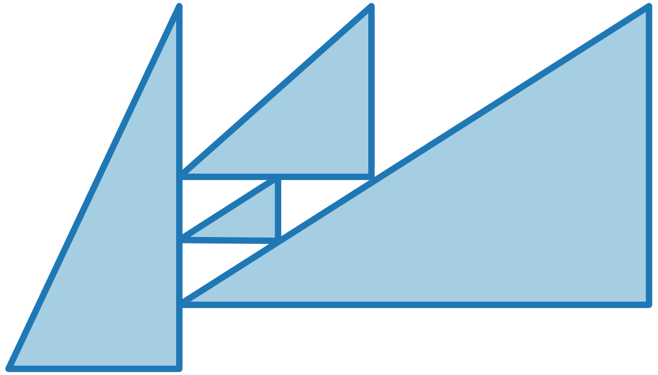
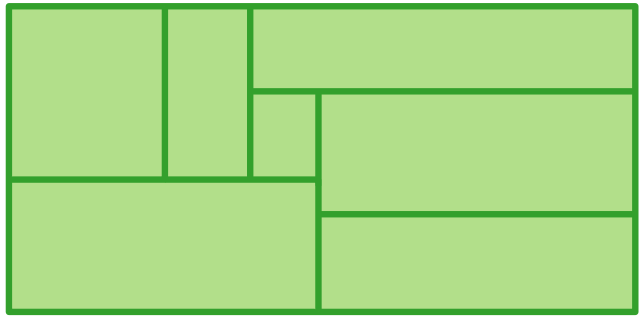
Universität Trier

Visualization of Graphs

Lecture 9:
Contact Representations of Planar Graphs: Triangle Contacts and Rectangular Duals



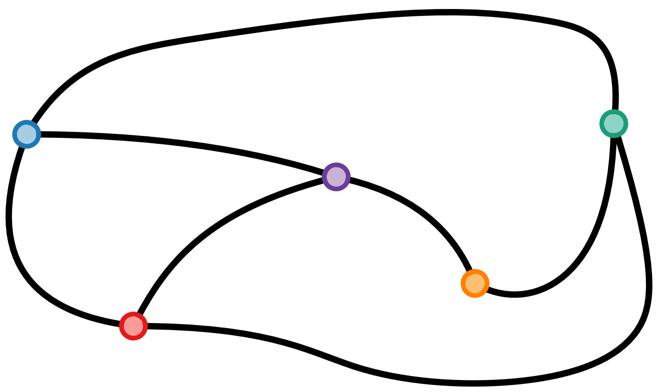
Part I:
Geometric Representations



Philipp Kindermann

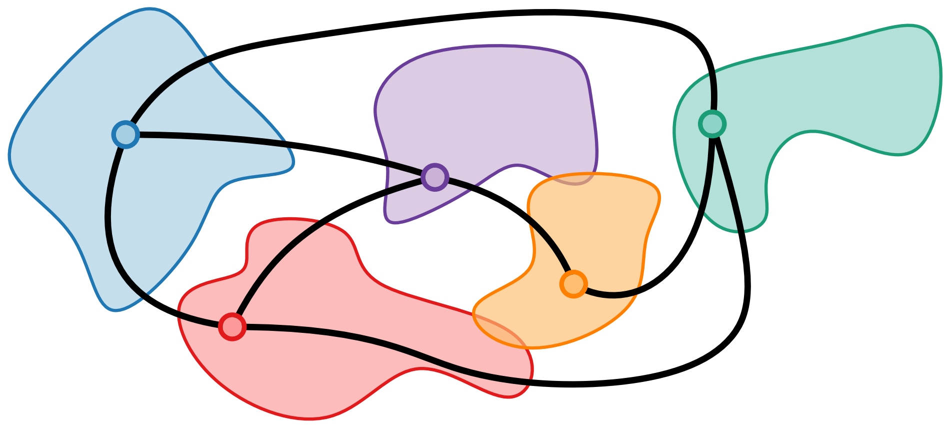
Intersection Representation

In an **intersection representation** of a graph each vertex is represented as a set



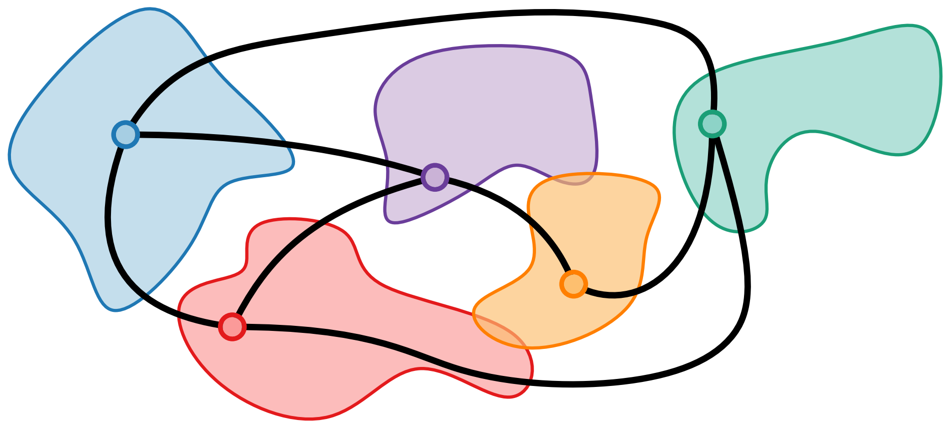
Intersection Representation

In an **intersection representation** of a graph each vertex is represented as a set



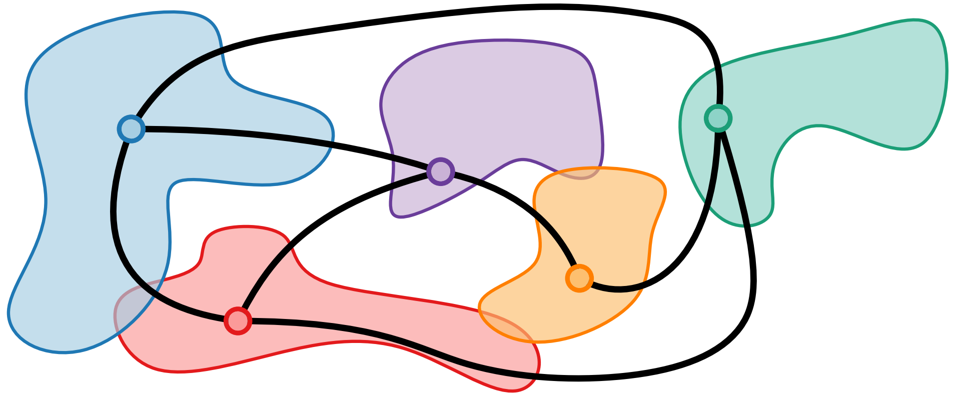
Intersection Representation

In an **intersection representation** of a graph each vertex is represented as a set such that two sets intersect if and only if the corresponding vertices are adjacent.



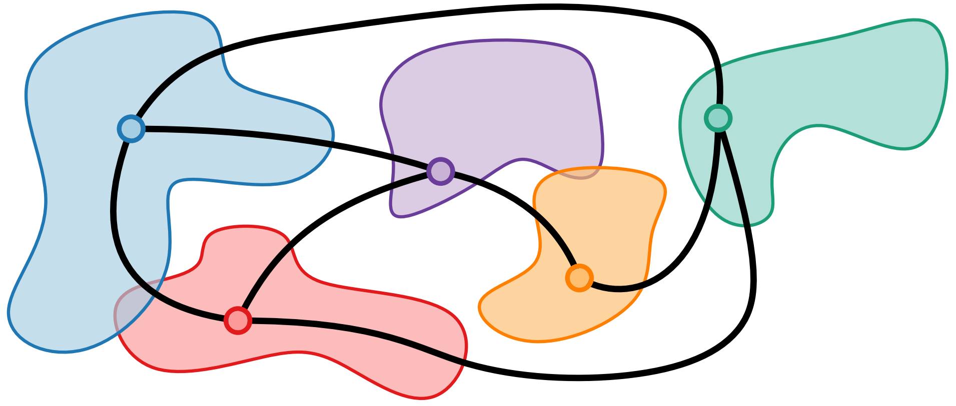
Intersection Representation

In an **intersection representation** of a graph each vertex is represented as a set such that two sets intersect if and only if the corresponding vertices are adjacent.



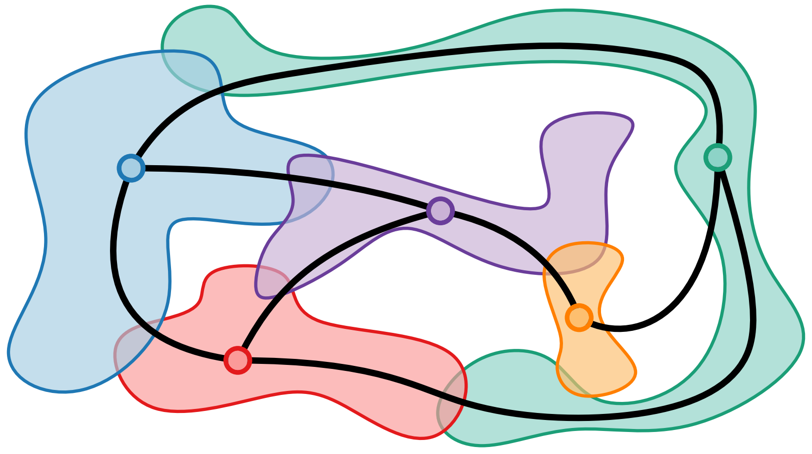
Intersection Representation

In an **intersection representation** of a graph each vertex is represented as a set such that two sets intersect if and only if the corresponding vertices are adjacent.



Intersection Representation

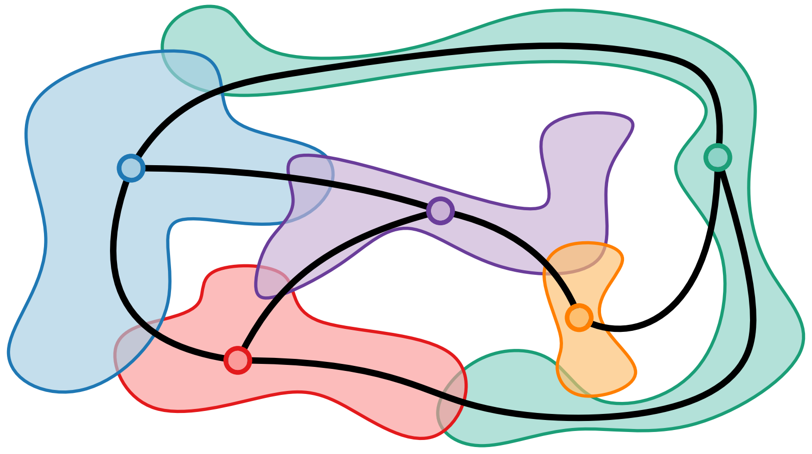
In an **intersection representation** of a graph each vertex is represented as a set such that two sets intersect if and only if the corresponding vertices are adjacent.



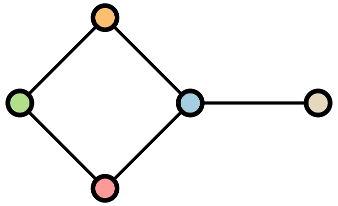
Intersection Representation

In an **intersection representation** of a graph each vertex is represented as a set such that two sets intersect if and only if the corresponding vertices are adjacent.

For a collection $\mathcal{S}$ of sets $S_1, \ldots, S_n$, the **intersection graph** $G(\mathcal{S})$ of $\mathcal{S}$ has vertex set $\mathcal{S}$ and edge set $\{S_iS_j \colon i, j \in \{1, \ldots, n\}, i \neq j$, and $S_i \cap S_j \neq \emptyset\}$.

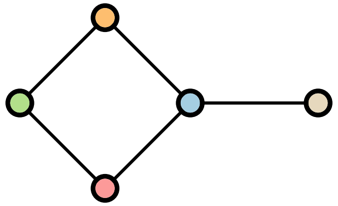


Contact Representation of Graphs

Let G be a graph. 

Contact Representation of Graphs

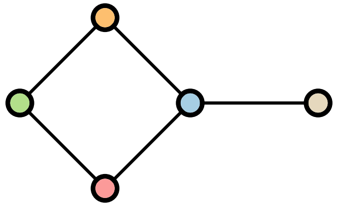
Let G be a graph.



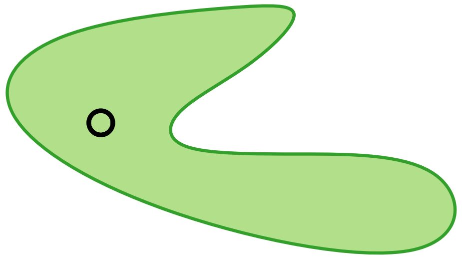
Represent each vertex v by a geometric object $S(v)$

Contact Representation of Graphs

Let G be a graph.

