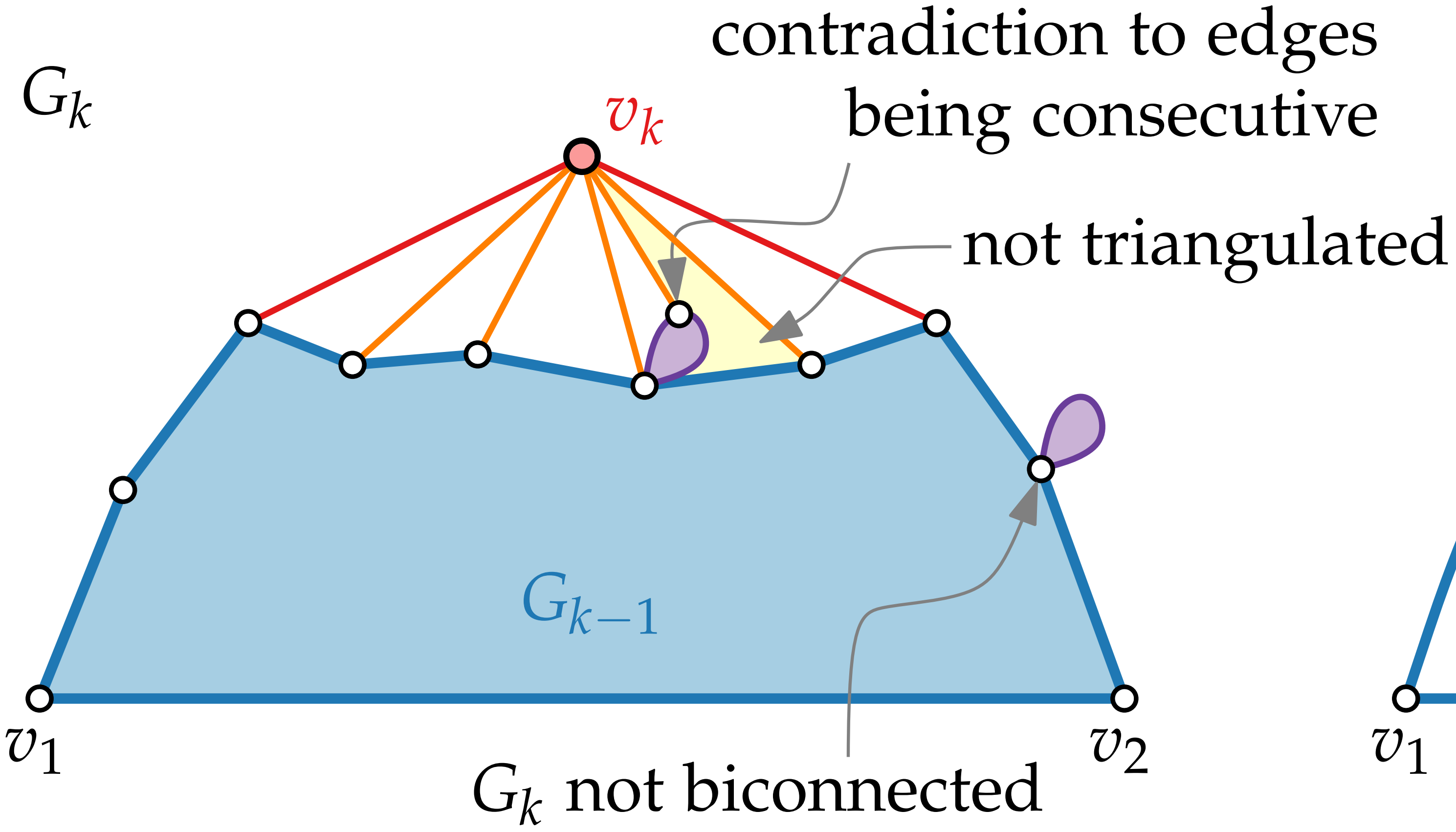

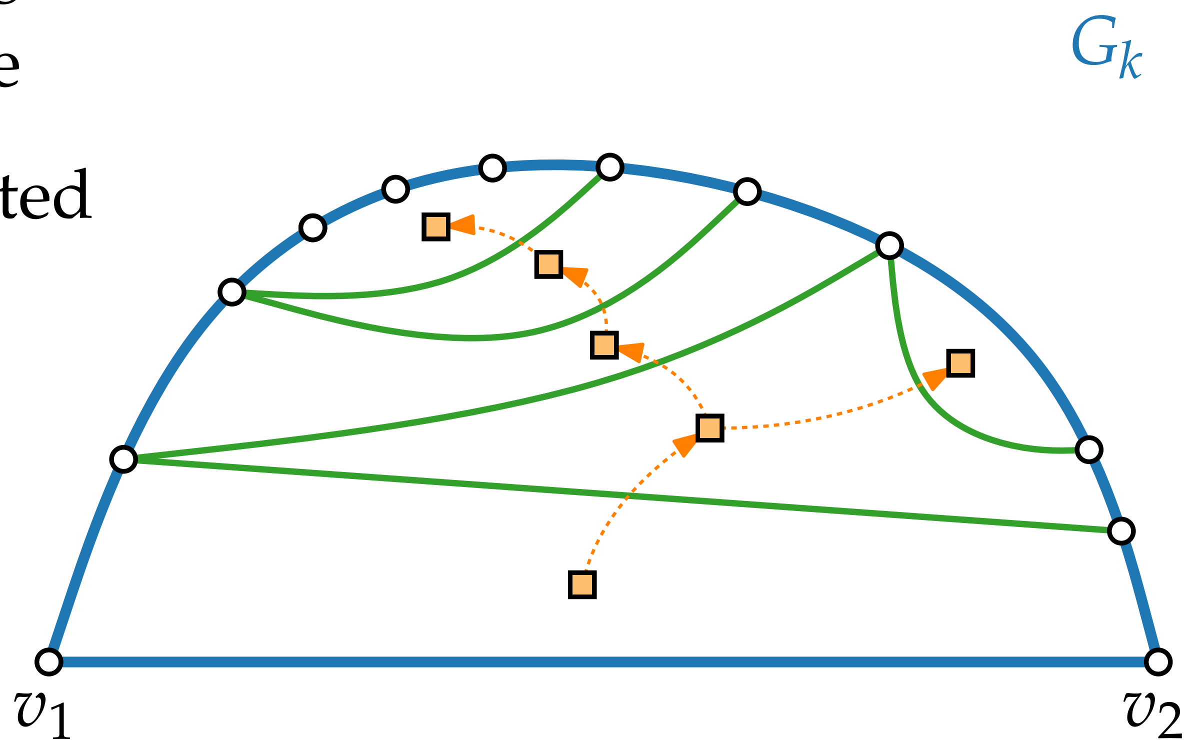

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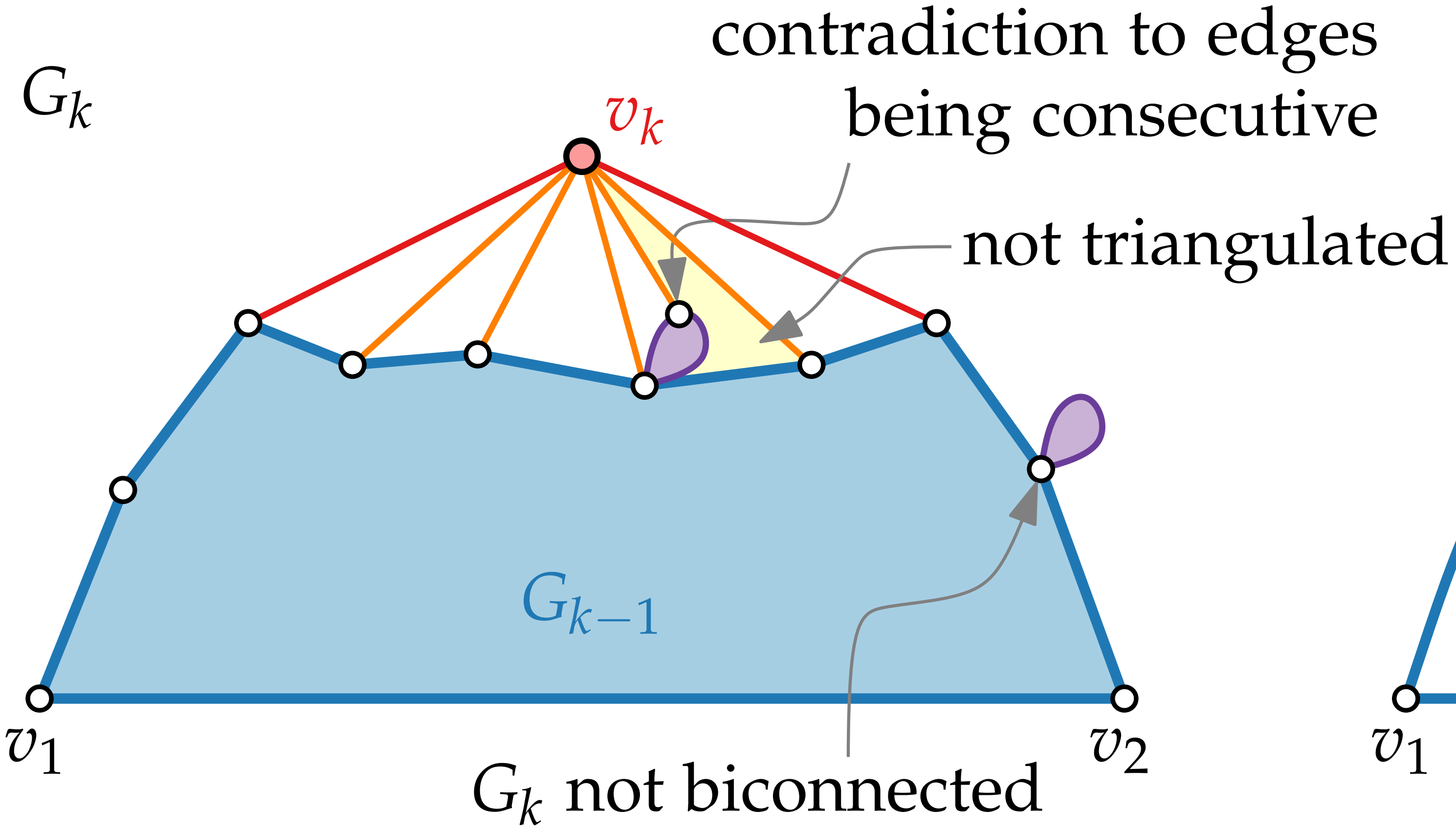

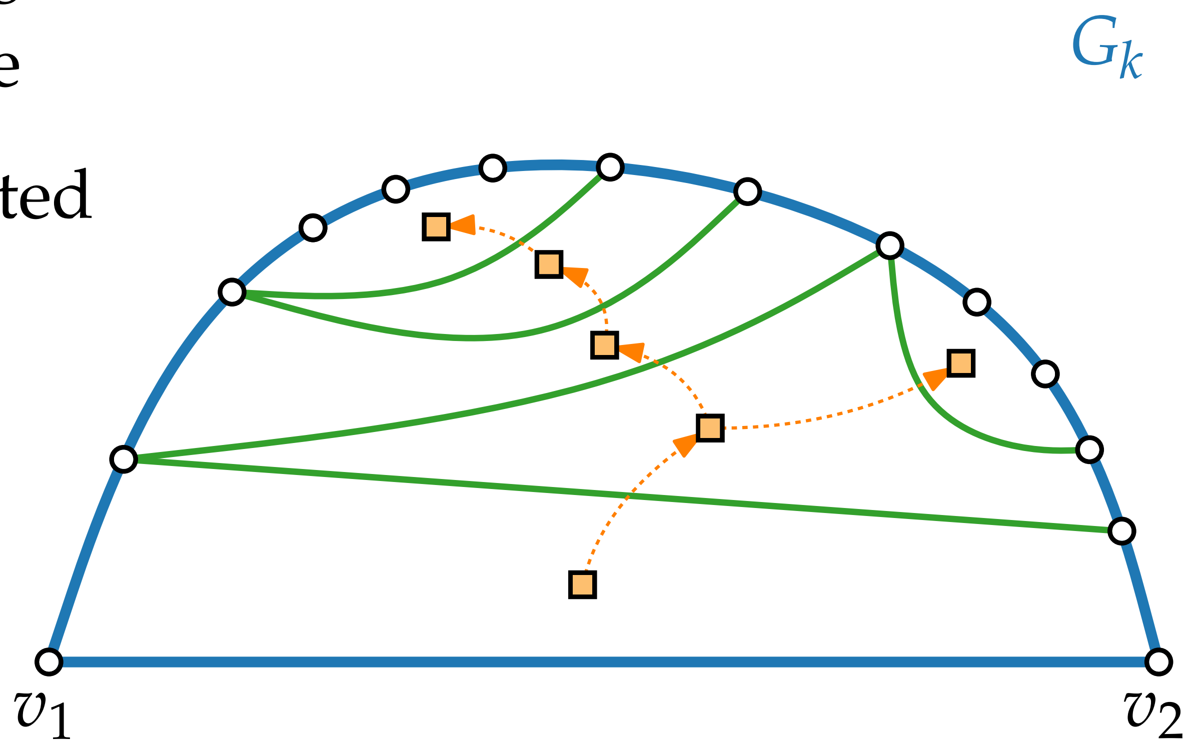

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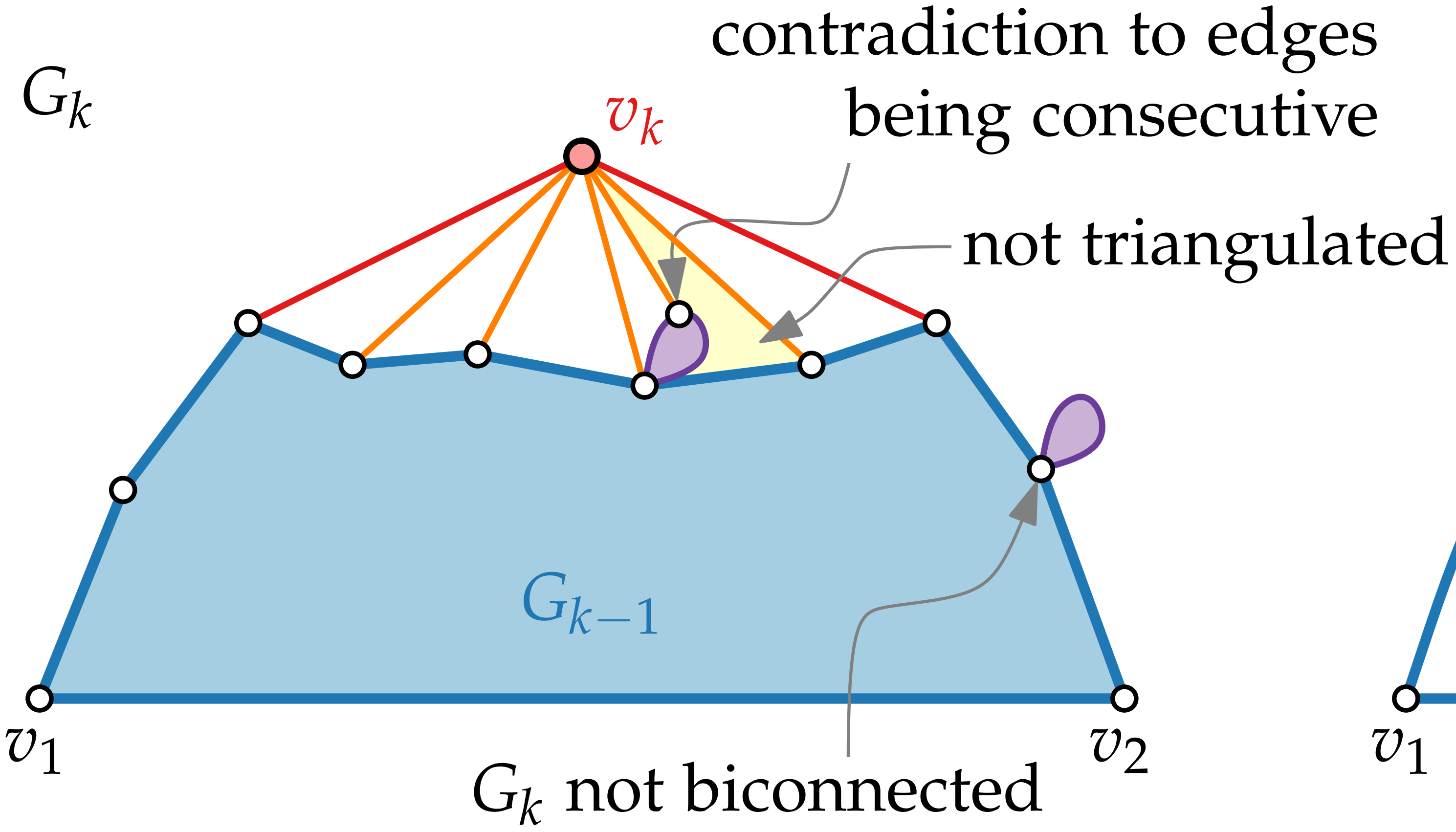

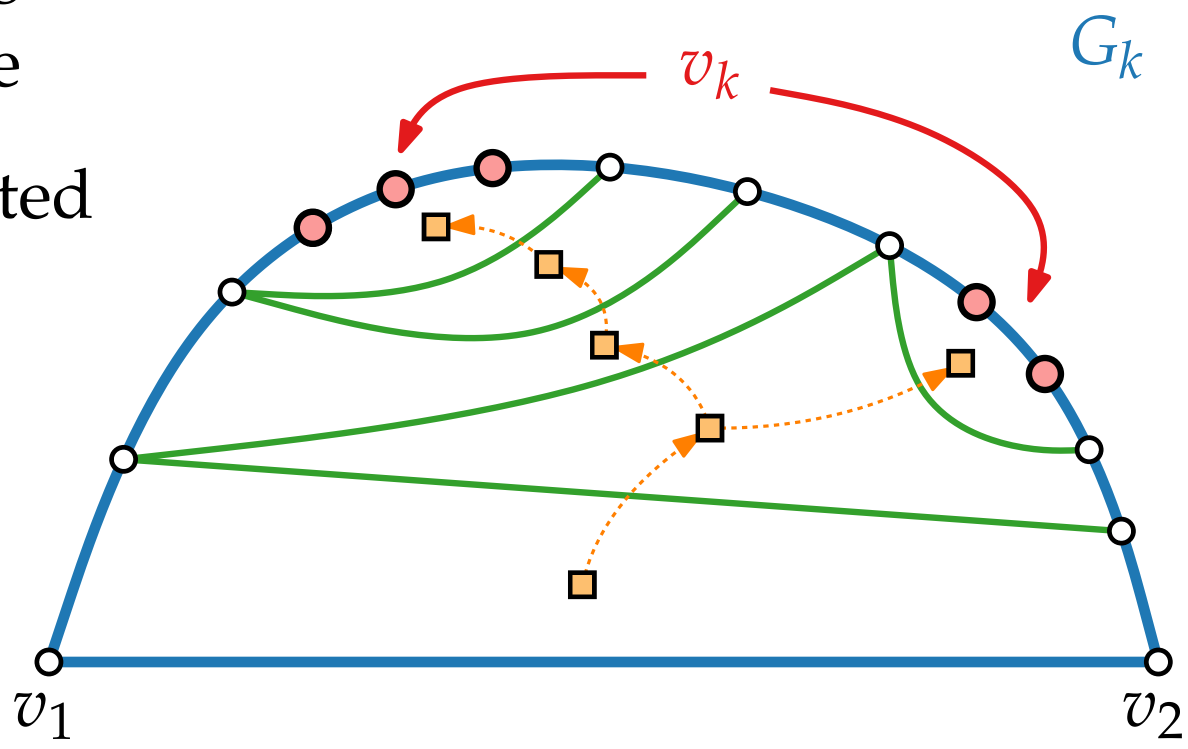

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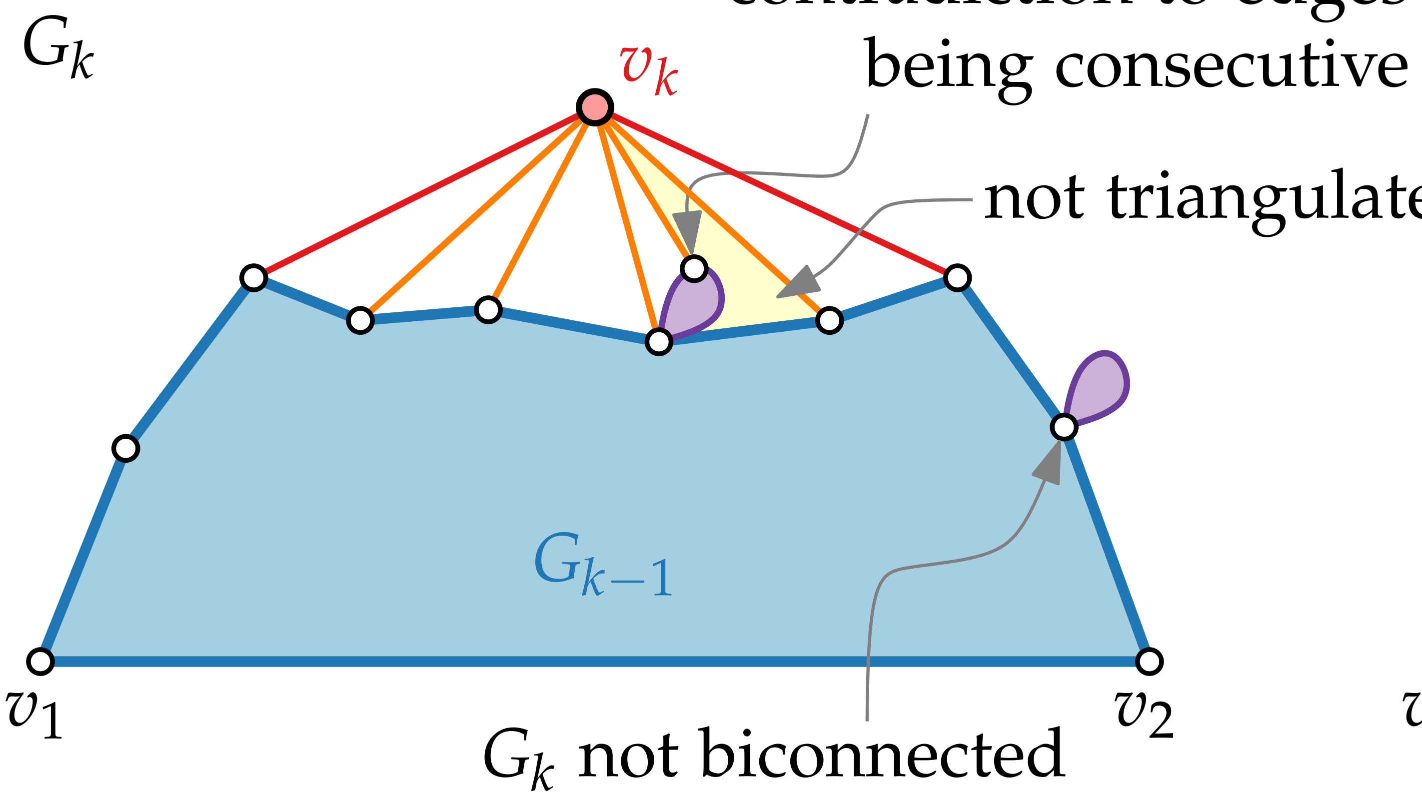

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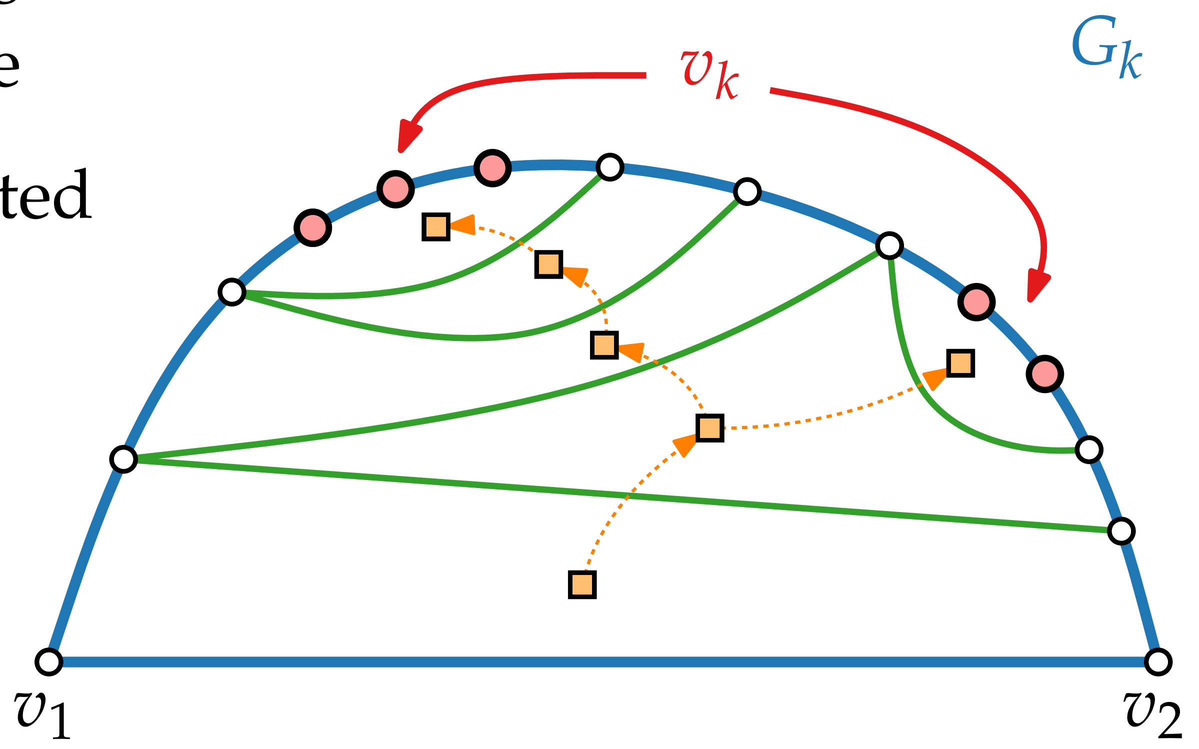

This completes proof of Lemma. □

Canonical Order – Implementation

CanonicalOrder($G = (V, E), (v_1, v_2, v_n)$)

Canonical Order – Implementation

outer face

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forall $v \in V$ **do**

Canonical Order – Implementation

outer face

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forall $v \in V$ **do**
  chords(v) $\leftarrow 0$;

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outer face

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Canonical Order – Implementation

outer face

CanonicalOrder($G = (V, E), (v_1, v_2, v_n)$)

forall $v \in V$ **do**

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- chord(v): # chords adjacent to v

Canonical Order – Implementation

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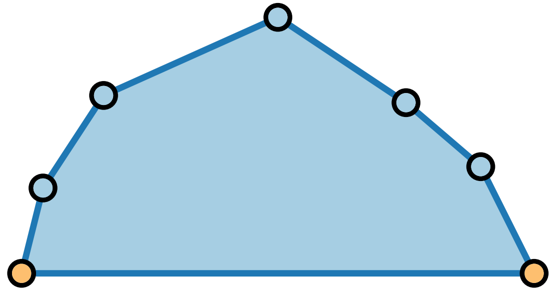

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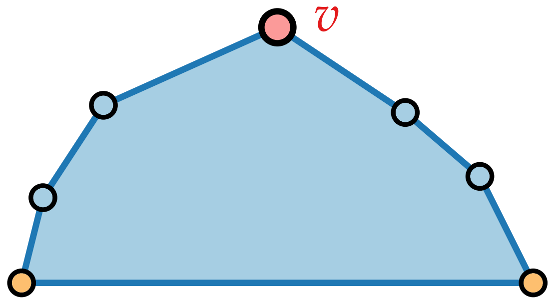

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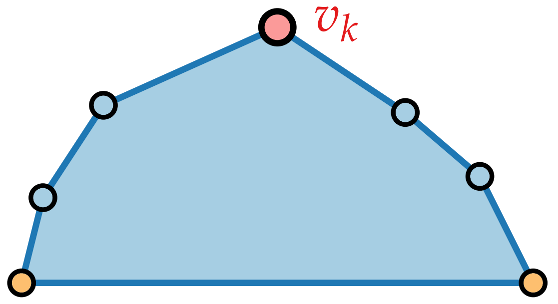

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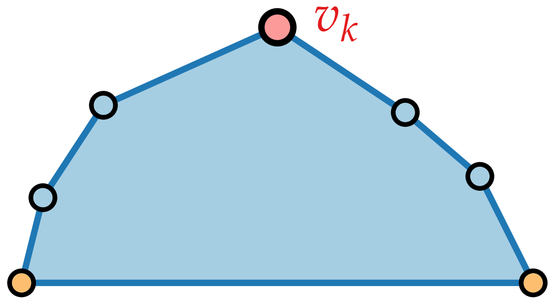

Canonical Order – Implementation

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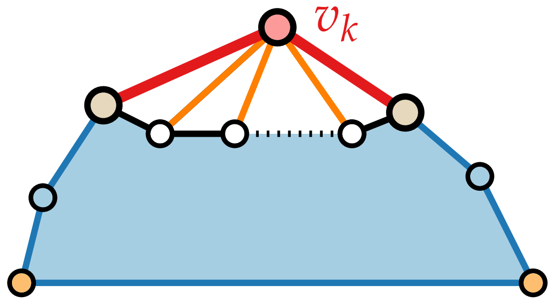

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outer face

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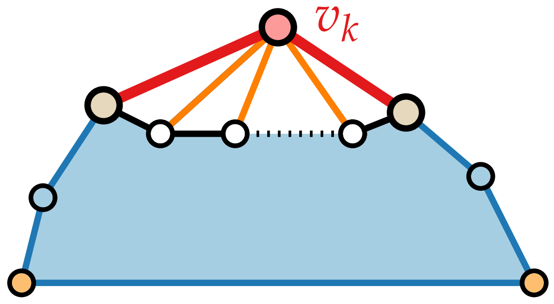

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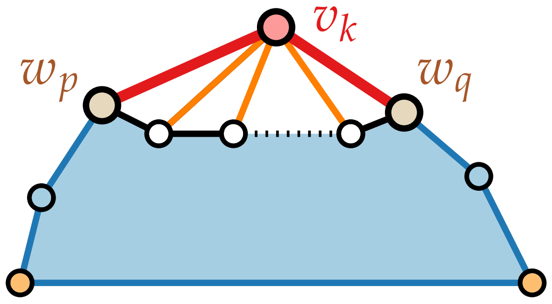

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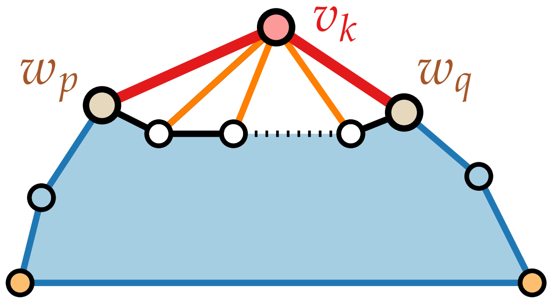

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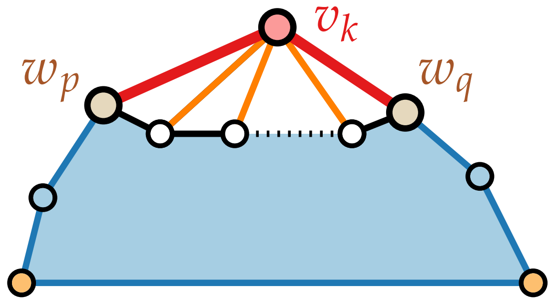

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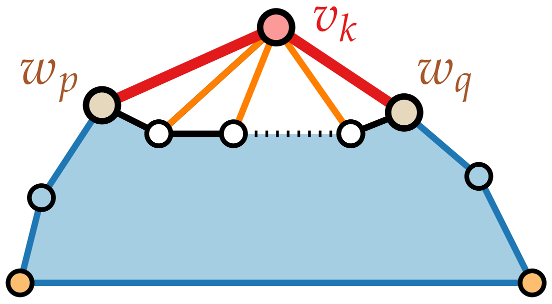

Lemma.
Algorithm CanonicalOrder computes a canonical order of a plane graph in $\mathcal{O}(n)$ time.

Canonical Order – Implementation

outer face

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for $k = n$ **to** 3 **do**
  choose v such that mark(v) = false, out(v) = true,
   and chords(v) = 0 *// keep list with candidates*
  $v_k \leftarrow v$; mark$(v) \leftarrow$ true
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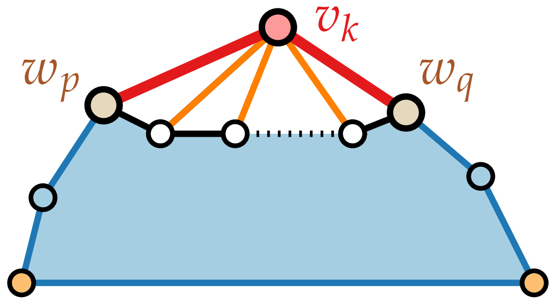

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 out(w_i) $\leftarrow$ true for all $p < i < q$ *// $O(n)$ in total*

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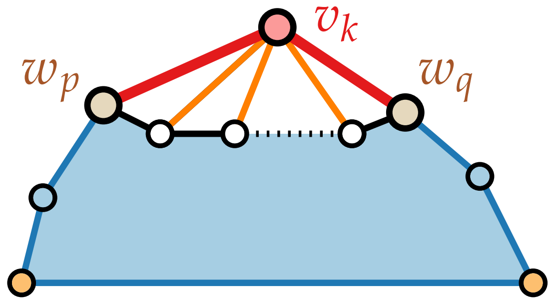

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  out(w_i) $\leftarrow$ true for all $p < i < q$ *// $O(n)$ in total*
  update number of chords for w_i
  and its neighbours   *// $O(m) = O(n)$ in total*

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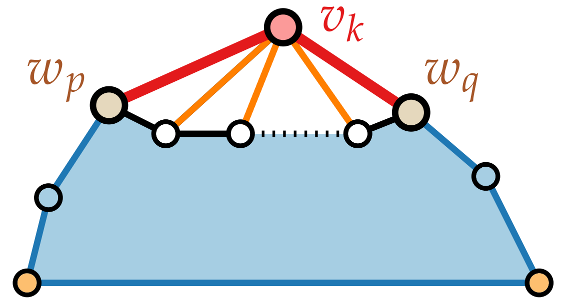

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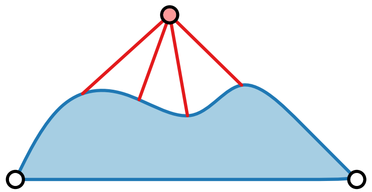
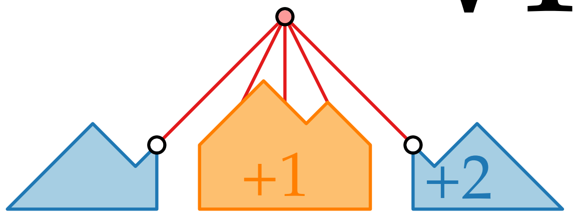
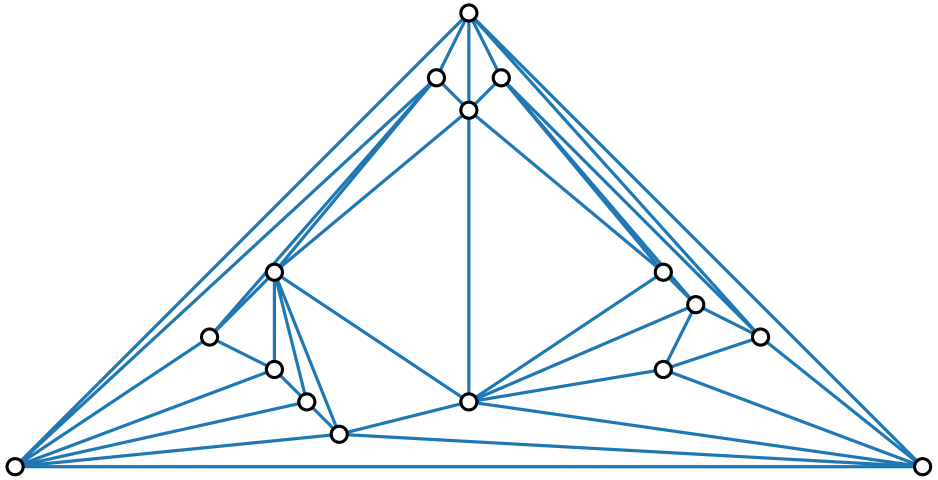


Visualization of Graphs



Lecture 4:
Straight-Line Drawings of Planar Graphs I: Canonical Ordering and Shift Method

Part IV:
Shift Method



Philipp Kindermann

Shift Method – Idea

Drawing invariants:

G_{k-1} is drawn such that

G_{k-1}

Shift Method – Idea

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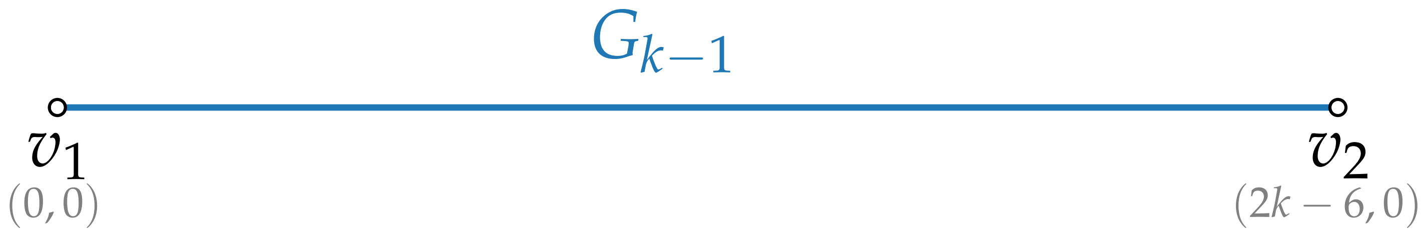

Shift Method – Idea

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G_{k-1} is drawn such that

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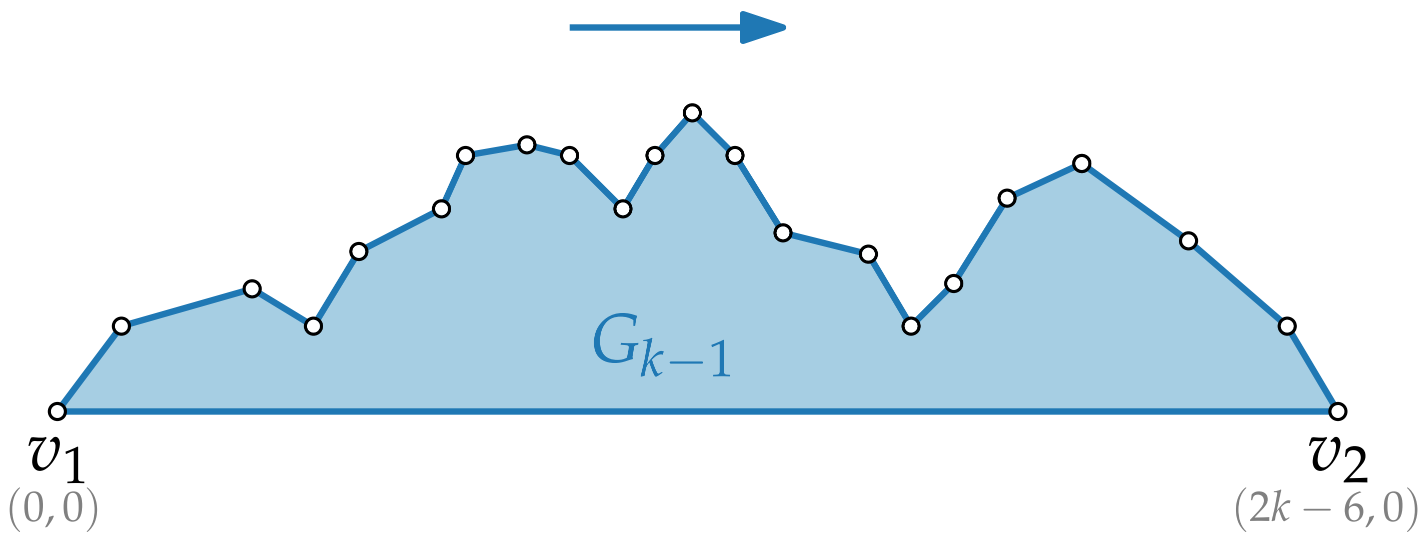

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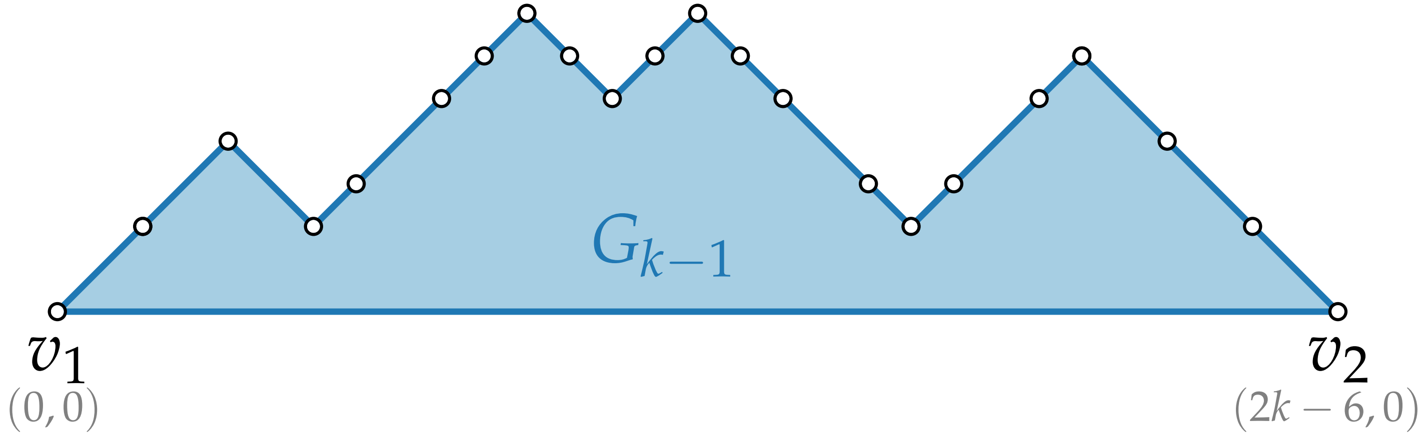

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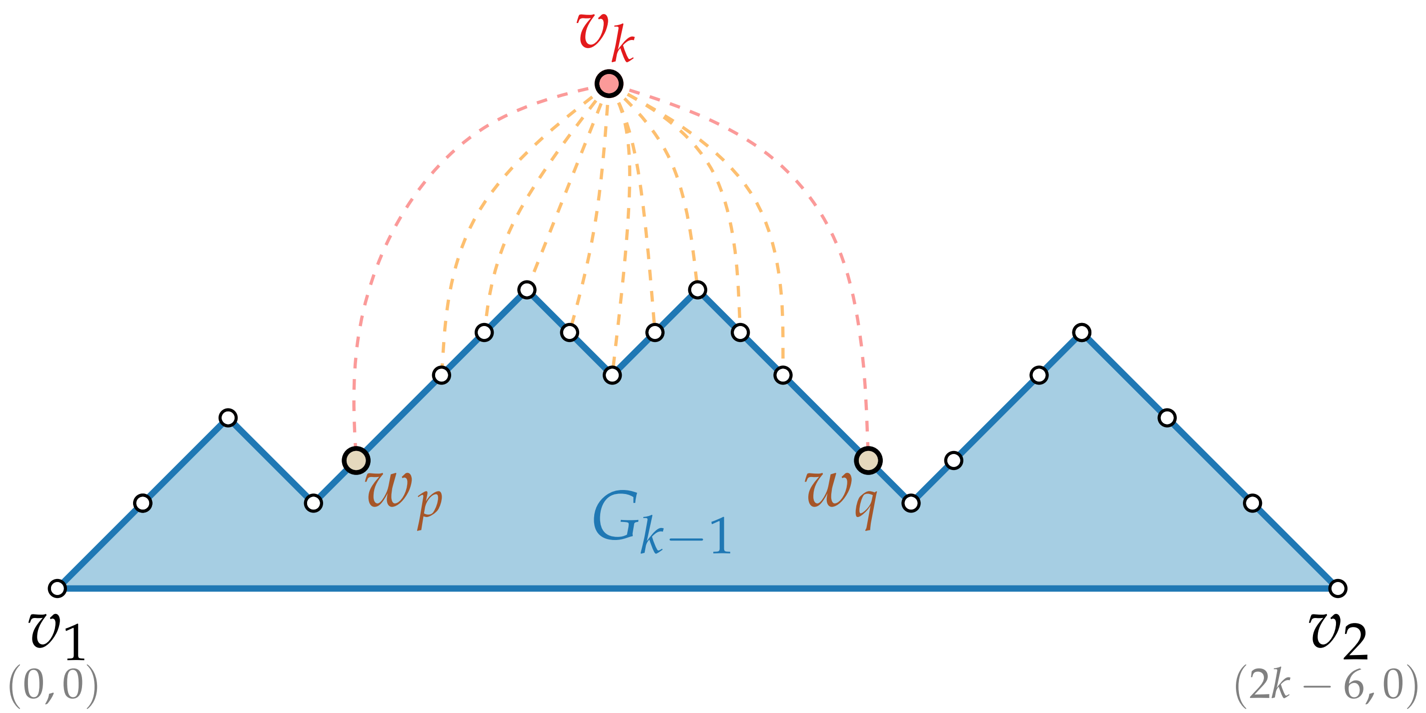

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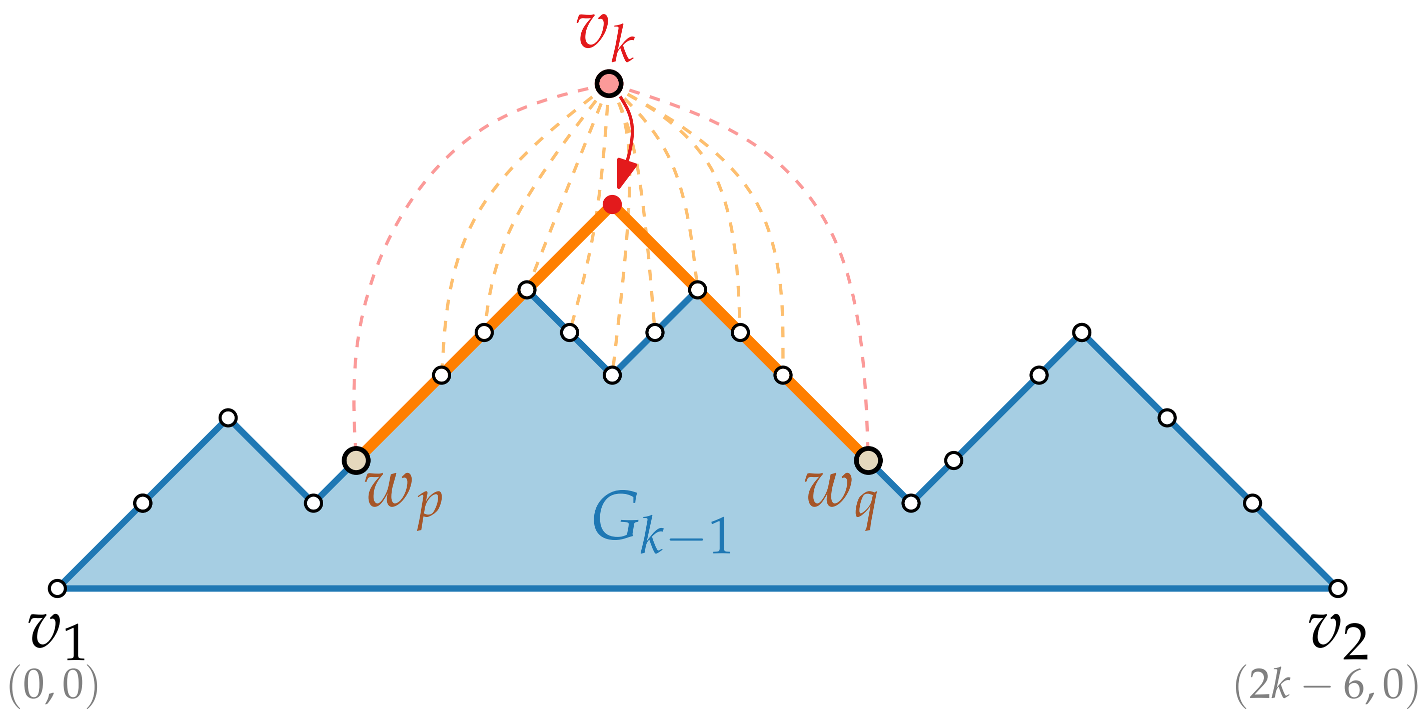

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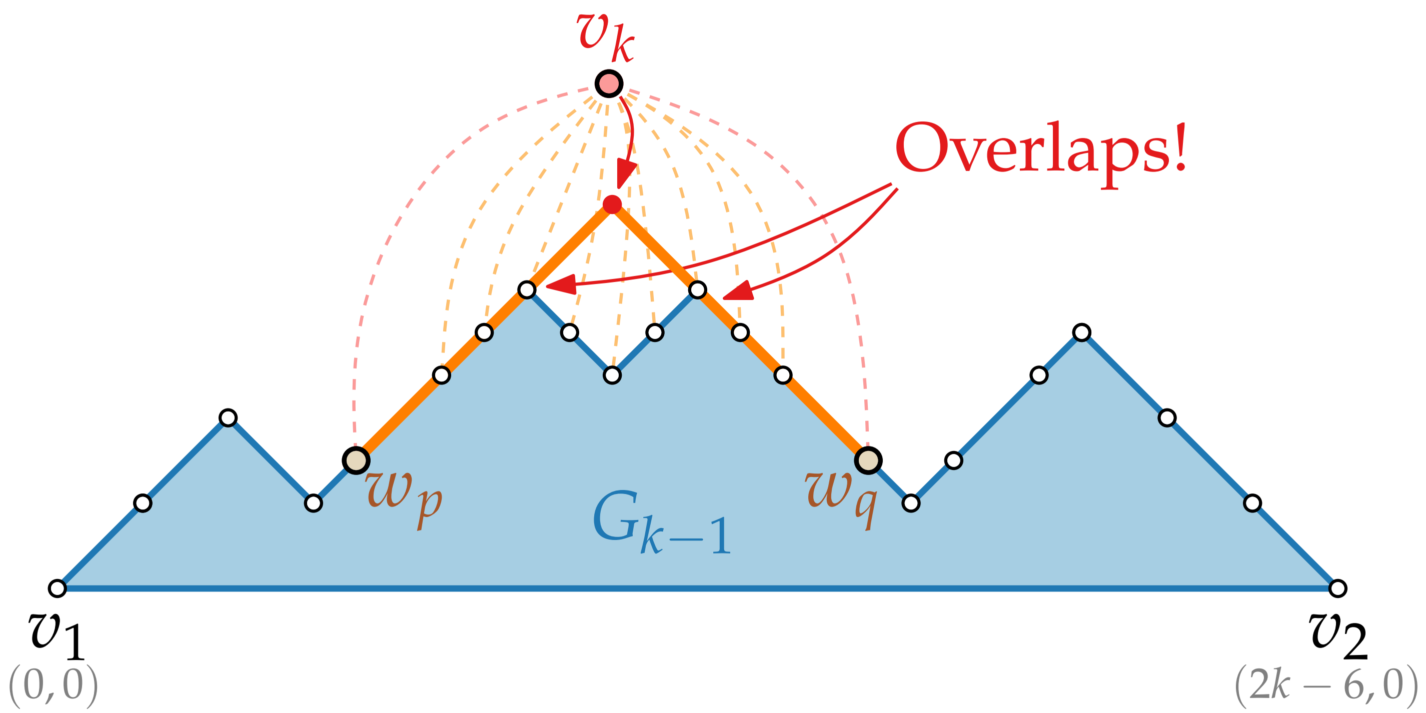

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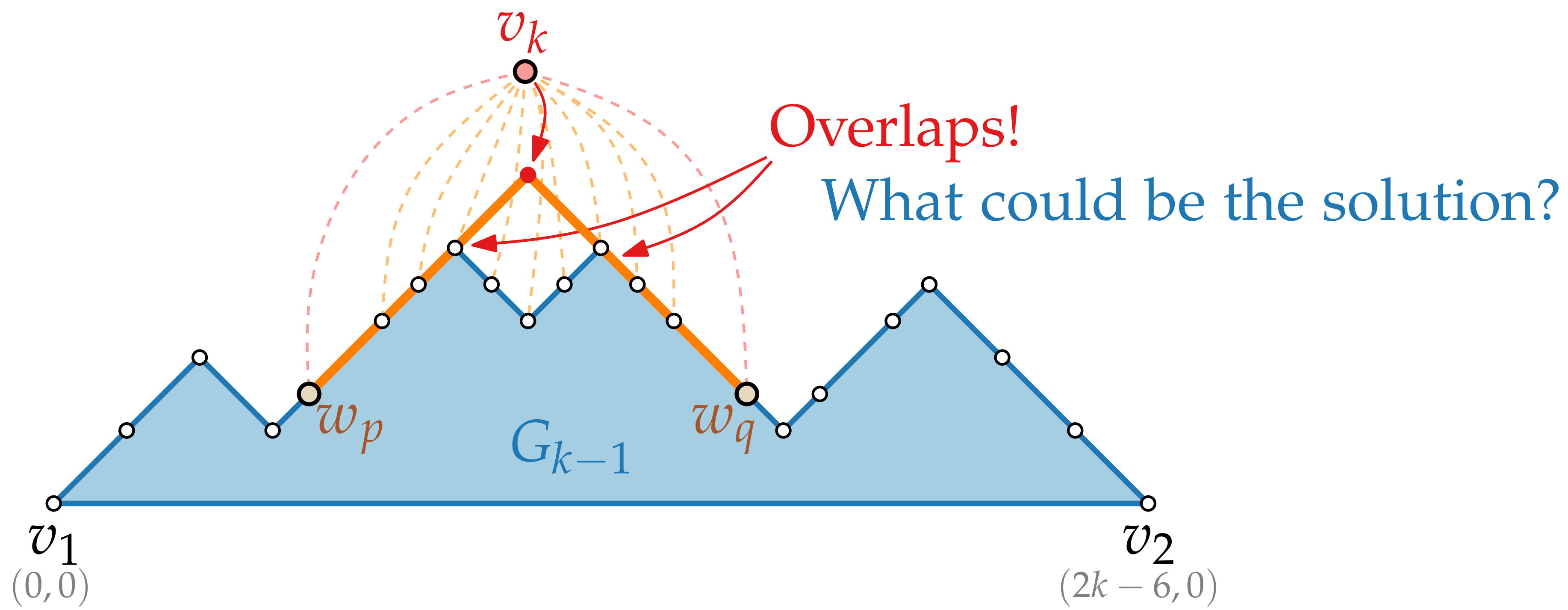

Shift Method – Idea

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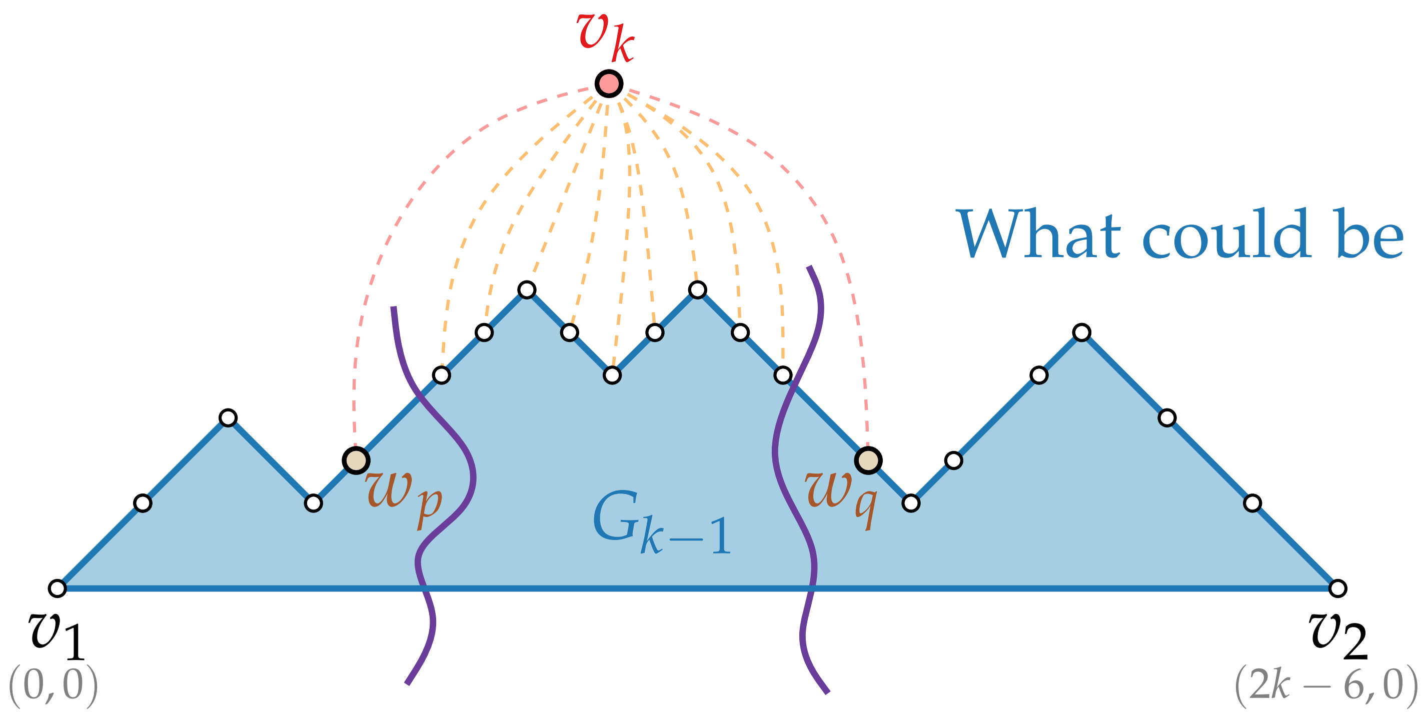

What could be the solution?

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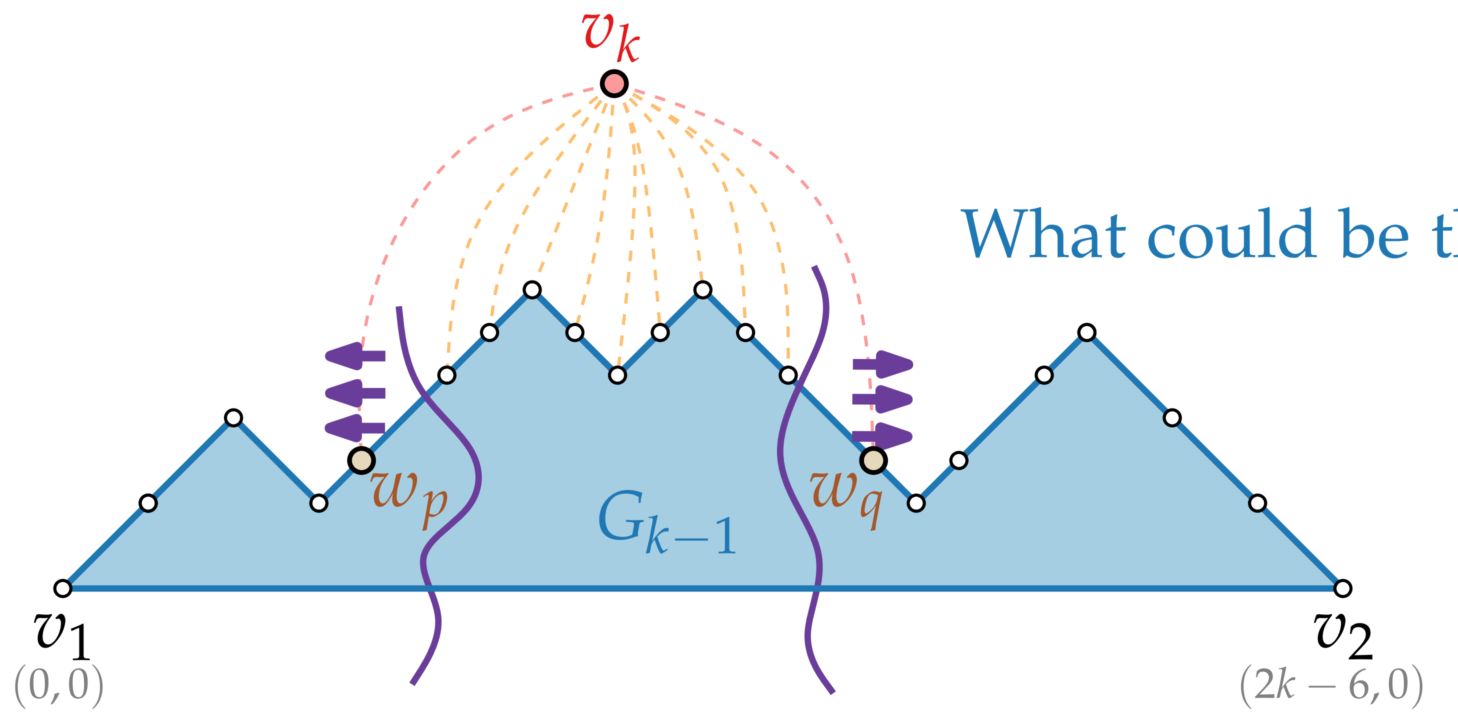

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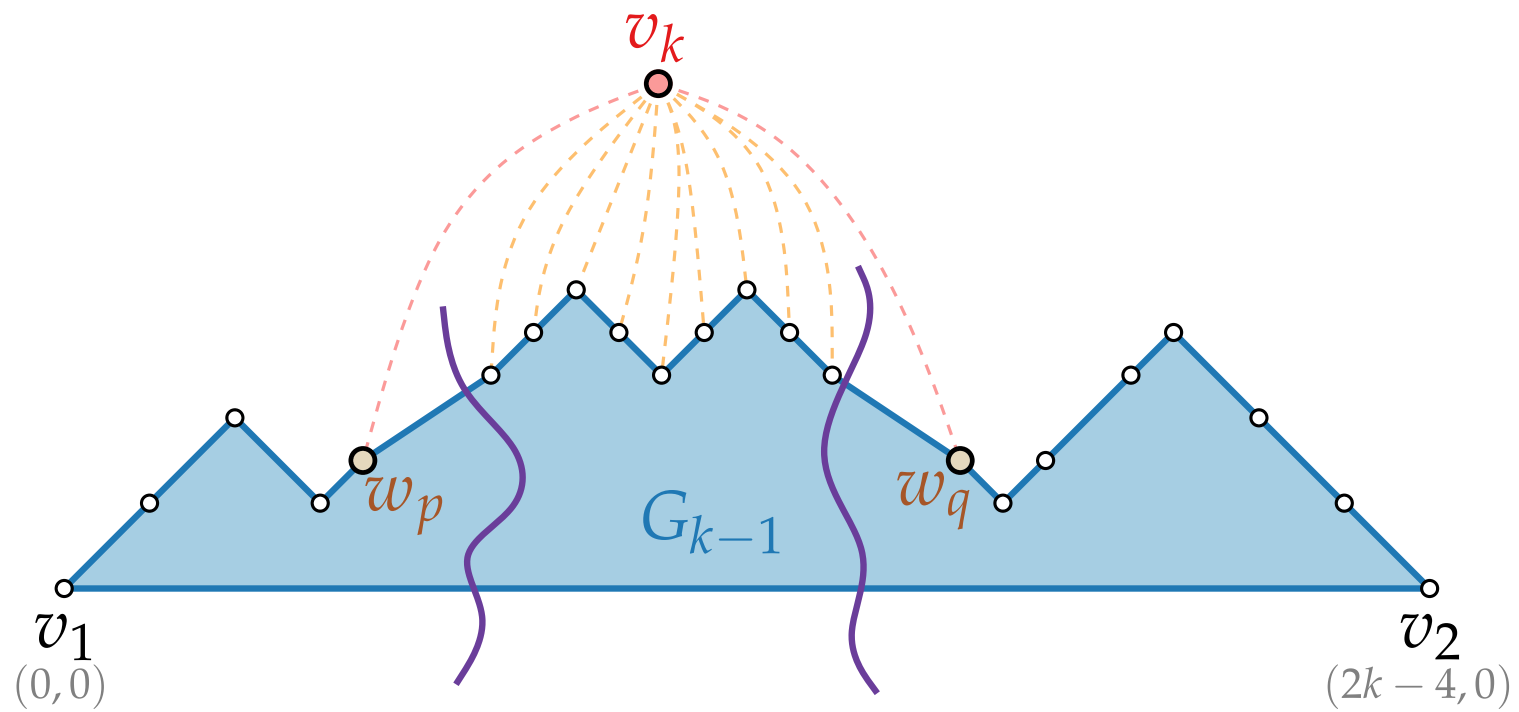

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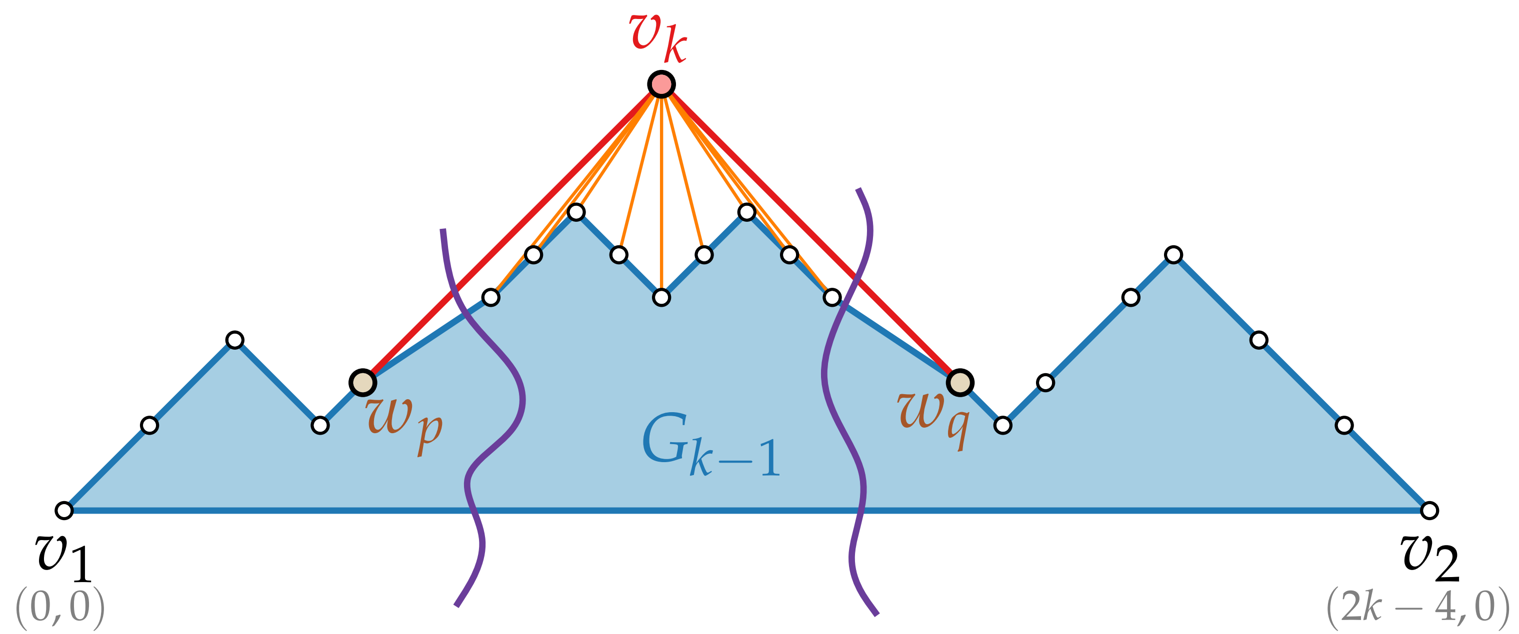

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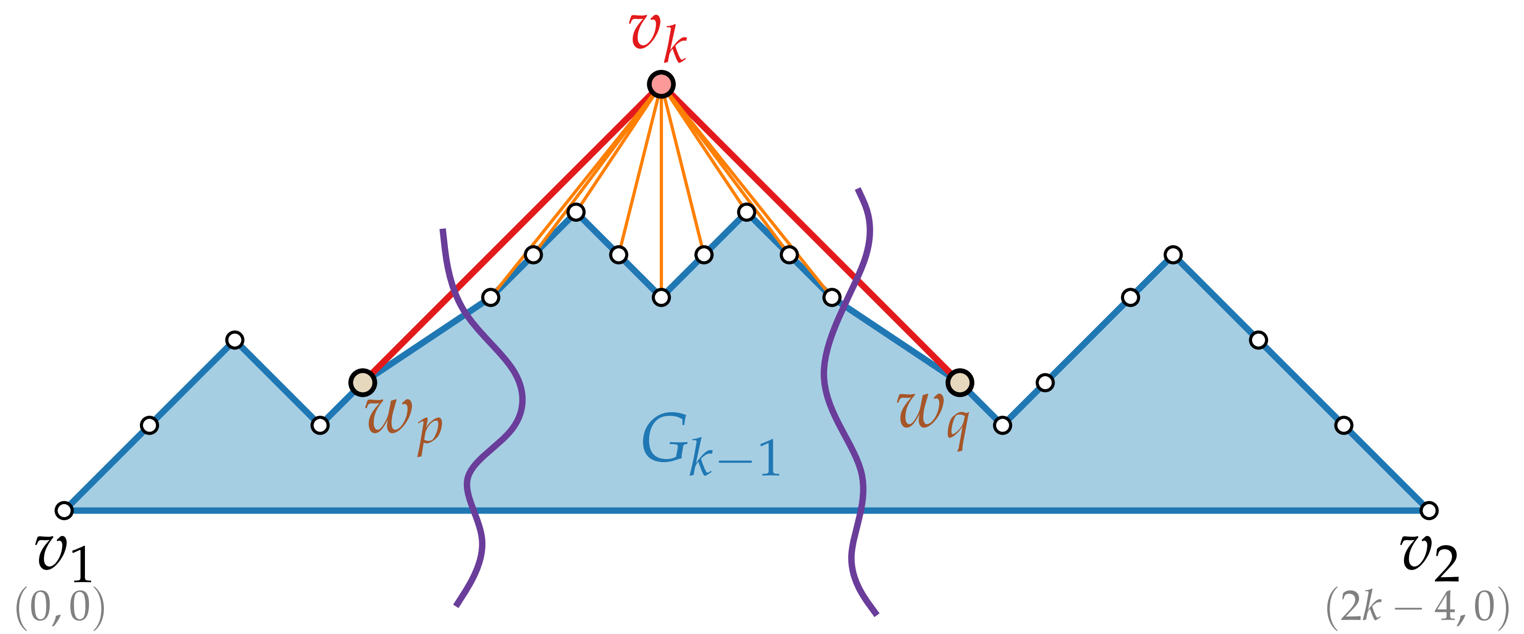

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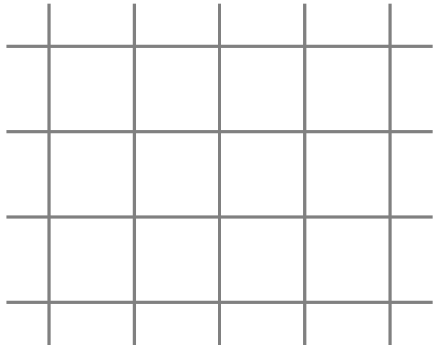

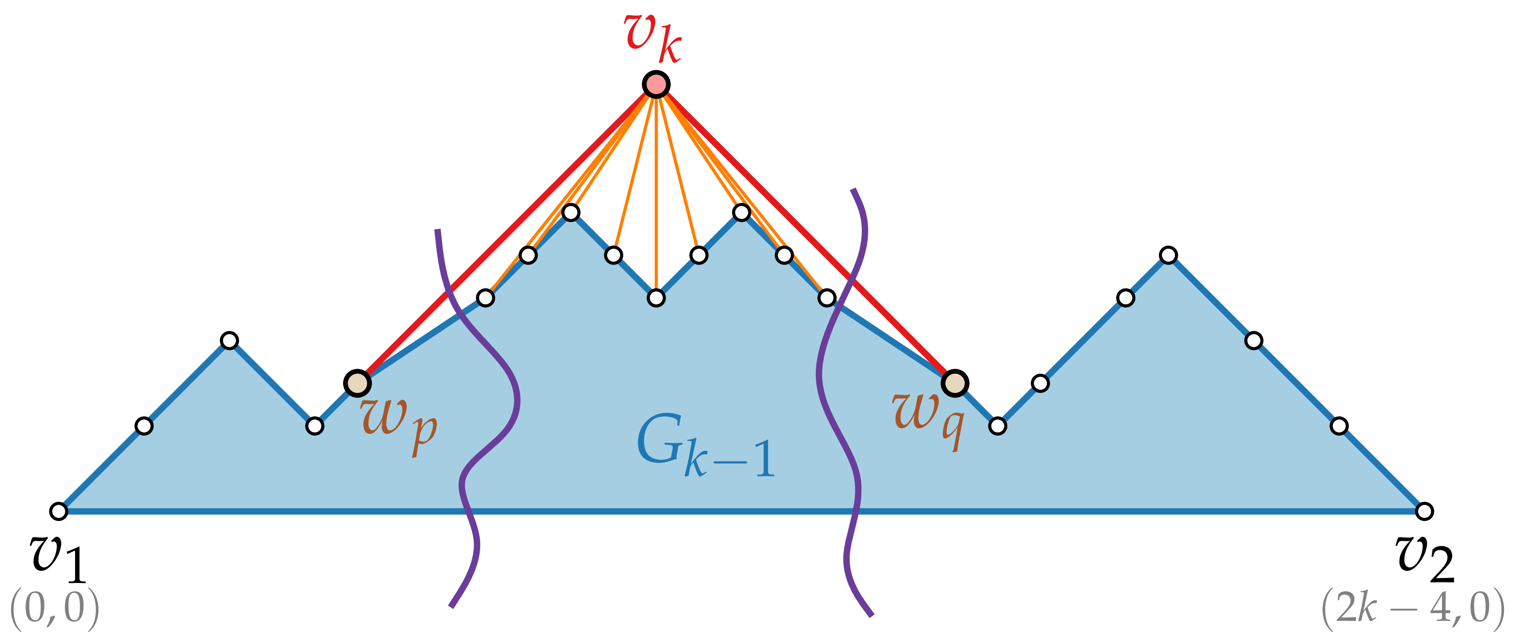

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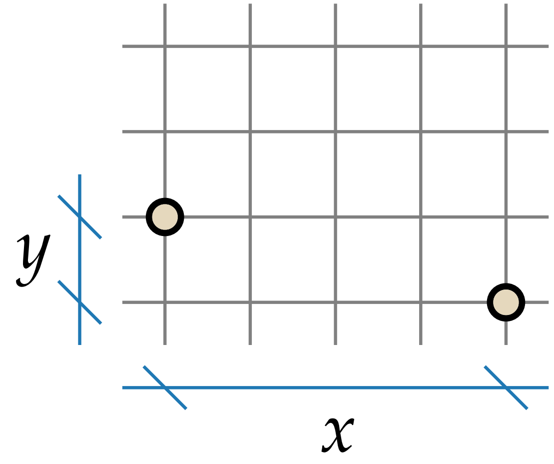

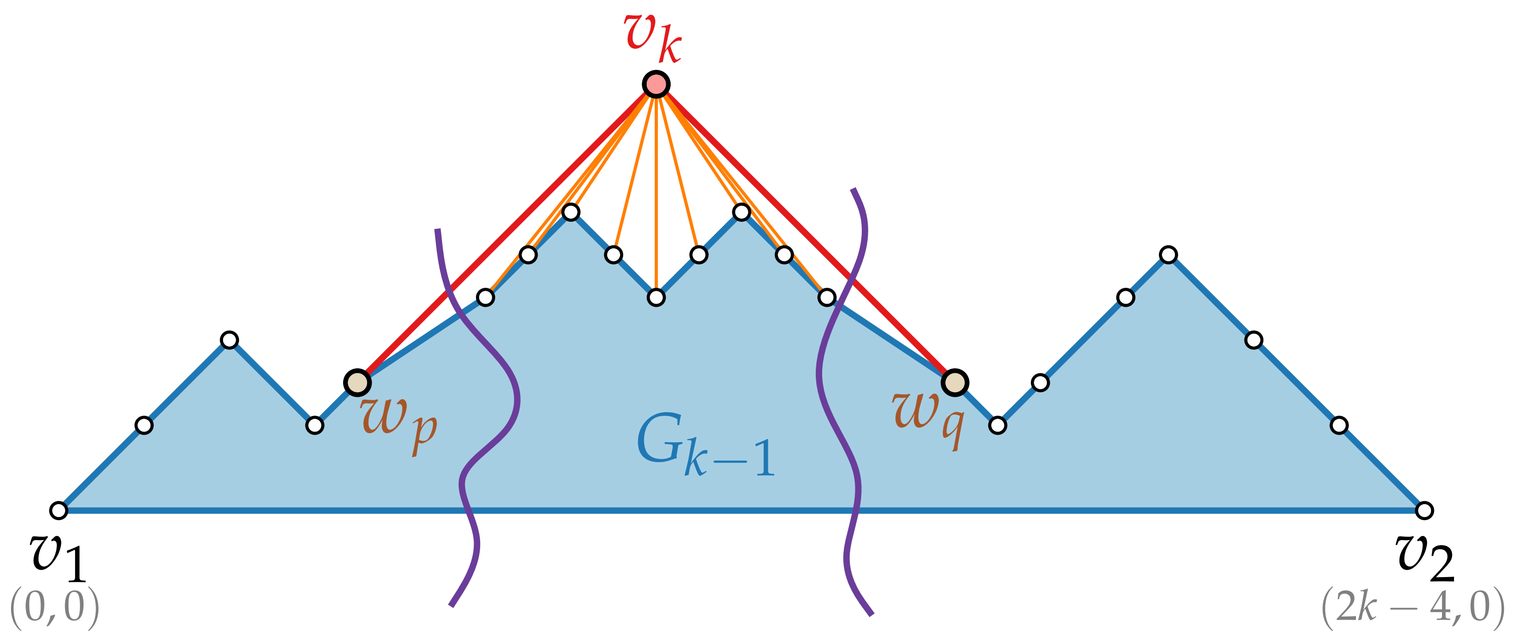

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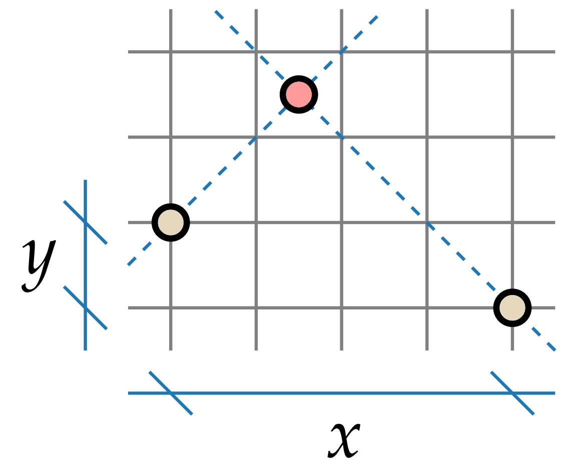

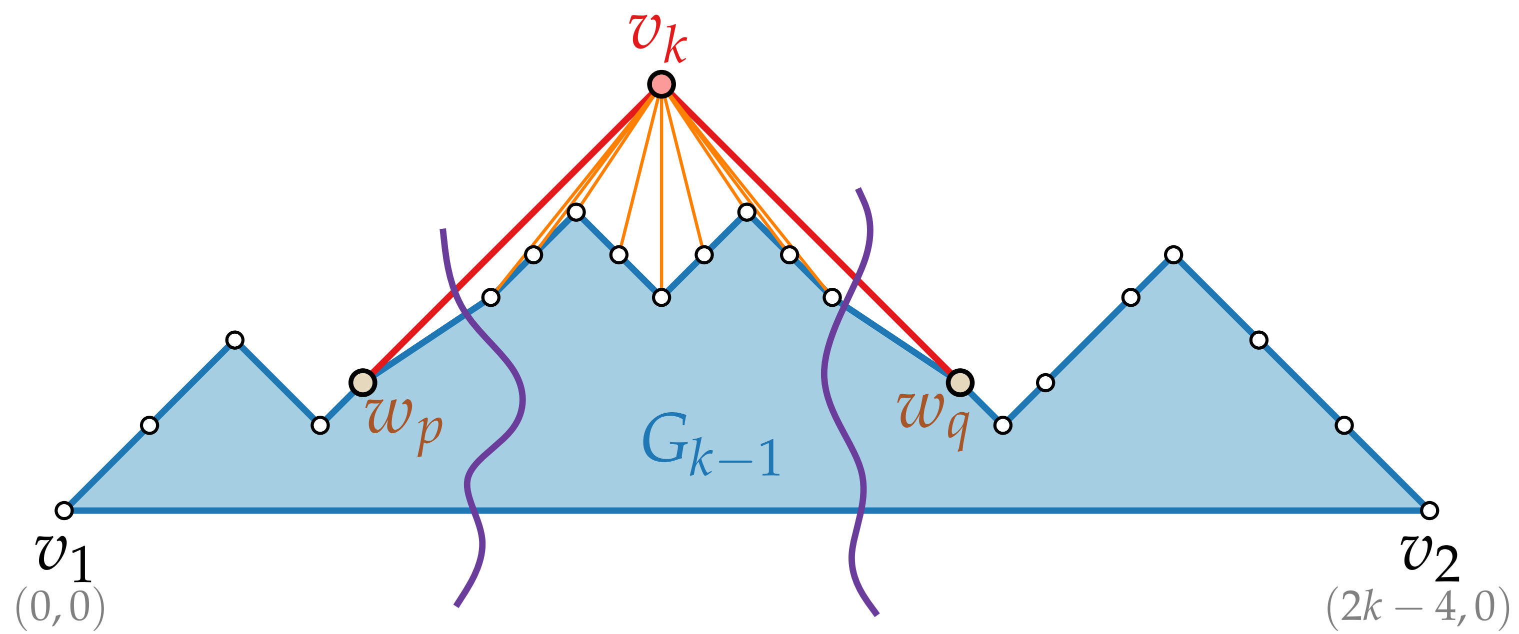

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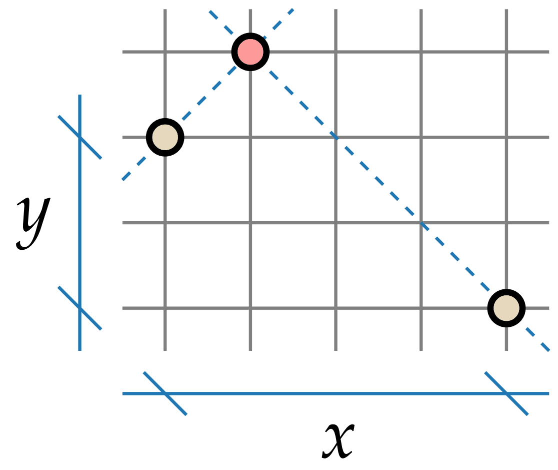

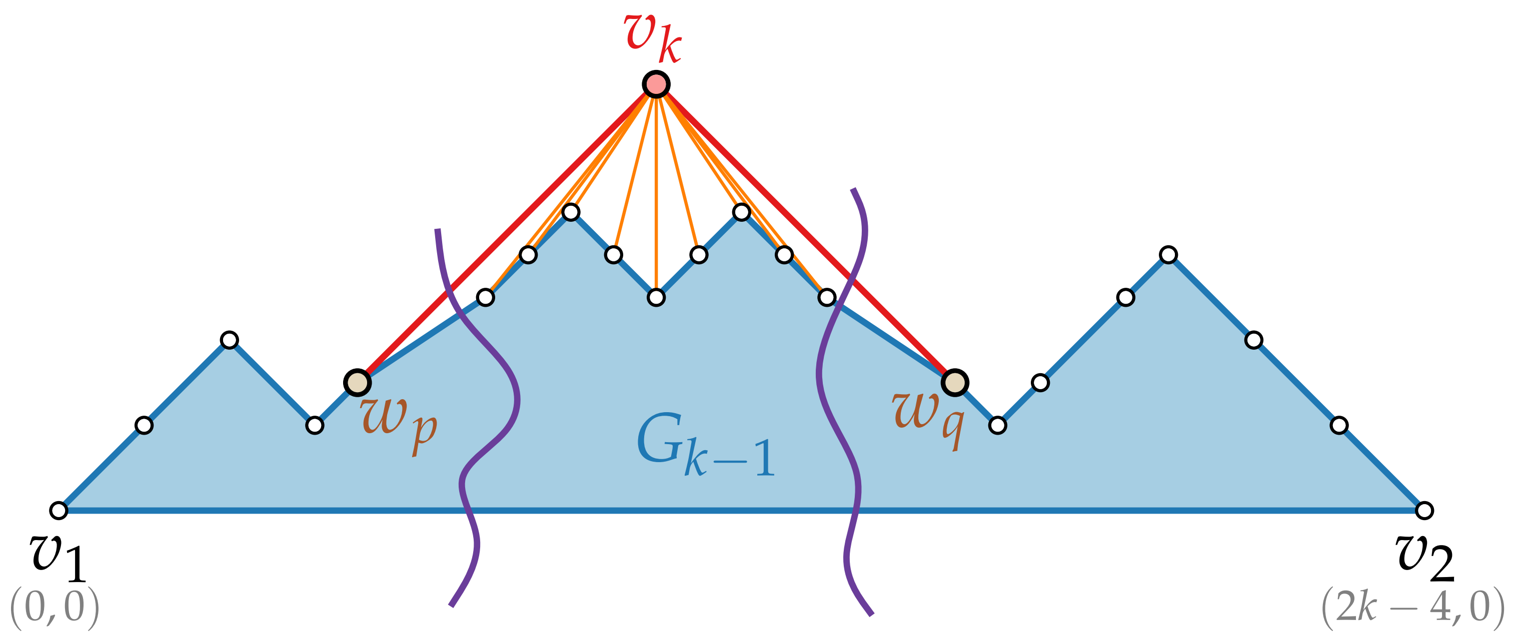

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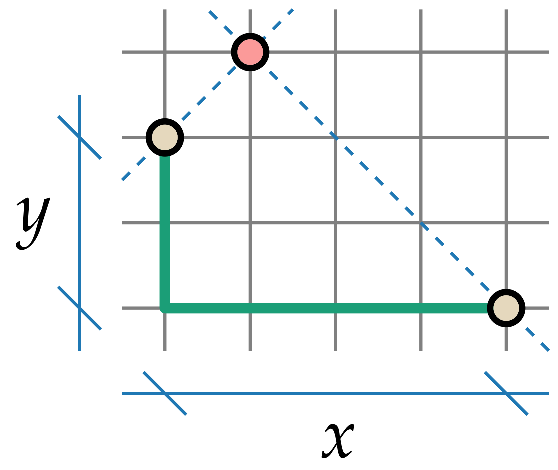