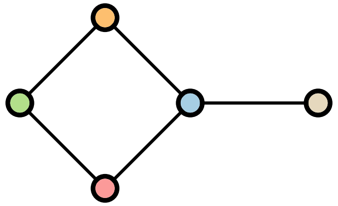


Represent each vertex v by a geometric object $S(v)$

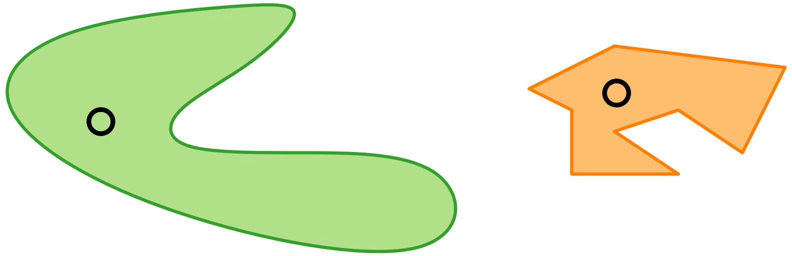


Contact Representation of Graphs

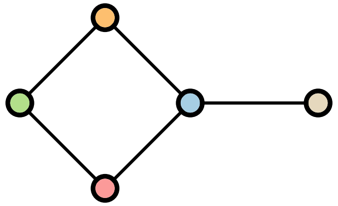
Let G be a graph.



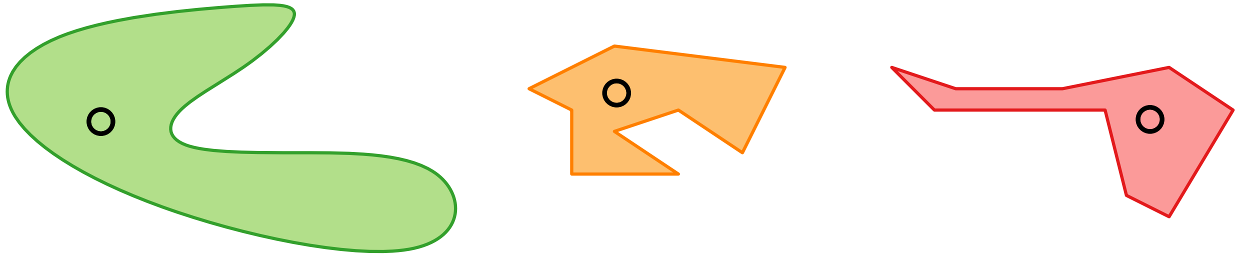
Represent each vertex v by a geometric object $S(v)$



Contact Representation of Graphs

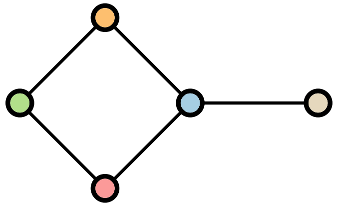
Let G be a graph. 

Represent each vertex v by a geometric object $S(v)$

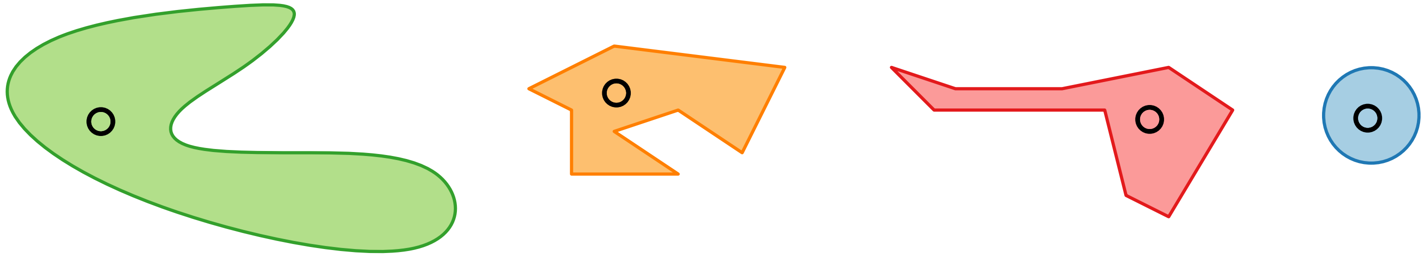


Contact Representation of Graphs

Let G be a graph.

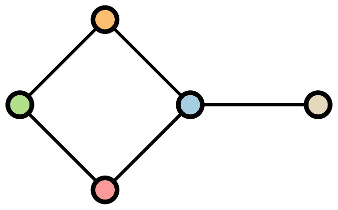


Represent each vertex v by a geometric object $S(v)$

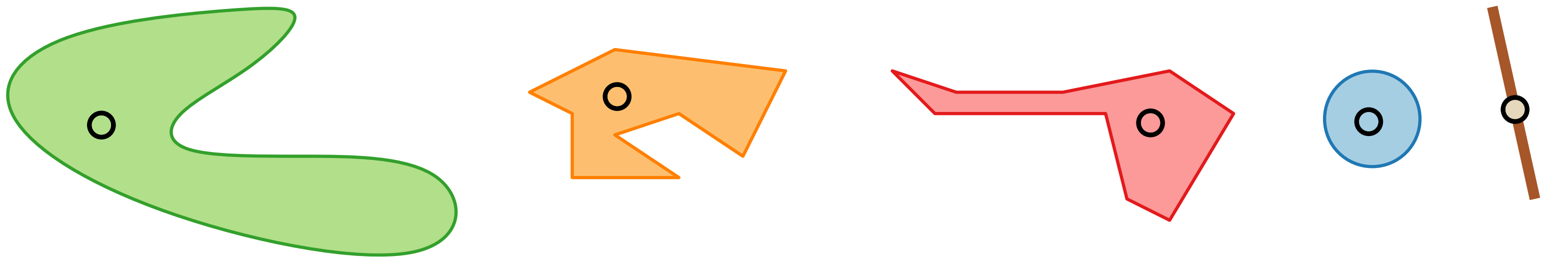


Contact Representation of Graphs

Let G be a graph.

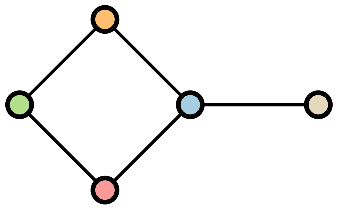


Represent each vertex v by a geometric object $S(v)$

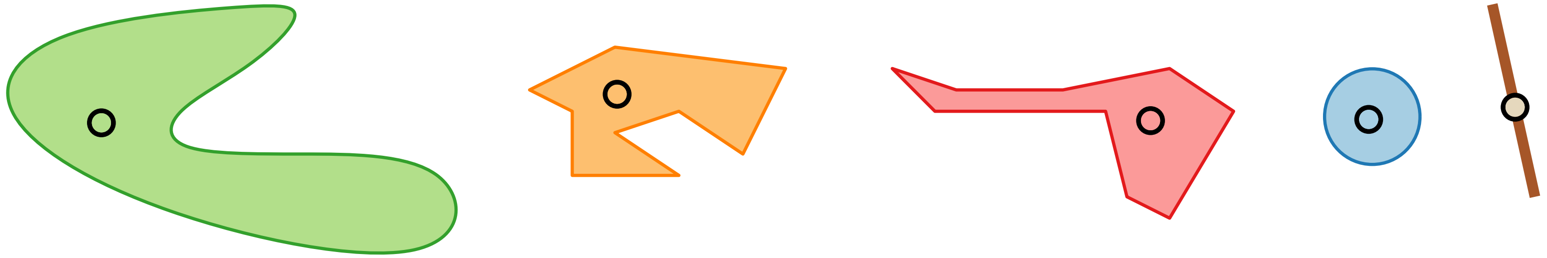


Contact Representation of Graphs

Let G be a graph.



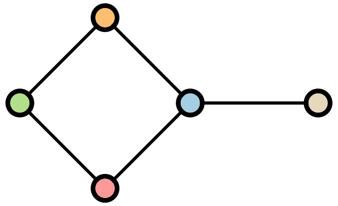
Represent each vertex v by a geometric object $S(v)$



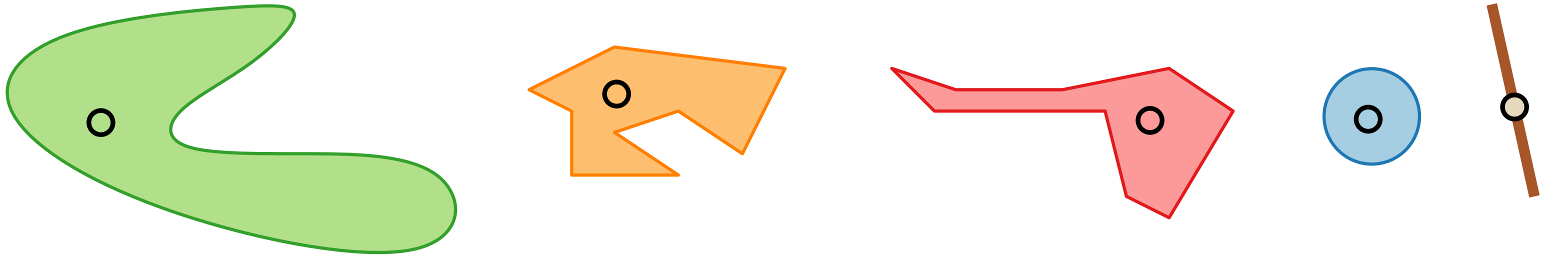
In a **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$

Contact Representation of Graphs

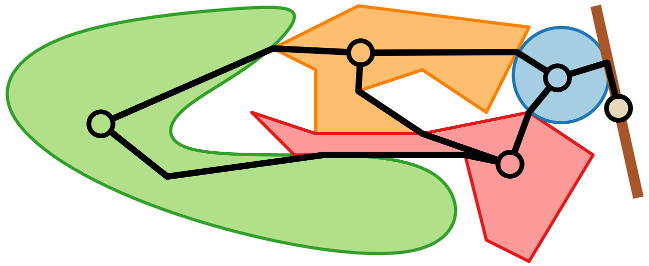
Let G be a graph.



Represent each vertex v by a geometric object $S(v)$

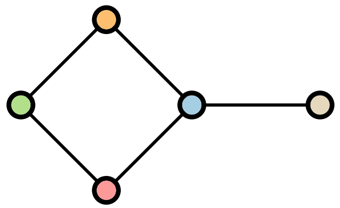


In a **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$



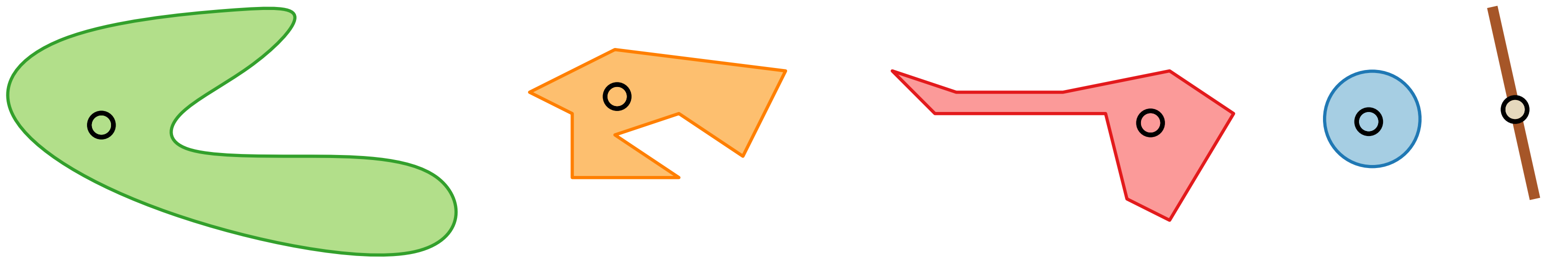
Contact Representation of Graphs

Let G be a graph.