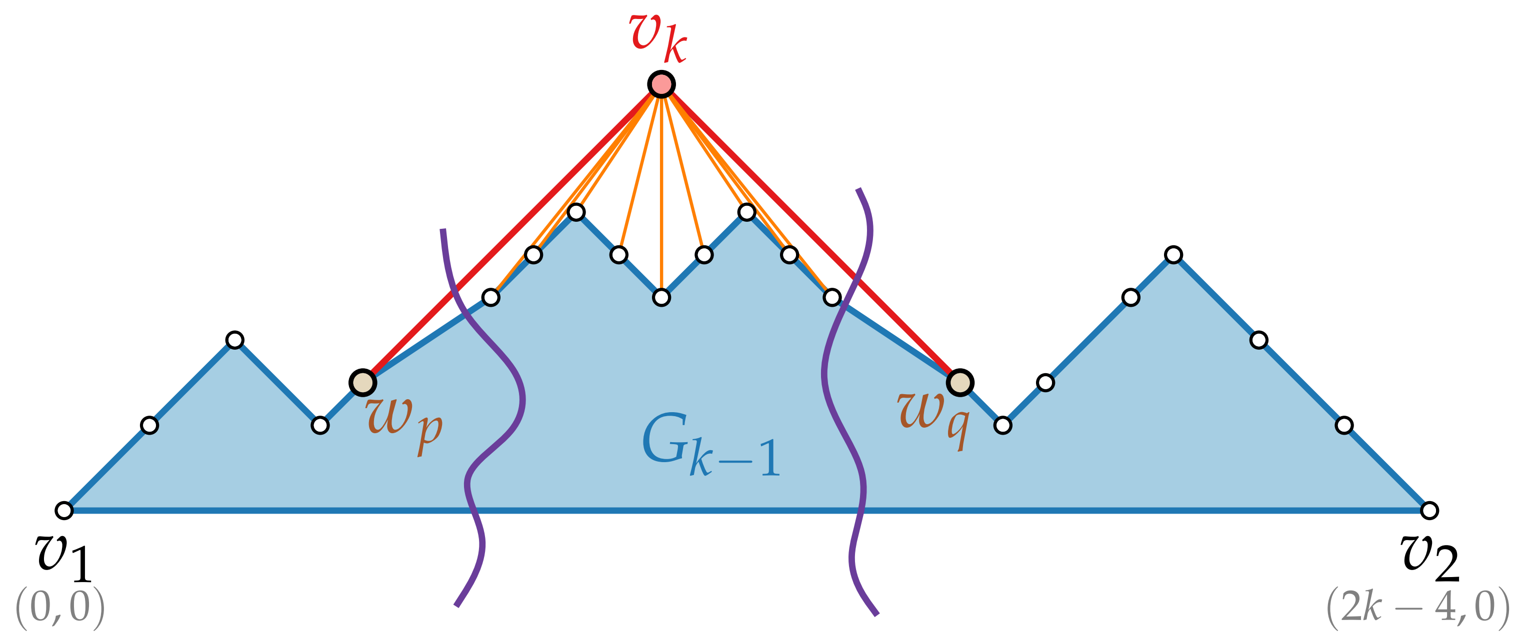

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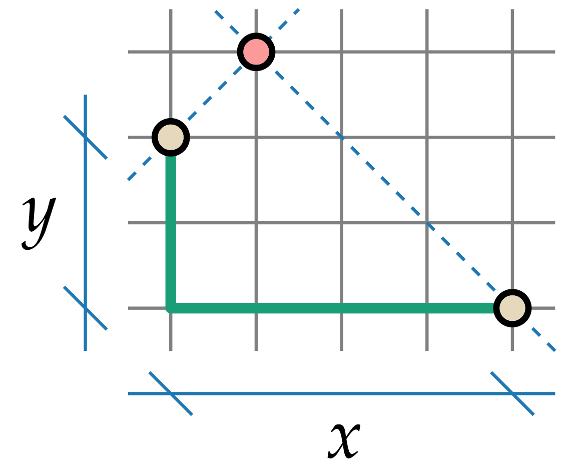
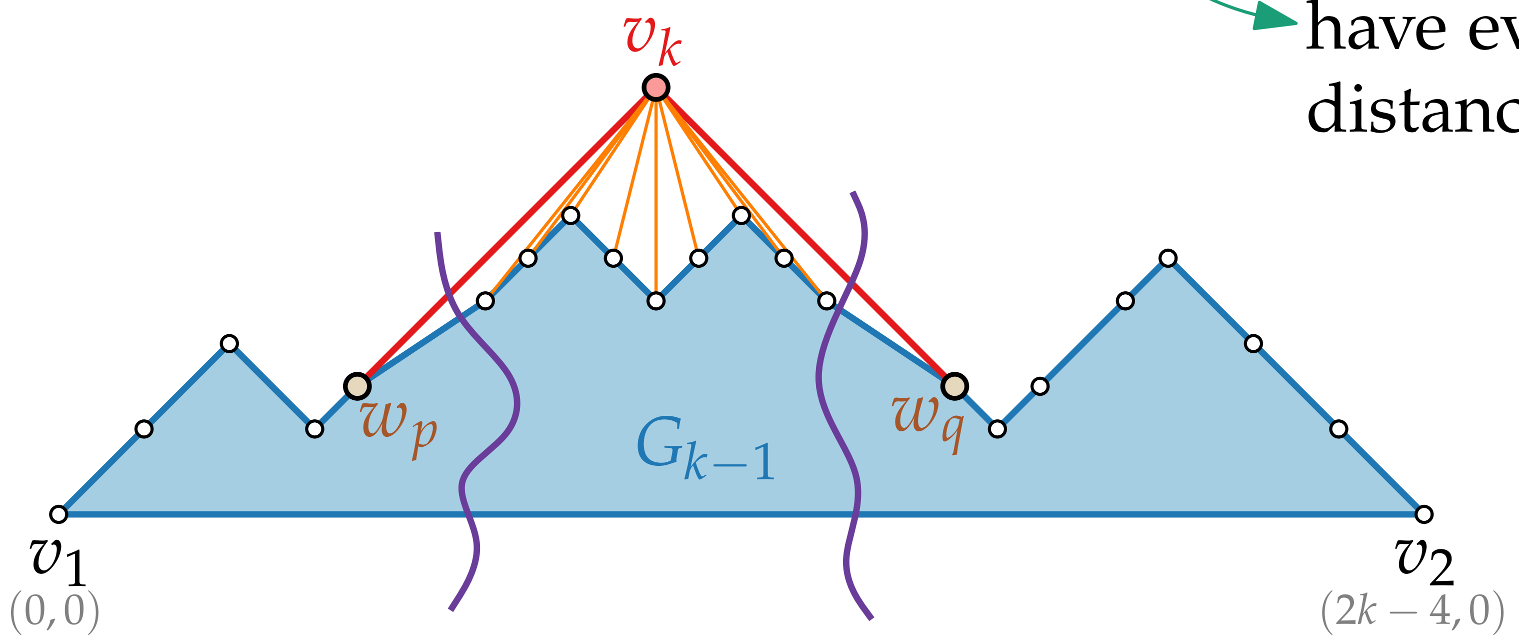


yes, beause w_p and w_q have even Manhattan distance



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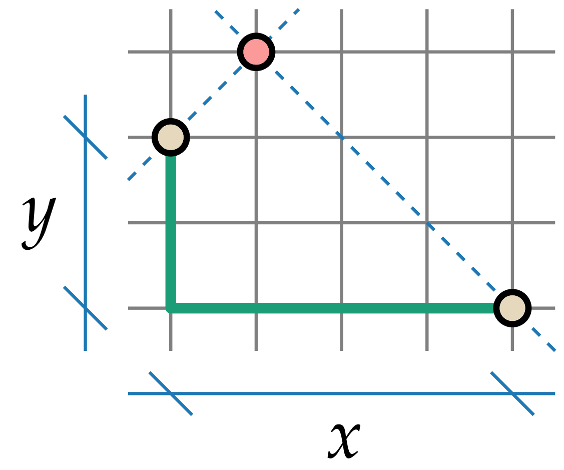

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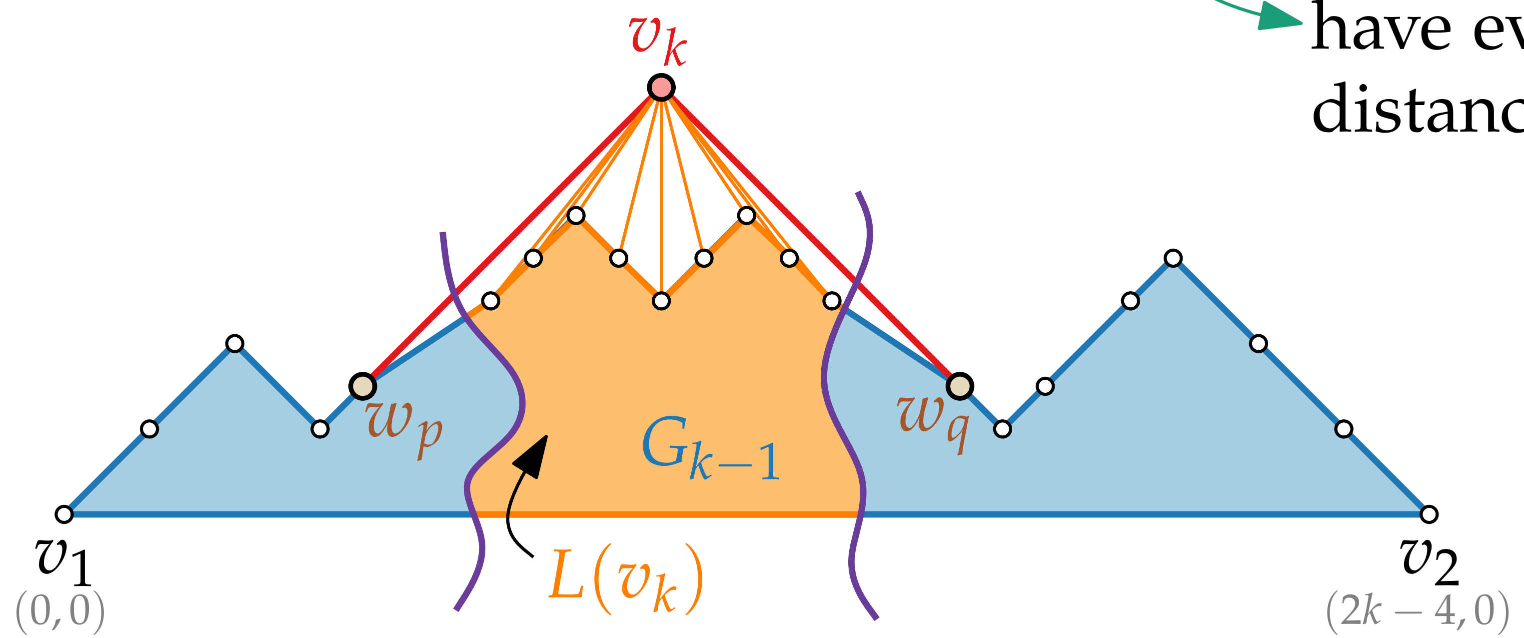

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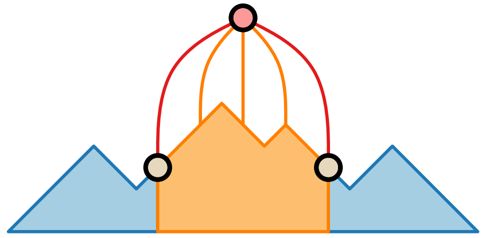

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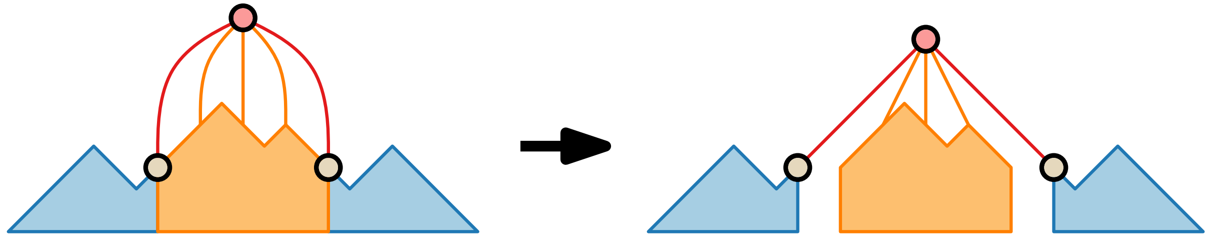

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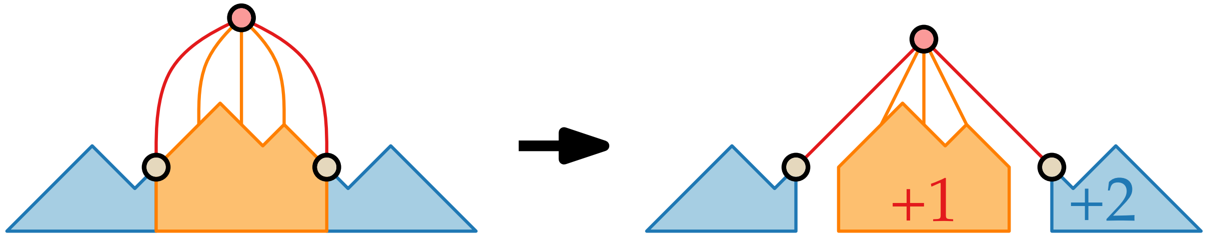

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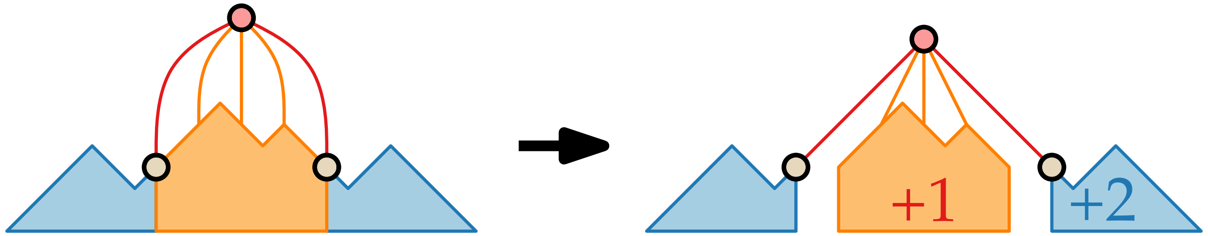

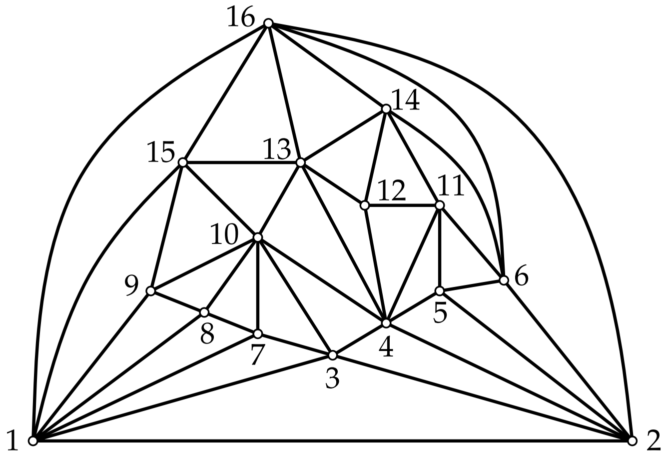

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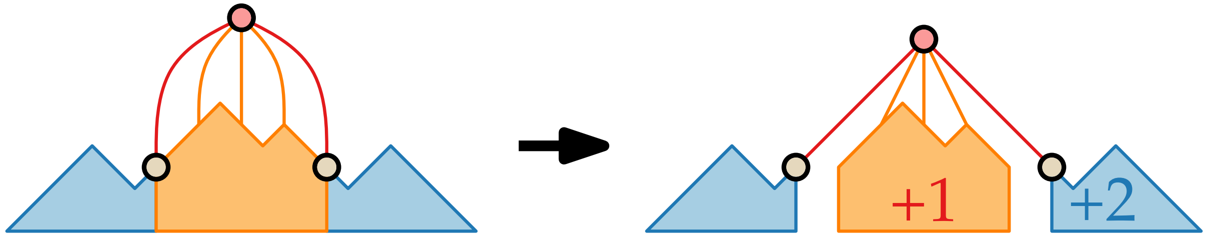

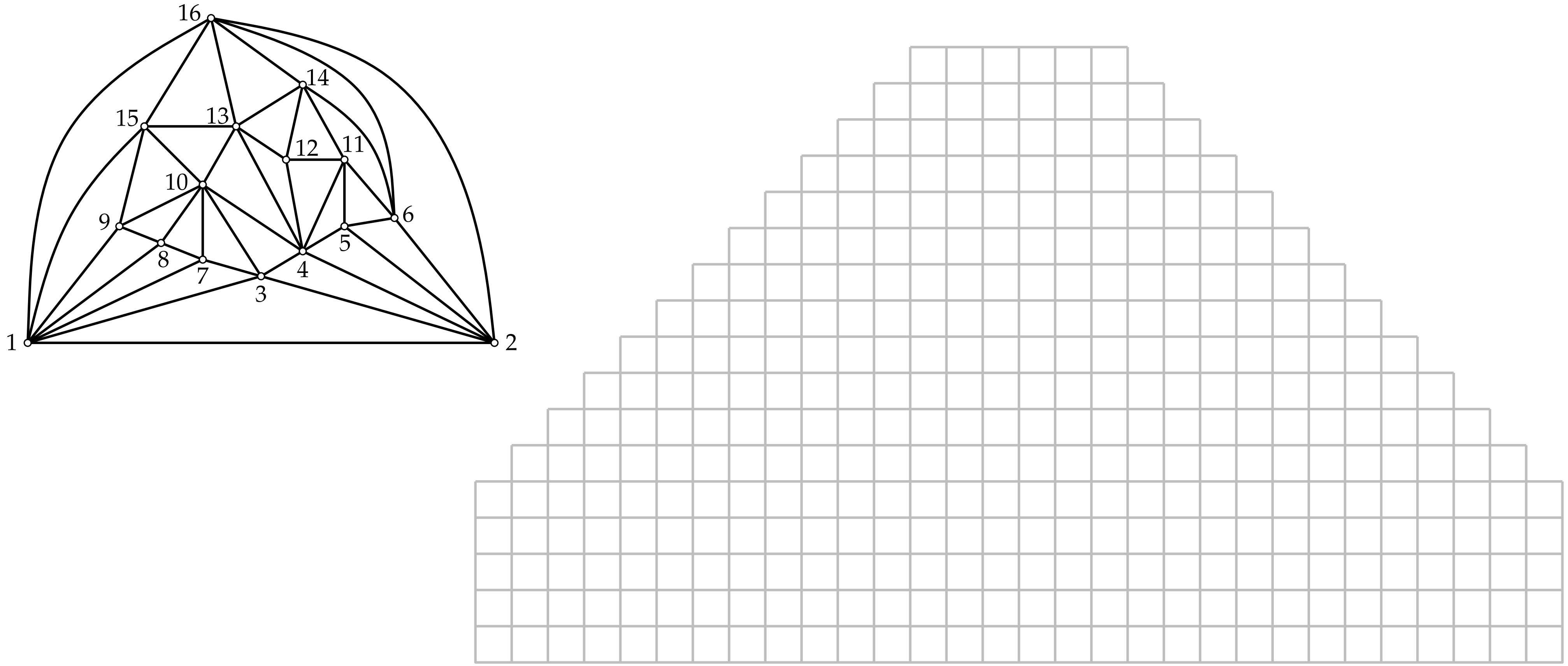

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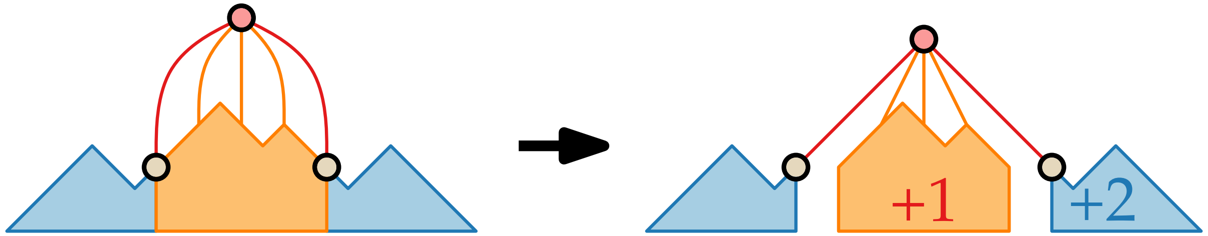

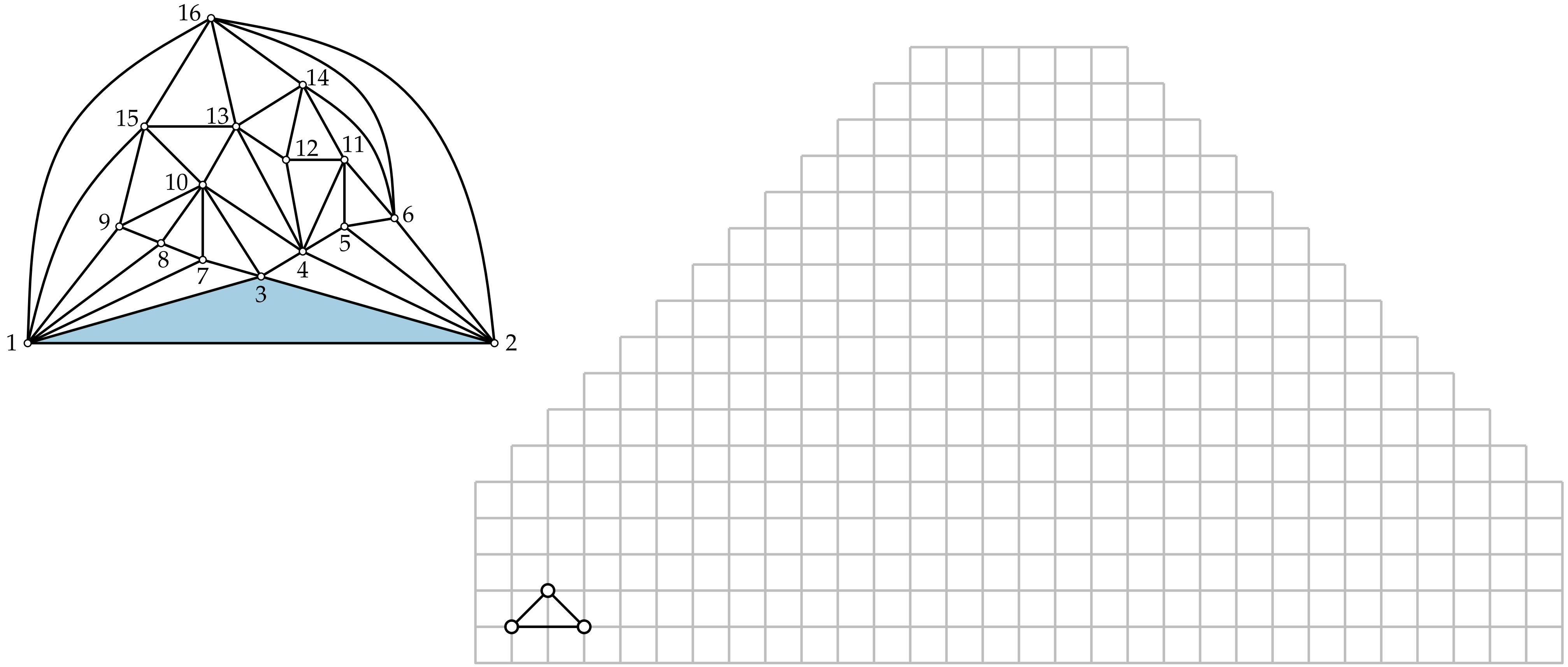

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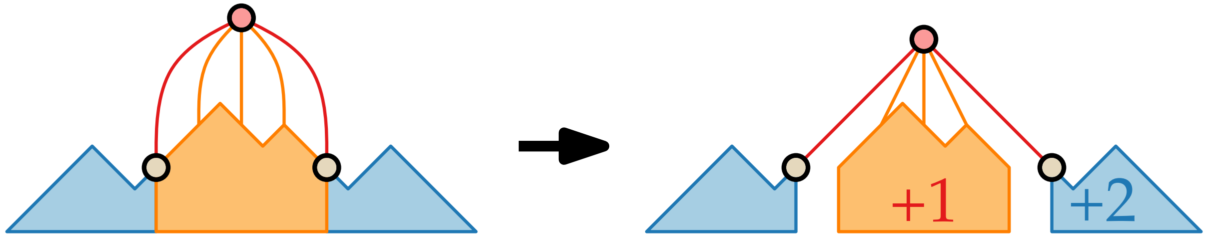

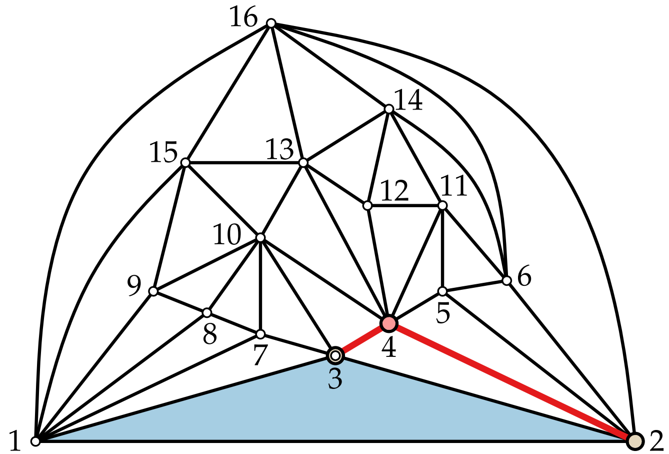

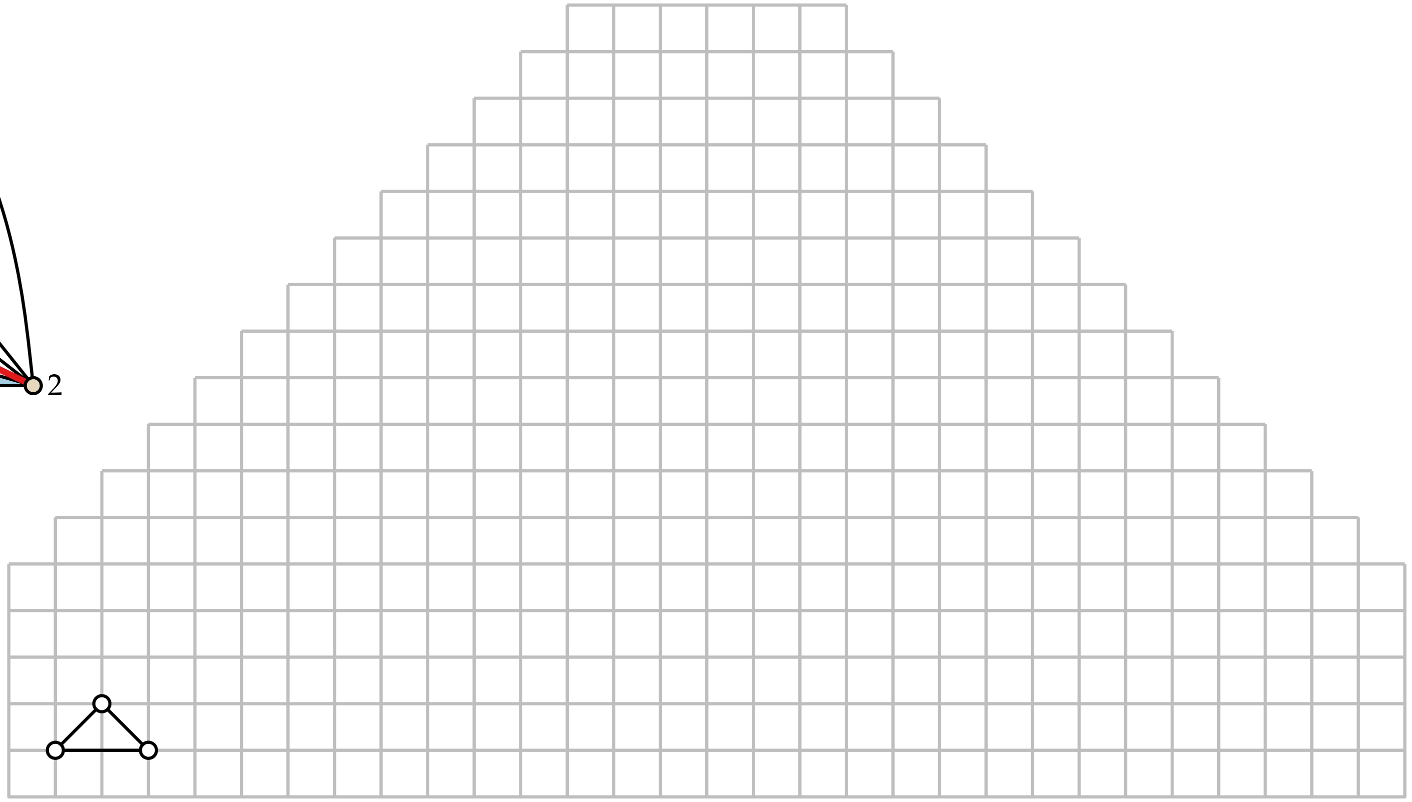

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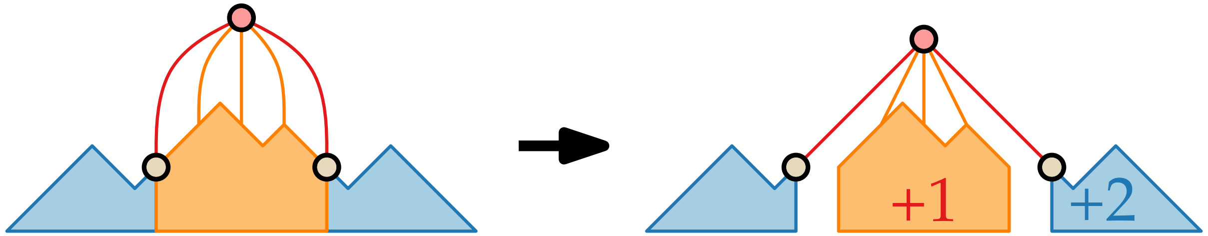

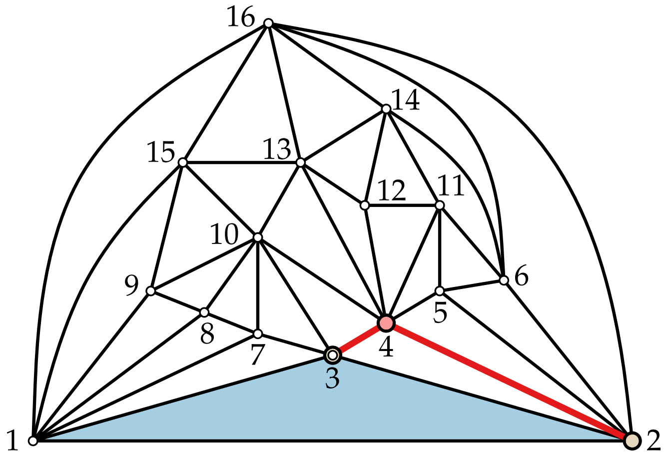

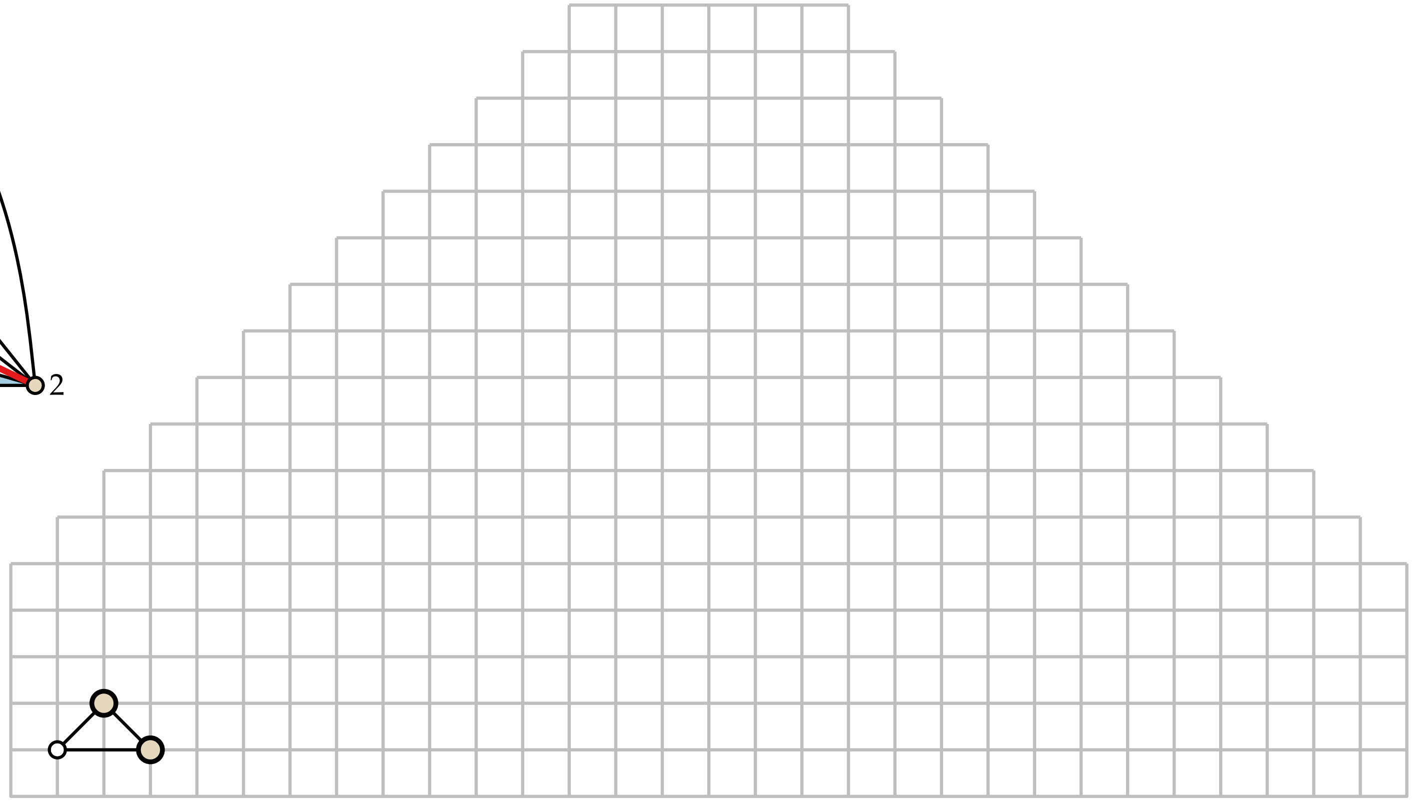

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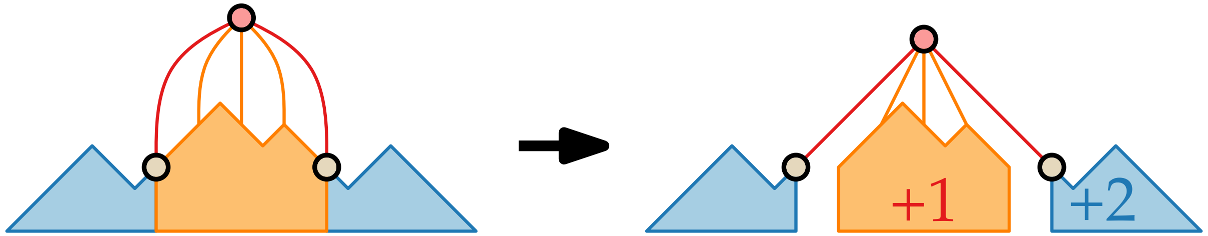

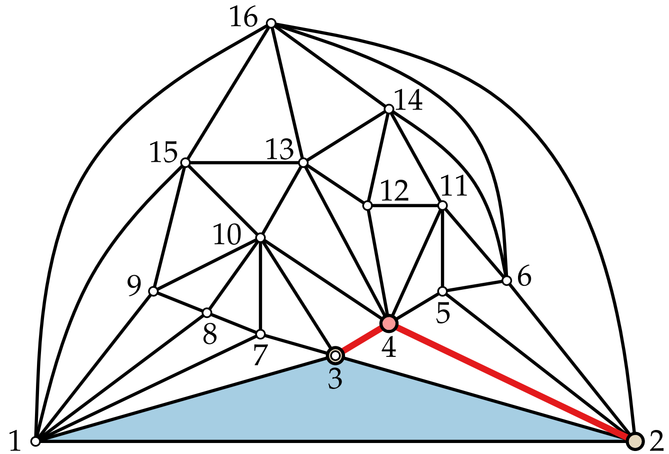

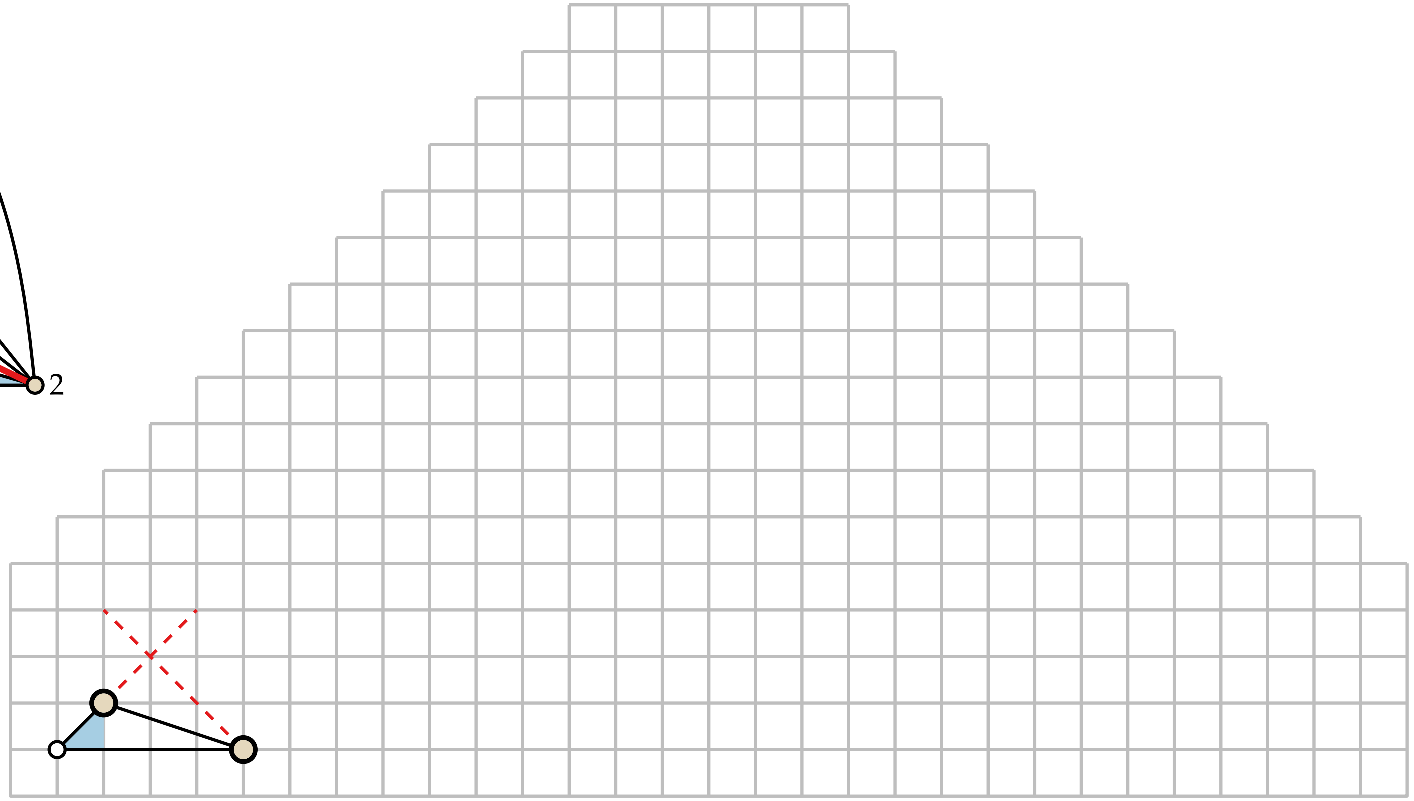

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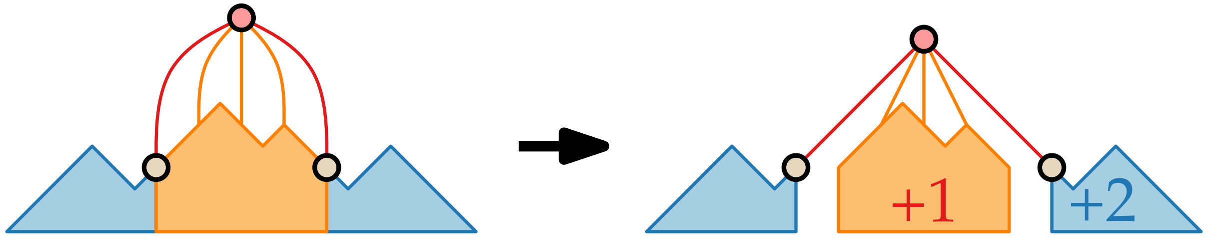

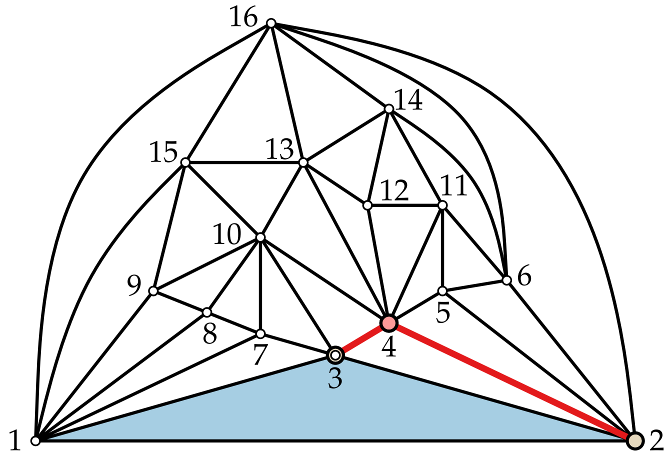

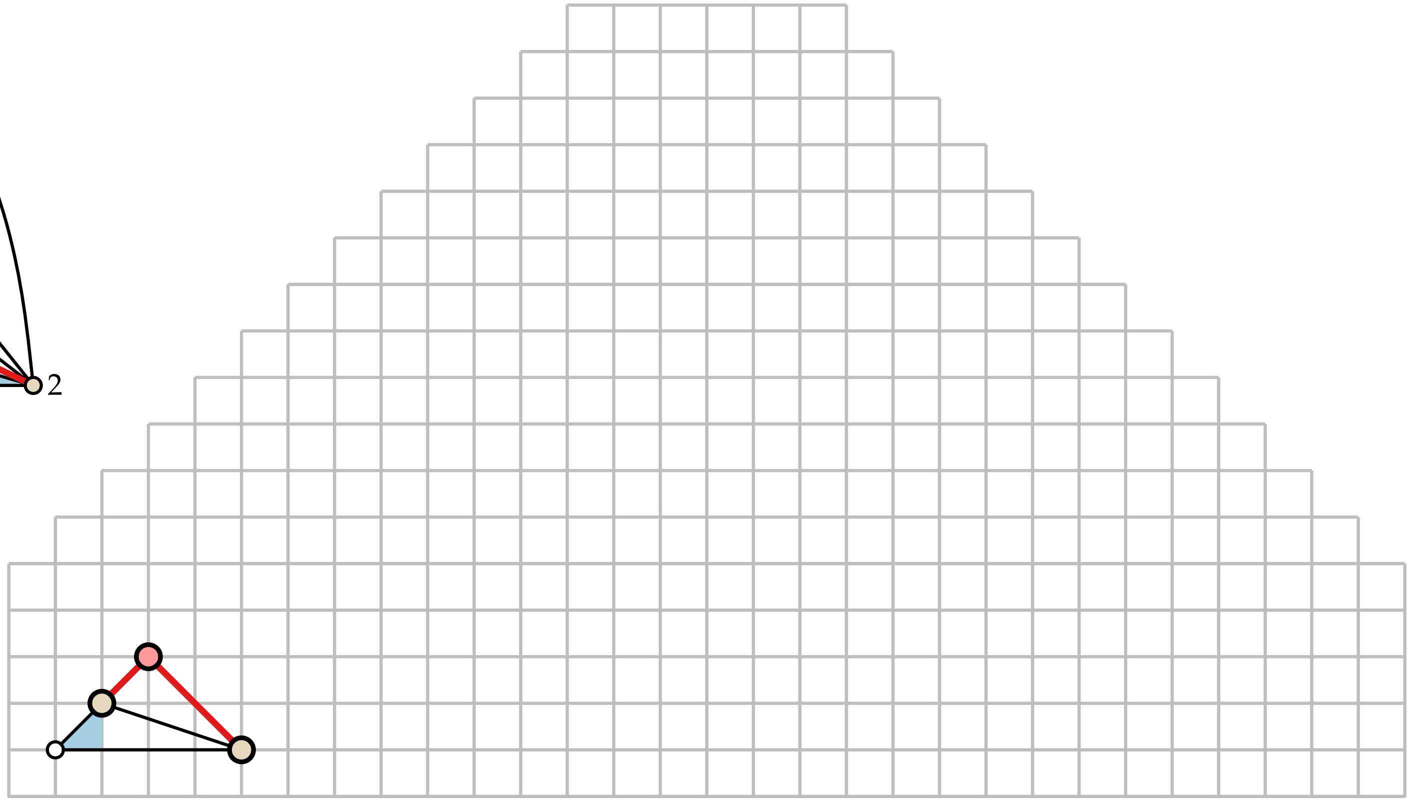

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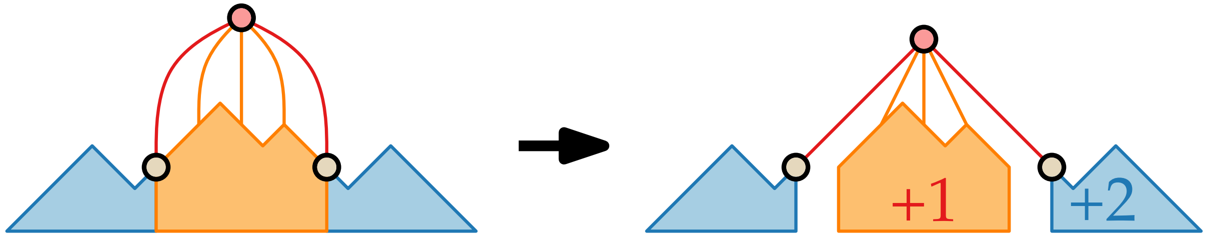

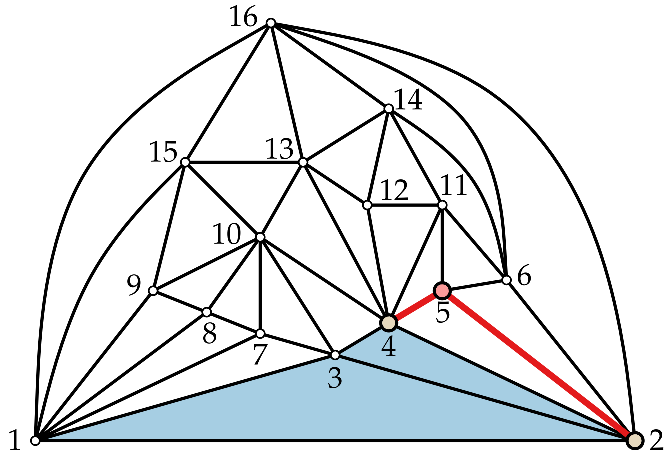

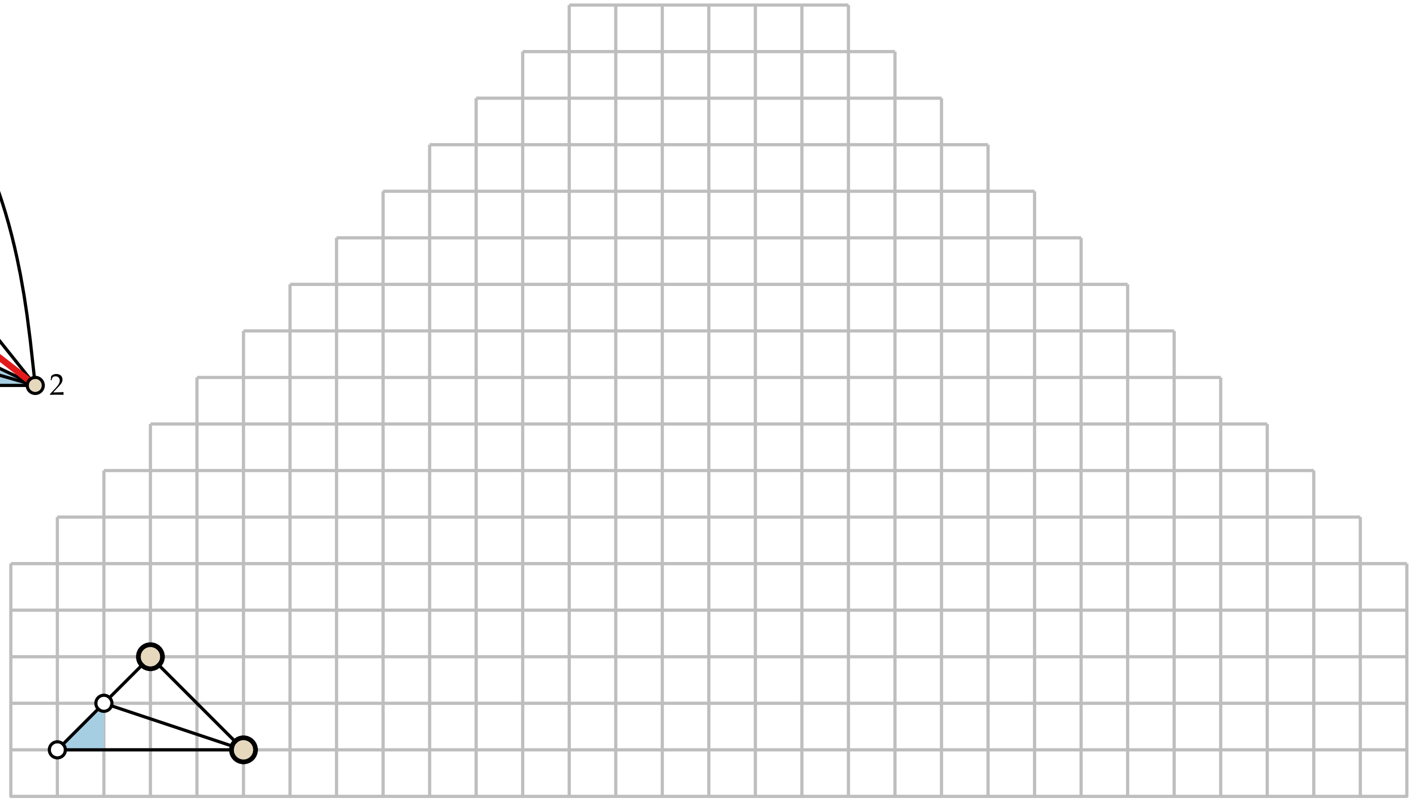

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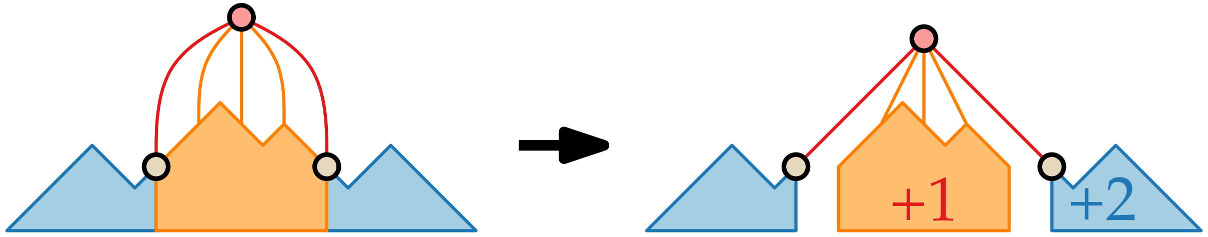

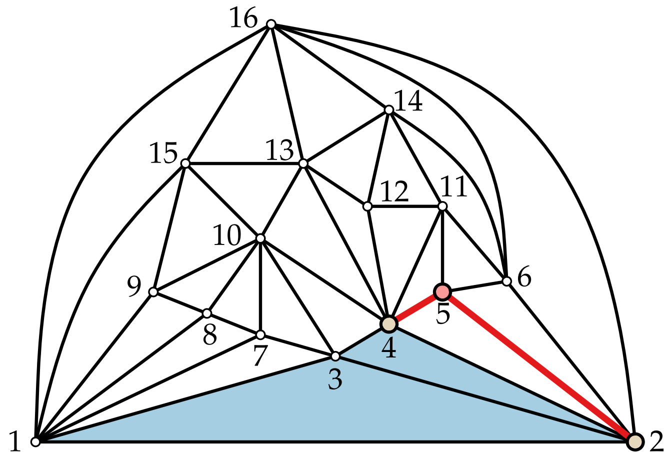

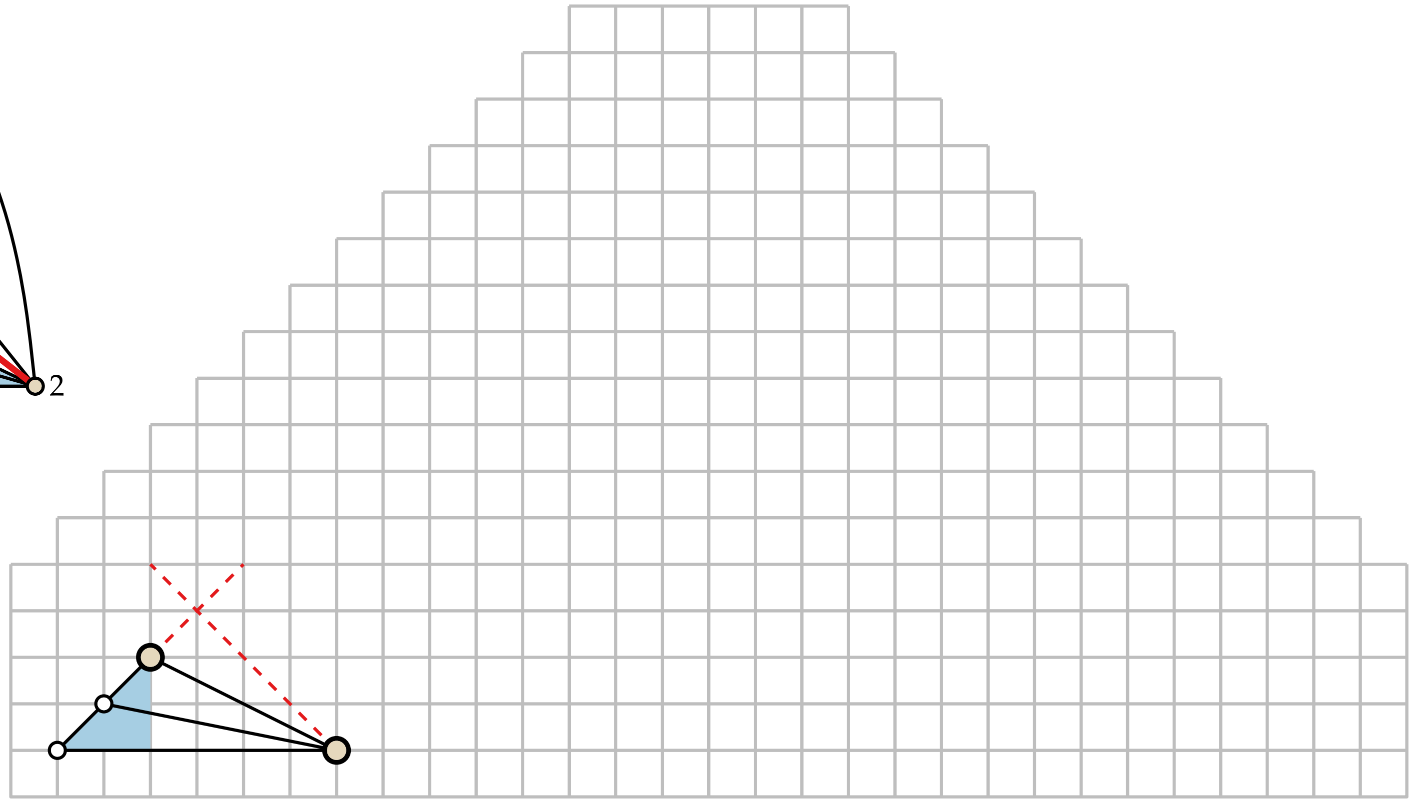

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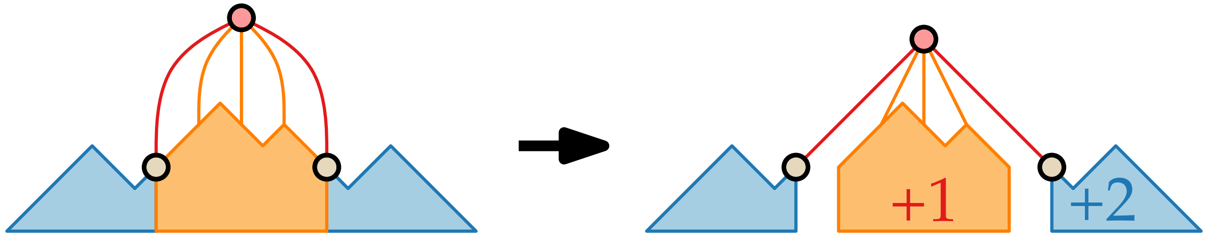

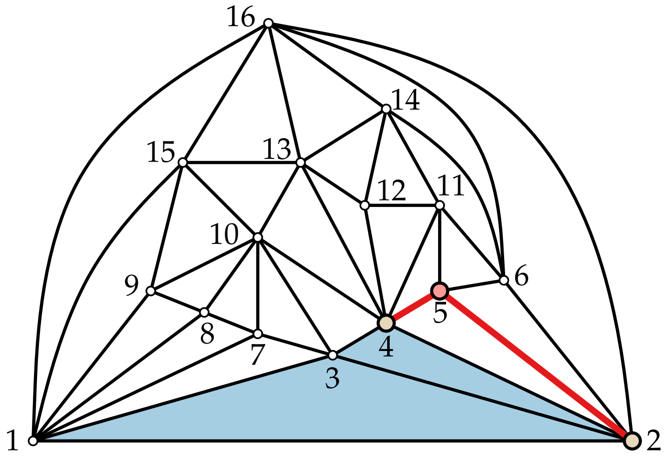

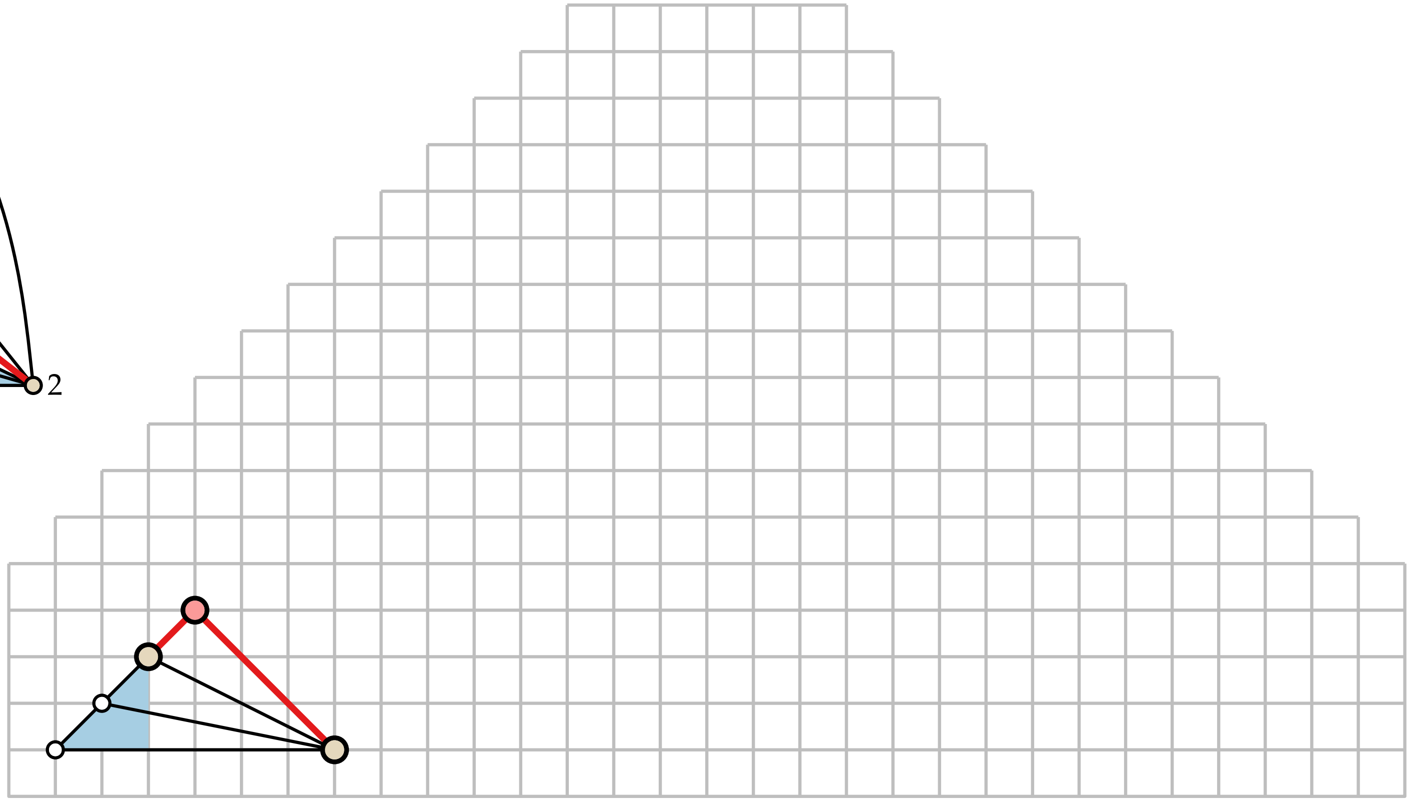

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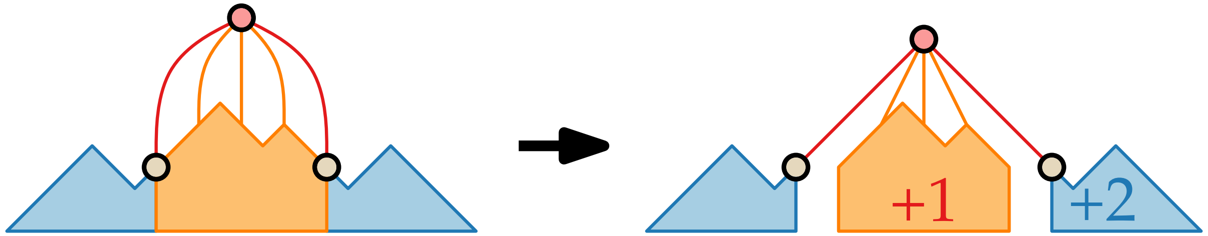

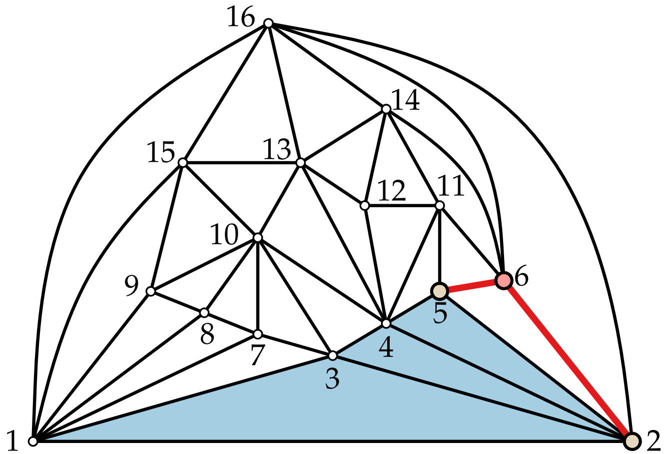

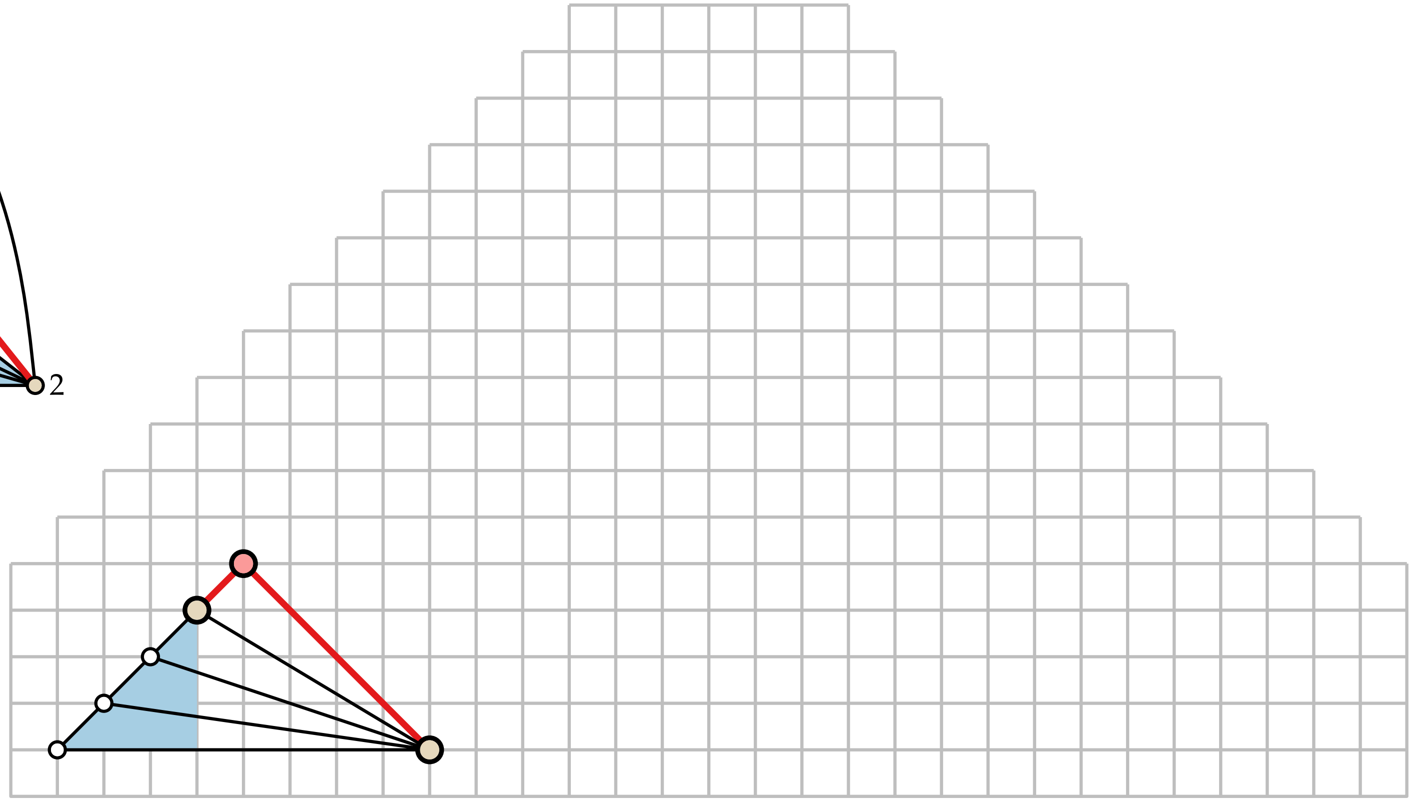

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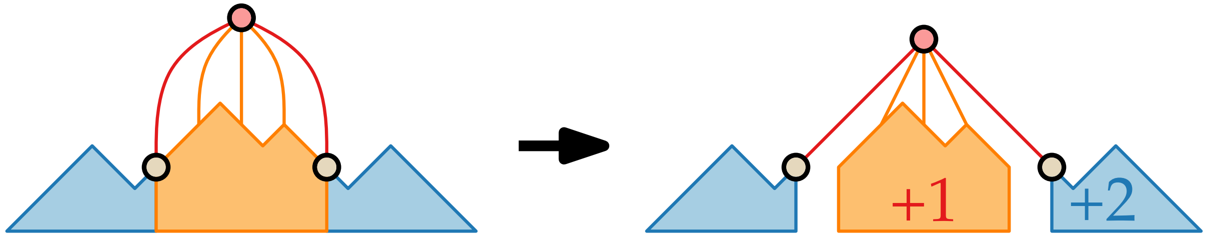

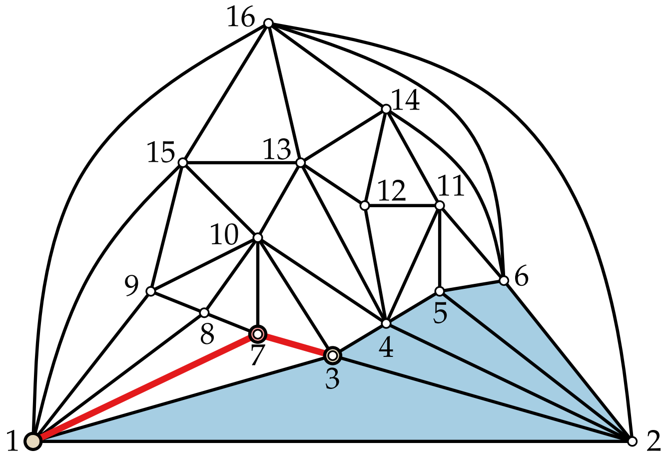

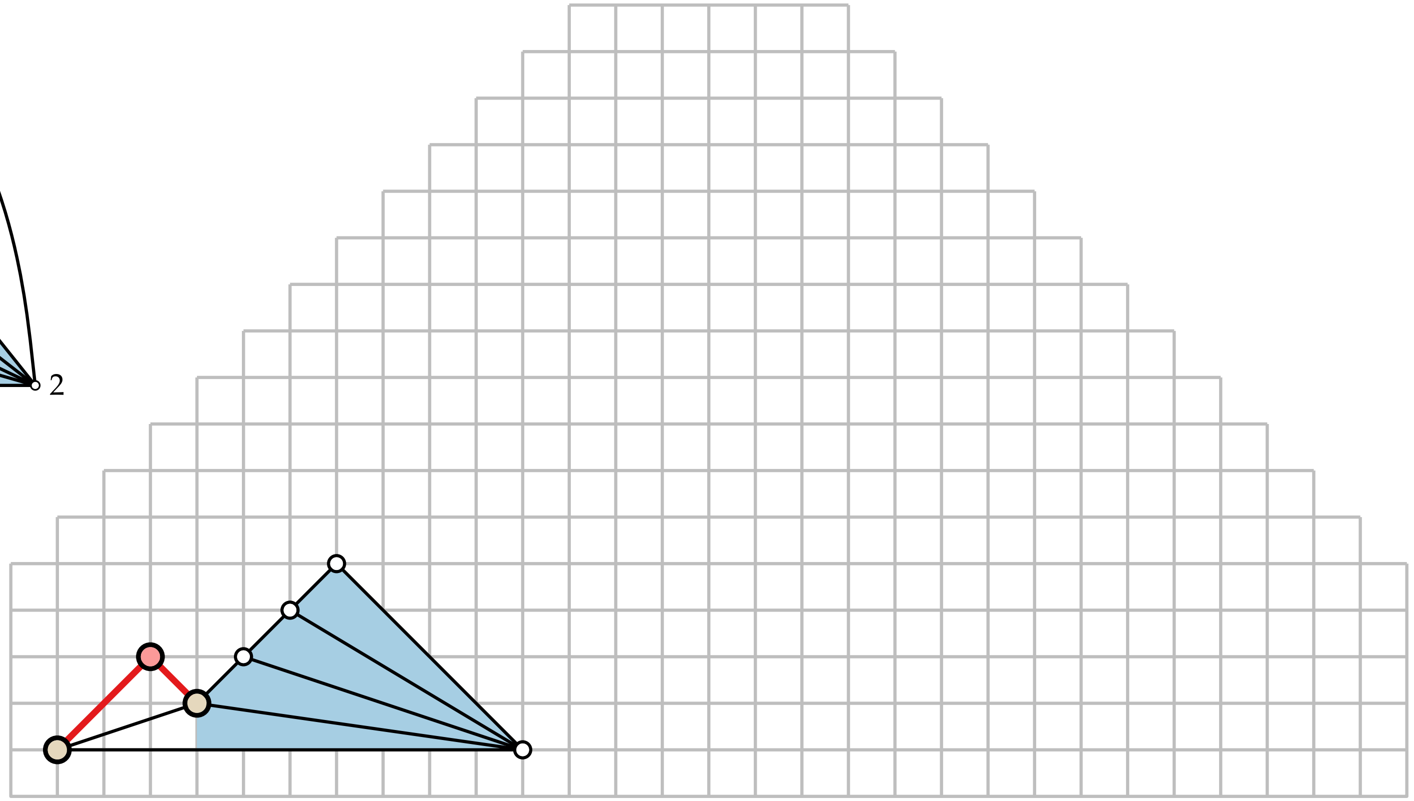

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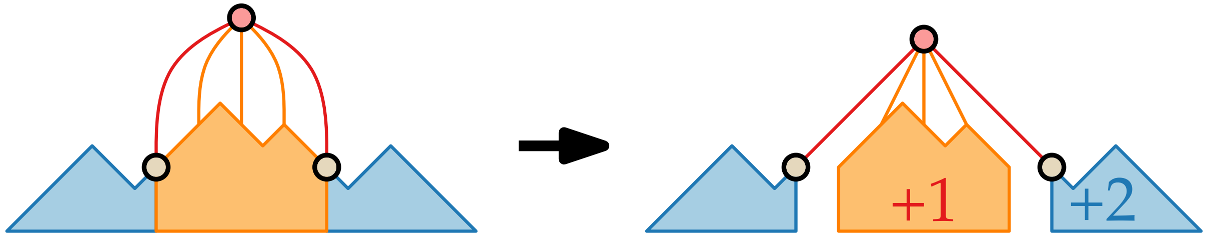

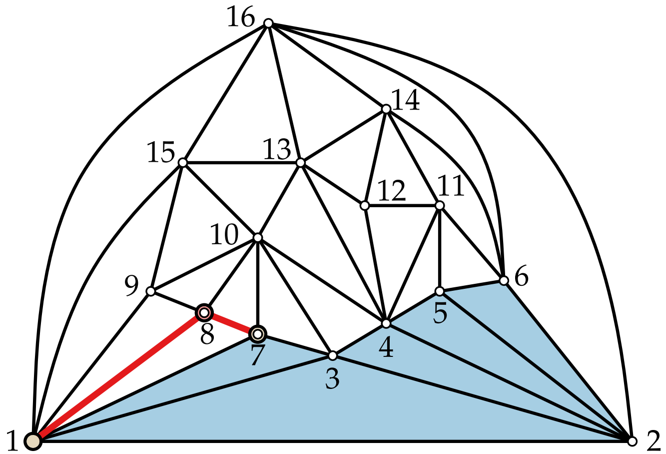

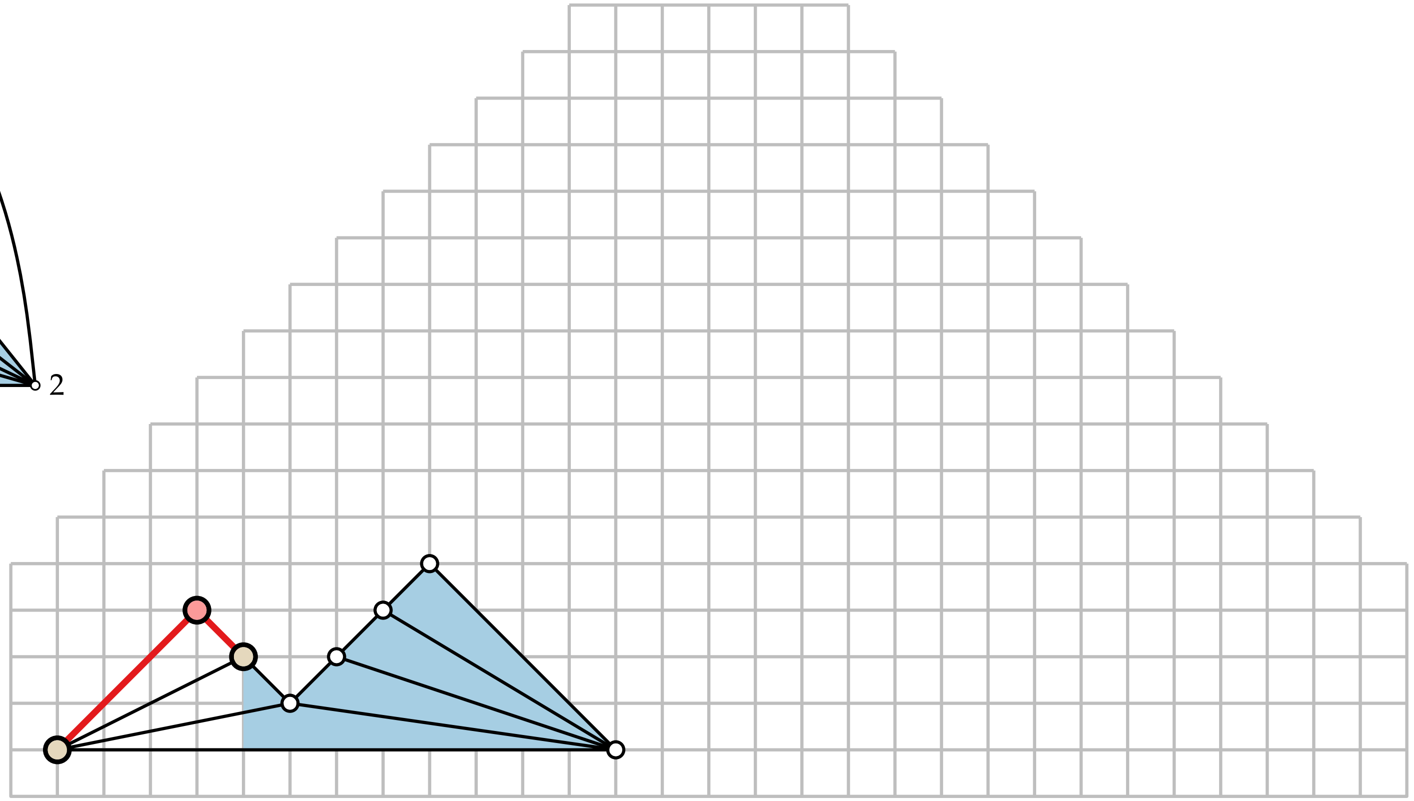

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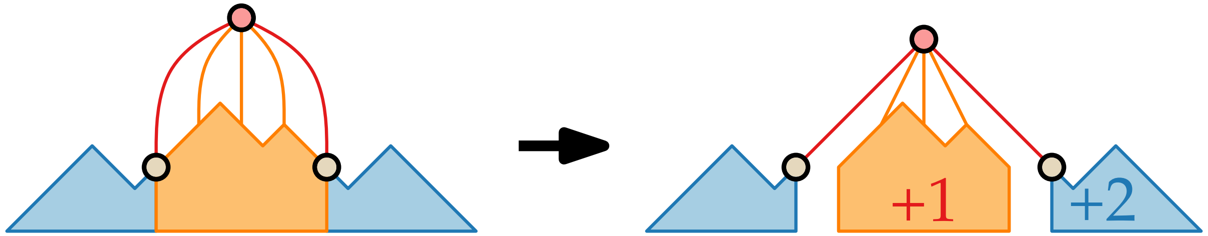
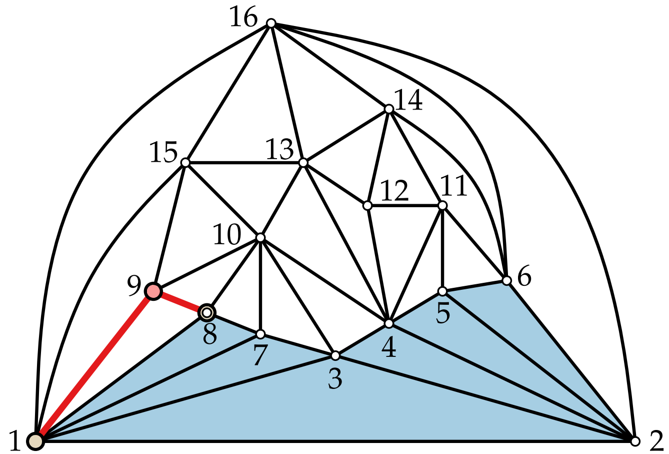
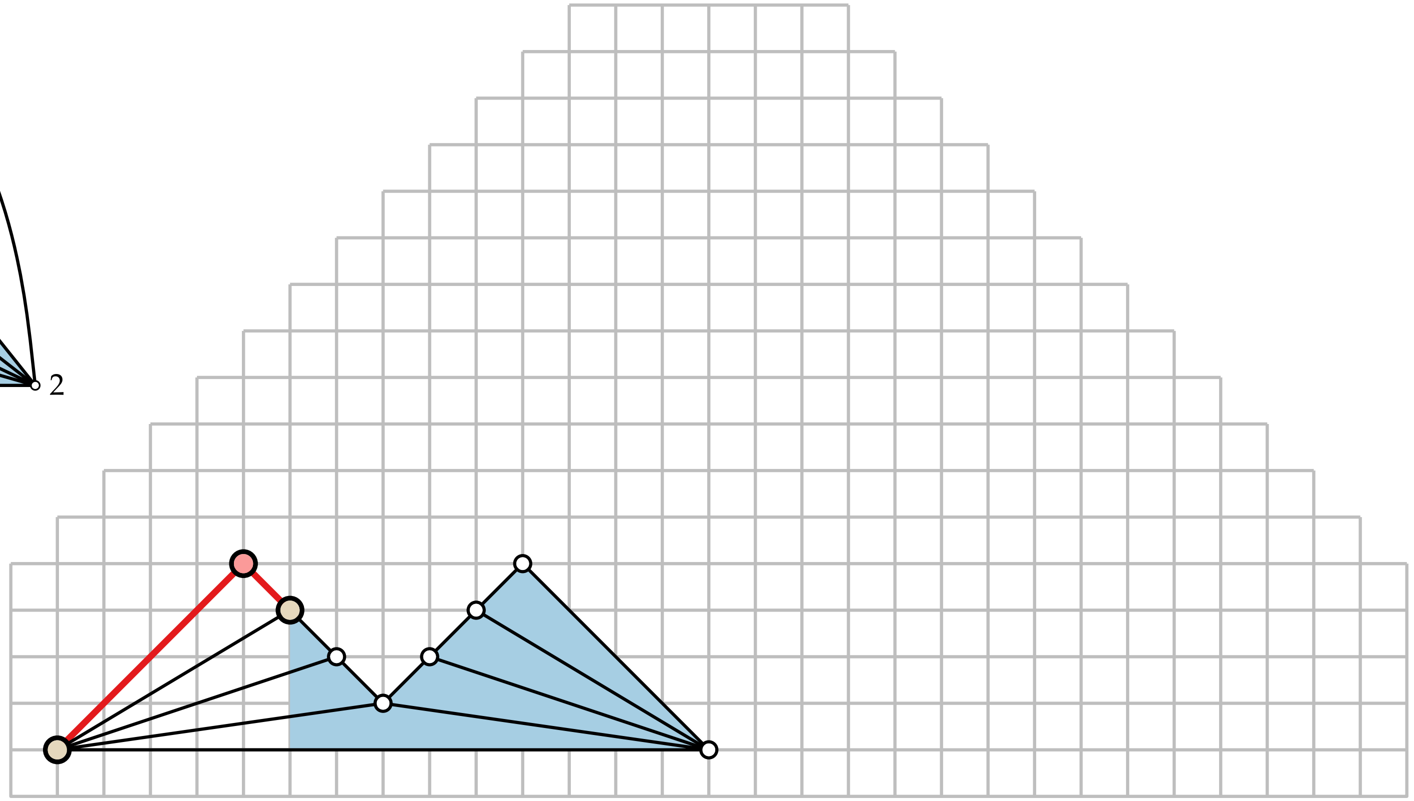
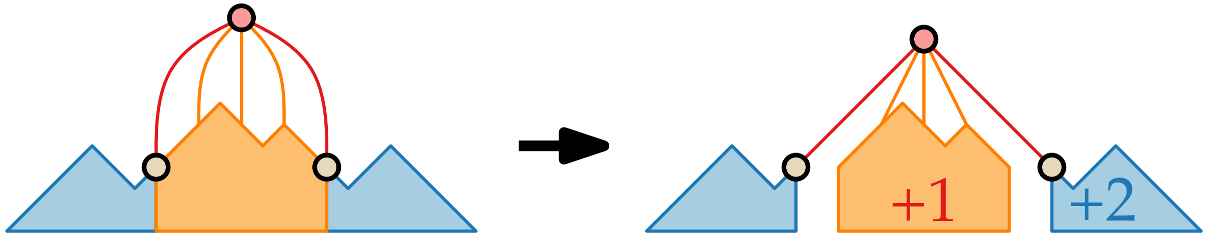
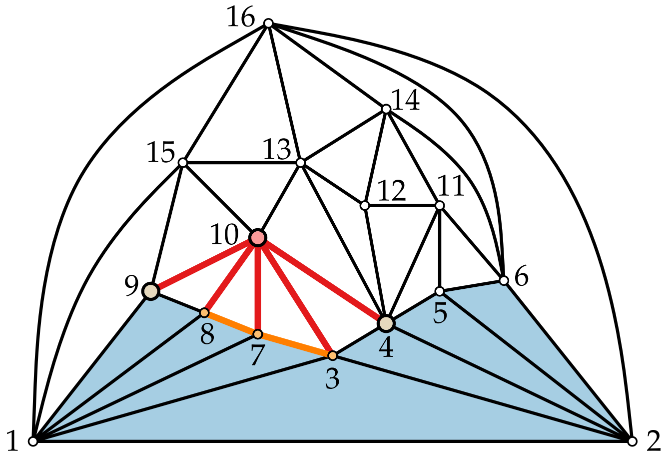