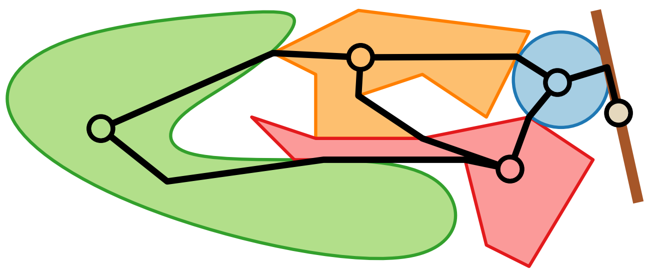


Let $\mathcal{S}$ be a set of geometric objects

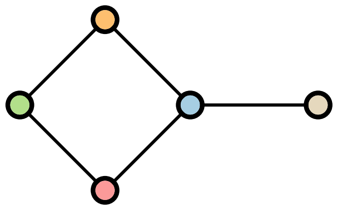
Represent each vertex v by a geometric object $S(v)$



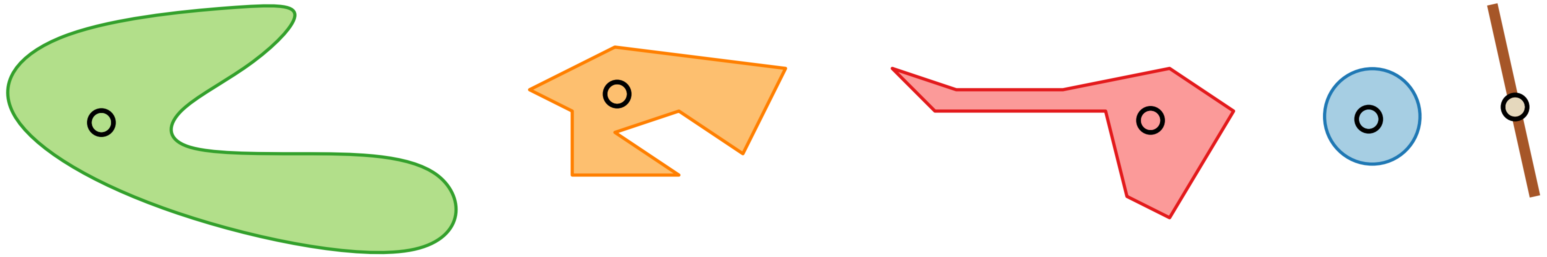
In a **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$



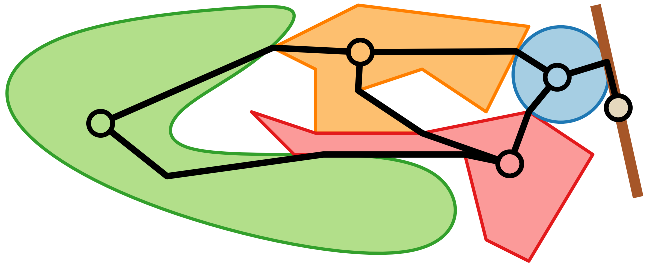
Contact Representation of Graphs

Let G be a graph.  Let $\mathcal{S}$ be a set of geometric objects

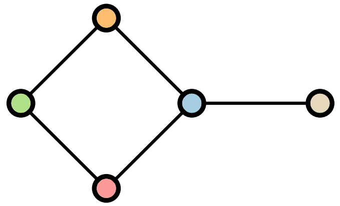
Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$



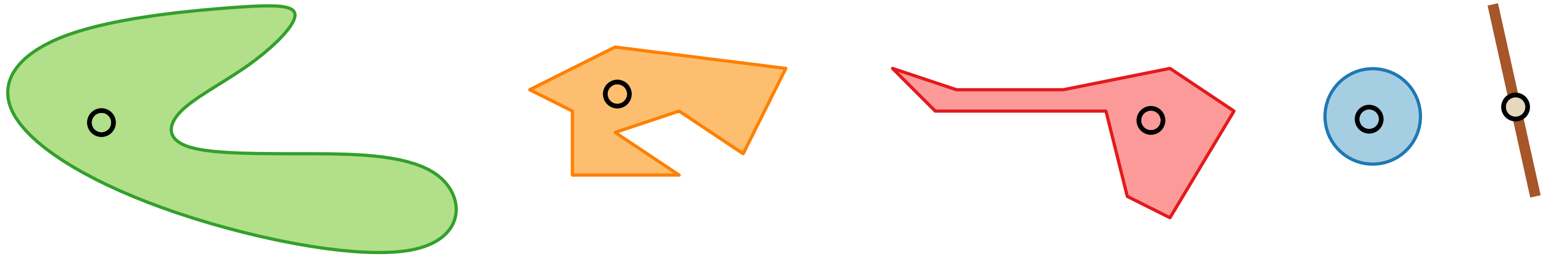
In a **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$



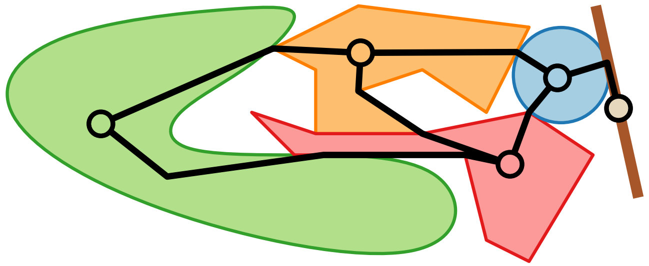
Contact Representation of Graphs

Let G be a graph.  Let $\mathcal{S}$ be a set of geometric objects

Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$

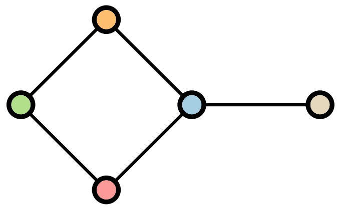


In an $\mathcal{S}$ **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$



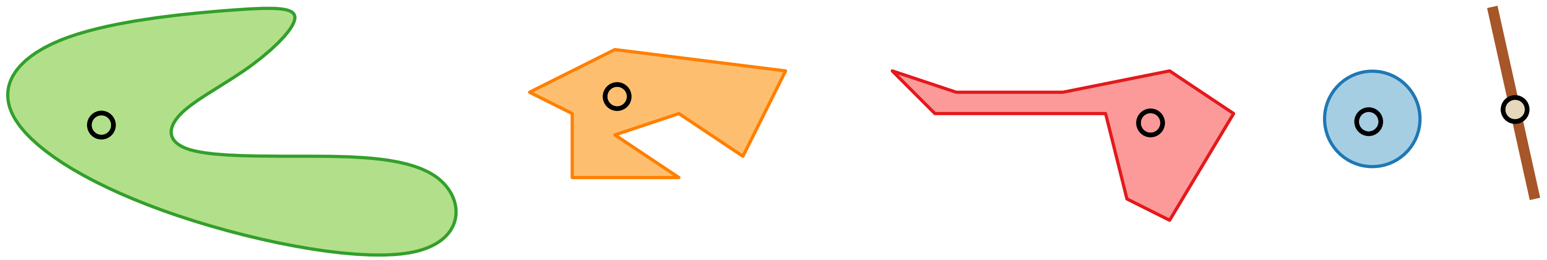
Contact Representation of Graphs

Let G be a graph.

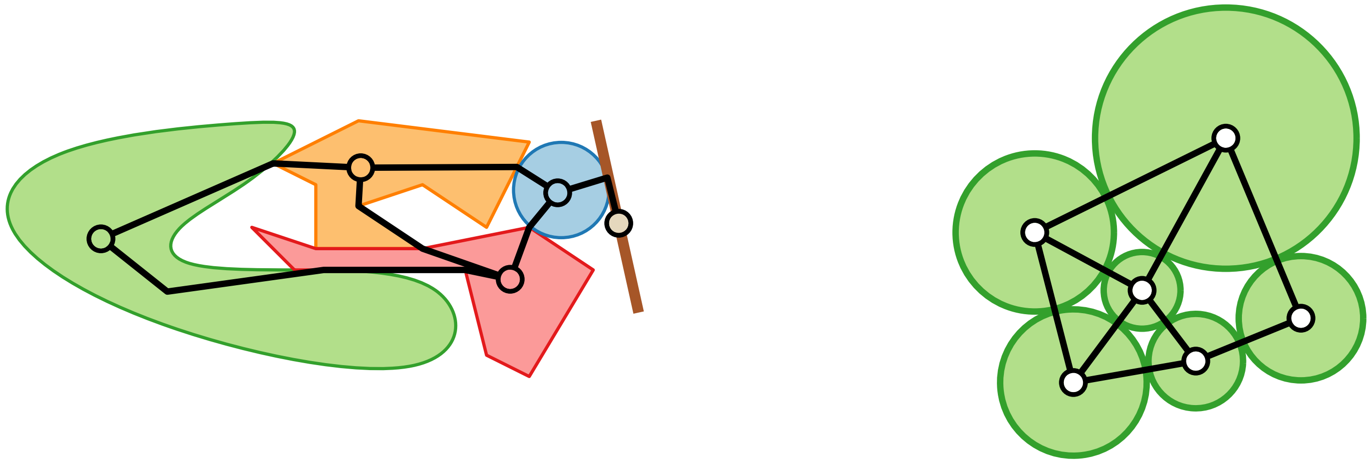


Let $\mathcal{S}$ be a set of geometric objects

Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$



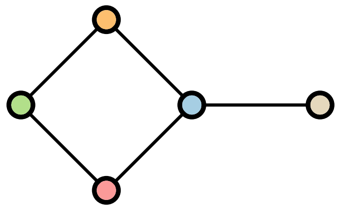
In an $\mathcal{S}$ **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$



disks

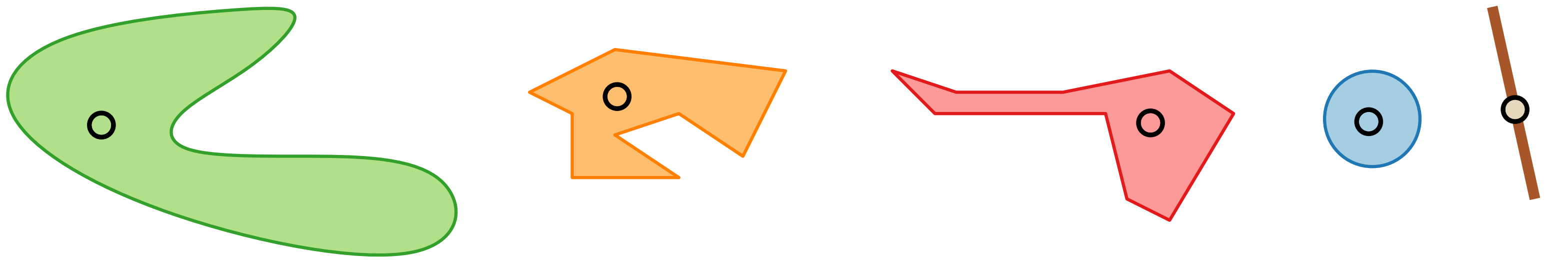
Contact Representation of Graphs

Let G be a graph.

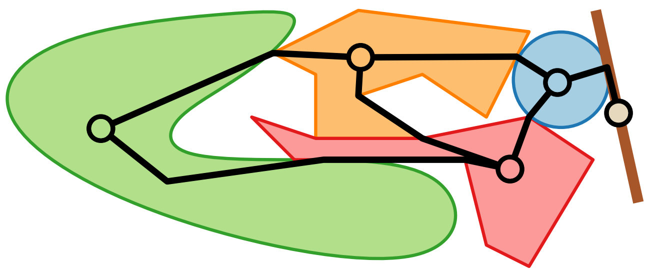


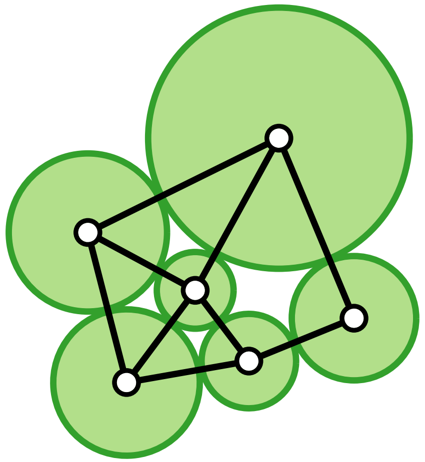
Let $\mathcal{S}$ be a set of geometric objects

Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$

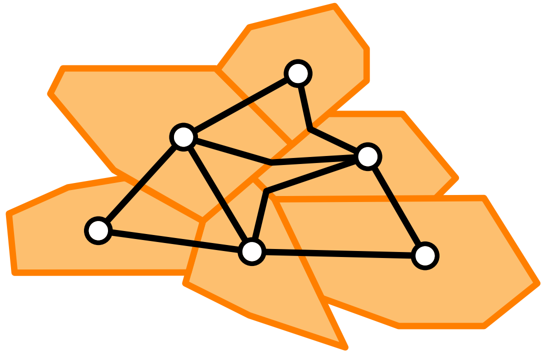


In an $\mathcal{S}$ **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$





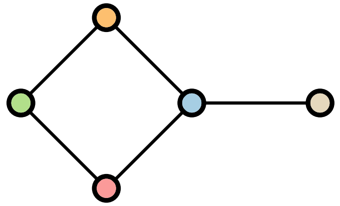
disks



polygons

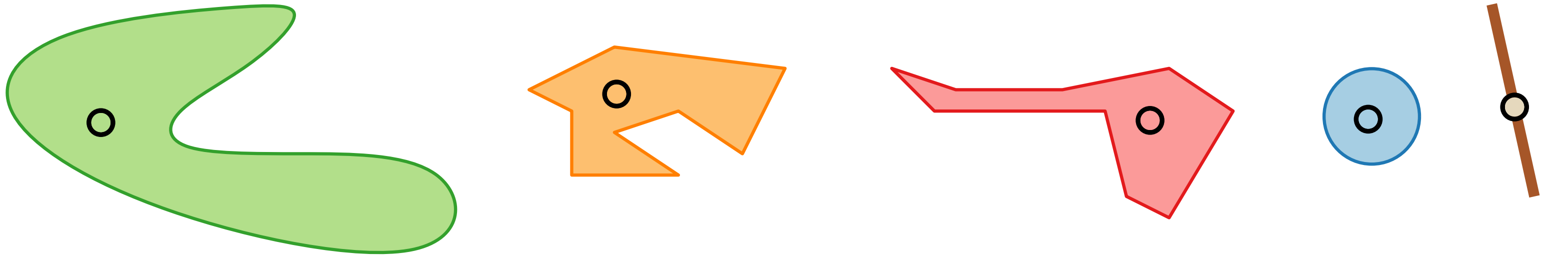
Contact Representation of Graphs

Let G be a graph.