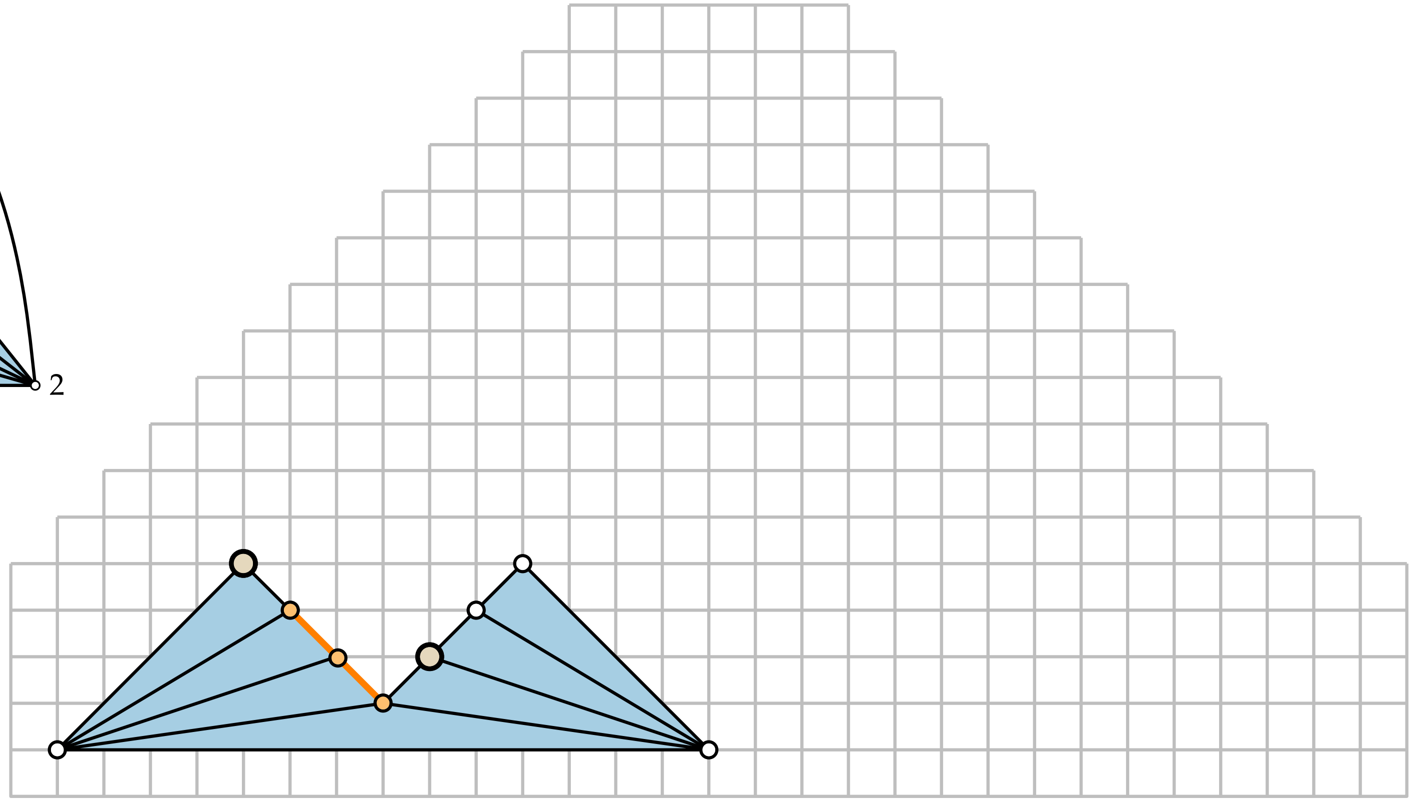




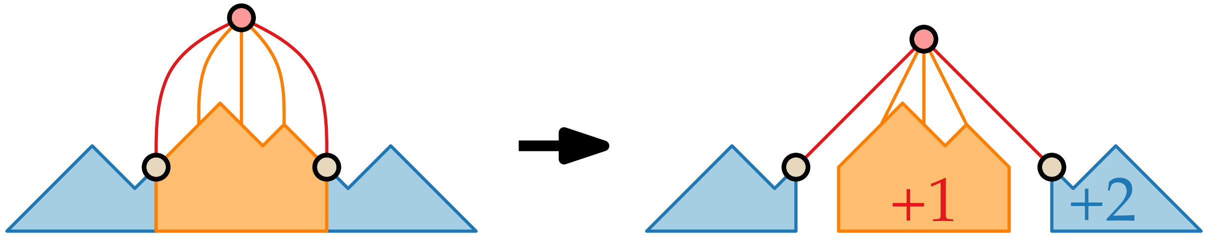
Shift Method – Example

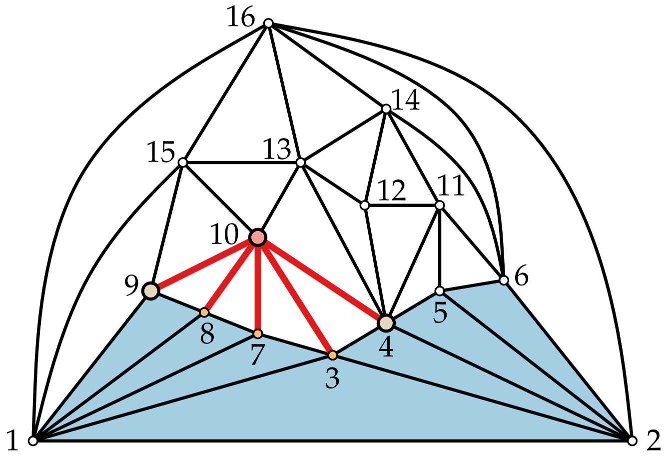


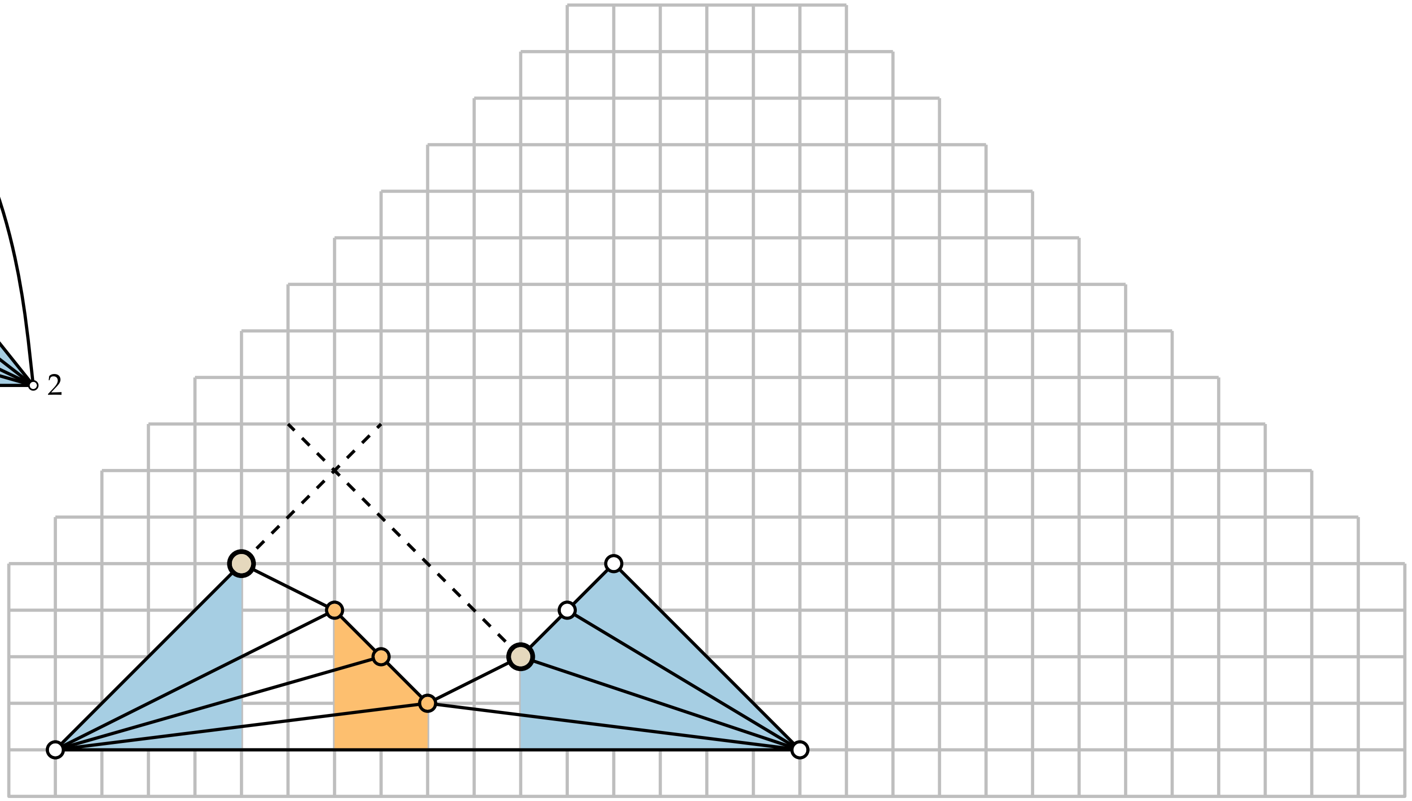




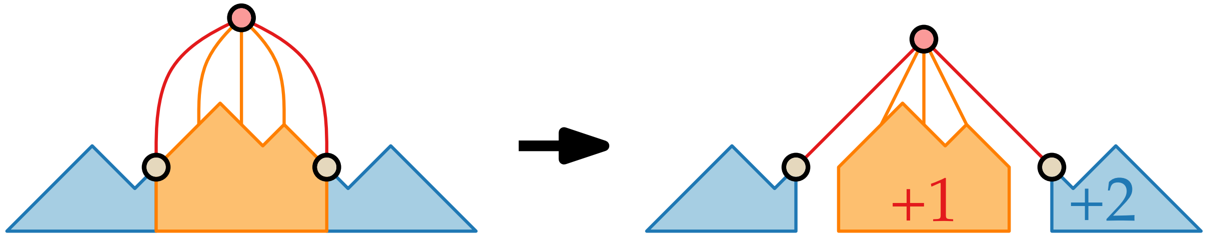
Shift Method – Example

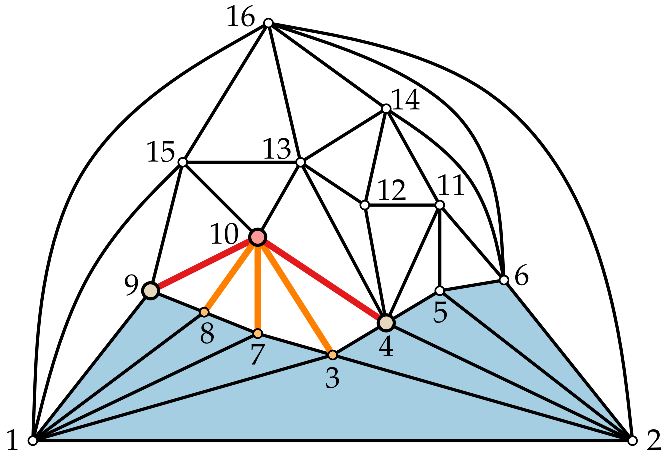


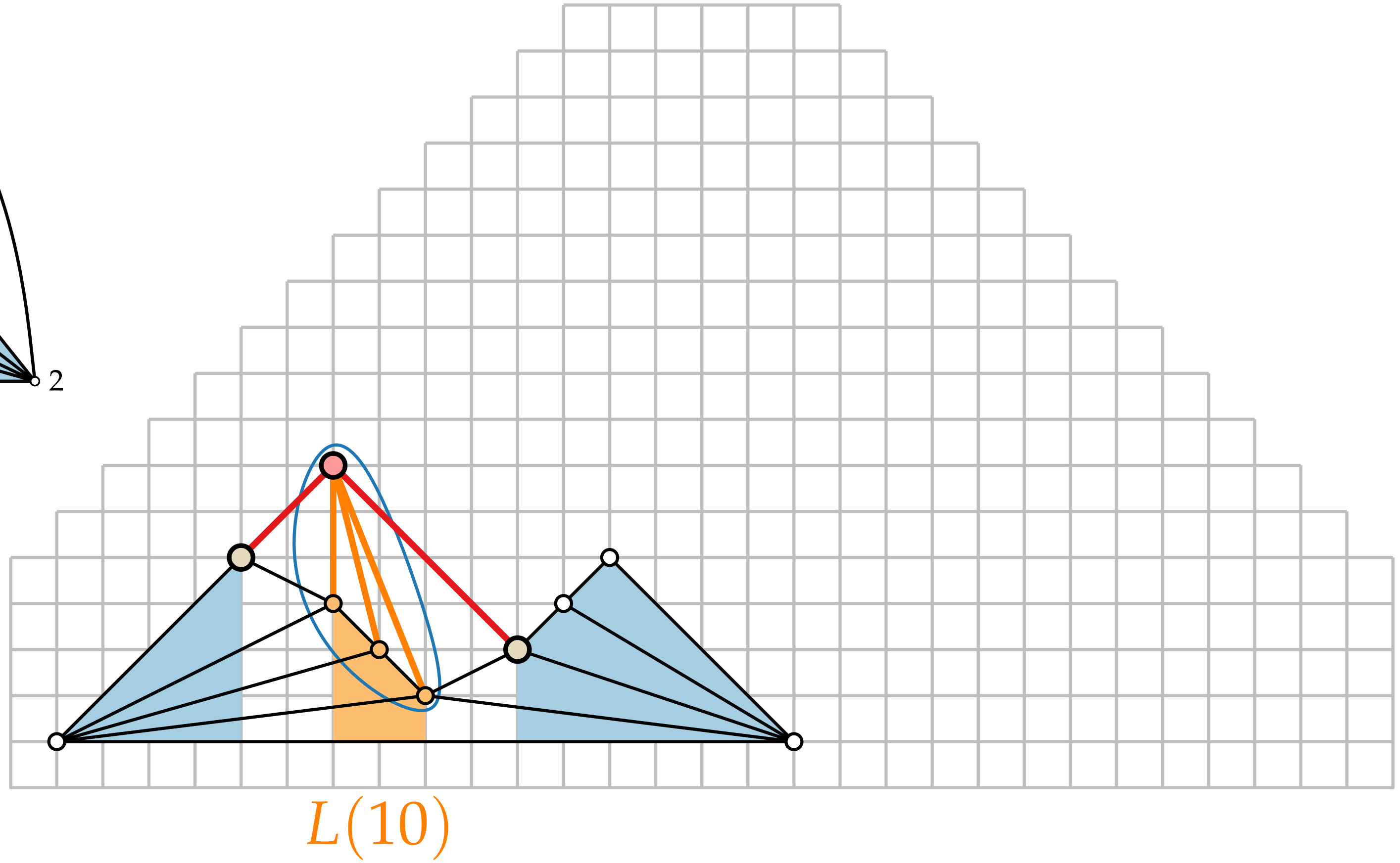




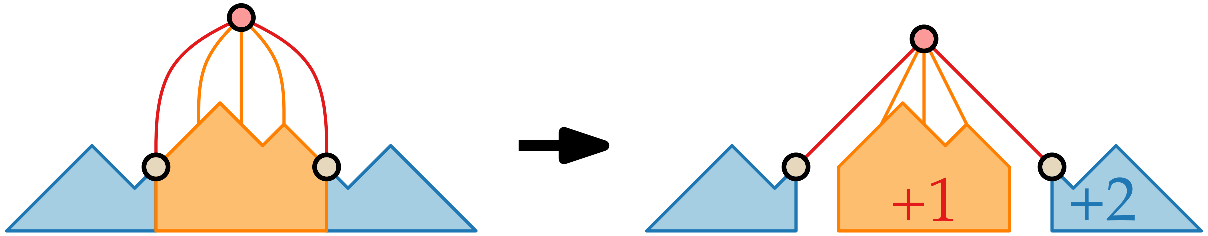
Shift Method – Example

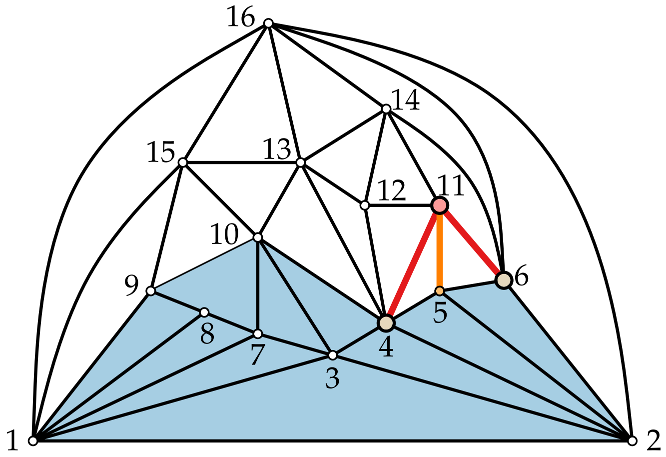


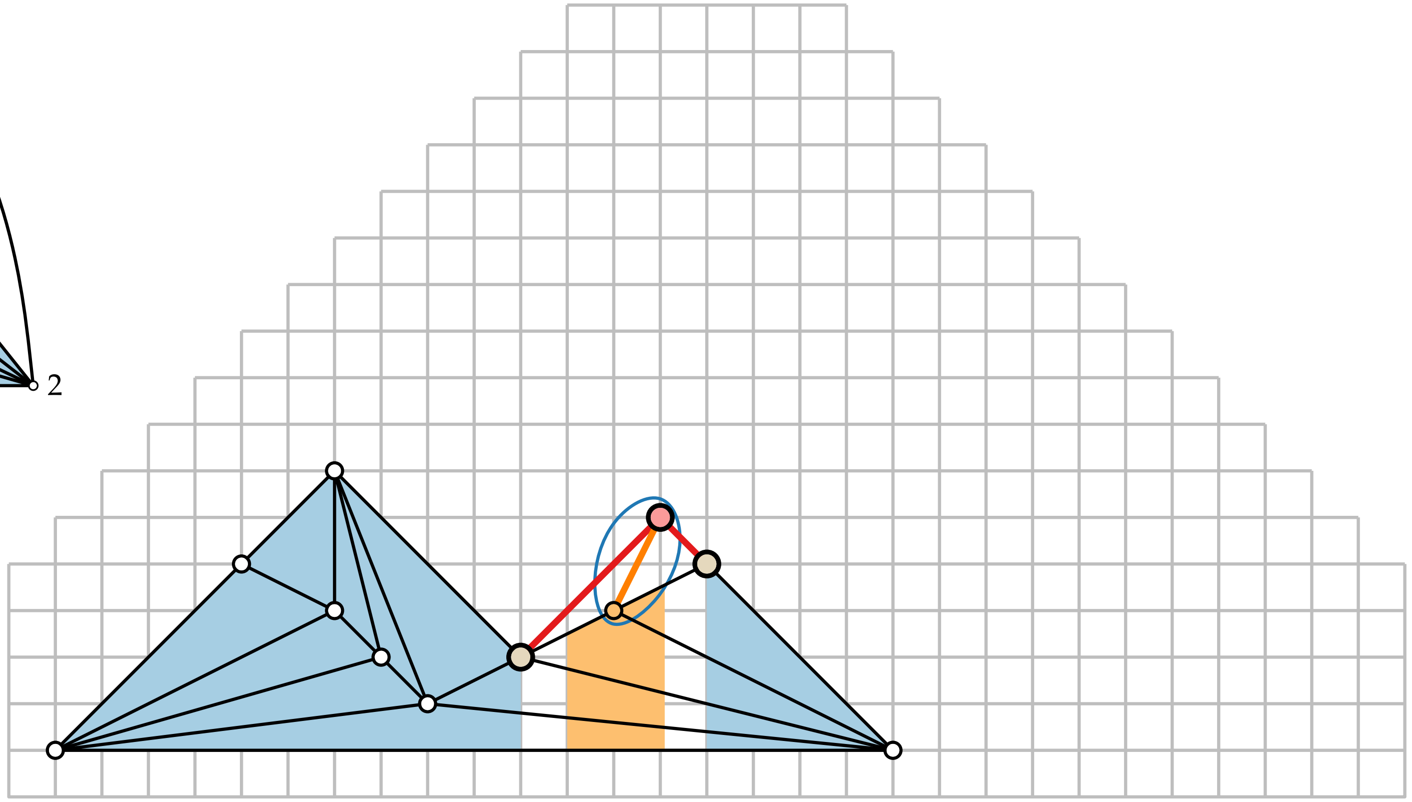




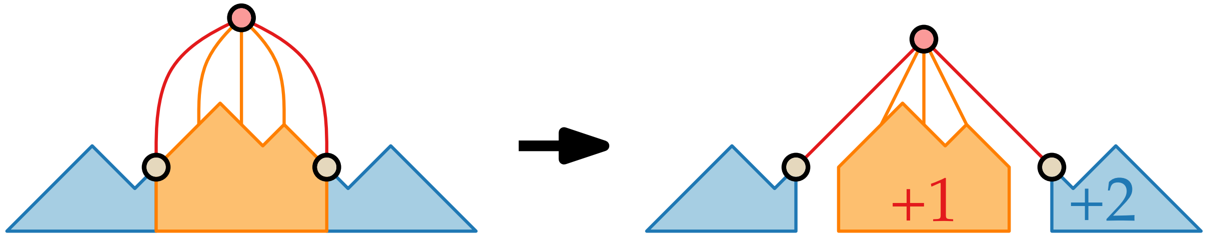
Shift Method – Example

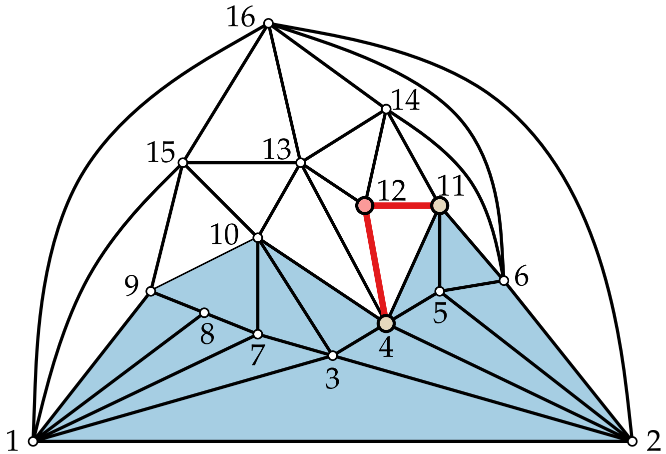


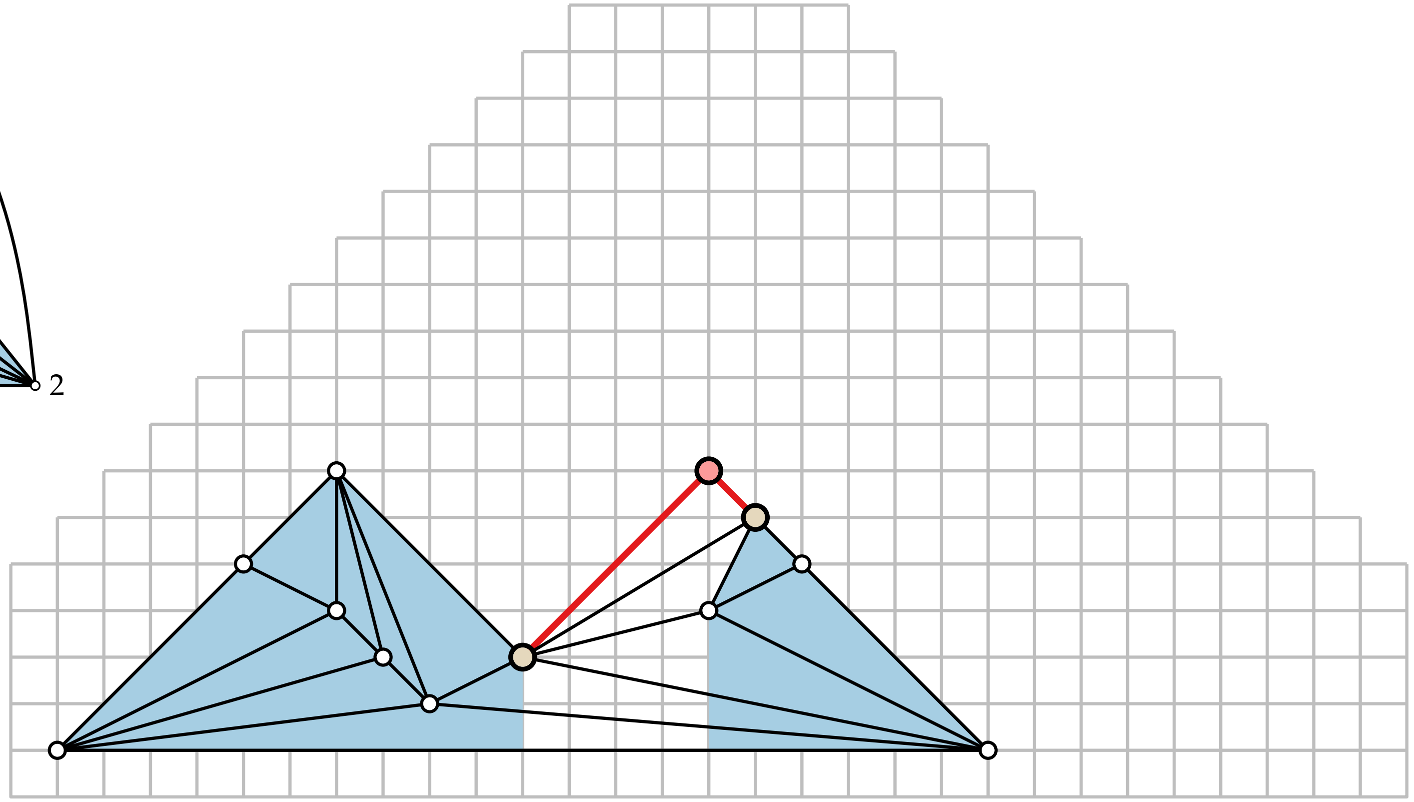




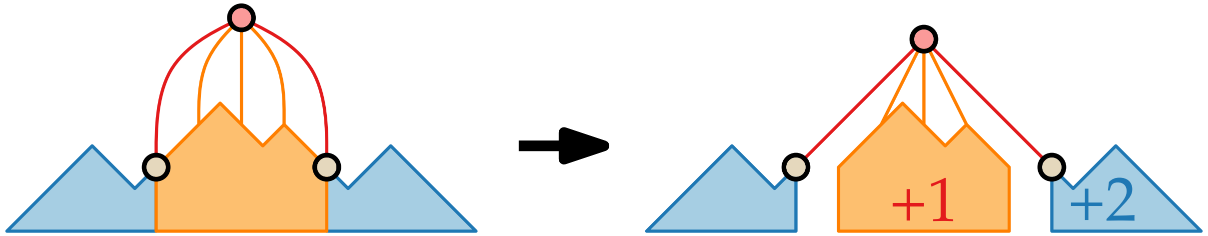
Shift Method – Example

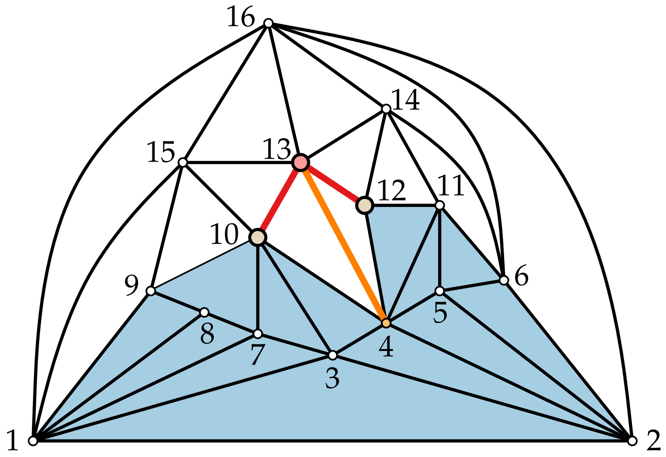


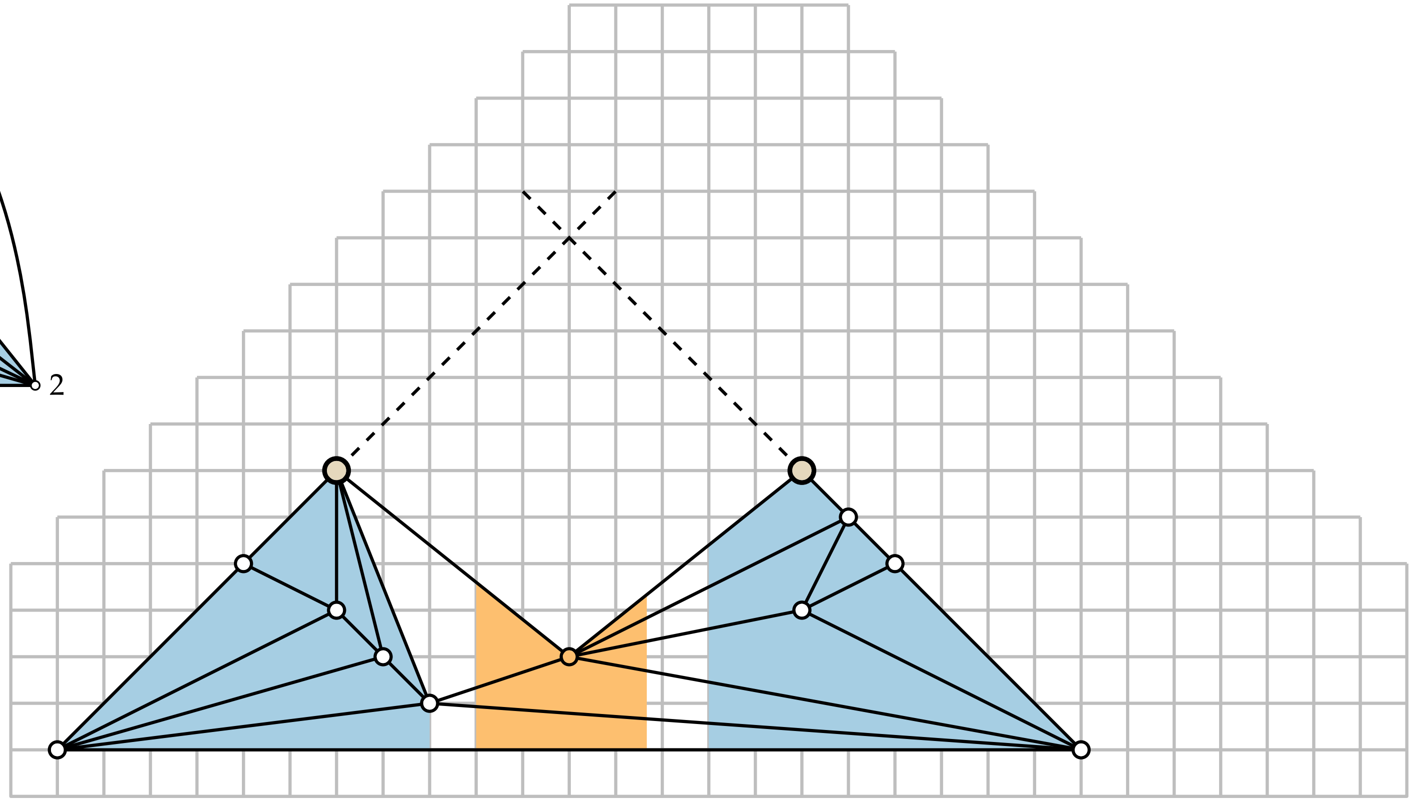




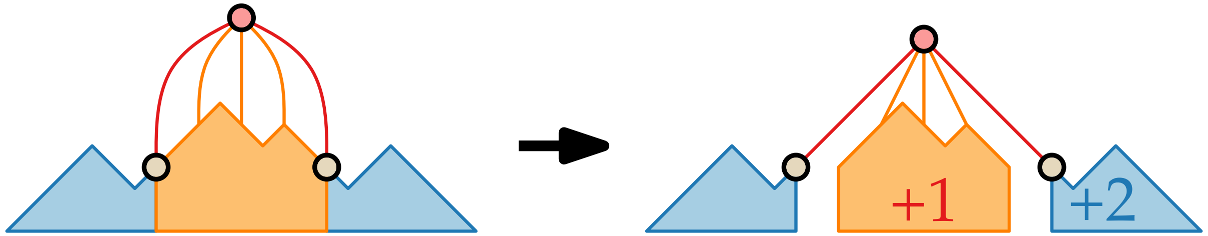
Shift Method – Example

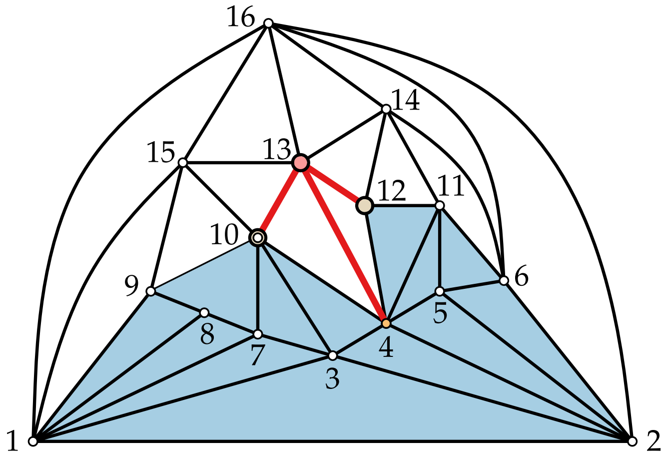


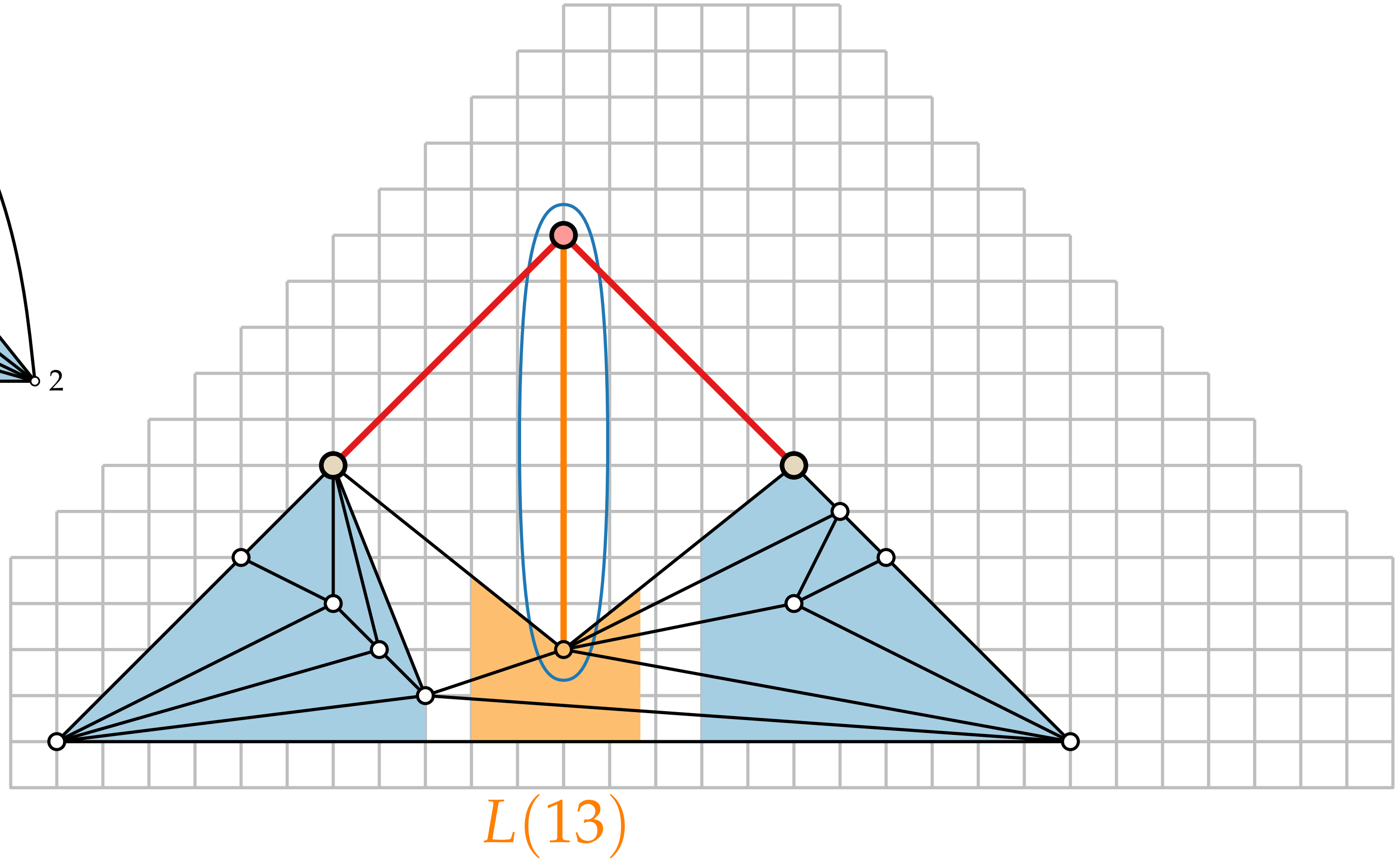




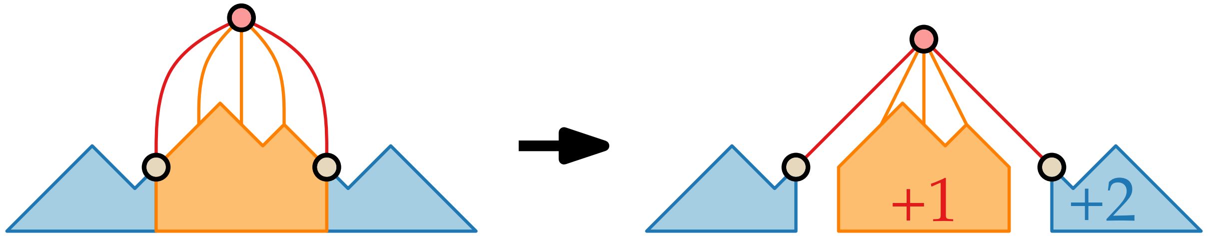
Shift Method – Example

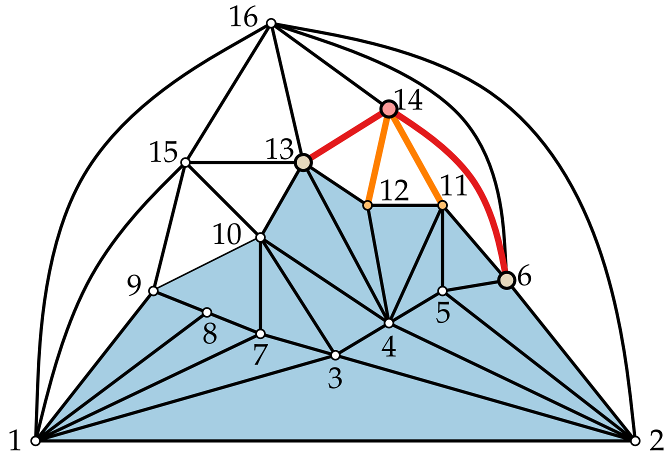


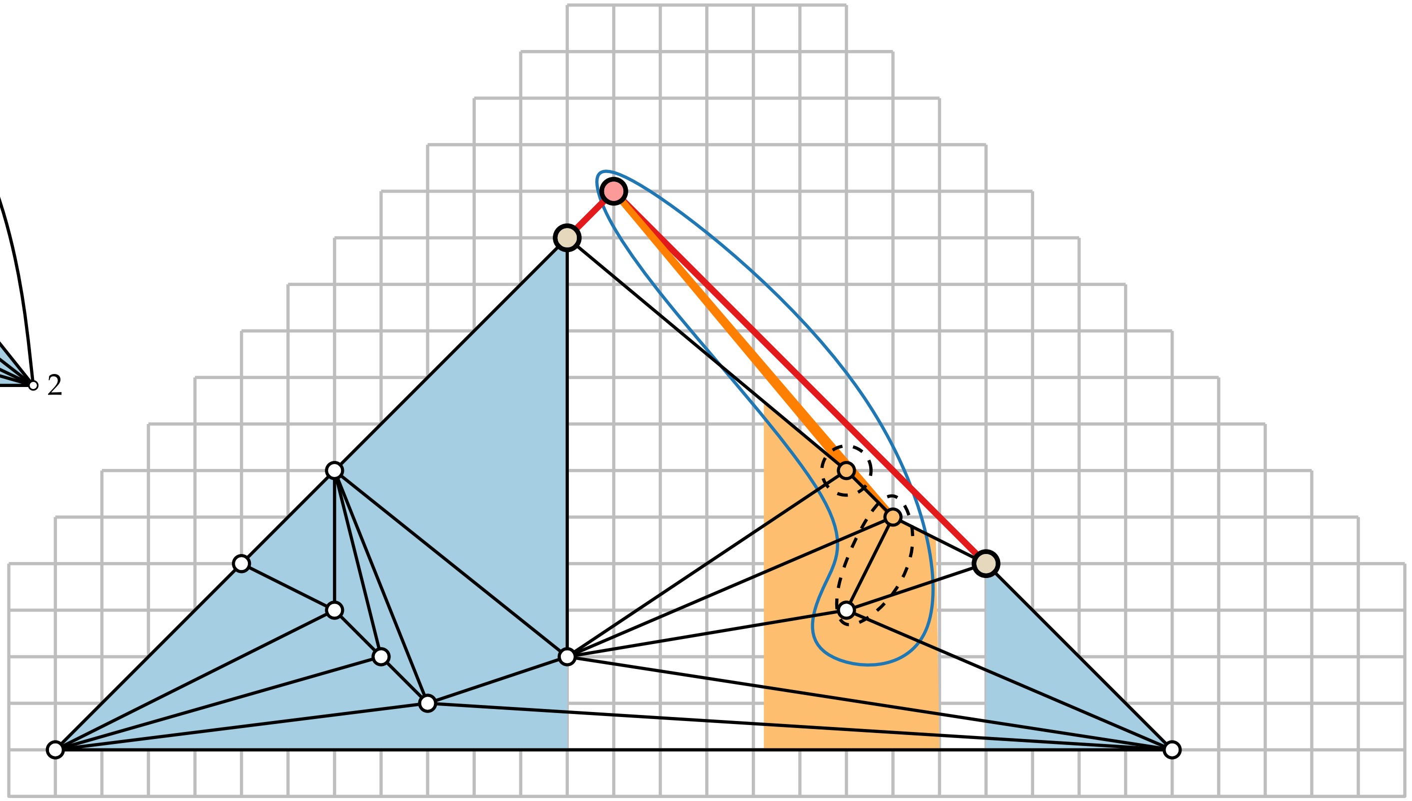




Shift Method – Example







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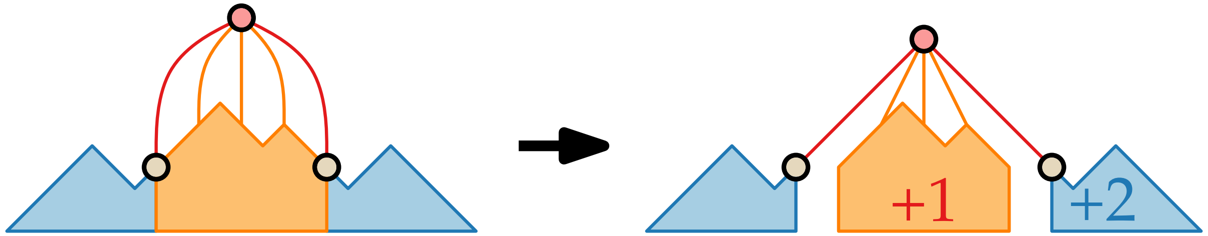
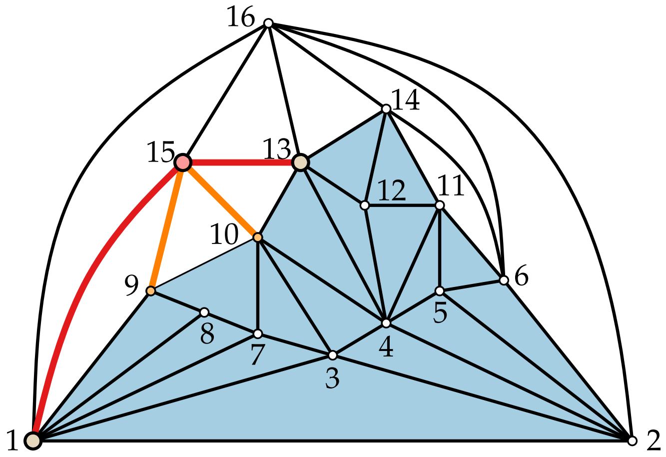
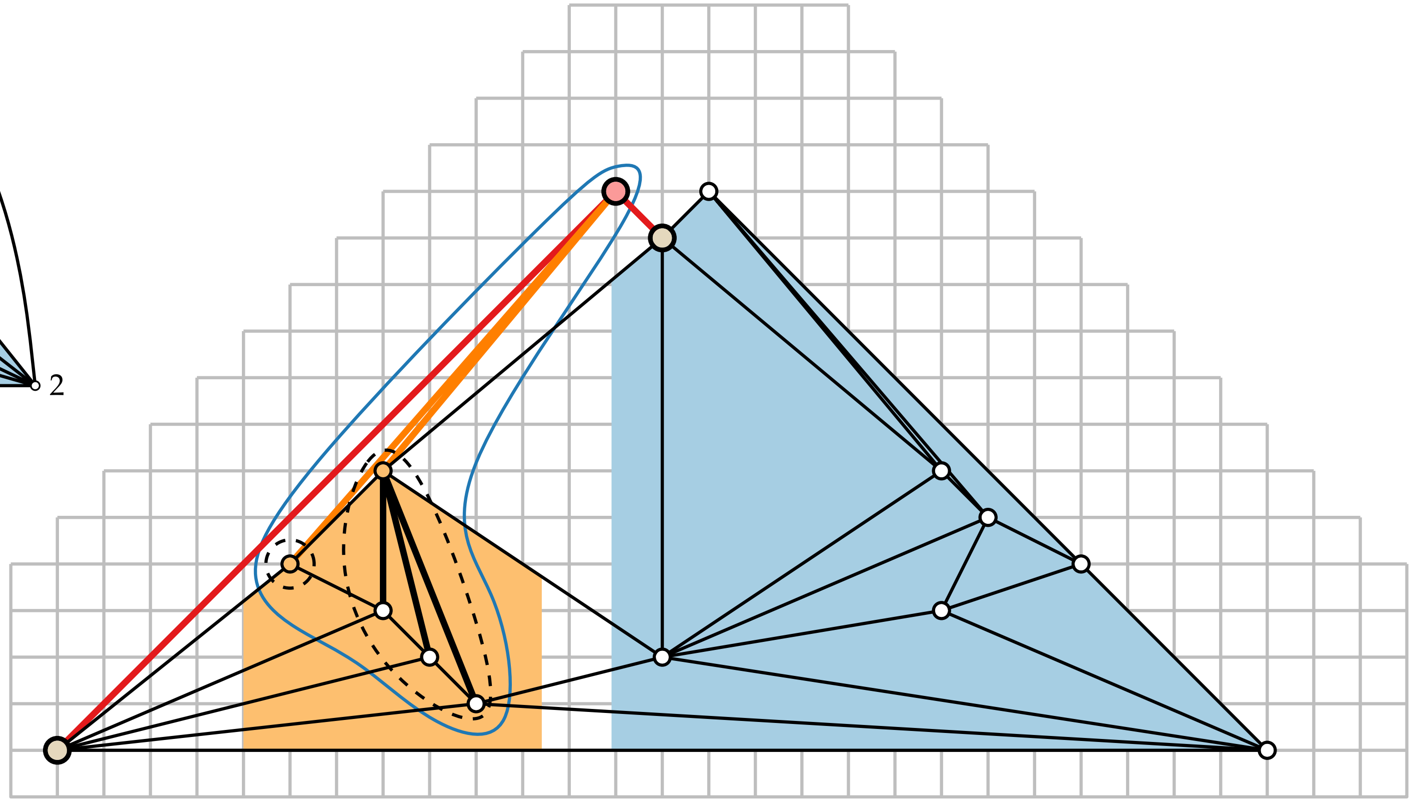
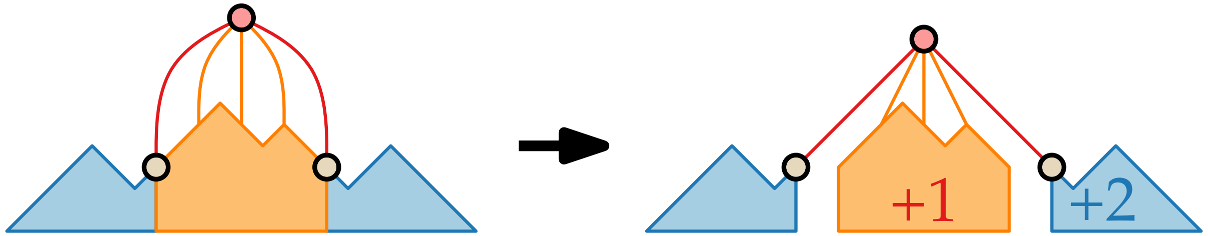
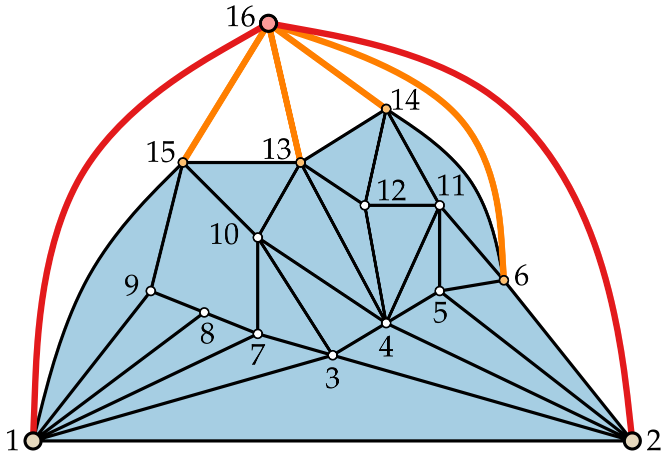


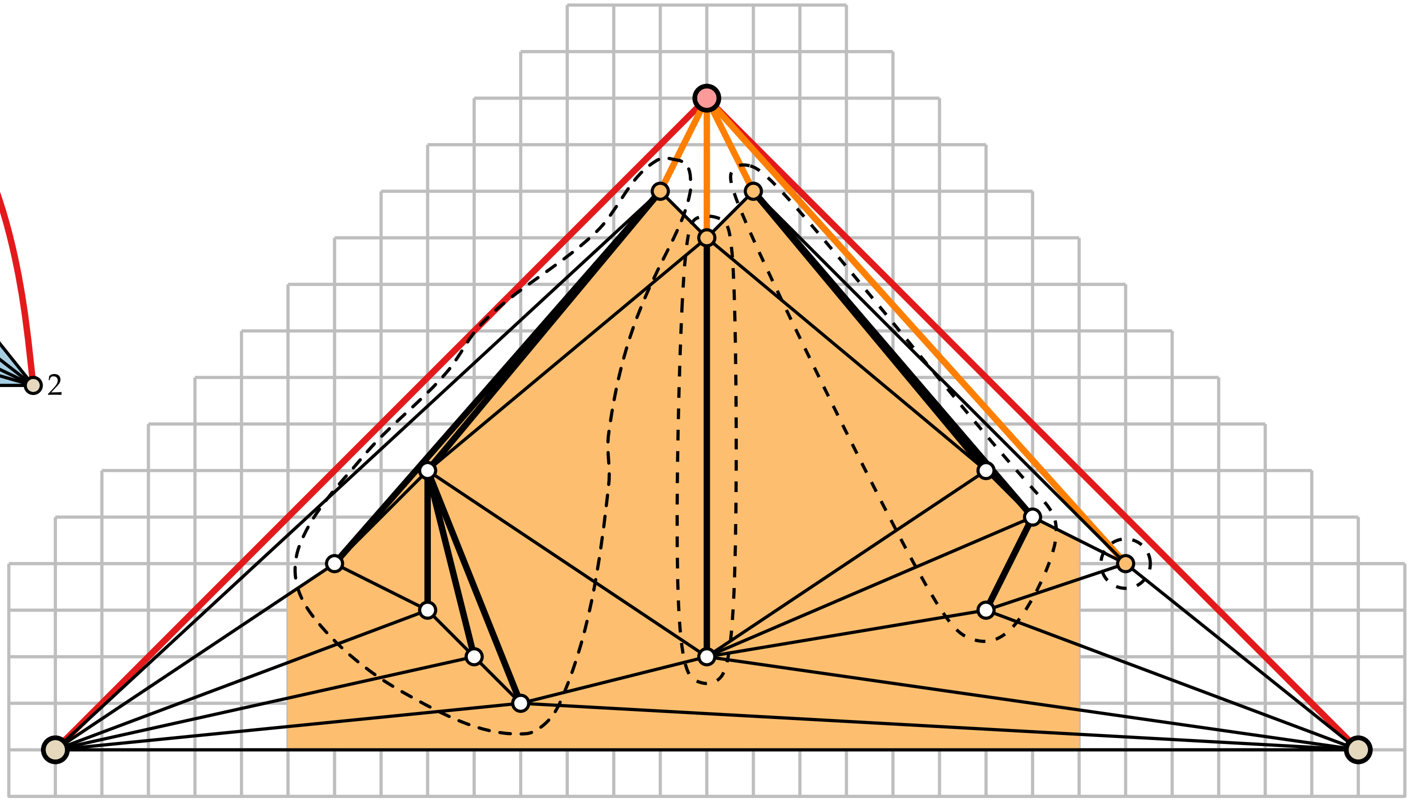




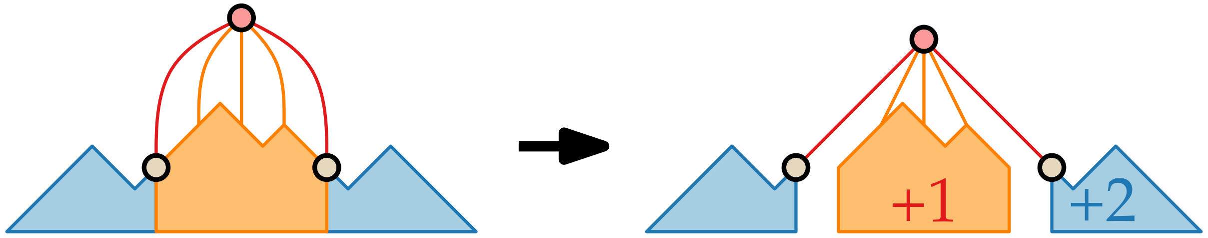
Shift Method – Example

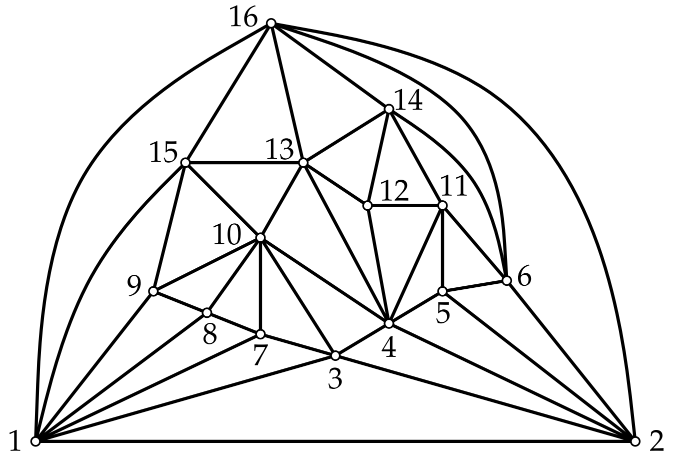


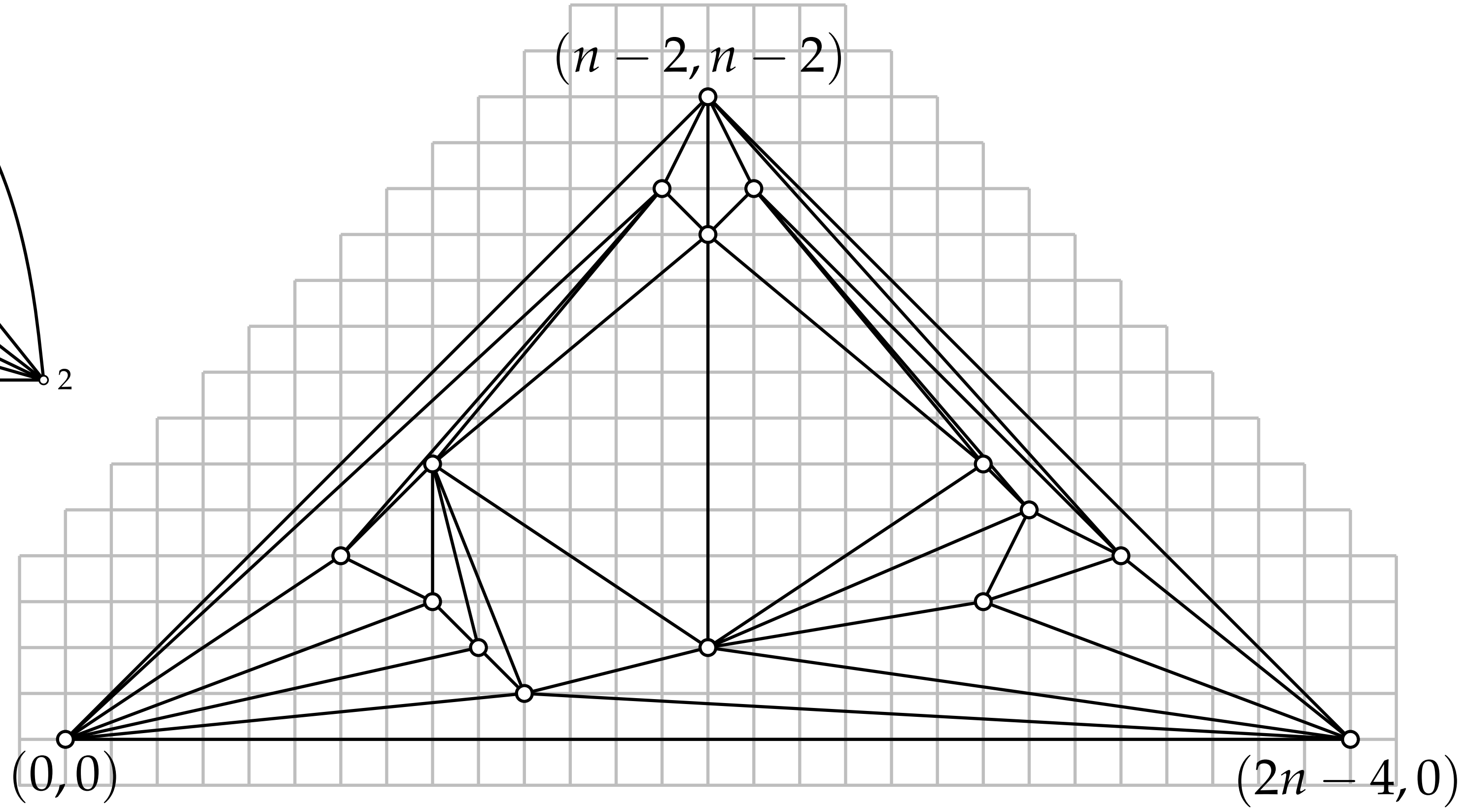




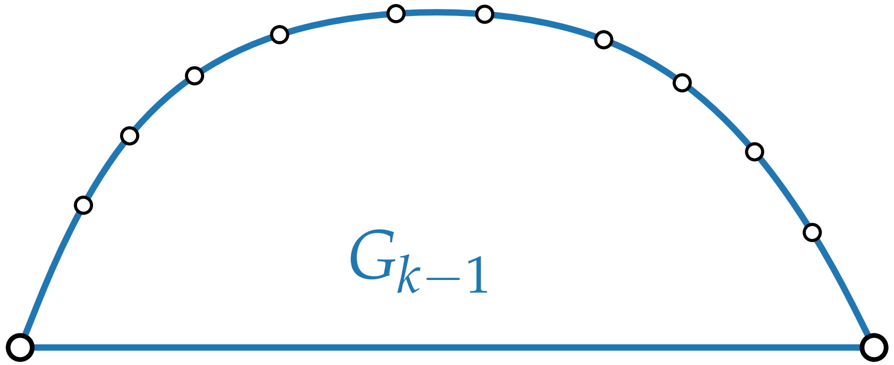
Shift Method – Example







Shift Method – Planarity



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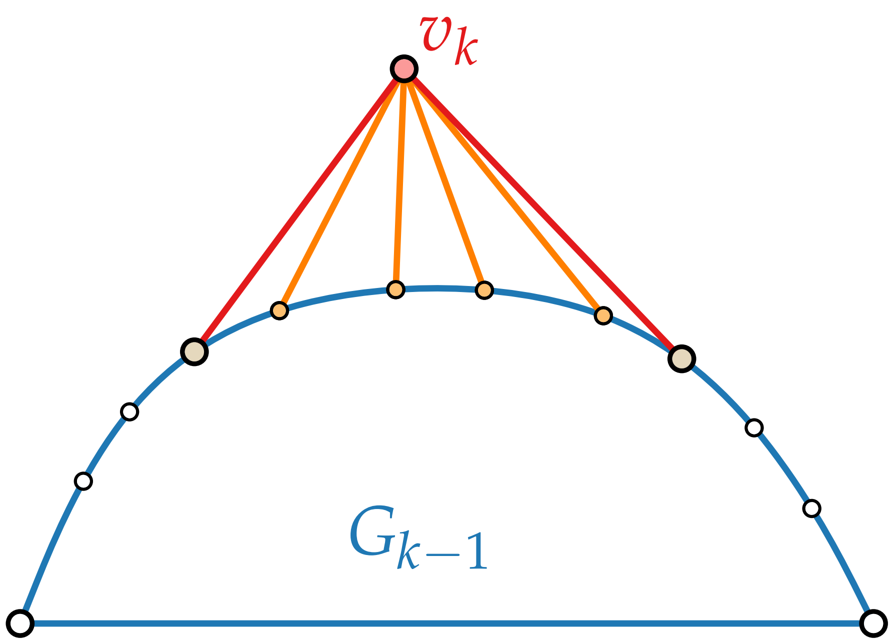
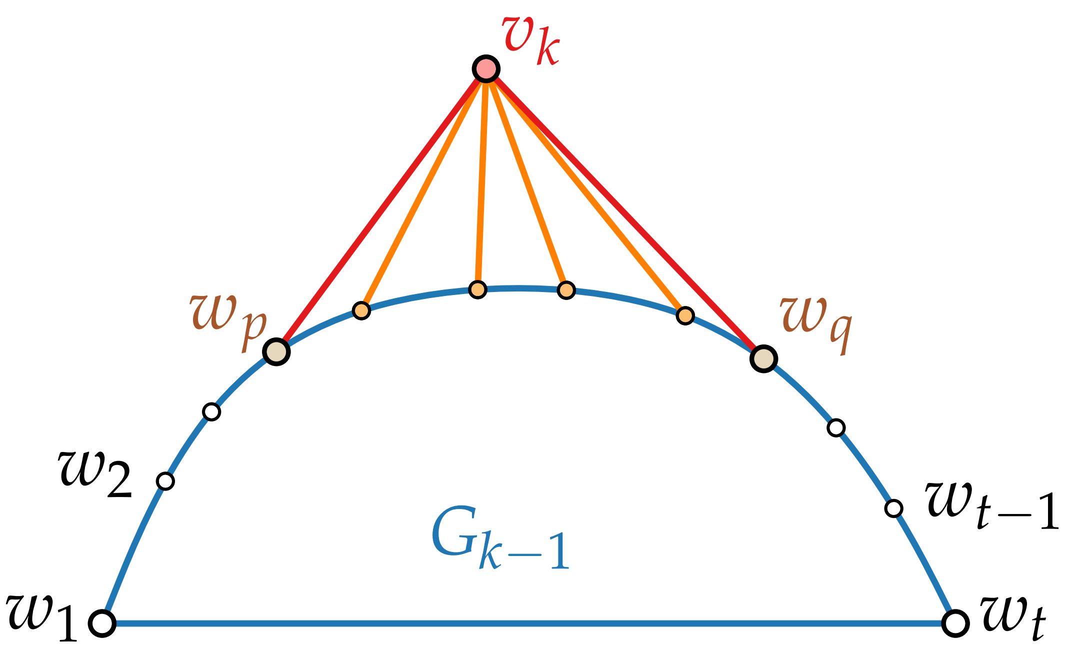
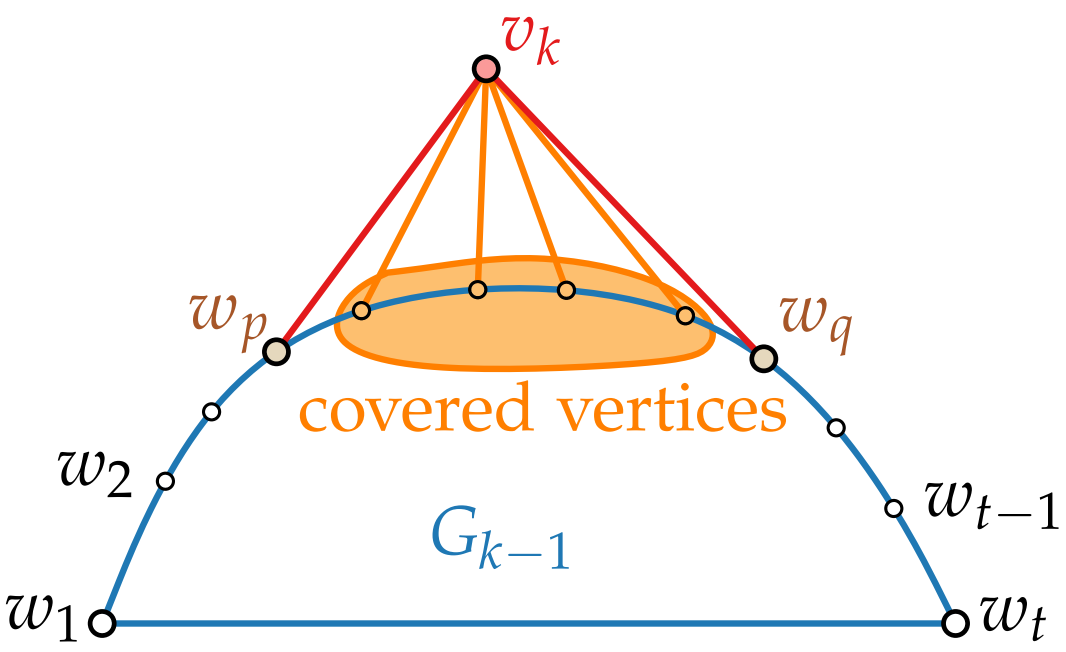


Shift Method – Planarity



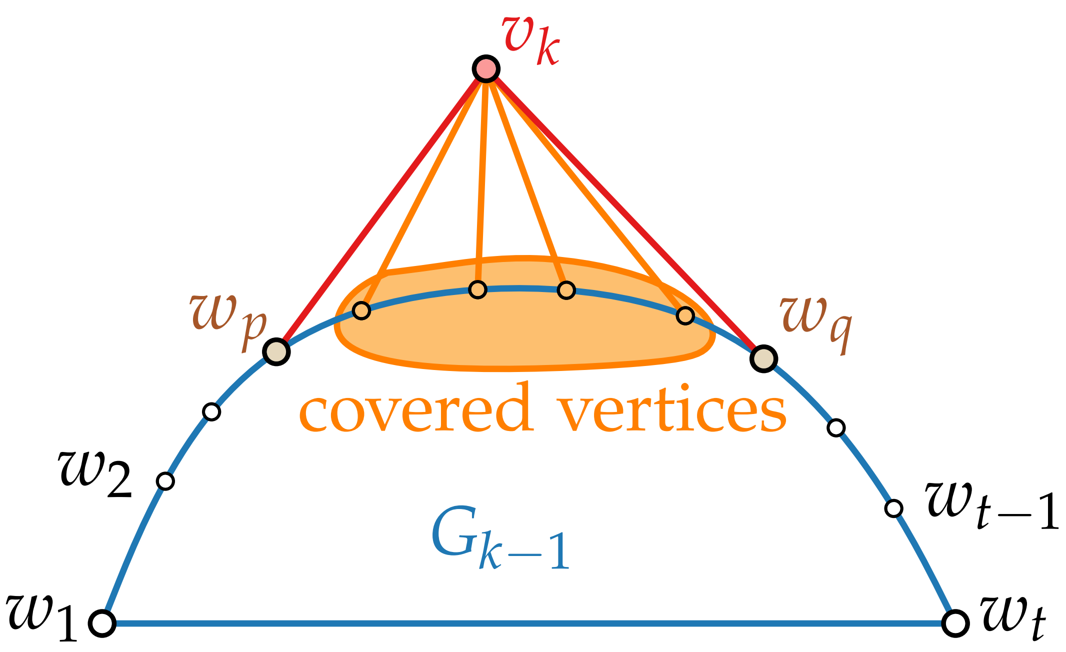
Shift Method – Planarity



Shift Method – Planarity

Observations.

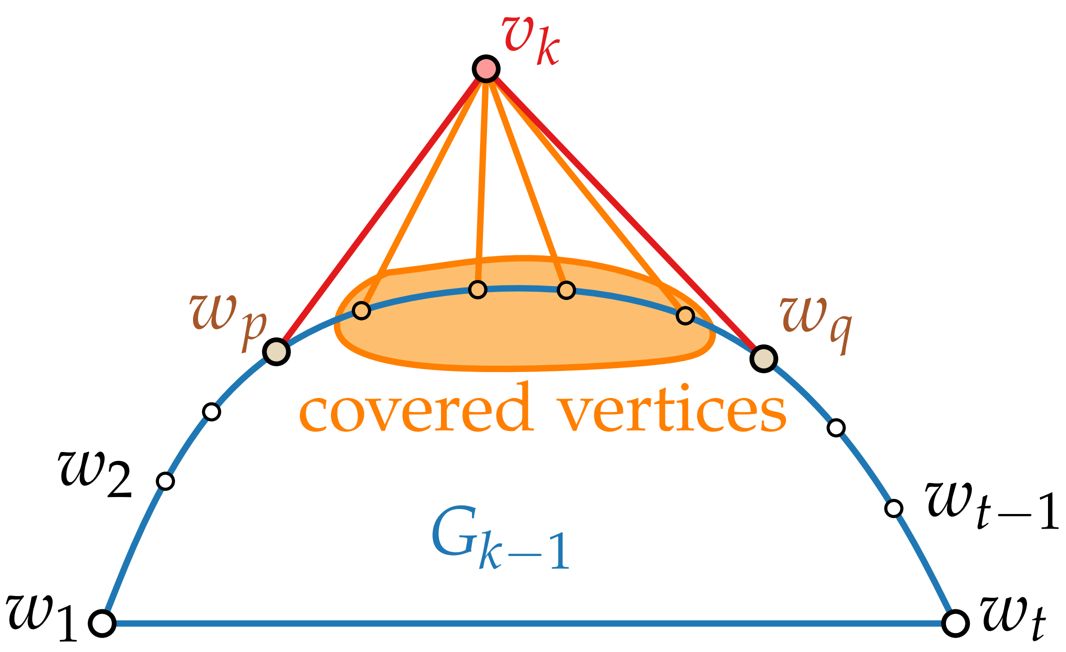
- Each internal vertex is covered exactly once.



Shift Method – Planarity

Observations.

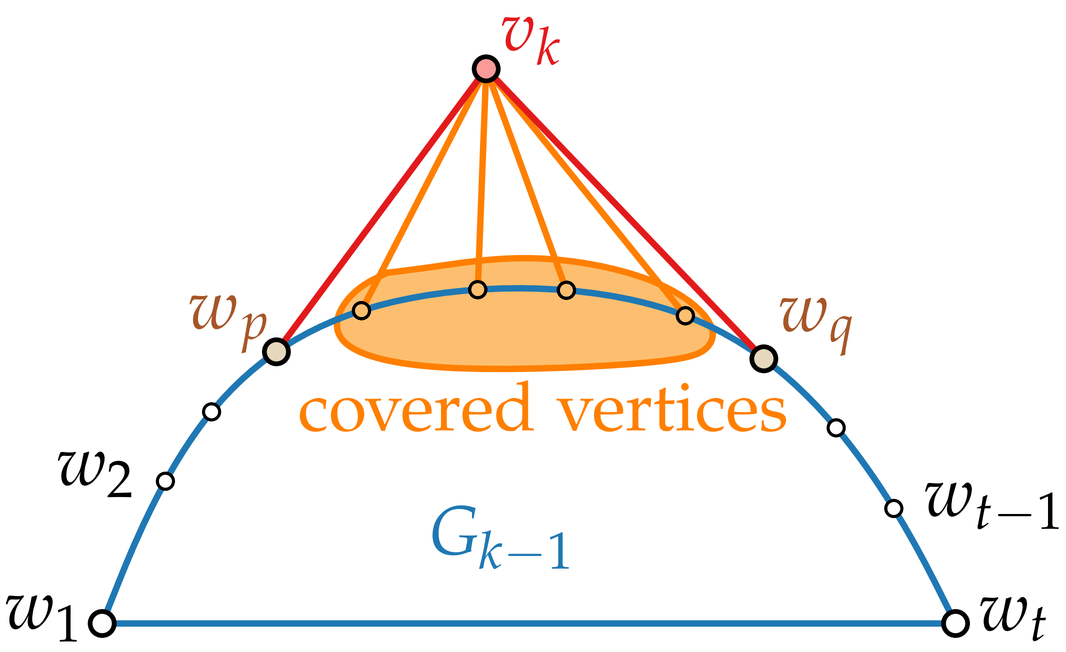
- Each internal vertex is covered exactly once.
- Covering relation defines a tree in G



Shift Method – Planarity

Observations.

- Each internal vertex is covered exactly once.
- Covering relation defines a tree in G
- and a forest in G_i, $1 \leq i \leq n-1$.



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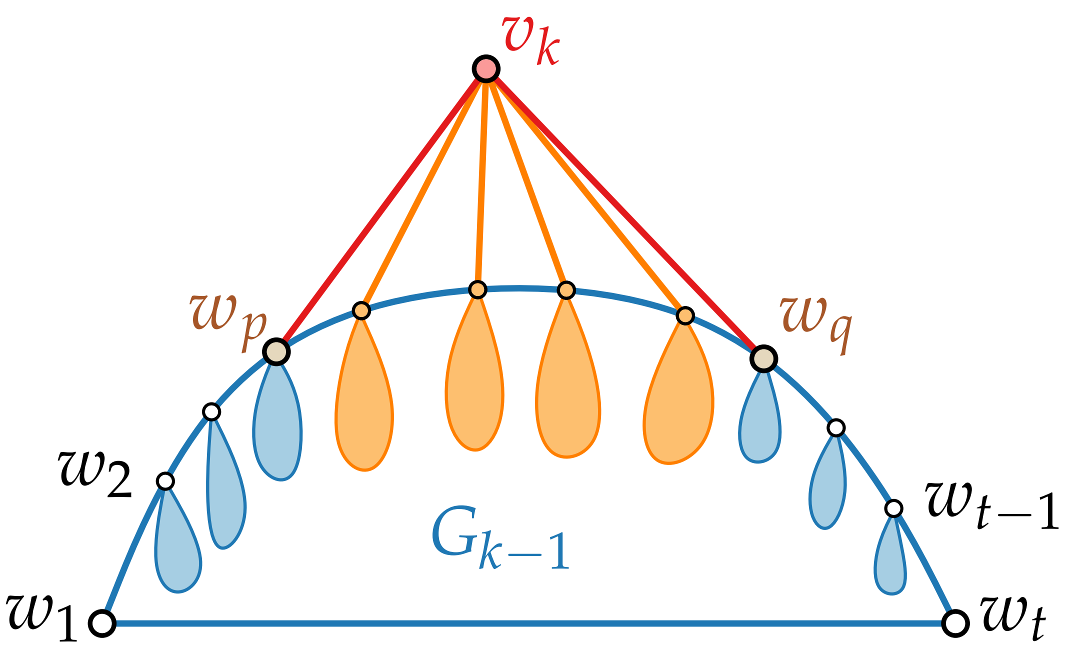

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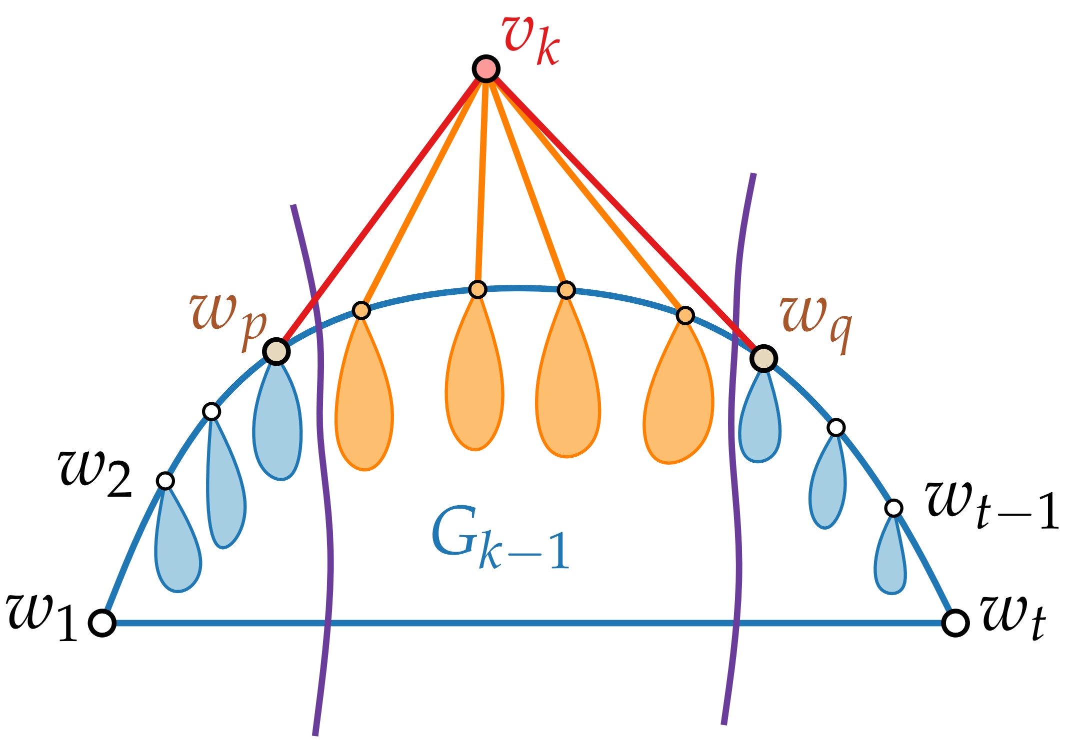

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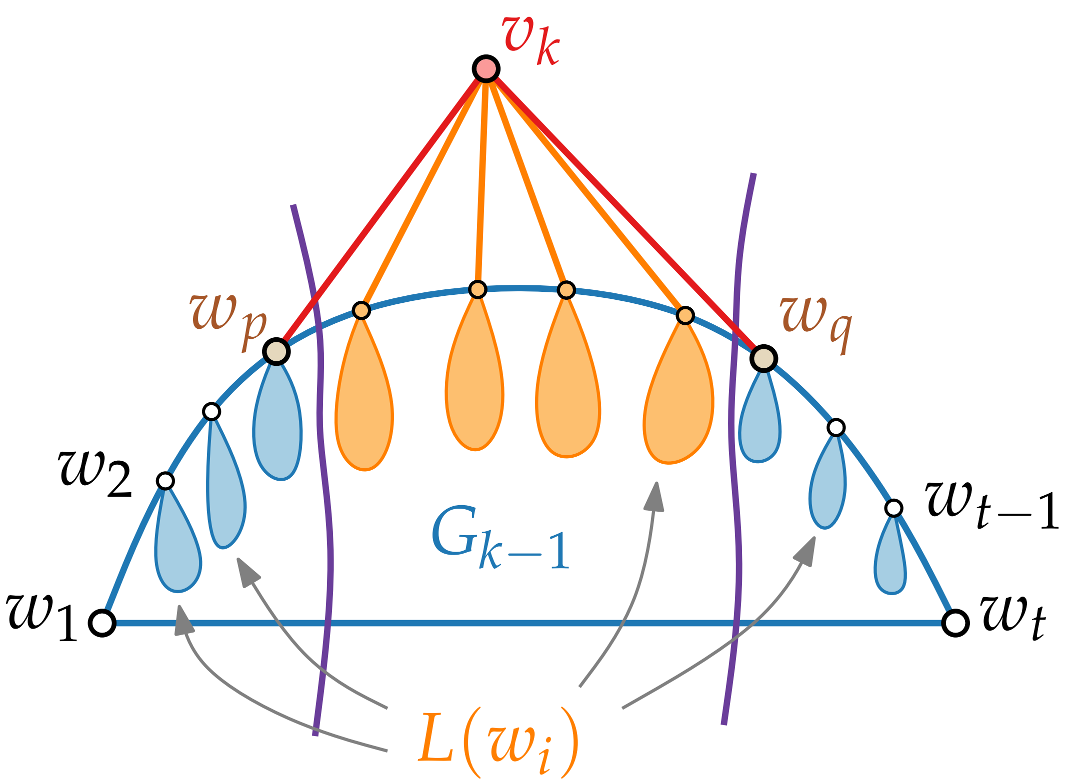
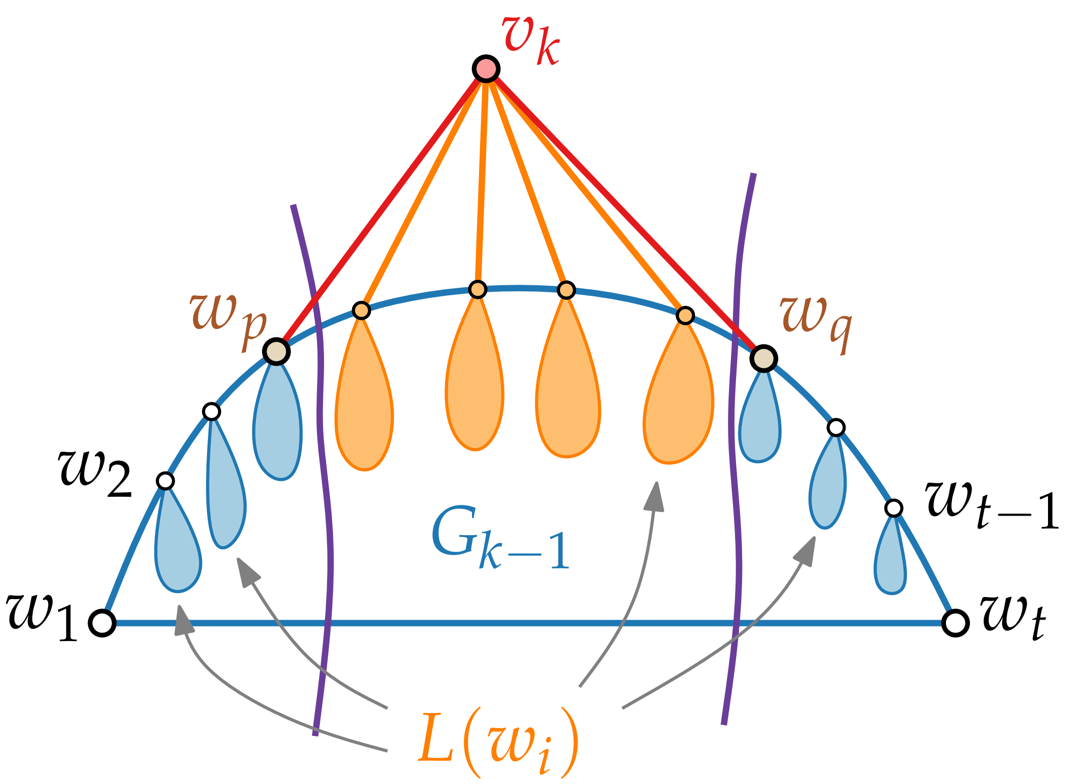


Shift Method – Planarity

Observations.

- Each internal vertex is covered exactly once.
- Covering relation defines a tree in G
- and a forest in G_i, $1 \le i \le n-1$.

Lemma.
Let $0 < \delta_1 \le \delta_2 \le \cdots \le \delta_t \in \mathbb{N}$, such that $\delta_q - \delta_p \ge 2$ and even.



Shift Method – Planarity

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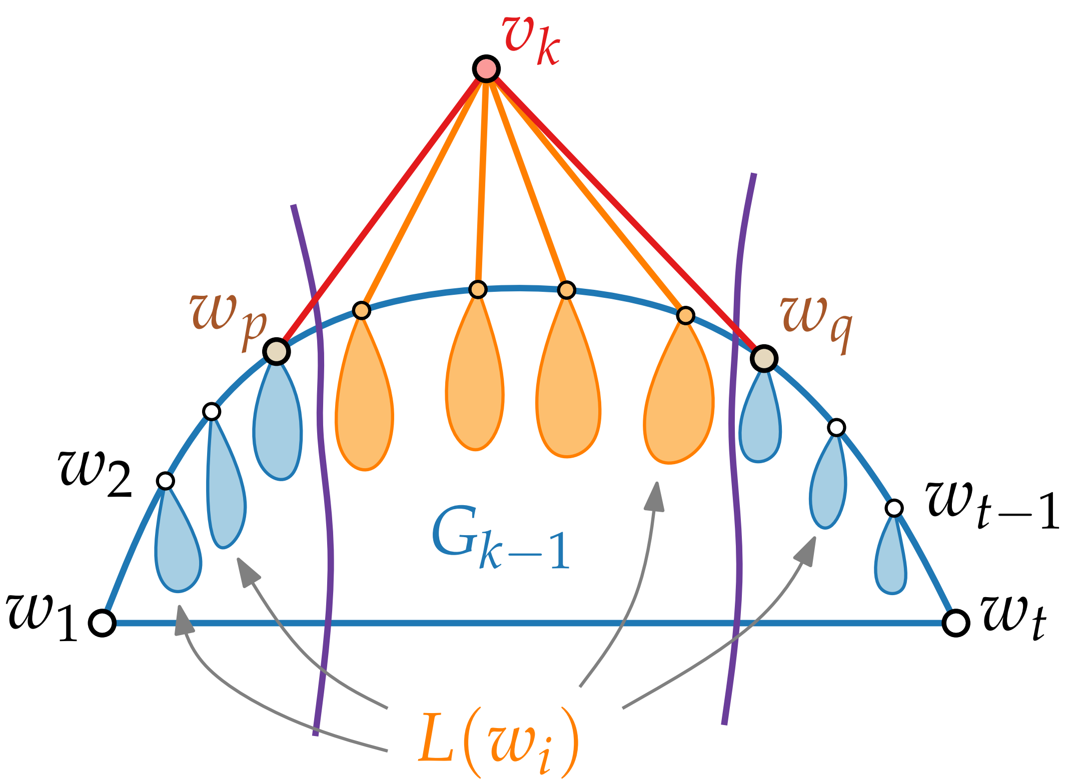
Lemma.
Let $0 < \delta_1 \leq \delta_2 \leq \cdots \leq \delta_t \in \mathbb{N}$, such that $\delta_q - \delta_p \geq 2$ and even. If we shift $L(w_i)$ by δ_i to the right, then we get a planar straight-line drawing.



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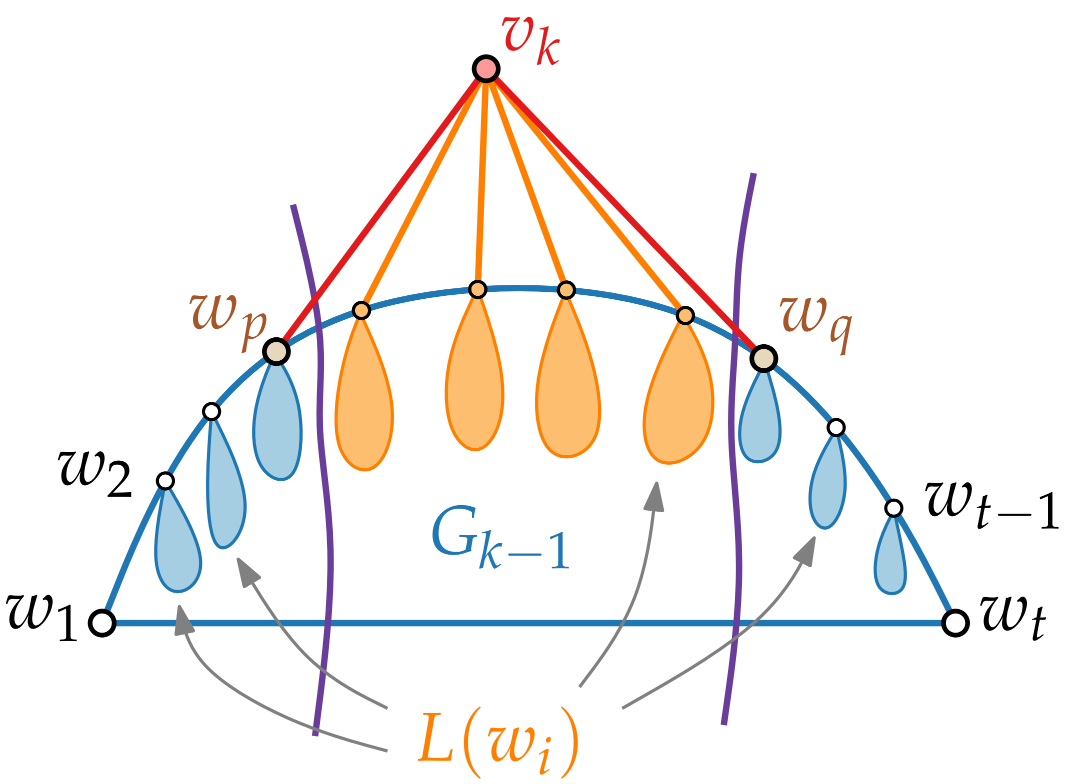

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Let $0 < \delta_1 \leq \delta_2 \leq \cdots \leq \delta_t \in \mathbb{N}$, such that $\delta_q - \delta_p \geq 2$ and even. If we shift $L(w_i)$ by δ_i to the right, then we get a planar straight-line drawing.

Proof by induction:
If G_{k-1} is drawn planar and straight-line, then so is G_k.