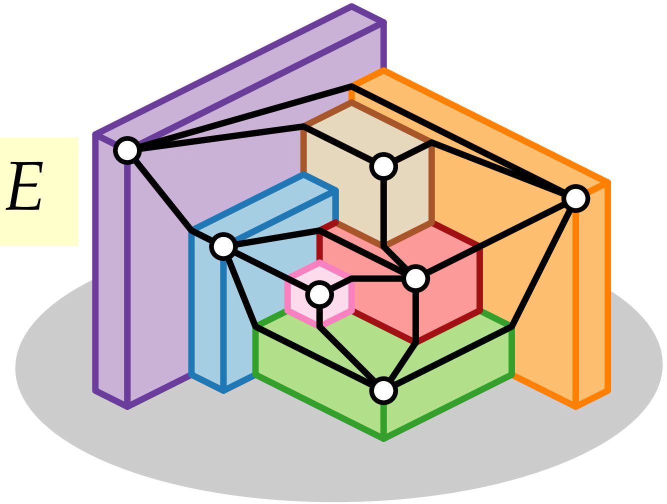


Let $\mathcal{S}$ be a set of geometric objects

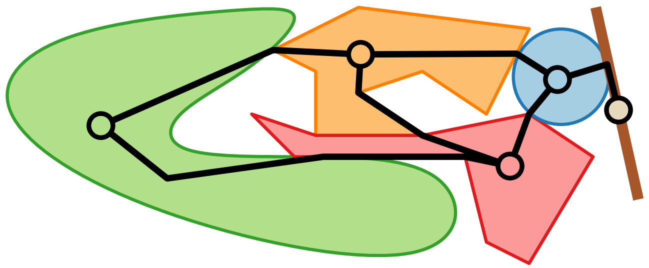
Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$

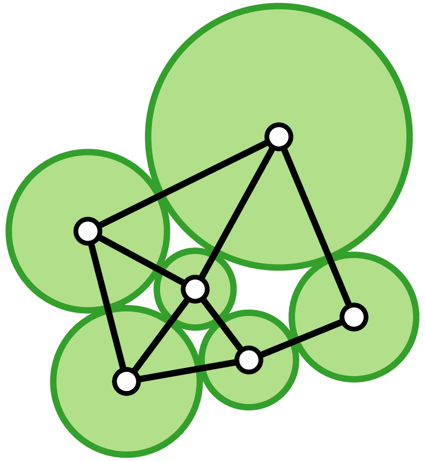


rectangular cuboids

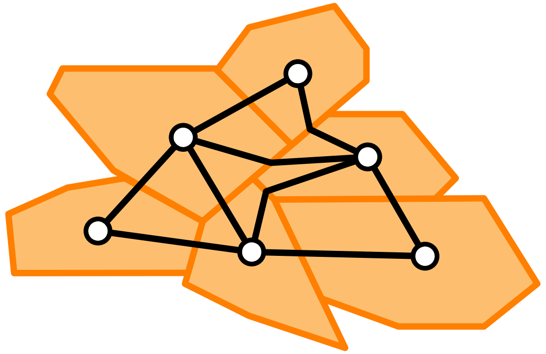


In an $\mathcal{S}$ **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$





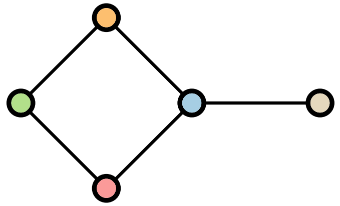
disks



polygons

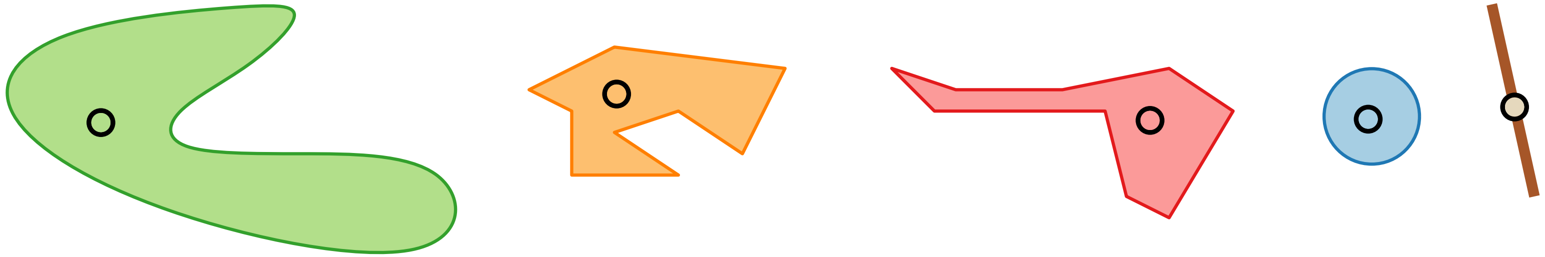
Contact Representation of Graphs

Let G be a graph.

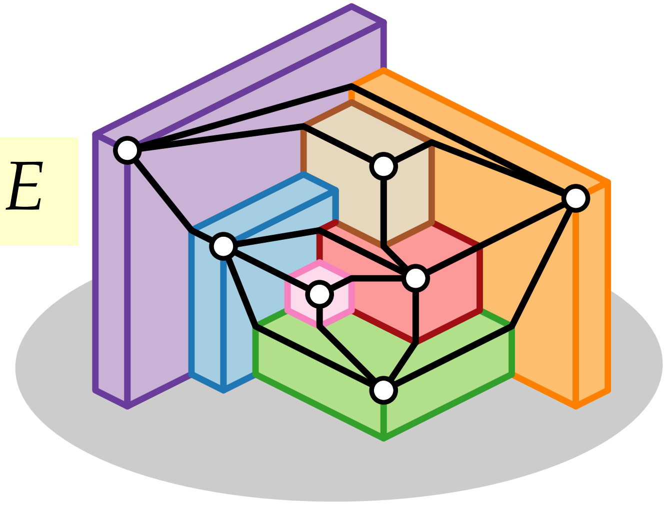


Let $\mathcal{S}$ be a set of geometric objects

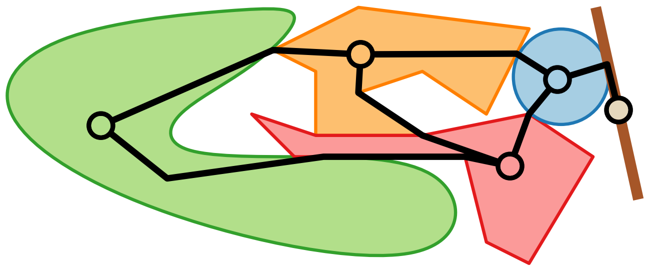
Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$



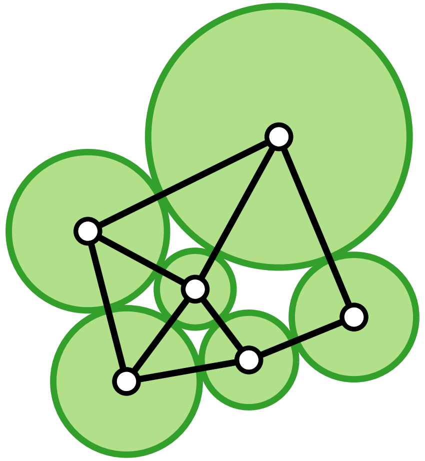
rectangular cuboids



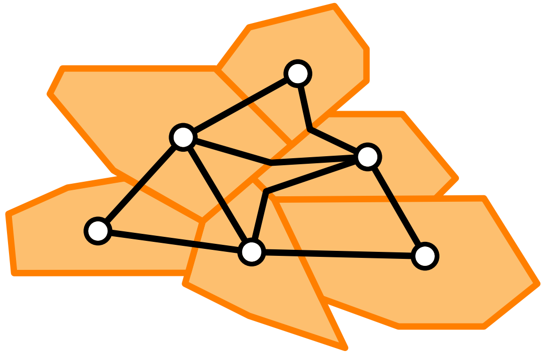
In an $\mathcal{S}$ **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$



G is planar



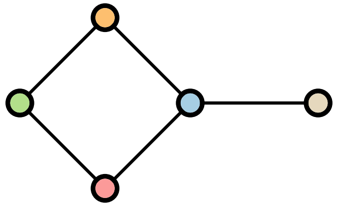
disks



polygons

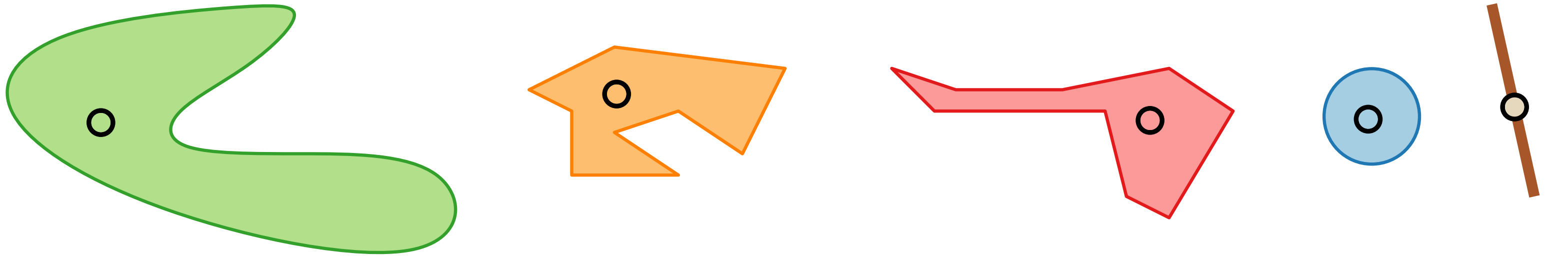
Contact Representation of Graphs

Let G be a graph.

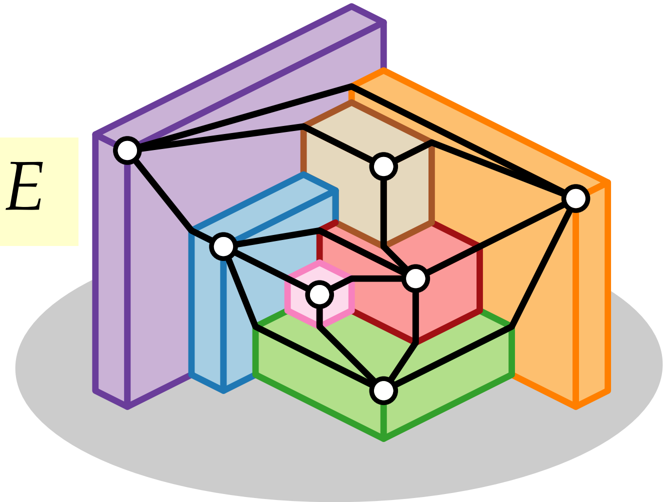


Let $\mathcal{S}$ be a set of geometric objects

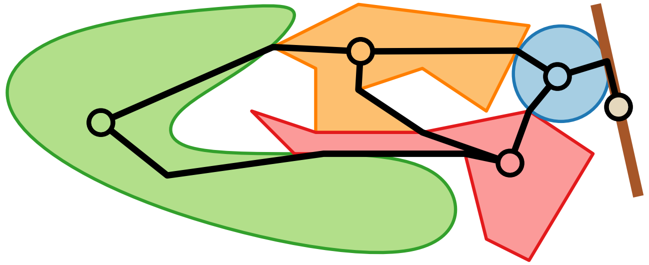
Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$

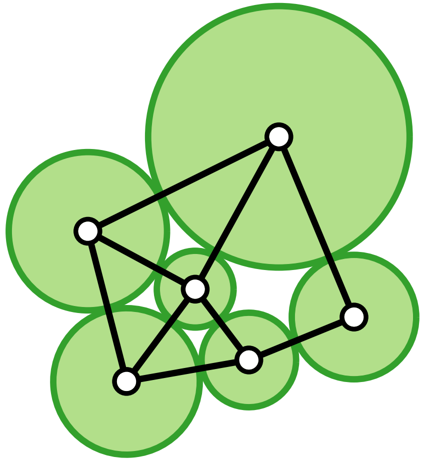


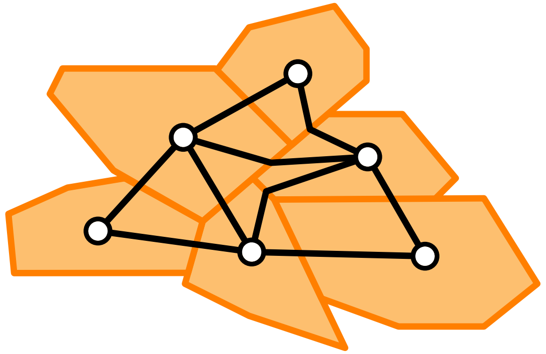
rectangular cuboids



In an $\mathcal{S}$ **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$





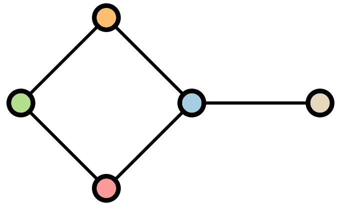


G is planar → disks [Koebe 1936]

polygons

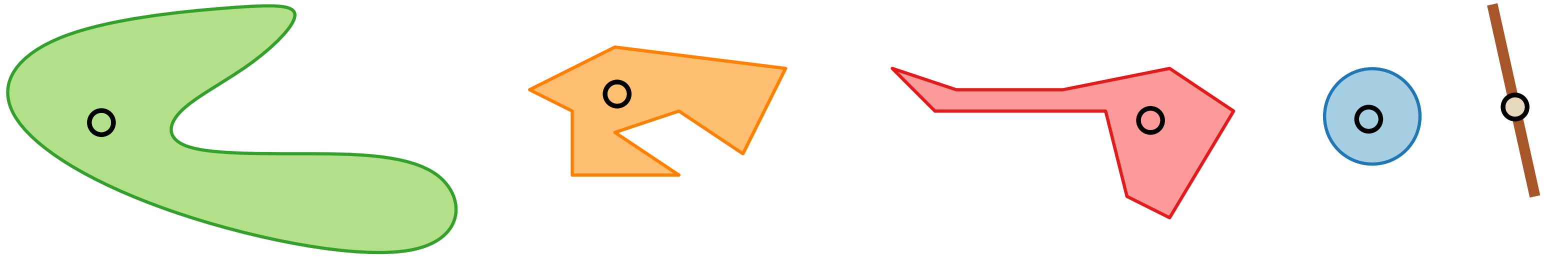
Contact Representation of Graphs

Let G be a graph.