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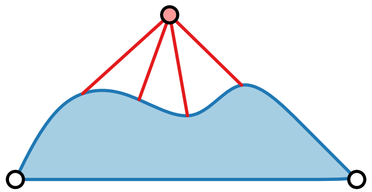
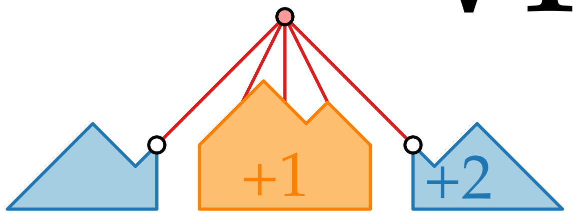
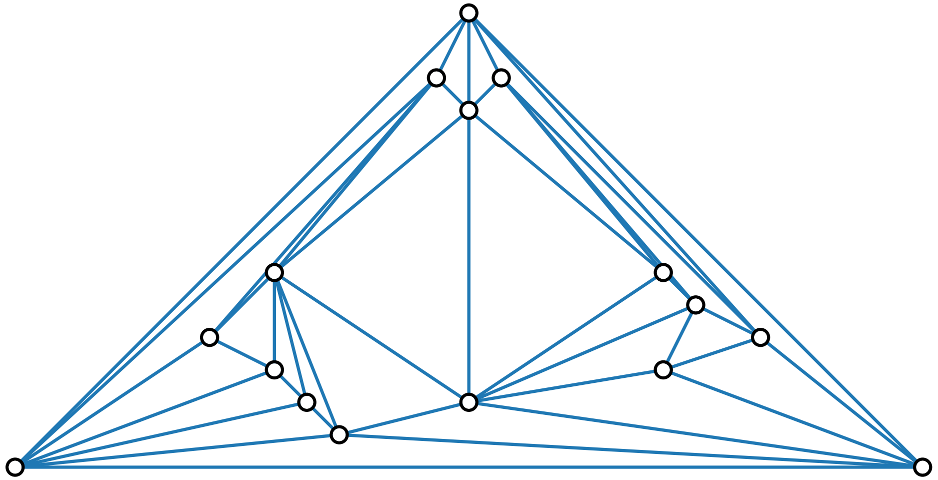


Visualization of Graphs



Lecture 4:
Straight-Line Drawings of Planar Graphs I: Canonical Ordering and Shift Method



Part V:
Linear Time

Philipp Kindermann

Shift Method – Pseudocode

Let $v_1, \ldots, v_n$ be a canonical order of G

for $i = 1$ to 3 **do**

for $i = 4$ to n **do**

Shift Method – Pseudocode

Let $v_1, \ldots, v_n$ be a canonical order of G

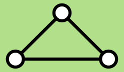
for $i = 1$ to 3 **do**

  $L(v_i) \leftarrow \{v_i\}$

for $i = 4$ to n **do**

Shift Method – Pseudocode

Let $v_1, \ldots, v_n$ be a canonical order of G
for $i = 1$ to 3 **do**
  $L(v_i) \leftarrow \{v_i\}$
$P(v_1) \leftarrow (0,0); P(v_2) \leftarrow (2,0), P(v_3) \leftarrow (1,1)$



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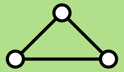
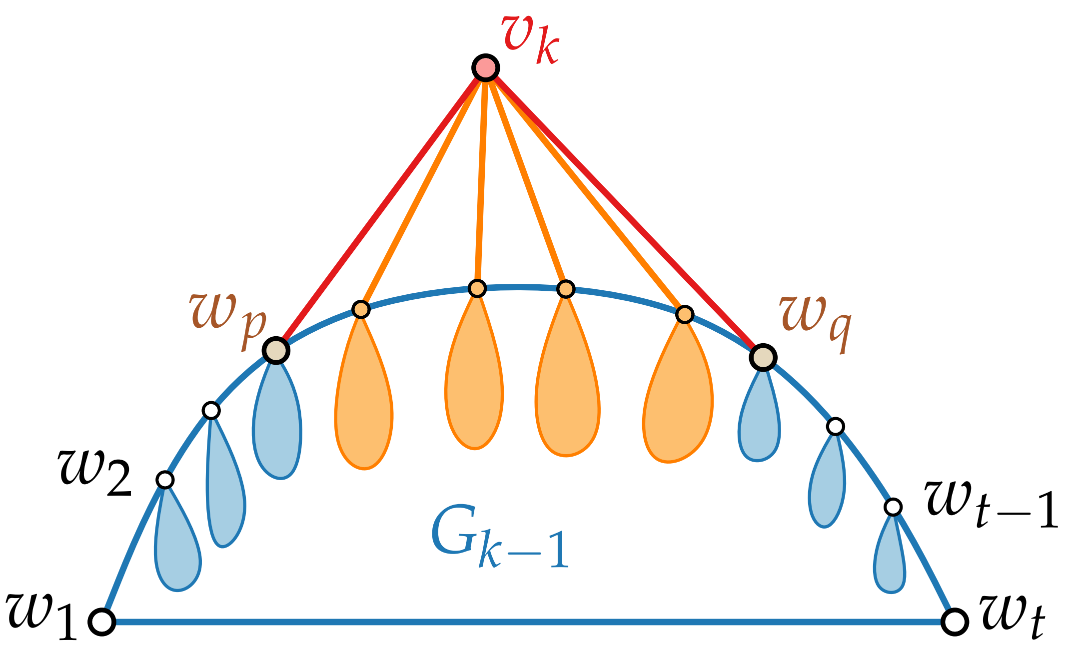


for $i = 4$ to n **do**
  Let $w_1 = v_1, w_2, \ldots, w_{t-1}, w_t = v_2$
  denote the boundary of G_{i-1}
  and let $w_p, \ldots, w_q$ be the neighbours of v_i



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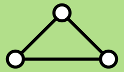

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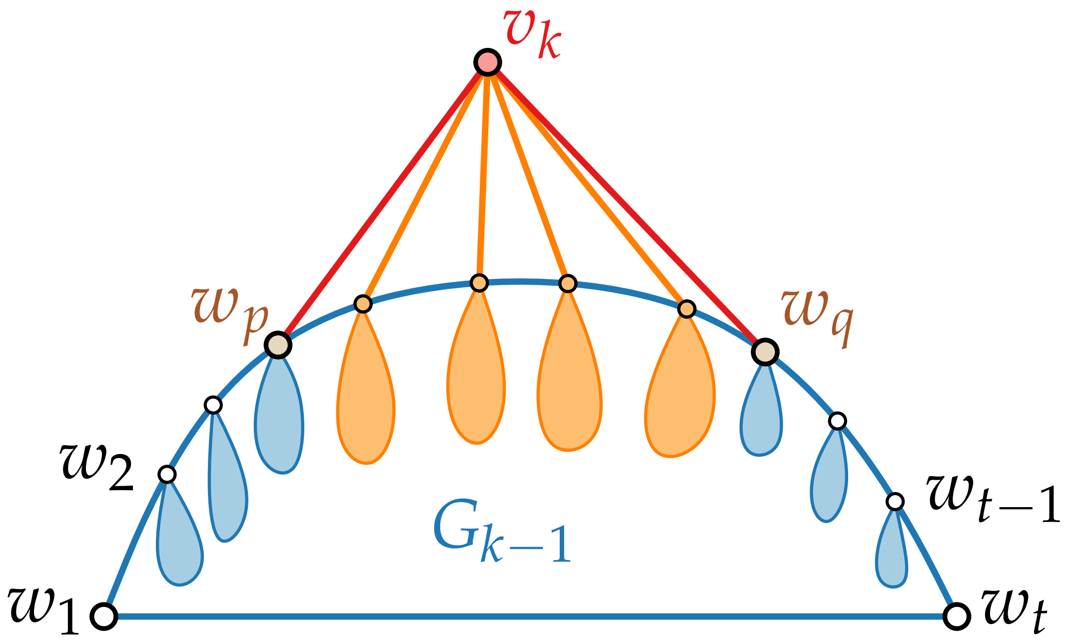

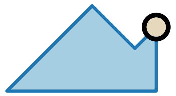

Shift Method – Pseudocode

Let $v_1, \ldots, v_n$ be a canonical order of G

for $i = 1$ to 3 **do**

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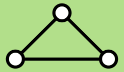

for $i = 4$ to n **do**

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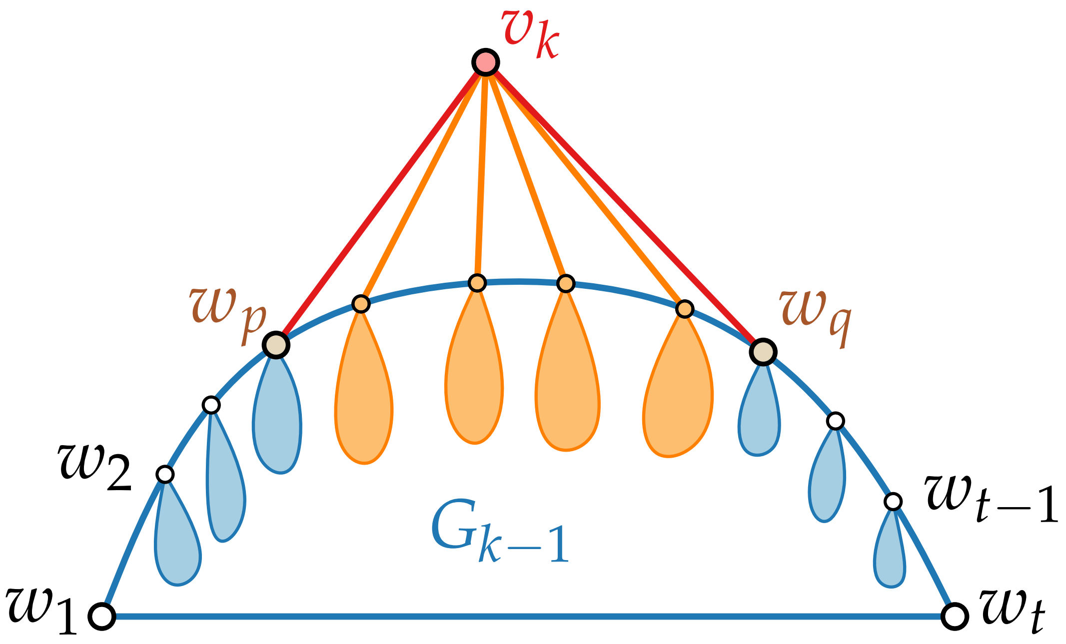
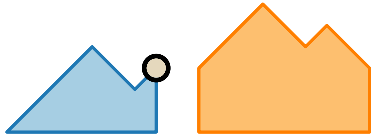
  denote the boundary of G_{i-1}
  and let $w_p, \ldots, w_q$ be the neighbours of v_i

  for $\forall v \in \cup_{j=p+1}^{q-1} L(w_j)$ **do**





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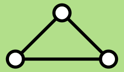
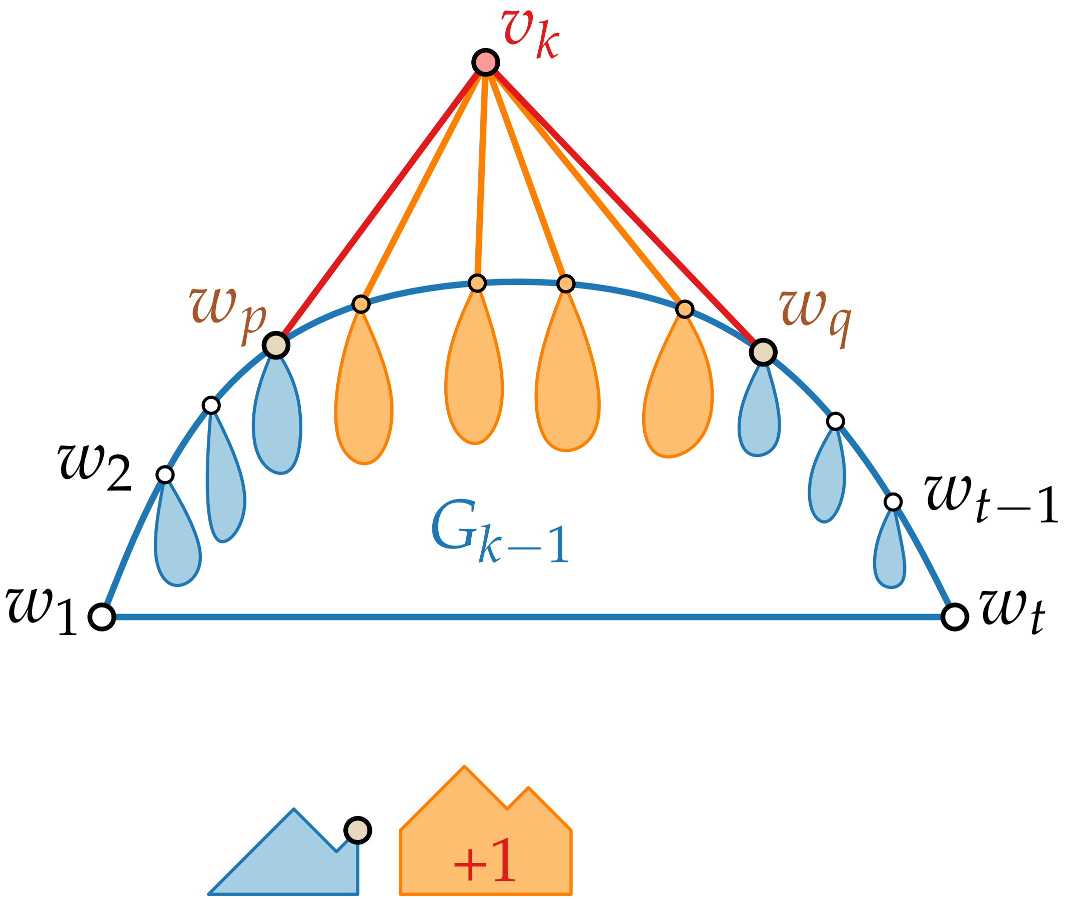
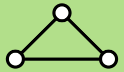


for $i = 4$ to n **do**
  Let $w_1 = v_1, w_2, \ldots, w_{t-1}, w_t = v_2$ denote the boundary of G_{i-1} and let $w_p, \ldots, w_q$ be the neighbours of v_i
  for $\forall v \in \cup_{j=p+1}^{q-1} L(w_j)$ **do**
    $x(v) \leftarrow x(v) + 1$

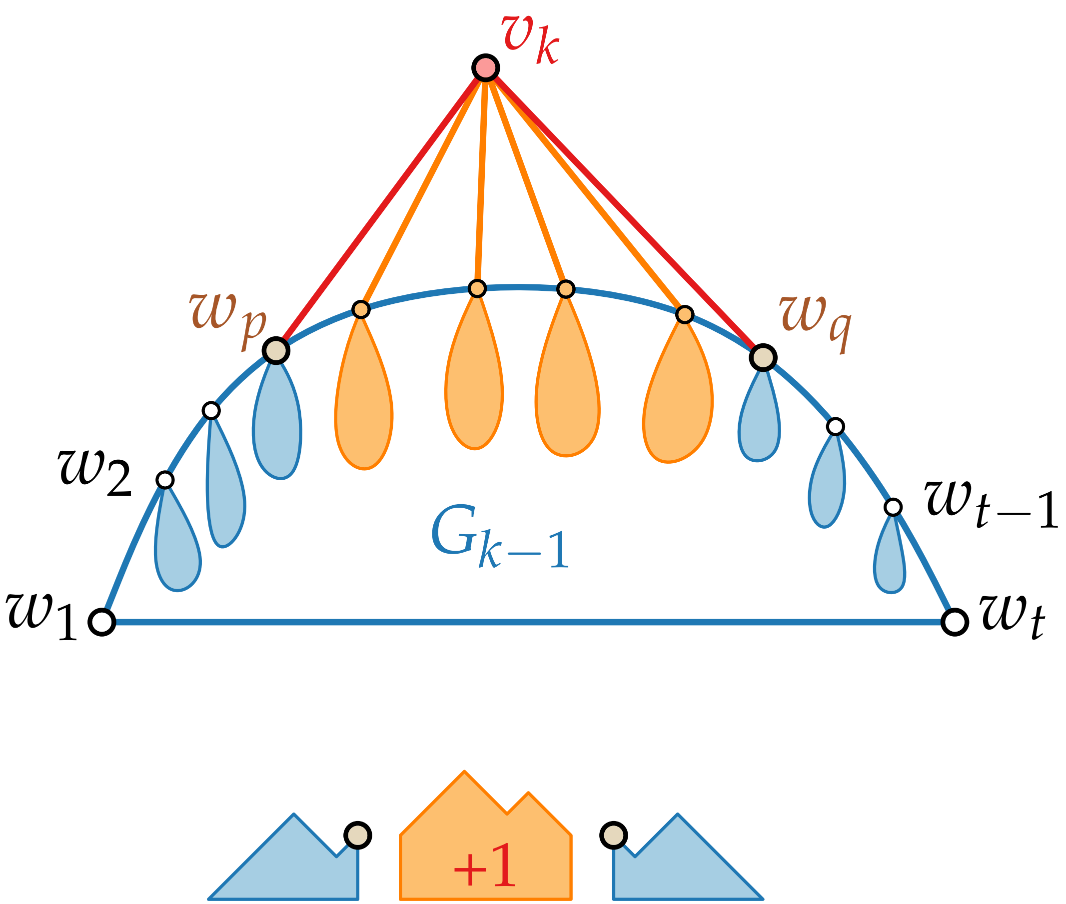


Shift Method – Pseudocode

Let $v_1, \ldots, v_n$ be a canonical order of G
for $i = 1$ to 3 **do**
  $L(v_i) \leftarrow \{v_i\}$
$P(v_1) \leftarrow (0,0); P(v_2) \leftarrow (2,0), P(v_3) \leftarrow (1,1)$



for $i = 4$ to n **do**
  Let $w_1 = v_1, w_2, \ldots, w_{t-1}, w_t = v_2$
  denote the boundary of G_{i-1}
  and let $w_p, \ldots, w_q$ be the neighbours of v_i
  for $\forall v \in \cup_{j=p+1}^{q-1} L(w_j)$ **do**
    $x(v) \leftarrow x(v) + 1$
  for $\forall v \in \cup_{j=q}^{t} L(w_j)$ **do**



Shift Method – Pseudocode

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  $L(v_i) \leftarrow \{v_i\}$

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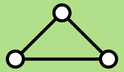


for $i = 4$ to n **do**

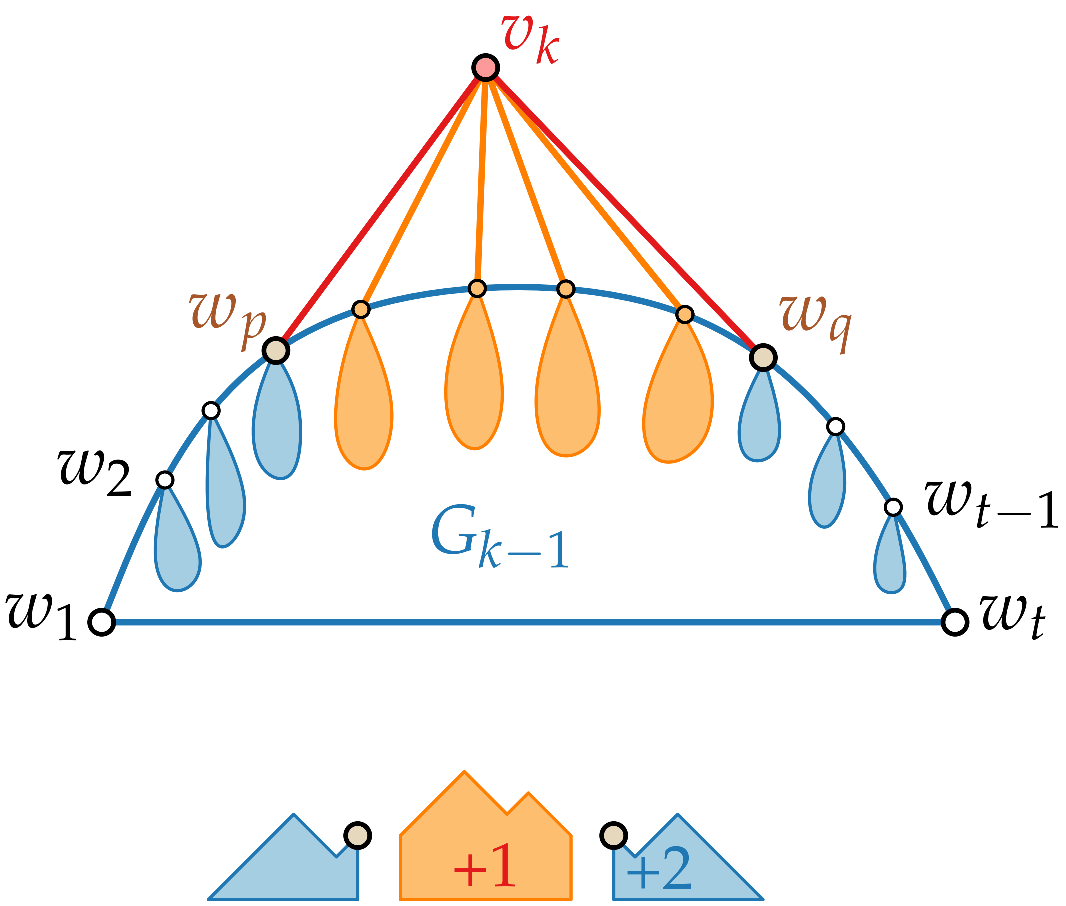
  Let $w_1 = v_1, w_2, \ldots, w_{t-1}, w_t = v_2$ denote the boundary of G_{i-1} and let $w_p, \ldots, w_q$ be the neighbours of v_i

  for $\forall v \in \cup_{j=p+1}^{q-1} L(w_j)$ **do**

    $x(v) \leftarrow x(v) + 1$

  for $\forall v \in \cup_{j=q}^{t} L(w_j)$ **do**

    $x(v) \leftarrow x(v) + 2$



Shift Method – Pseudocode

Let $v_1, \ldots, v_n$ be a canonical order of G

for $i = 1$ to 3 **do**
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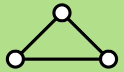
$P(v_1) \leftarrow (0,0); P(v_2) \leftarrow (2,0), P(v_3) \leftarrow (1,1)$ 

for $i = 4$ to n **do**
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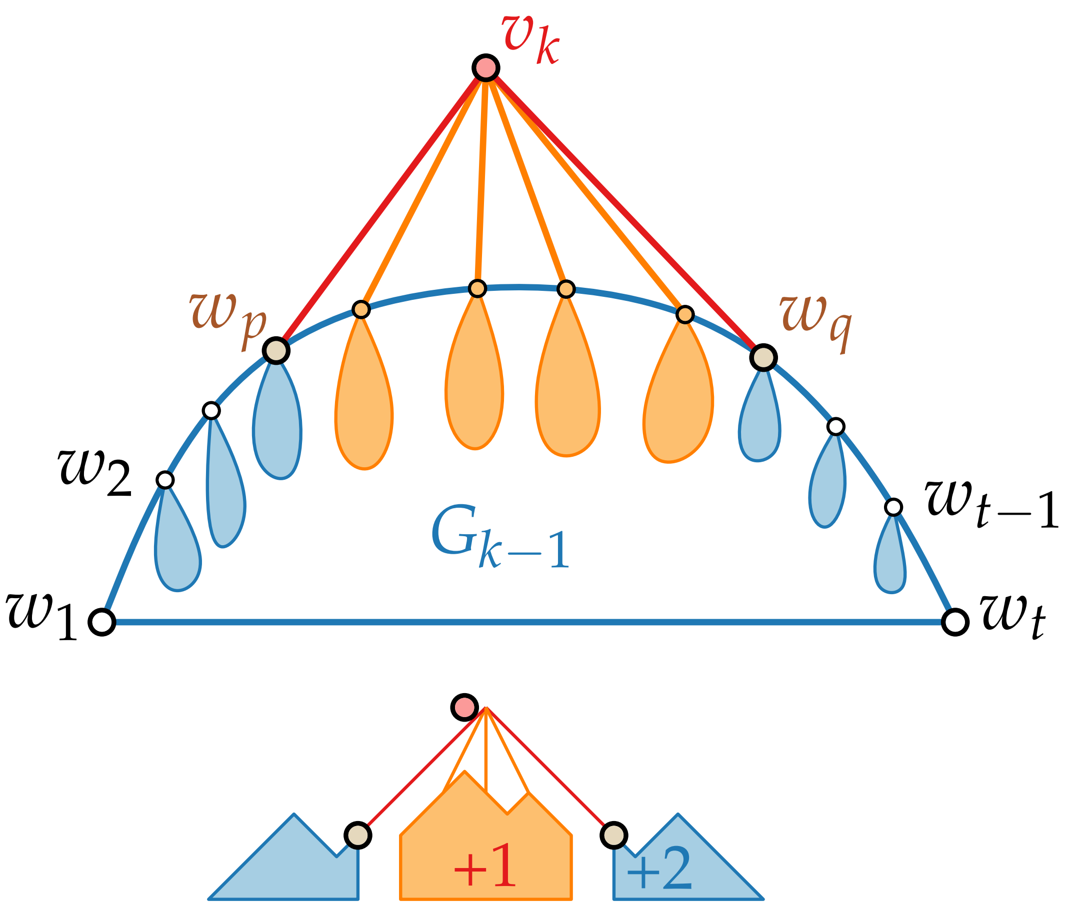
  $P(v_i) \leftarrow$ intersection of $+1/-1$ diagonals
    through $P(w_p)$ and $P(w_q)$



Shift Method – Pseudocode

Let $v_1, \ldots, v_n$ be a canonical order of G

for $i = 1$ to 3 **do**

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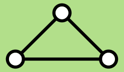

for $i = 4$ to n **do**

  Let $w_1 = v_1, w_2, \ldots, w_{t-1}, w_t = v_2$
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  and let $w_p, \ldots, w_q$ be the neighbours of v_i

  for $\forall v \in \cup_{j=p+1}^{q-1} L(w_j)$ **do**

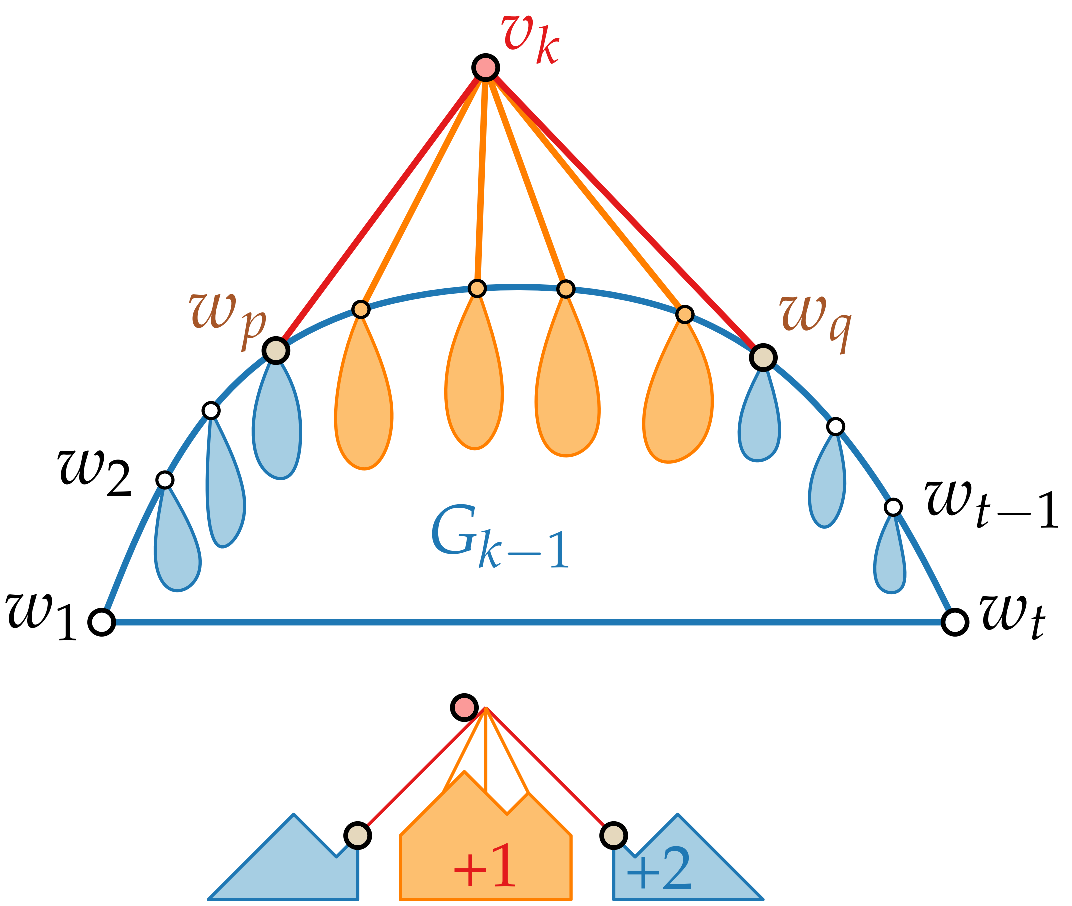
    $x(v) \leftarrow x(v) + 1$

  for $\forall v \in \cup_{j=q}^{t} L(w_j)$ **do**

    $x(v) \leftarrow x(v) + 2$

  $P(v_i) \leftarrow$ intersection of $+1/-1$ diagonals
  through $P(w_p)$ and $P(w_q)$

  $L(v_i) \leftarrow \cup_{j=p+1}^{q-1} L(w_j) \cup \{v_i\}$



Shift Method – Pseudocode

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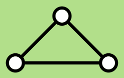
$P(v_1) \leftarrow (0,0); P(v_2) \leftarrow (2,0), P(v_3) \leftarrow (1,1)$ 
for $i = 4$ to n **do**
  Let $w_1 = v_1, w_2, \ldots, w_{t-1}, w_t = v_2$
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  for $\forall v \in \cup_{j=q}^{t} L(w_j)$ **do**
    $x(v) \leftarrow x(v) + 2$
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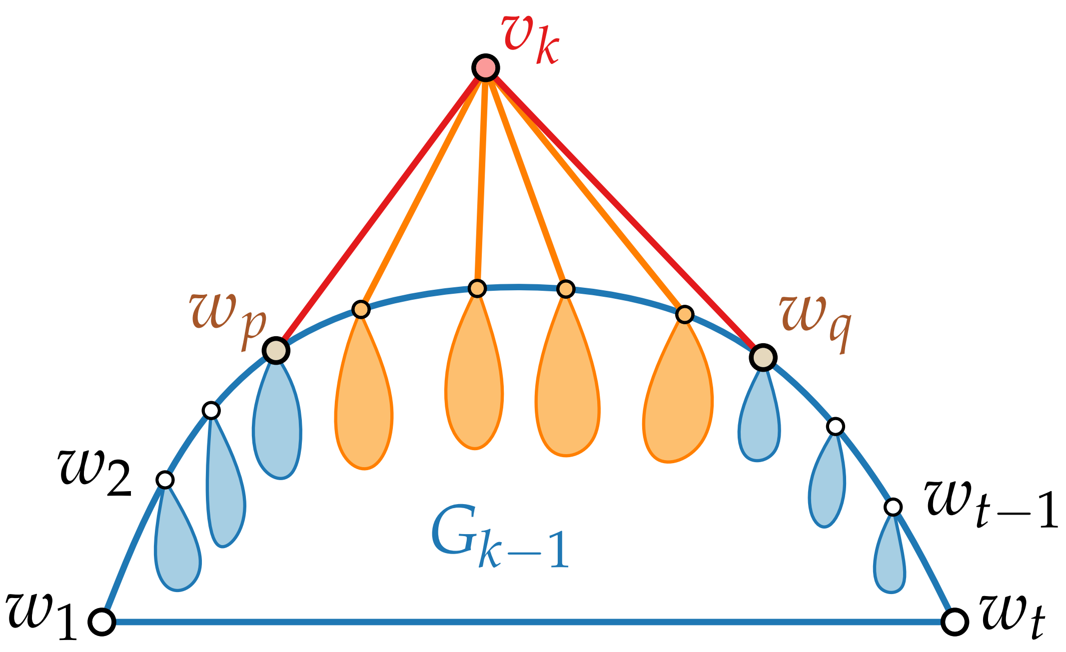
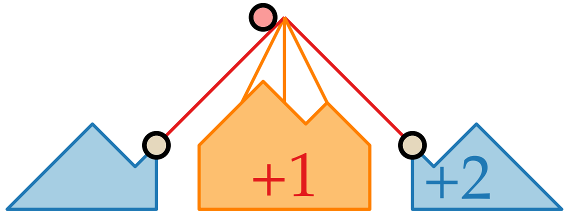
  $L(v_i) \leftarrow \cup_{j=p+1}^{q-1} L(w_j) \cup \{v_i\}$





Running Time?

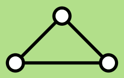
Shift Method – Pseudocode

Let $v_1, \ldots, v_n$ be a canonical order of G

for $i = 1$ to 3 **do**

  $L(v_i) \leftarrow \{v_i\}$

$P(v_1) \leftarrow (0,0); P(v_2) \leftarrow (2,0), P(v_3) \leftarrow (1,1)$



for $i = 4$ to n **do**

  Let $w_1 = v_1, w_2, \ldots, w_{t-1}, w_t = v_2$ denote the boundary of G_{i-1} and let $w_p, \ldots, w_q$ be the neighbours of v_i

  for $\forall v \in \cup_{j=p+1}^{q-1} L(w_j)$ **do** // $\mathcal{O}(n^2)$ *in total*

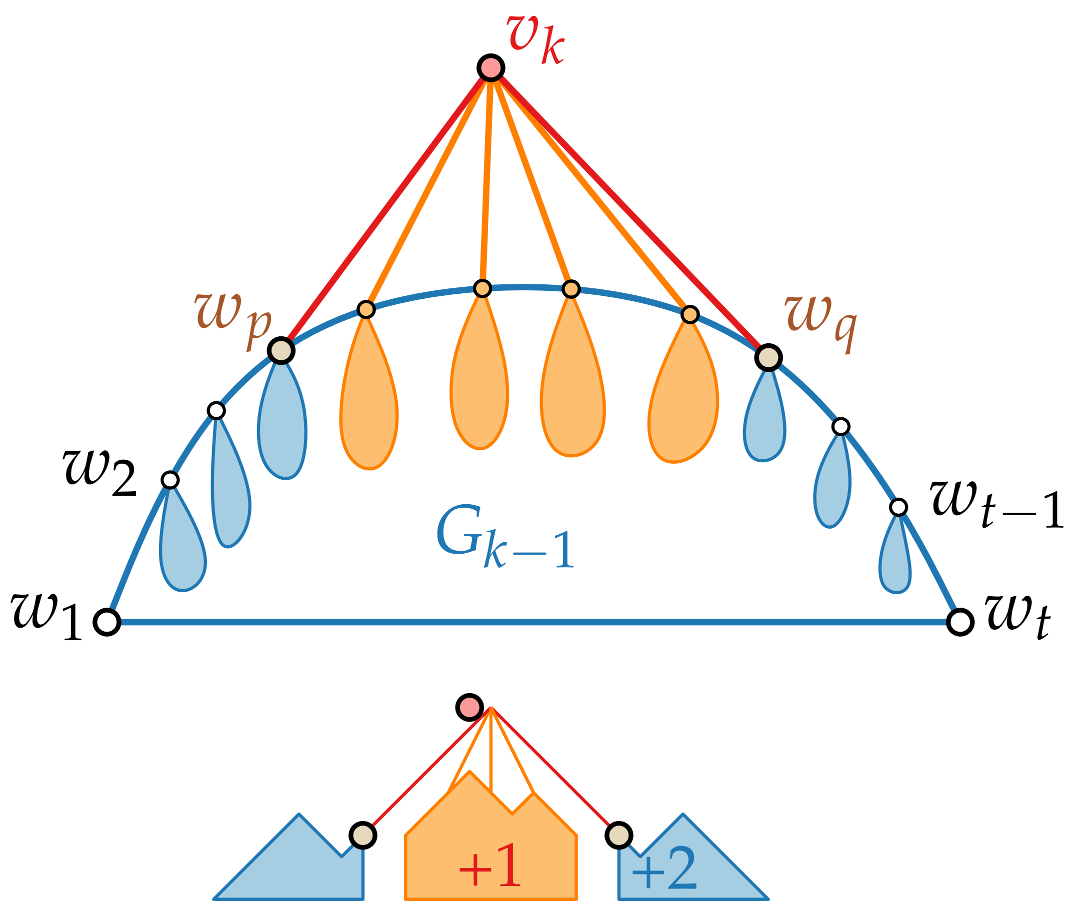
    $x(v) \leftarrow x(v) + 1$

  for $\forall v \in \cup_{j=q}^{t} L(w_j)$ **do** // $\mathcal{O}(n^2)$ *in total*

    $x(v) \leftarrow x(v) + 2$

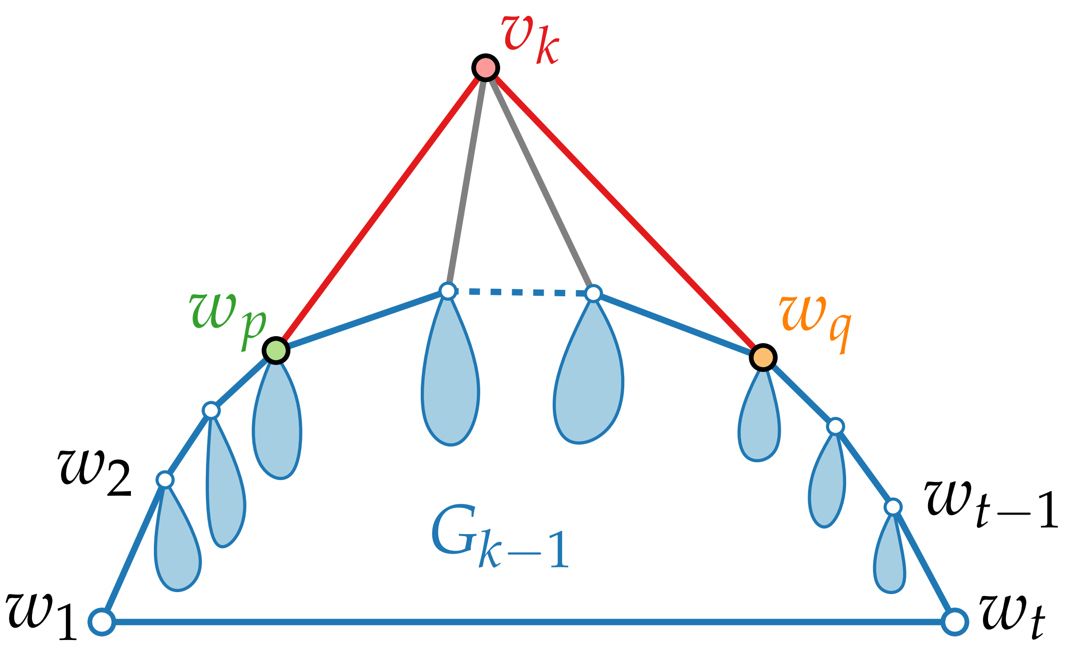
  $P(v_i) \leftarrow$ intersection of $+1/-1$ diagonals through $P(w_p)$ and $P(w_q)$

  $L(v_i) \leftarrow \cup_{j=p+1}^{q-1} L(w_j) \cup \{v_i\}$



Running Time?

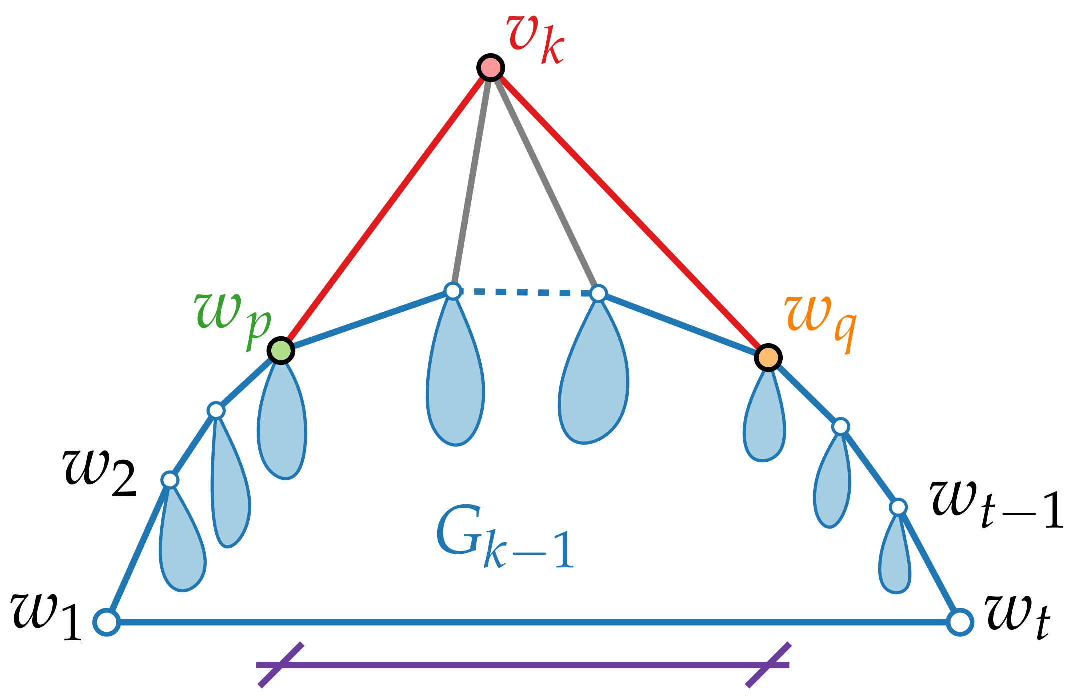
Shift Method – Linear Time Implementation



Shift Method – Linear Time Implementation

Idea 1.

To compute $x(v_k)$ & $y(v_k)$,
we only need $y(w_p)$ and $y(w_q)$ and $x(w_q) - x(w_p)$



Shift Method – Linear Time Implementation

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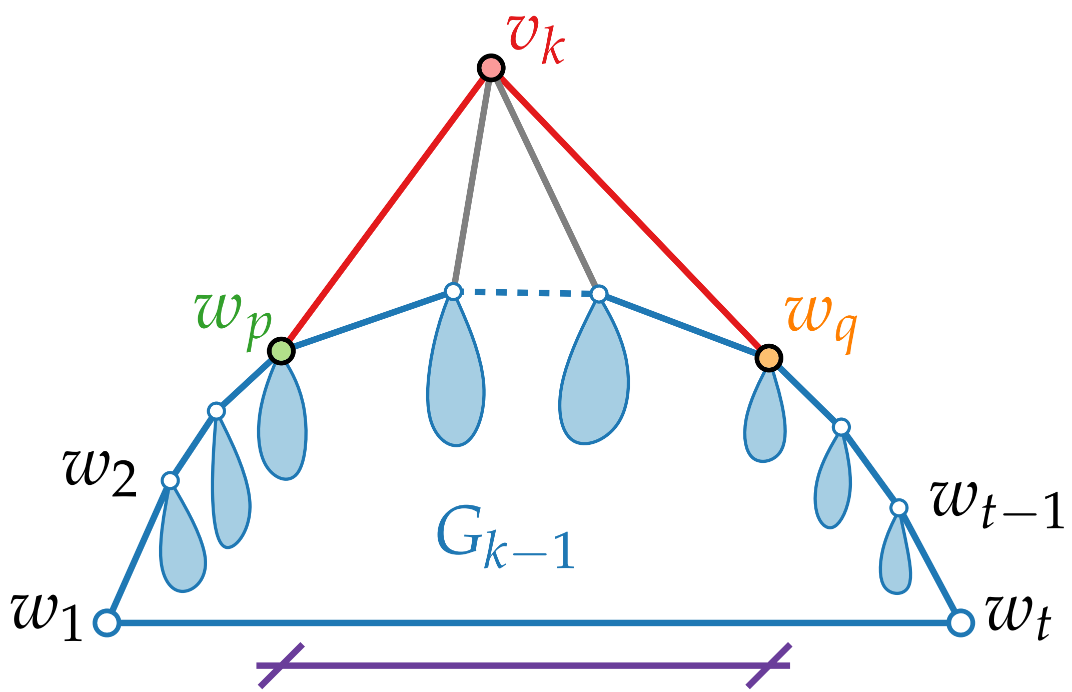


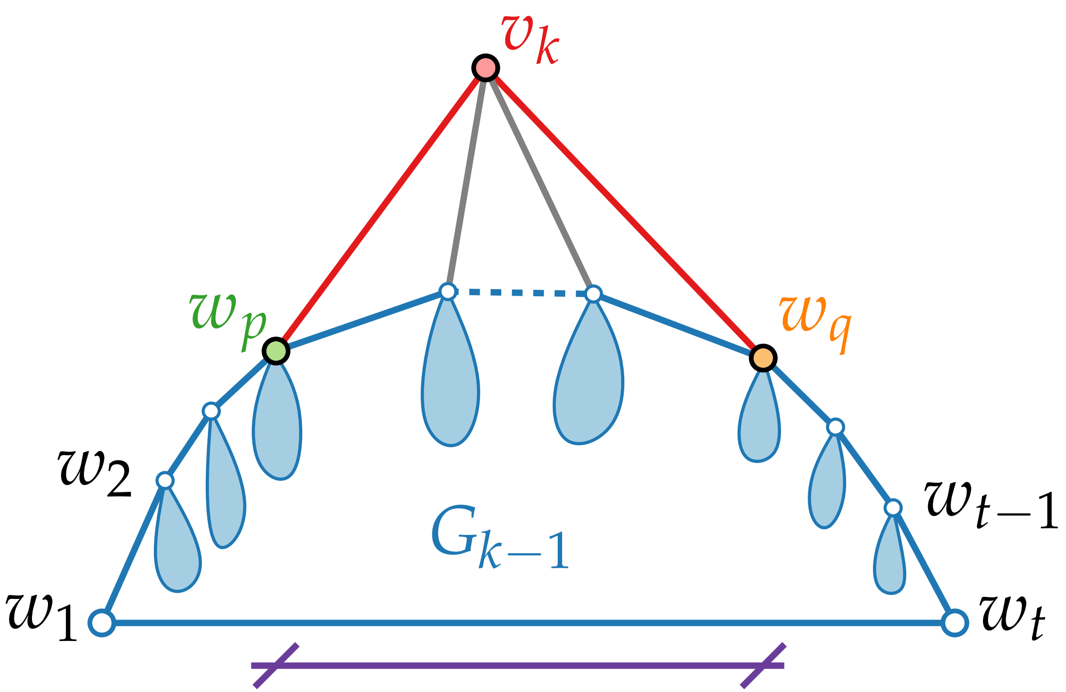
(1) $x(v_k) =$

(2) $y(v_k) =$

Shift Method – Linear Time Implementation

Idea 1.