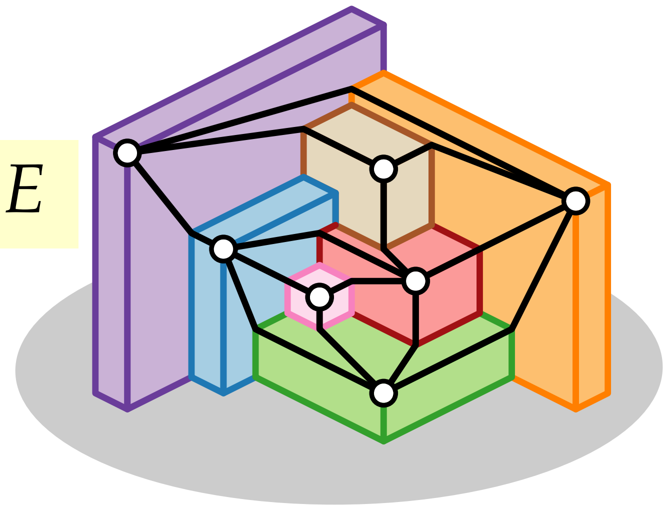


Let $\mathcal{S}$ be a set of geometric objects

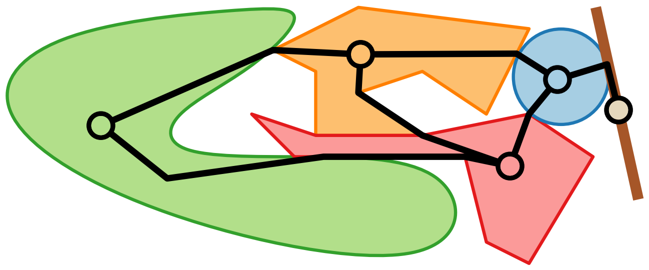
Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$

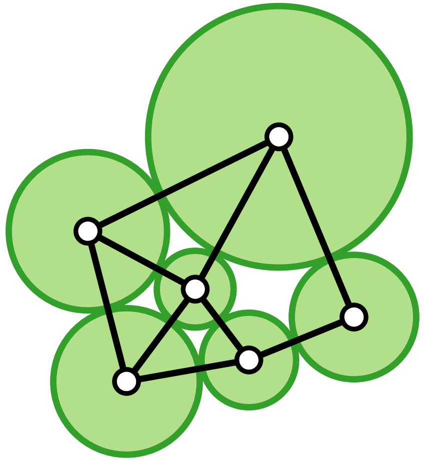


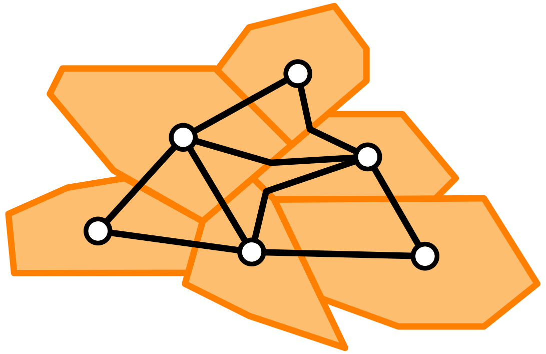
rectangular cuboids



In an $\mathcal{S}$ **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$







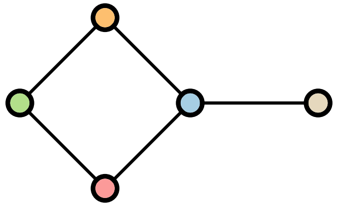
G is planar → disks → polygons

[Koebe 1936]

Contact Representation of Graphs

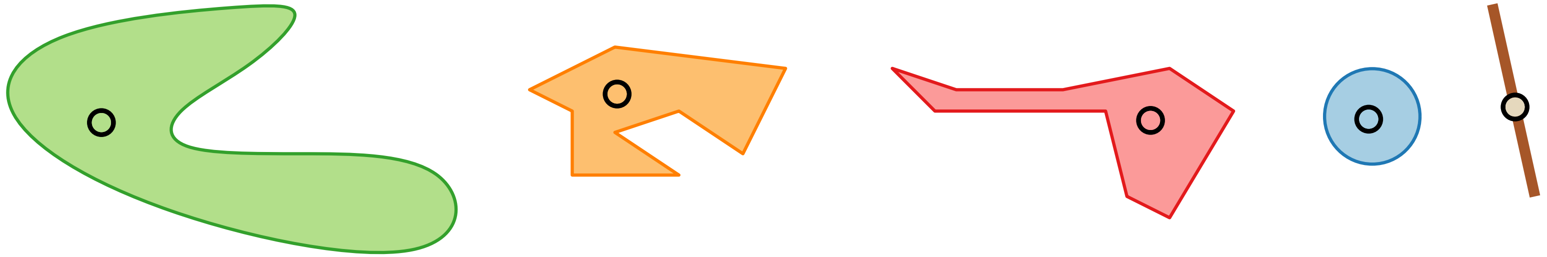
A contact representation is an intersection representation with interior-disjoint sets.

Let G be a graph.

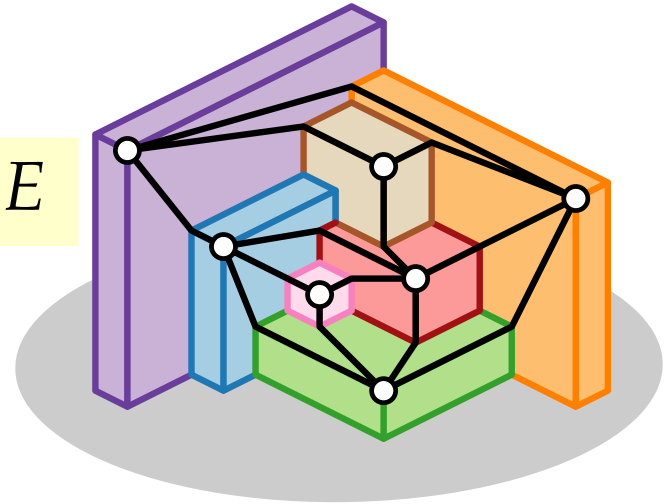


Let $\mathcal{S}$ be a set of geometric objects

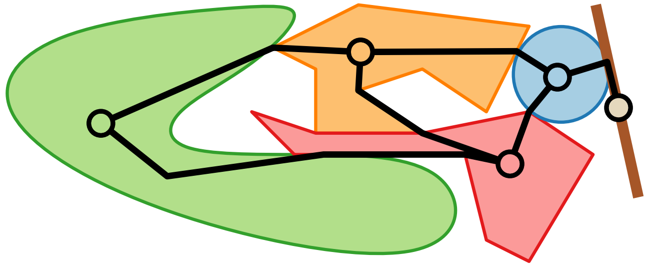
Represent each vertex v by a geometric object $S(v) \in \mathcal{S}$

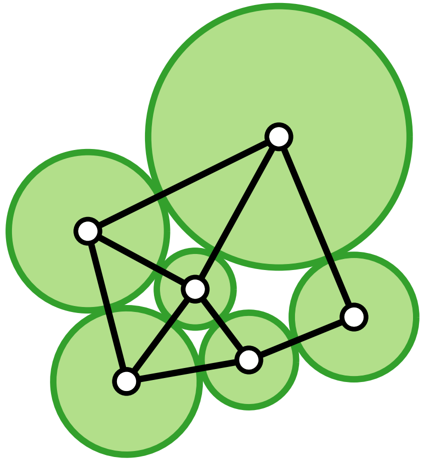


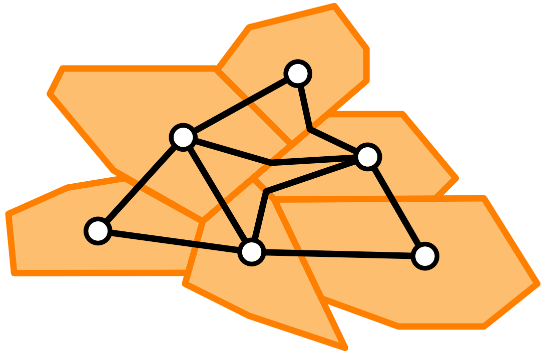
rectangular cuboids



In an $\mathcal{S}$ **contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$







G is planar → disks → polygons

[Koebe 1936]

Contact Representation of Planar Graphs

Is the intersection graph of a contact representation always planar?

Contact Representation of Planar Graphs

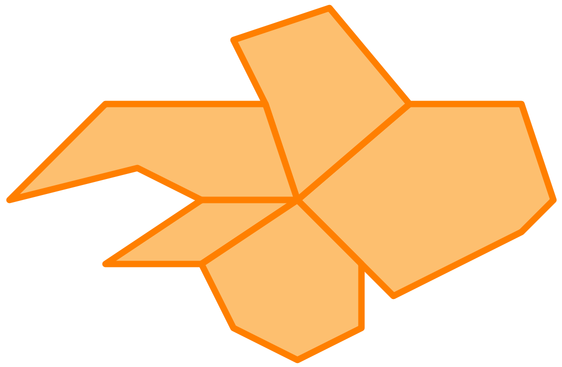
Is the intersection graph of a contact representation always planar?

- No, not even for connected object types.

Contact Representation of Planar Graphs

Is the intersection graph of a contact representation always planar?

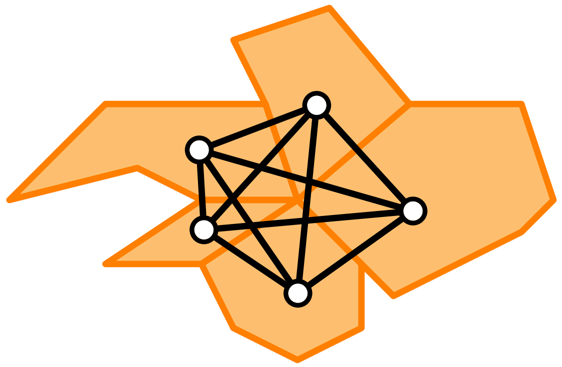
- No, not even for connected object types.



Contact Representation of Planar Graphs

Is the intersection graph of a contact representation always planar?

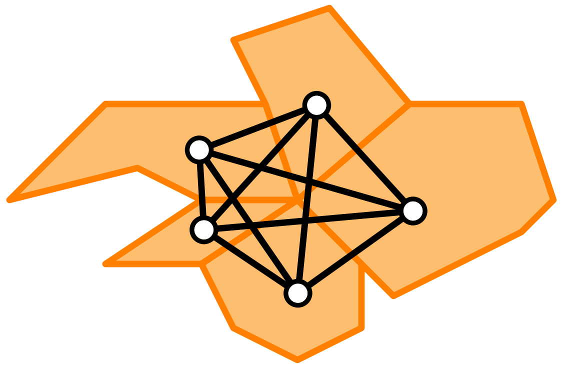
- No, not even for connected object types.



Contact Representation of Planar Graphs

Is the intersection graph of a contact representation always planar?

- No, not even for connected object types.