To compute $x(v_k)$ & $y(v_k)$,
we only need $y(w_p)$ and $y(w_q)$ and $x(w_q) - x(w_p)$

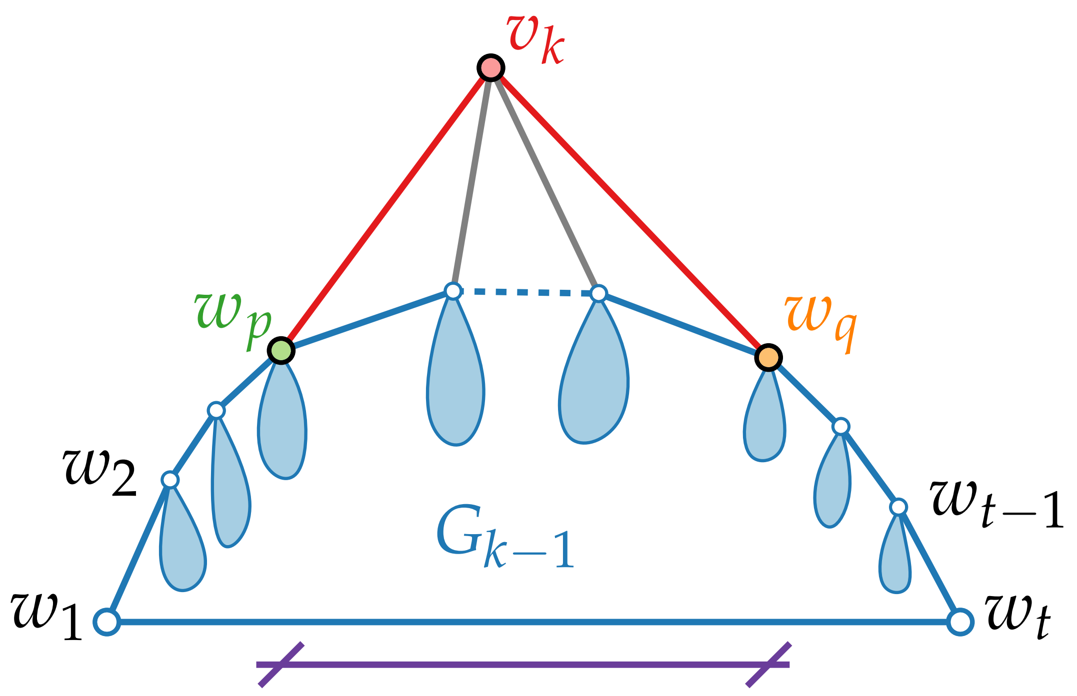


(1) $x(v_k) = \frac{1}{2}(x(w_q) + x(w_p) + y(w_q) - y(w_p))$

(2) $y(v_k) =$

Shift Method – Linear Time Implementation

Idea 1.
To compute $x(v_k)$ & $y(v_k)$,
we only need $y(w_p)$ and $y(w_q)$ and $x(w_q) - x(w_p)$



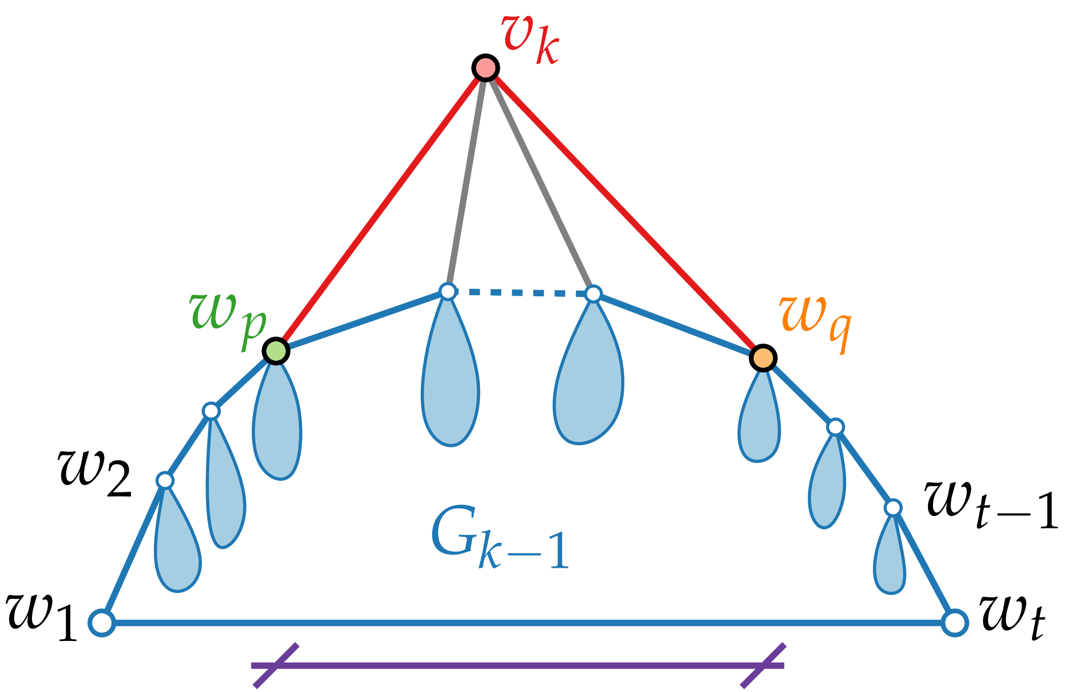
(1) $x(v_k) = \frac{1}{2}(x(w_q) + x(w_p) + y(w_q) - y(w_p))$

(2) $y(v_k) = \frac{1}{2}(x(w_q) - x(w_p) + y(w_q) + y(w_p))$

Shift Method – Linear Time Implementation

Idea 1.
To compute $x(v_k)$ & $y(v_k)$,
we only need $y(w_p)$ and $y(w_q)$ and $x(w_q) - x(w_p)$

Idea 2.
Instead of storing explicit x-coordinates,
we store x-distances.



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(2) $y(v_k) = \frac{1}{2}(x(w_q) - x(w_p) + y(w_q) + y(w_p))$

Shift Method – Linear Time Implementation

Idea 1.
To compute $x(v_k)$ & $y(v_k)$,
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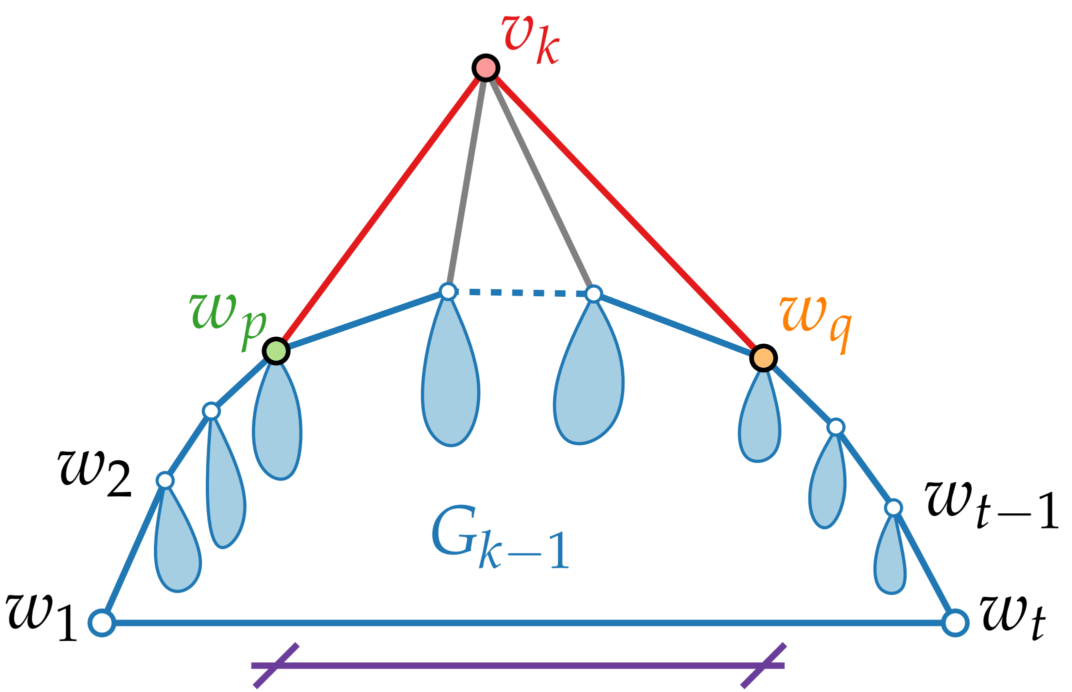

(1) $x(v_k) = \frac{1}{2}(x(w_q) + x(w_p) + y(w_q) - y(w_p))$

(2) $y(v_k) = \frac{1}{2}(x(w_q) - x(w_p) + y(w_q) + y(w_p))$

(3) $x(v_k) - x(w_p) =$

Shift Method – Linear Time Implementation

Idea 1.
To compute $x(v_k)$ & $y(v_k)$,
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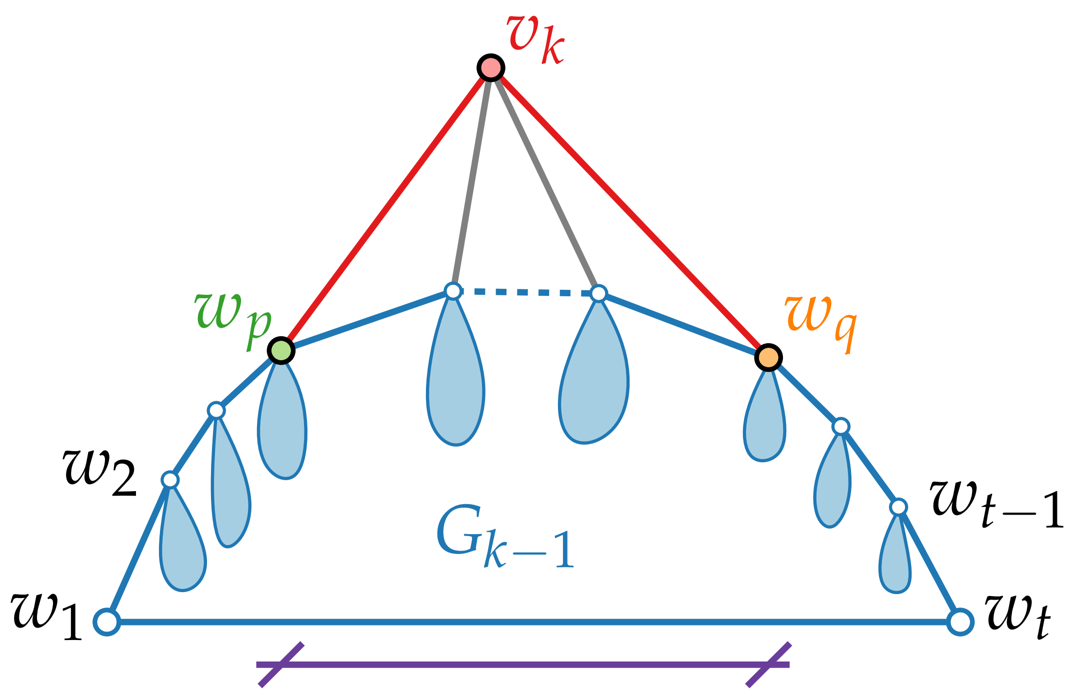

(1) $x(v_k) = \frac{1}{2}(x(w_q) + x(w_p) + y(w_q) - y(w_p))$

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Shift Method – Linear Time Implementation

Idea 1.
To compute $x(v_k)$ & $y(v_k)$,
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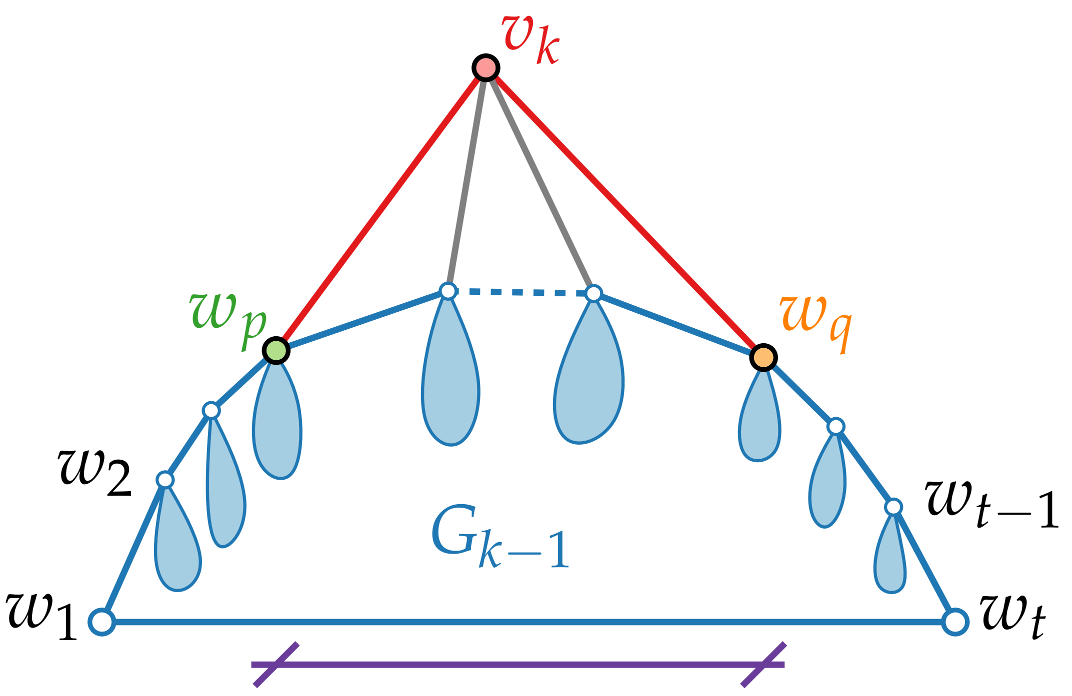
Idea 2.
Instead of storing explicit x-coordinates,
we store x-distances.

After x distance for v_n computed, use preorder traversal to compute all x-coordinates.



(1) $x(v_k) = \frac{1}{2}(x(w_q) + x(w_p) + y(w_q) - y(w_p))$

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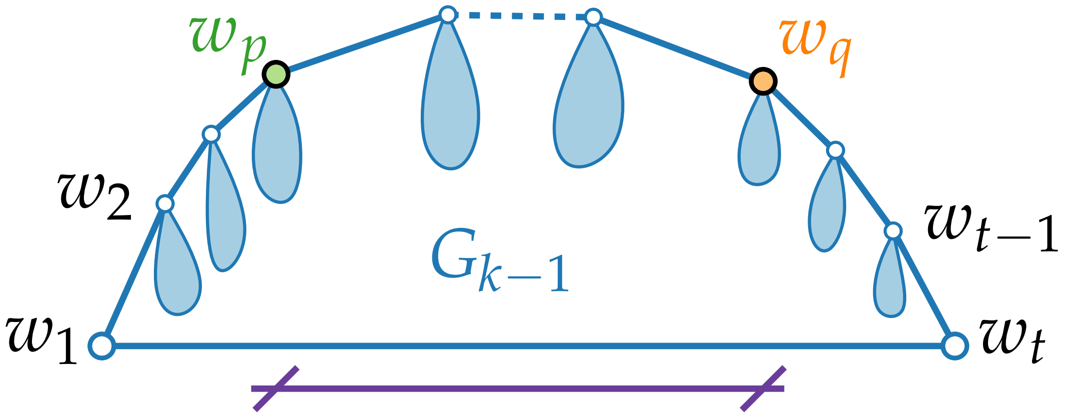
(3) $x(v_k) - x(w_p) = \frac{1}{2}(x(w_q) - x(w_p) + y(w_q) - y(w_p))$

Shift Method – Linear Time Implementation

Relative x-distance tree.

For each vertex v store

- x-offset $\Delta_x(v)$ from parent
- y-coordinate $y(v)$



(1) $x(v_k) = \frac{1}{2}(x(w_q) + x(w_p) + y(w_q) - y(w_p))$

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Shift Method – Linear Time Implementation

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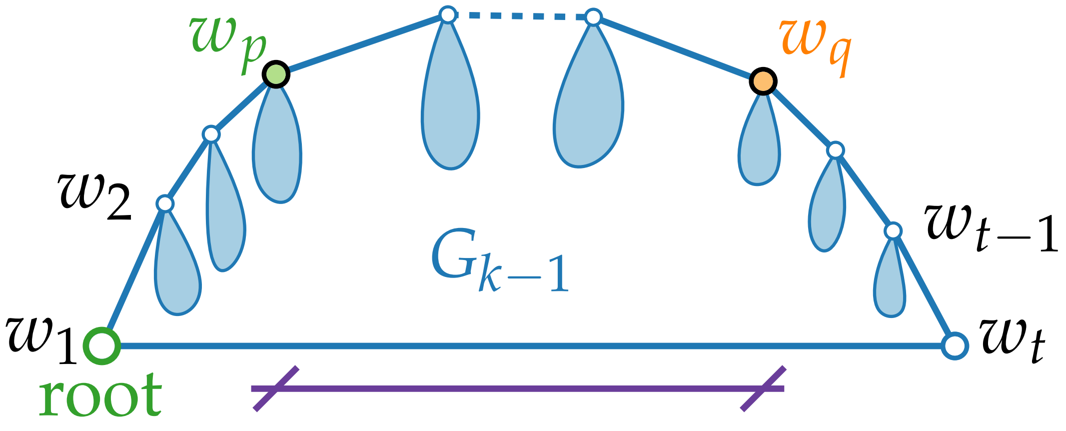

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Shift Method – Linear Time Implementation

Relative x-distance tree.

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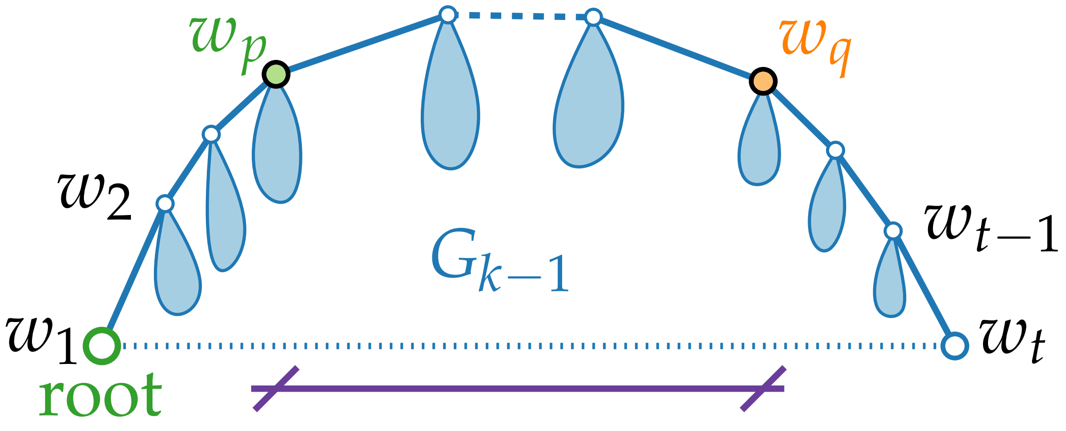

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Shift Method – Linear Time Implementation

Relative x-distance tree.

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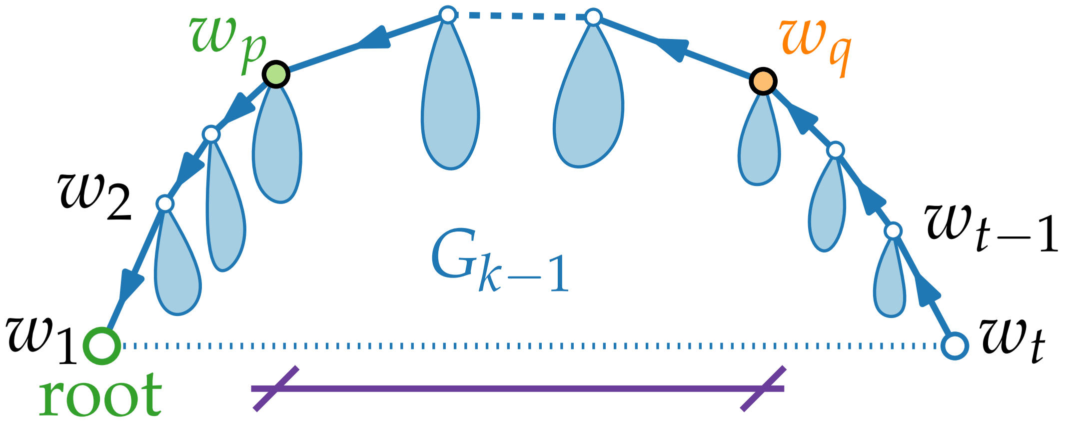

$$(1)\quad x(v_k) = \tfrac{1}{2}(x(w_q) + x(w_p) + y(w_q) - y(w_p))$$

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Shift Method – Linear Time Implementation

Relative x-distance tree.

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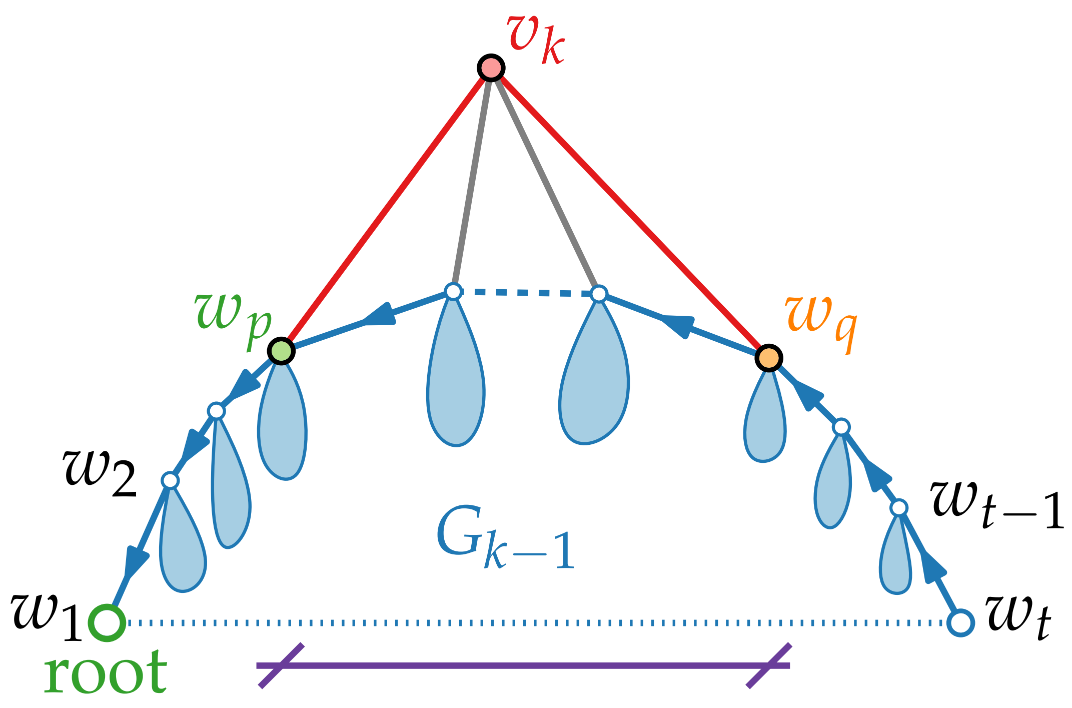

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Shift Method – Linear Time Implementation

Relative x-distance tree.

For each vertex v store

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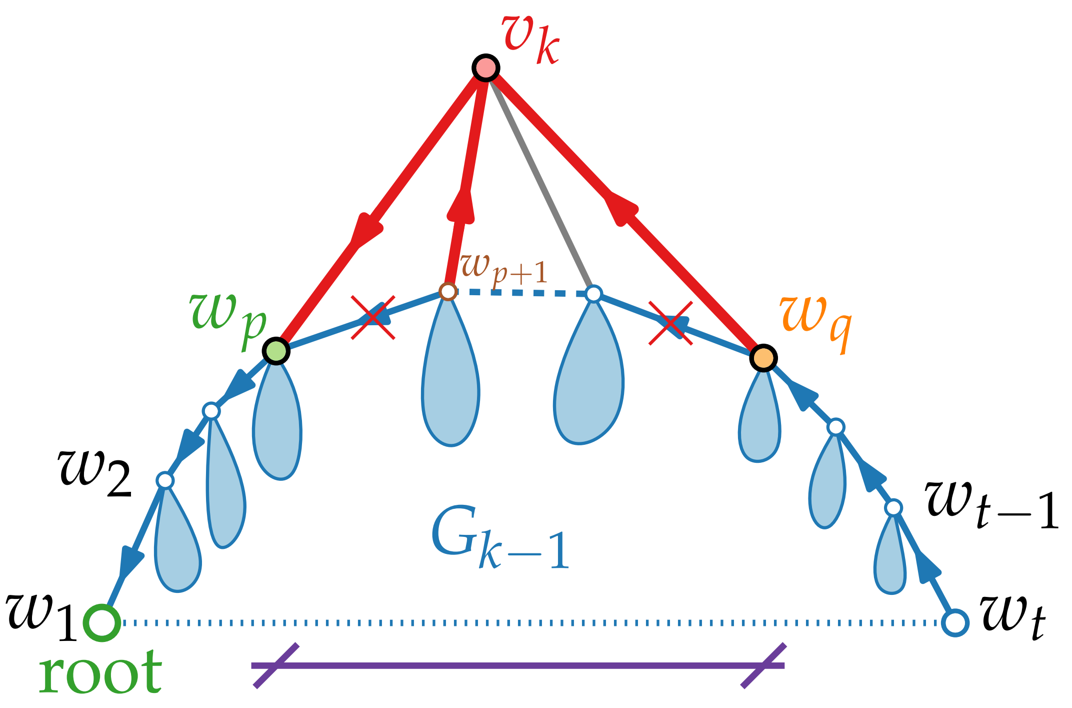

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Shift Method – Linear Time Implementation

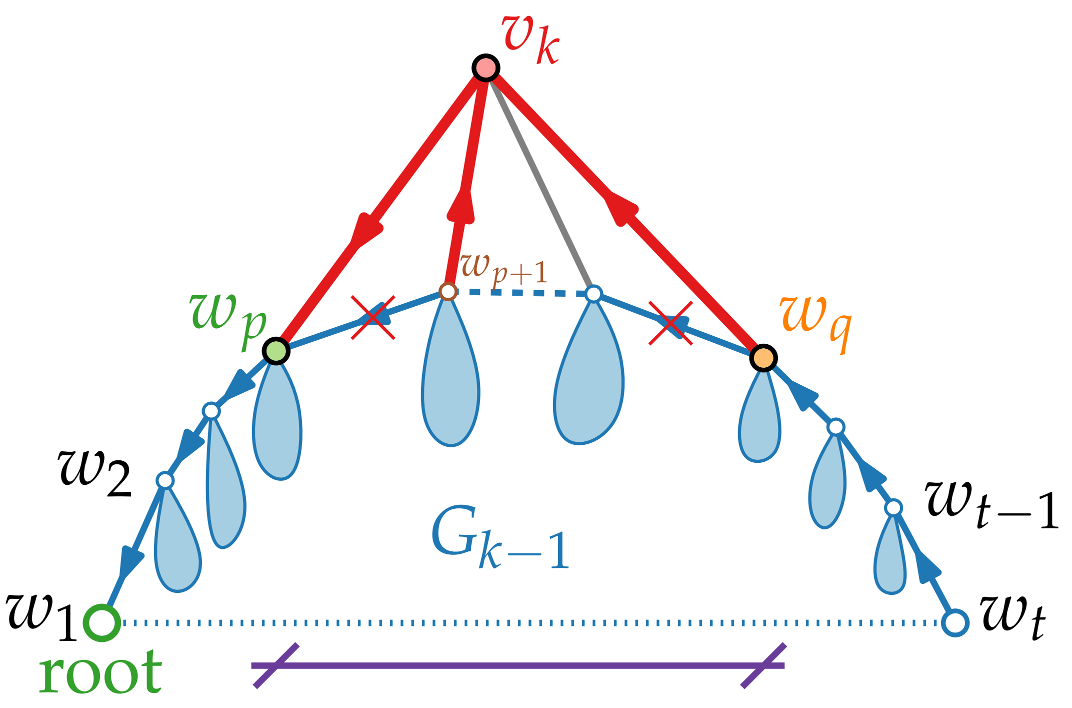
Relative x-distance tree.

For each vertex v store

- x-offset $\Delta_x(v)$ from parent
- y-coordinate $y(v)$

Calculations.

- $\Delta_x(w_{p+1})$++, $\Delta_x(w_q)$++



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(3) $x(v_k) - x(w_p) = \frac{1}{2}(x(w_q) - x(w_p) + y(w_q) - y(w_p))$

Shift Method – Linear Time Implementation

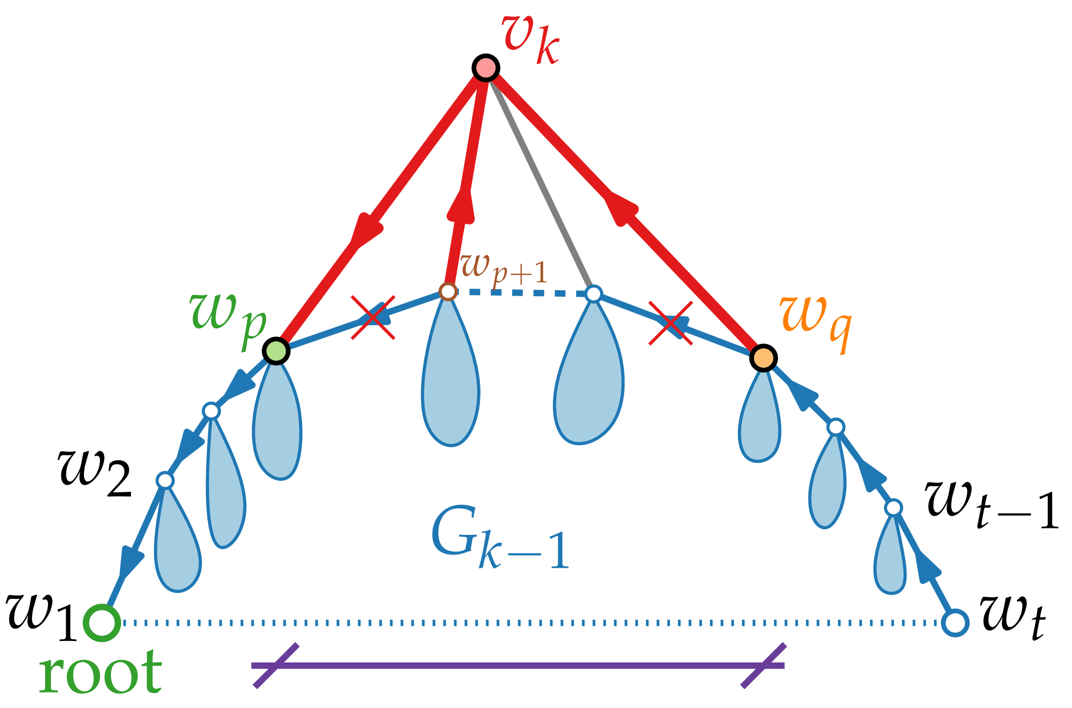
Relative x-distance tree.

For each vertex v store

- x-offset $\Delta_x(v)$ from parent
- y-coordinate $y(v)$

Calculations.

- $\Delta_x(w_{p+1})$++, $\Delta_x(w_q)$++
- $\Delta_x(w_p, w_q) = \Delta_x(w_{p+1}) + \ldots + \Delta_x(w_q)$



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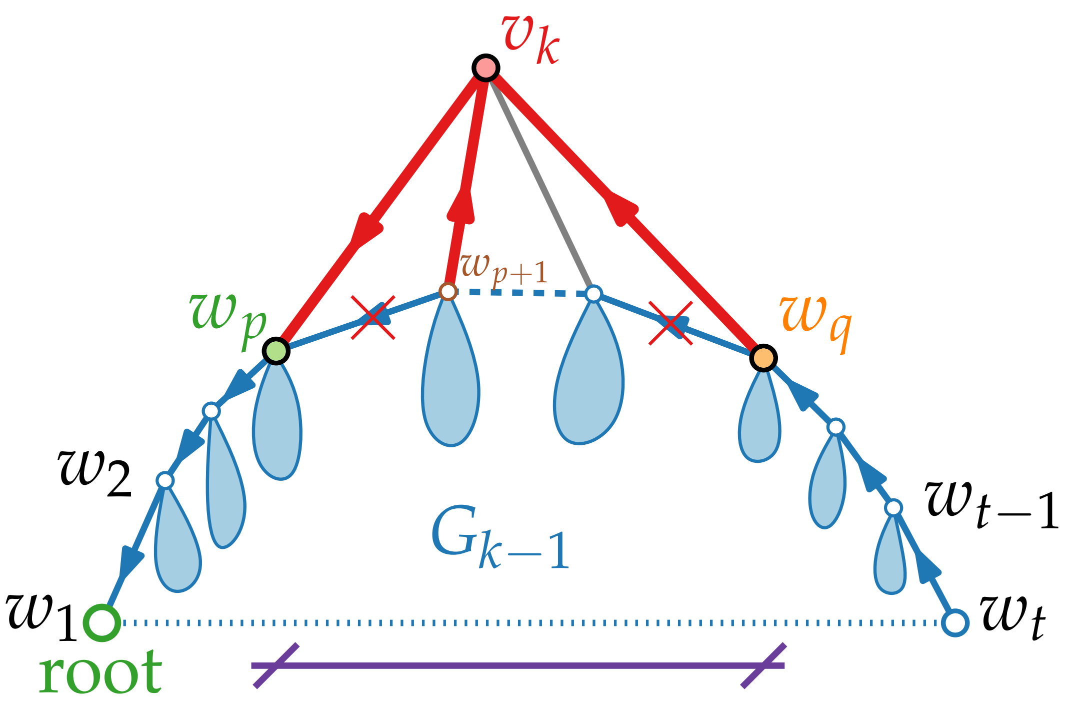
Shift Method – Linear Time Implementation

Relative x-distance tree.

For each vertex v store

- x-offset $\Delta_x(v)$ from parent
- y-coordinate $y(v)$



Calculations.

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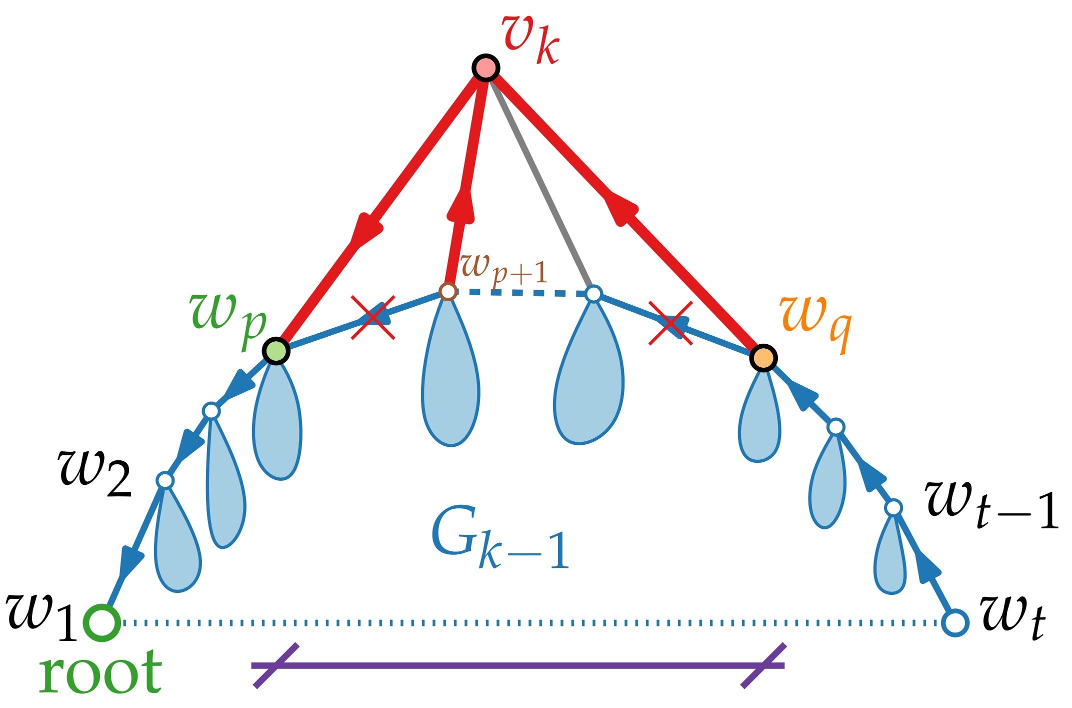

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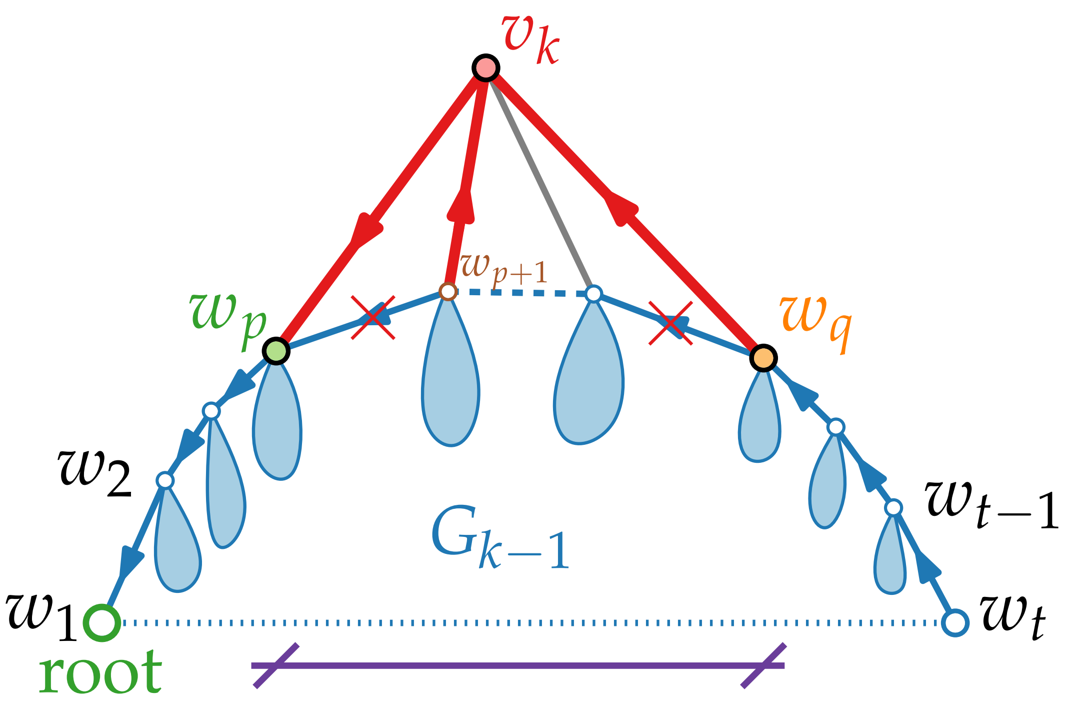

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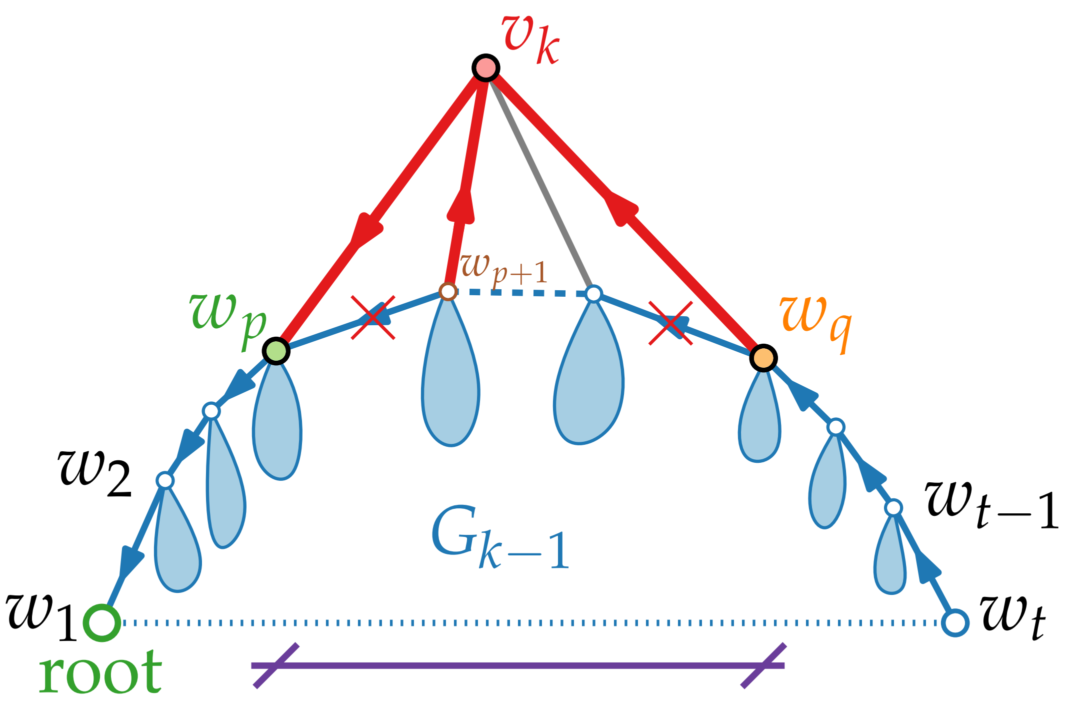

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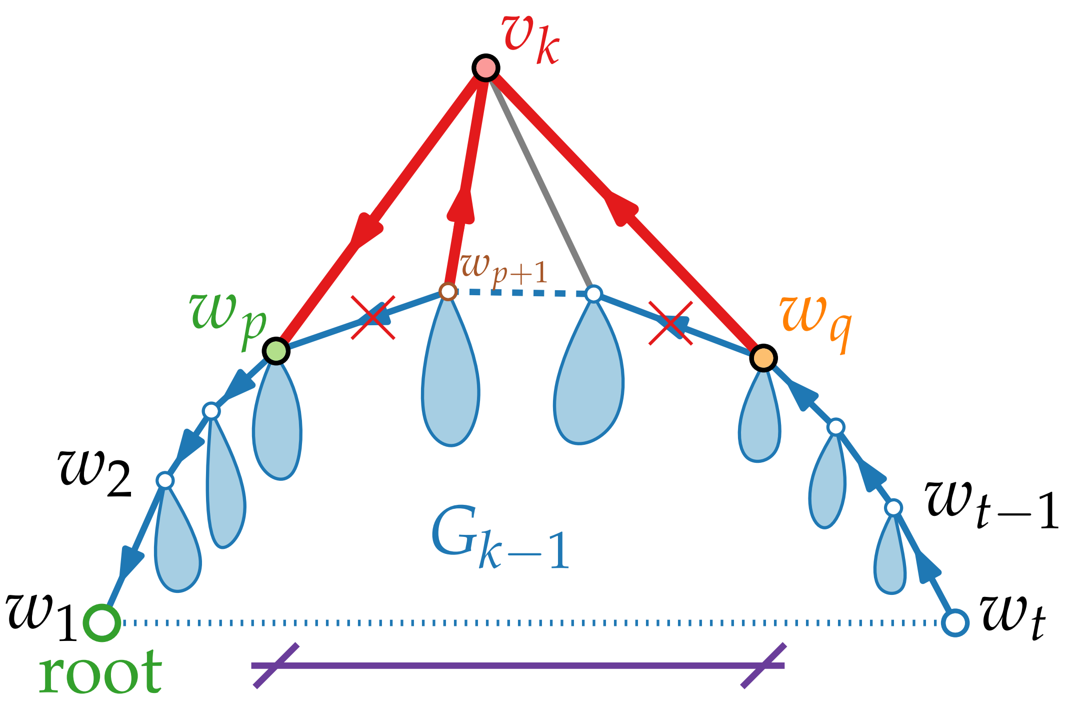
$\mathcal{O}(n)$ in total



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Every n-vertex planar graph has a planar straight-line drawing of size $\frac{4}{3}n \times \frac{2}{3}n$. Such a drawing can be computed in $O(n)$ time.