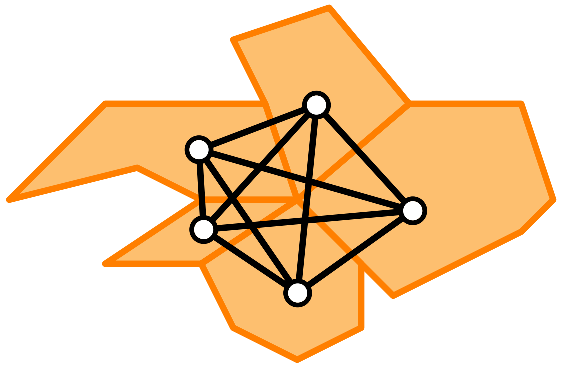


Some object types are used to represent special classes of planar graphs:

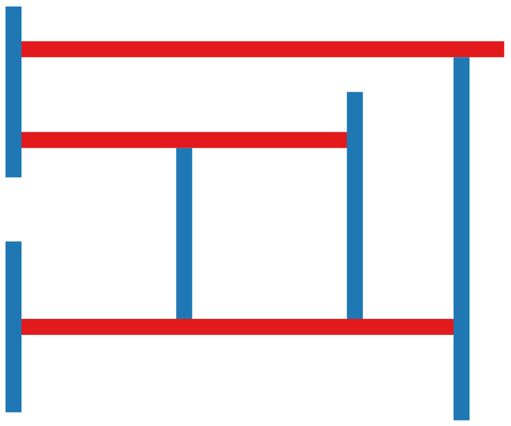
Contact Representation of Planar Graphs

Is the intersection graph of a contact representation always planar?

- No, not even for connected object types.



Some object types are used to represent special classes of planar graphs:

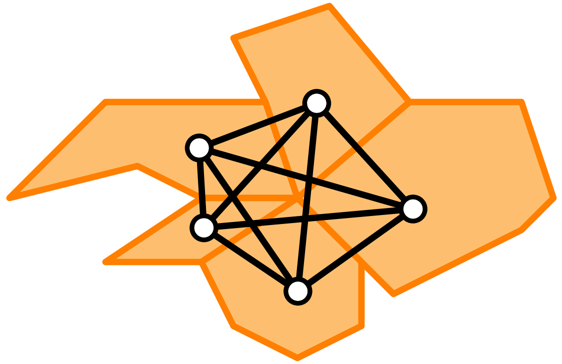


bipartite graphs

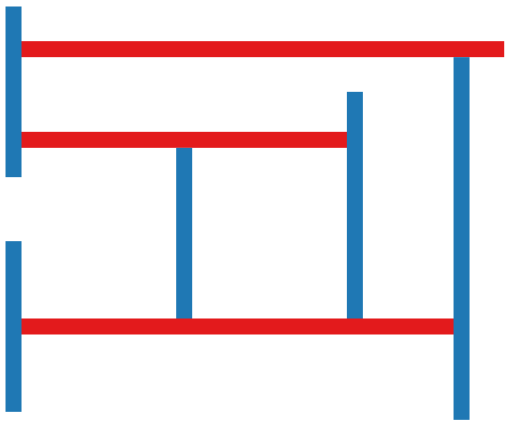
Contact Representation of Planar Graphs

Is the intersection graph of a contact representation always planar?

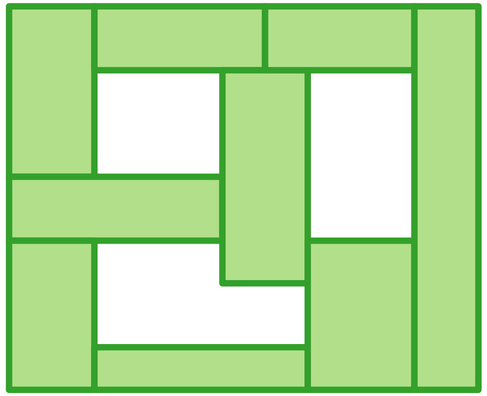
- No, not even for connected object types.



Some object types are used to represent special classes of planar graphs:



bipartite graphs

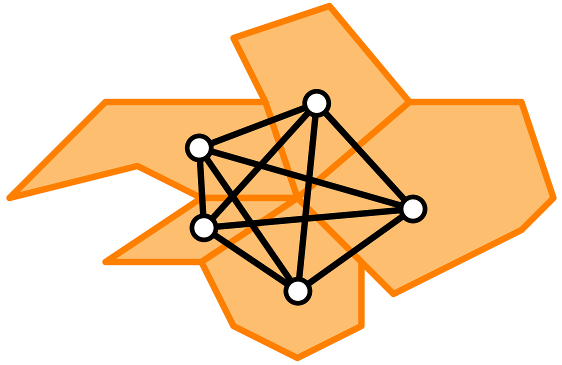


max. triangle-free graphs

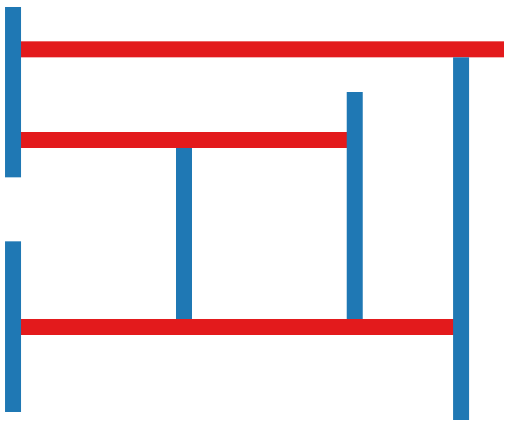
Contact Representation of Planar Graphs

Is the intersection graph of a contact representation always planar?

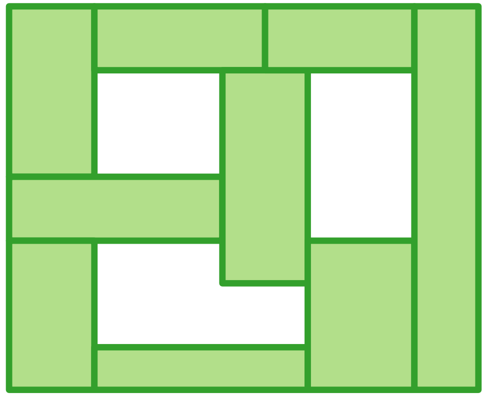
- No, not even for connected object types.



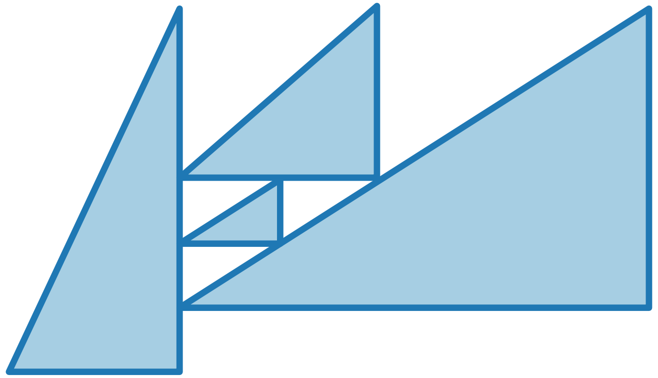
Some object types are used to represent special classes of planar graphs:



bipartite graphs



max. triangle-free graphs



planar triangulations

General Approach

How to compute a contact representation of a given graph G?

General Approach

How to compute a contact representation of a given graph G?

- Consider only inner triangulations
  (or maximally bipartite graphs, etc)

General Approach

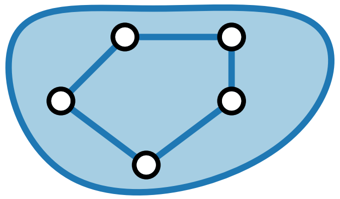
How to compute a contact representation of a given graph G?

- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges

General Approach

How to compute a contact representation of a given graph G?

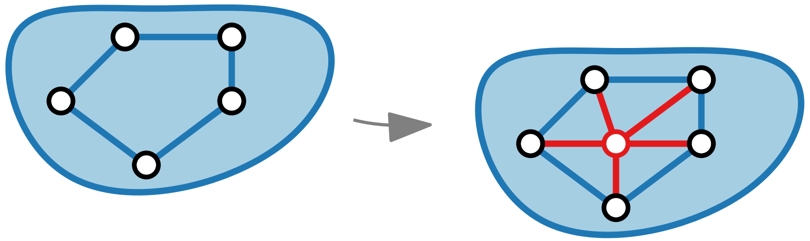
- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges



General Approach

How to compute a contact representation of a given graph G?

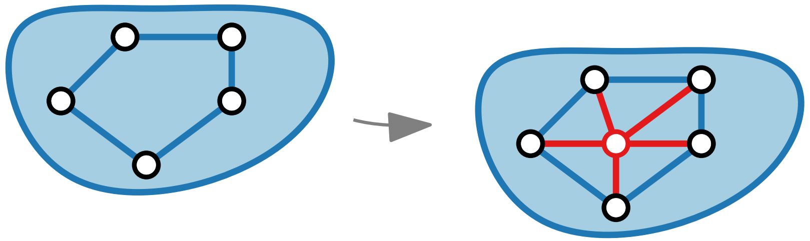
- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges



General Approach

How to compute a contact representation of a given graph G?

- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges

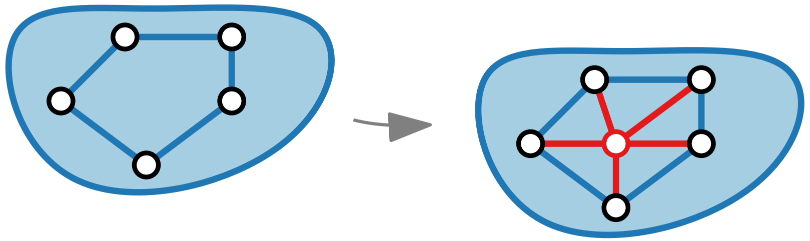


- Describe contact representation combinatorically.

General Approach

How to compute a contact representation of a given graph G?

- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges

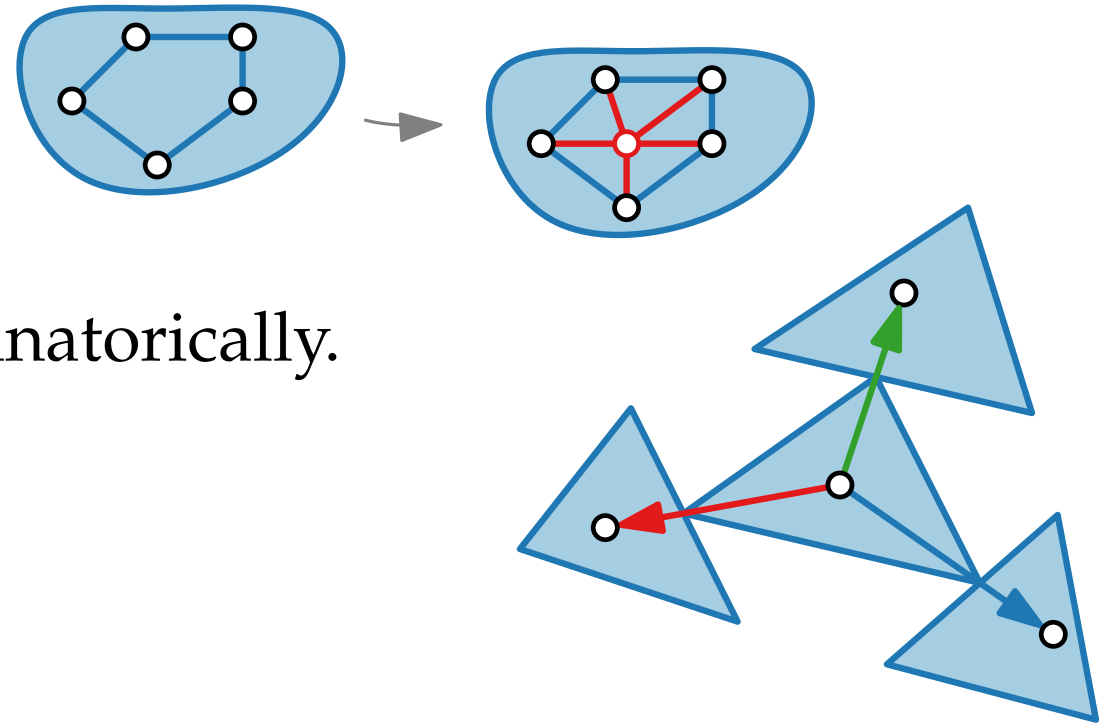


- Describe contact representation combinatorically.

General Approach

How to compute a contact representation of a given graph G?

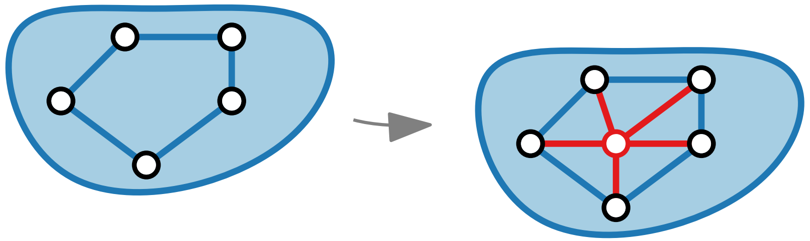
- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges
- Describe contact representation combinatorically.



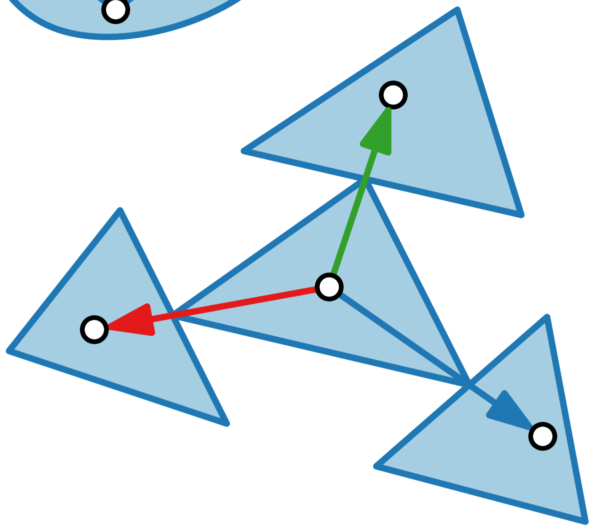
General Approach

How to compute a contact representation of a given graph G?

- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges



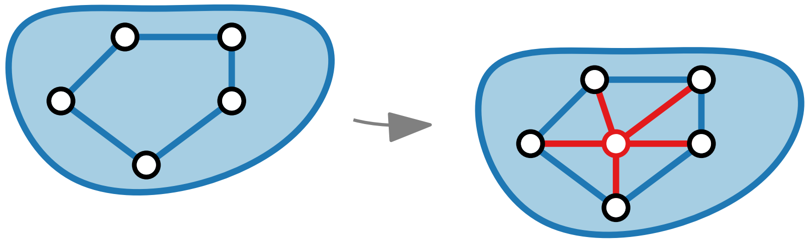
- Describe contact representation combinatorically.
  - Which objects contact each other in which way?



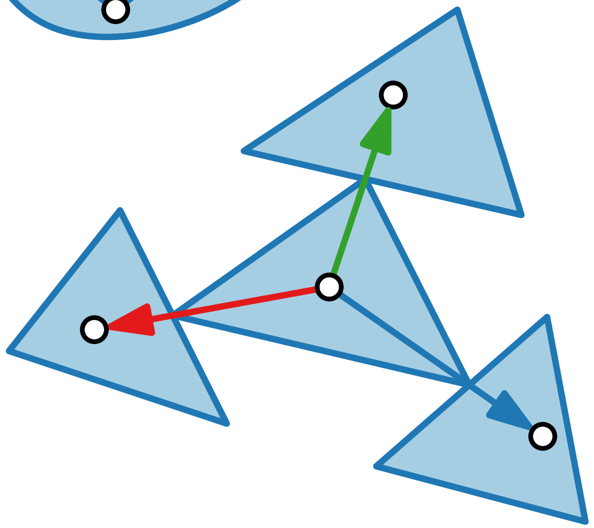
General Approach

How to compute a contact representation of a given graph G?

- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges



- Describe contact representation combinatorically.
  - Which objects contact each other in which way?

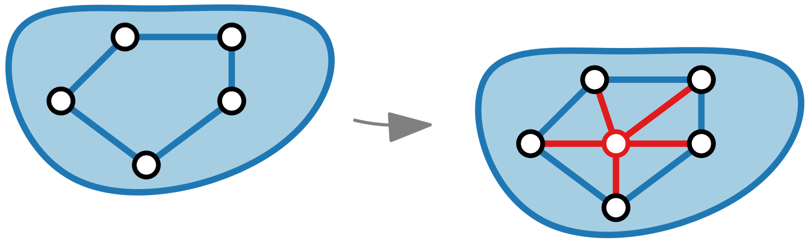


- Compute combinatorical description.

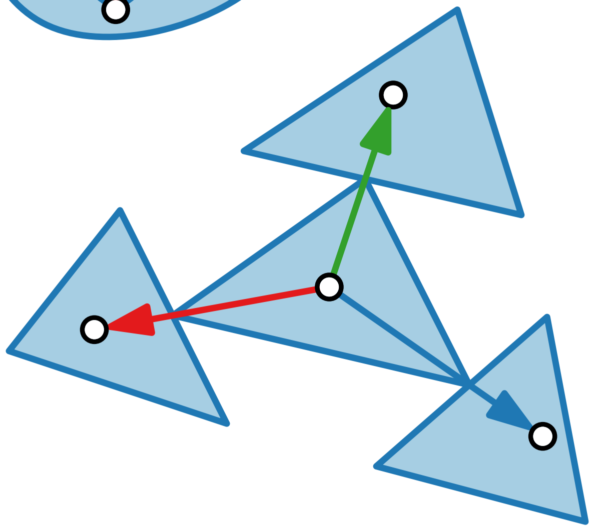
General Approach

How to compute a contact representation of a given graph G?

- Consider only inner triangulations (or maximally bipartite graphs, etc)
  - Triangulate by adding vertices, not by adding edges

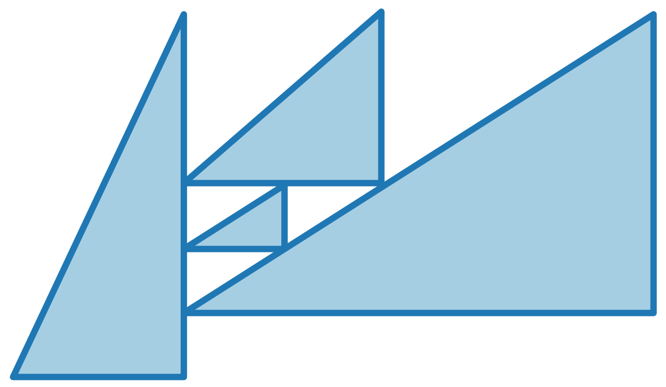


- Describe contact representation combinatorically.
  - Which objects contact each other in which way?
- Compute combinatorical description.
- Show that combinatorical description can be used to construct drawing.



In This Lecture

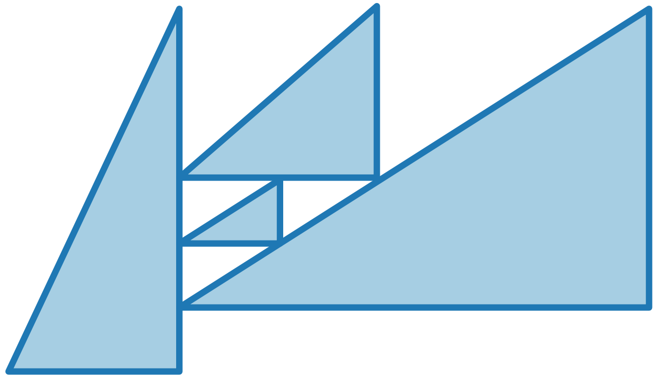
Representations with right-triangles and corner contact



In This Lecture

Representations with right-triangles and corner contact

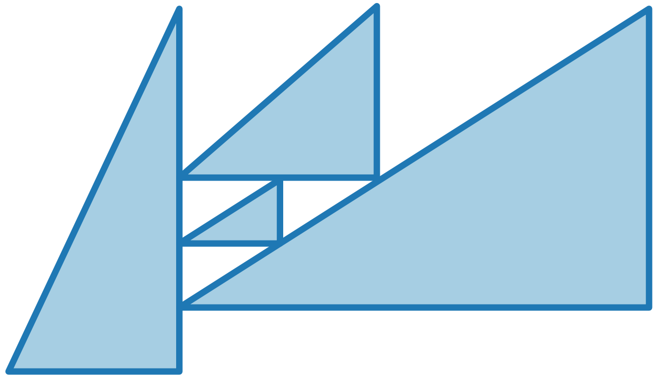
- Use Schnyder realizer to describe contacts between triangles



In This Lecture

Representations with right-triangles and corner contact

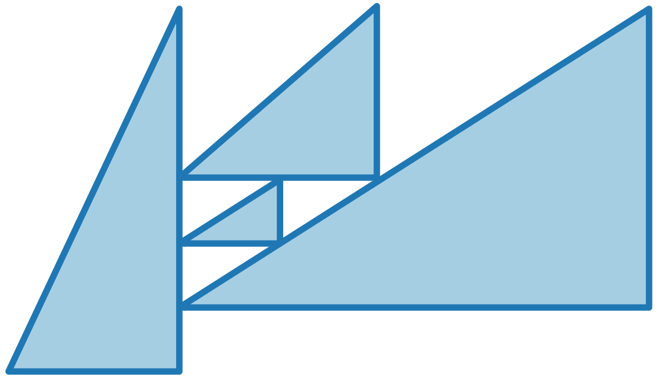
- Use Schnyder realizer to describe contacts between triangles
- Use canonical order to calculate drawing



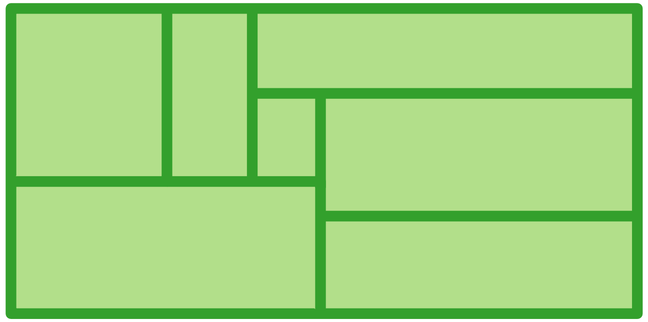
In This Lecture

Representations with right-triangles and corner contact

- Use Schnyder realizer to describe contacts between triangles
- Use canonical order to calculate drawing



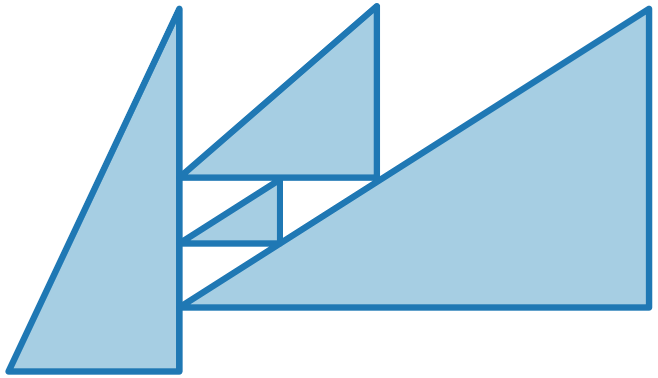
Representation with dissection of a rectangle, called **rectangular dual**



In This Lecture

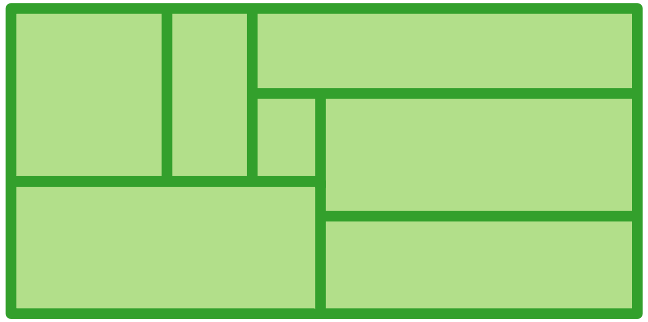
Representations with right-triangles and corner contact

- Use Schnyder realizer to describe contacts between triangles
- Use canonical order to calculate drawing



Representation with dissection of a rectangle, called **rectangular dual**

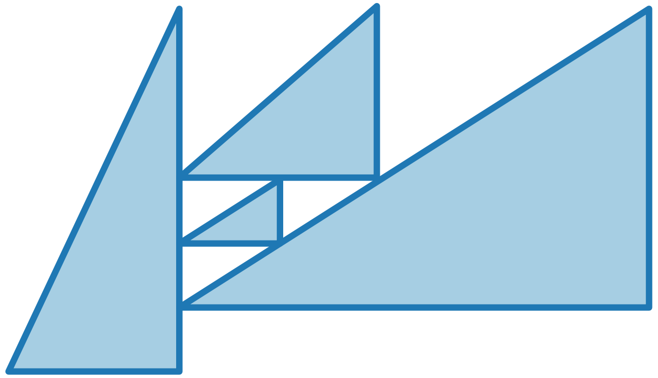
- Find similar description like Schnyder realizer for rectangles



In This Lecture

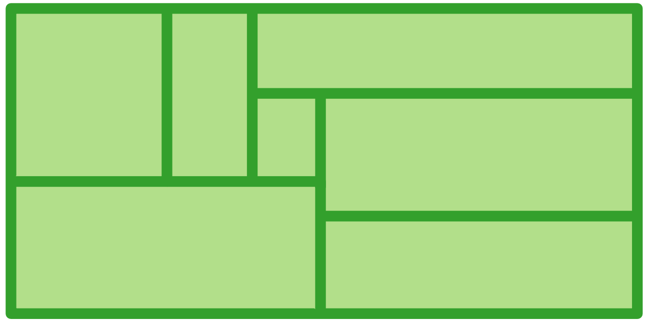
Representations with right-triangles and corner contact

- Use Schnyder realizer to describe contacts between triangles
- Use canonical order to calculate drawing



Representation with dissection of a rectangle, called **rectangular dual**

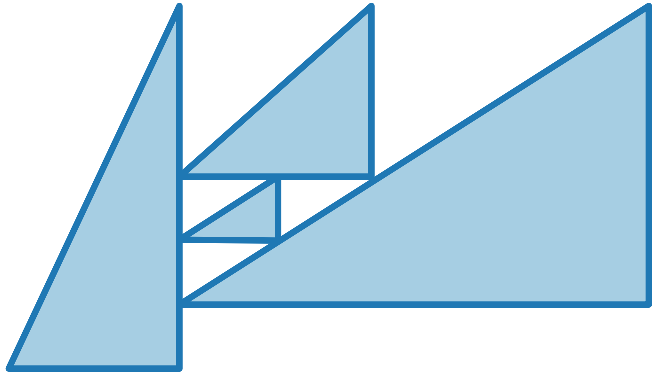
- Find similar description like Schnyder realizer for rectangles
- Construct drawing via st-digraphs, duals, and topological sorting



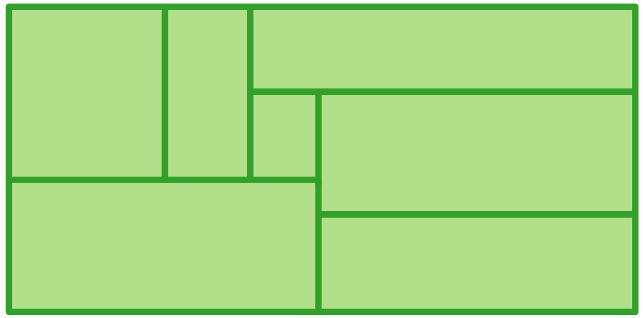
Universität Trier

Visualization of Graphs

Lecture 9:
Contact Representations of Planar Graphs: Triangle Contacts and Rectangular Duals



Part II: Triangle Contact Representations



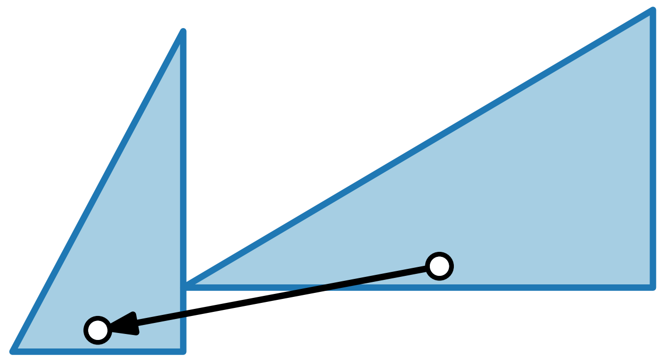
Philipp Kindermann

Triangle Corner Contact Representation

Idea.
Use canonical order and Schnyder realizer to find coordinates for triangles.

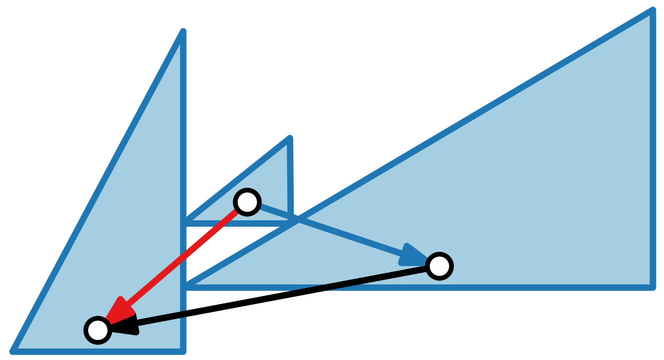
Triangle Corner Contact Representation

Idea.
Use canonical order and Schnyder realizer to find coordinates for triangles.



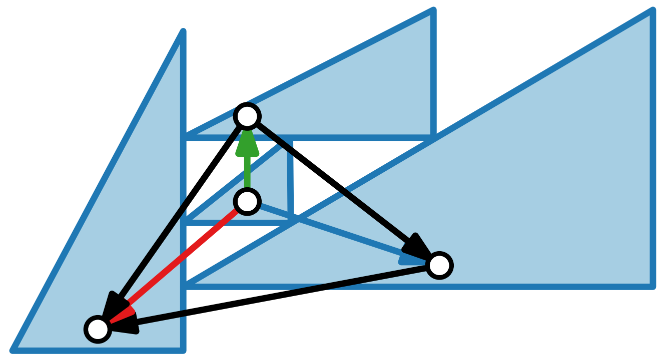
Triangle Corner Contact Representation

Idea.
Use canonical order and Schnyder realizer to find coordinates for triangles.



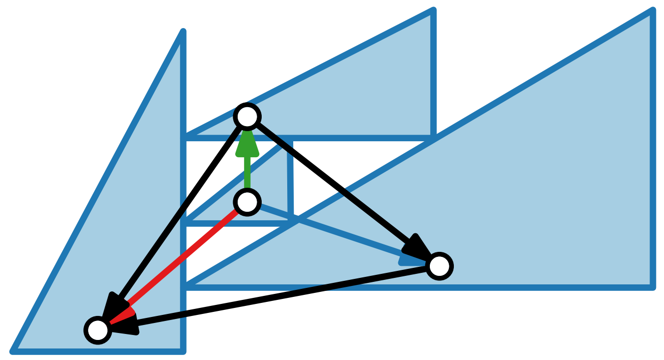
Triangle Corner Contact Representation

Idea.
Use canonical order and Schnyder realizer to find coordinates for triangles.



Triangle Corner Contact Representation

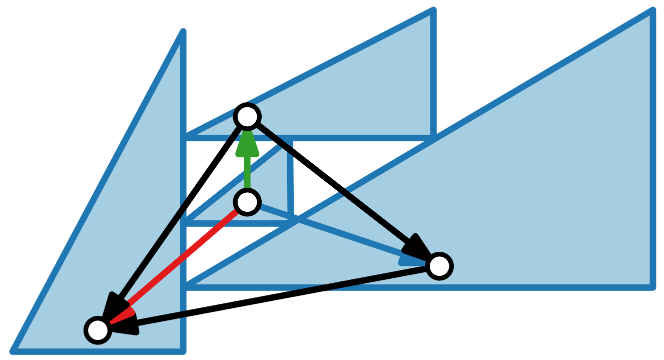
Idea.
Use canonical order and Schnyder realizer to find coordinates for triangles.



Observation.

- Can set base of triangle at height equal to position in canonical order.

Triangle Corner Contact Representation

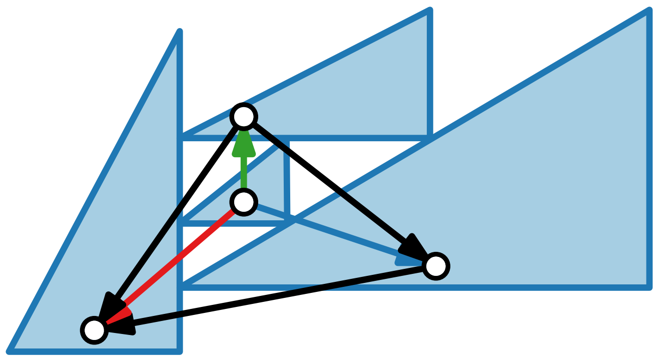


Idea.
Use canonical order and Schnyder realizer to find coordinates for triangles.

Observation.

- Can set base of triangle at height equal to position in canonical order.
- Triangle tip is precisely at base of triangle corresponding to cover neighbor.

Triangle Corner Contact Representation

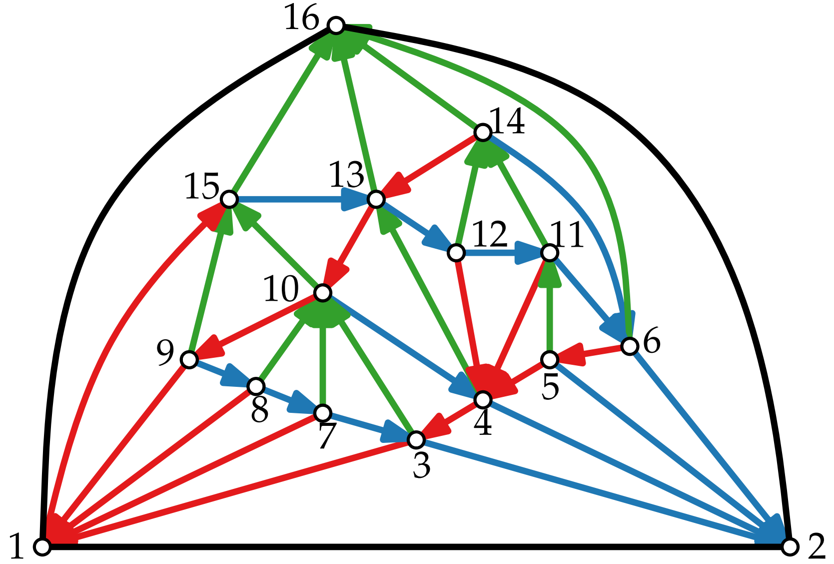


Idea.
Use canonical order and Schnyder realizer to find coordinates for triangles.

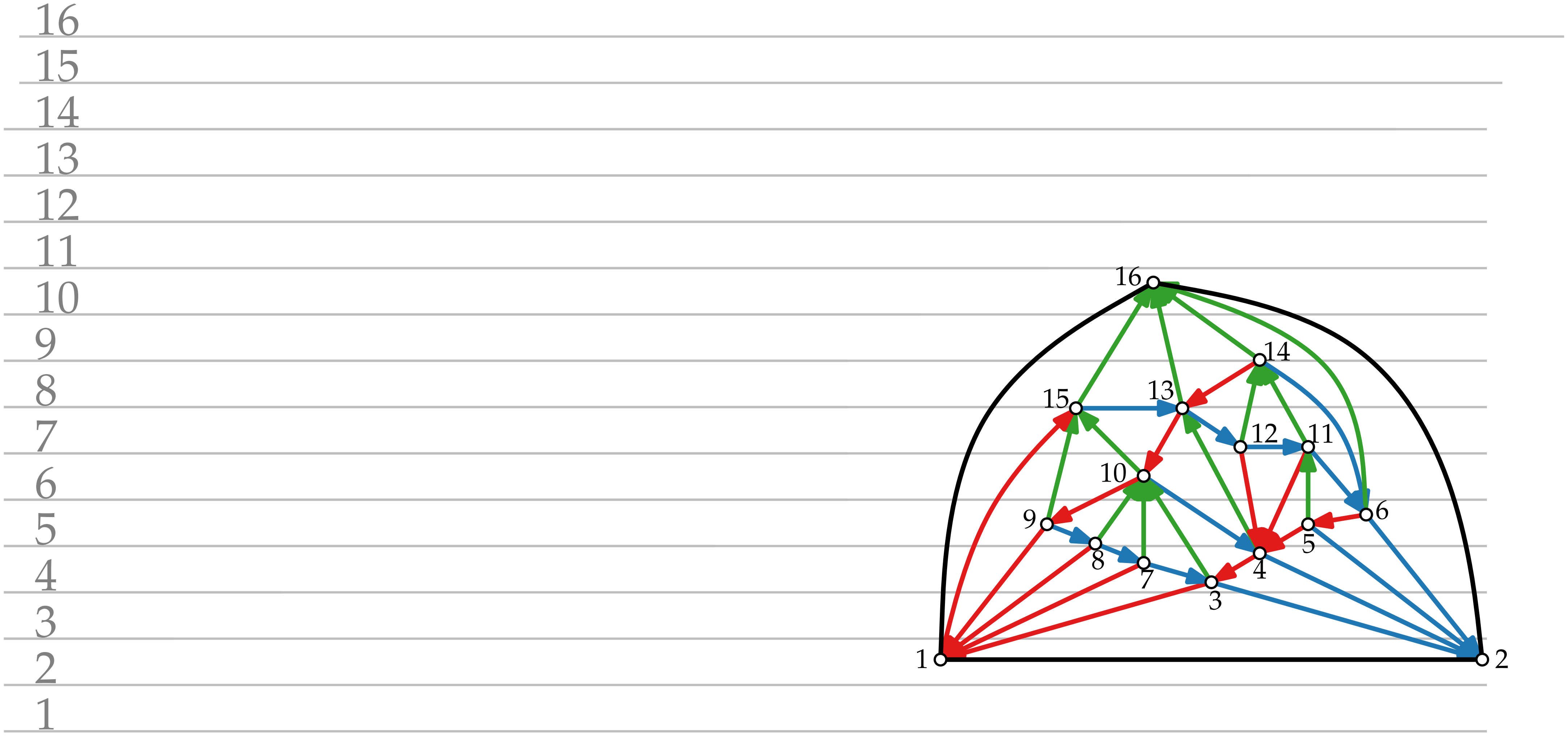
Observation.

- Can set base of triangle at height equal to position in canonical order.
- Triangle tip is precisely at base of triangle corresponding to cover neighbor.
- Outgoing edges in Schnyder forest indicate corner contacts.

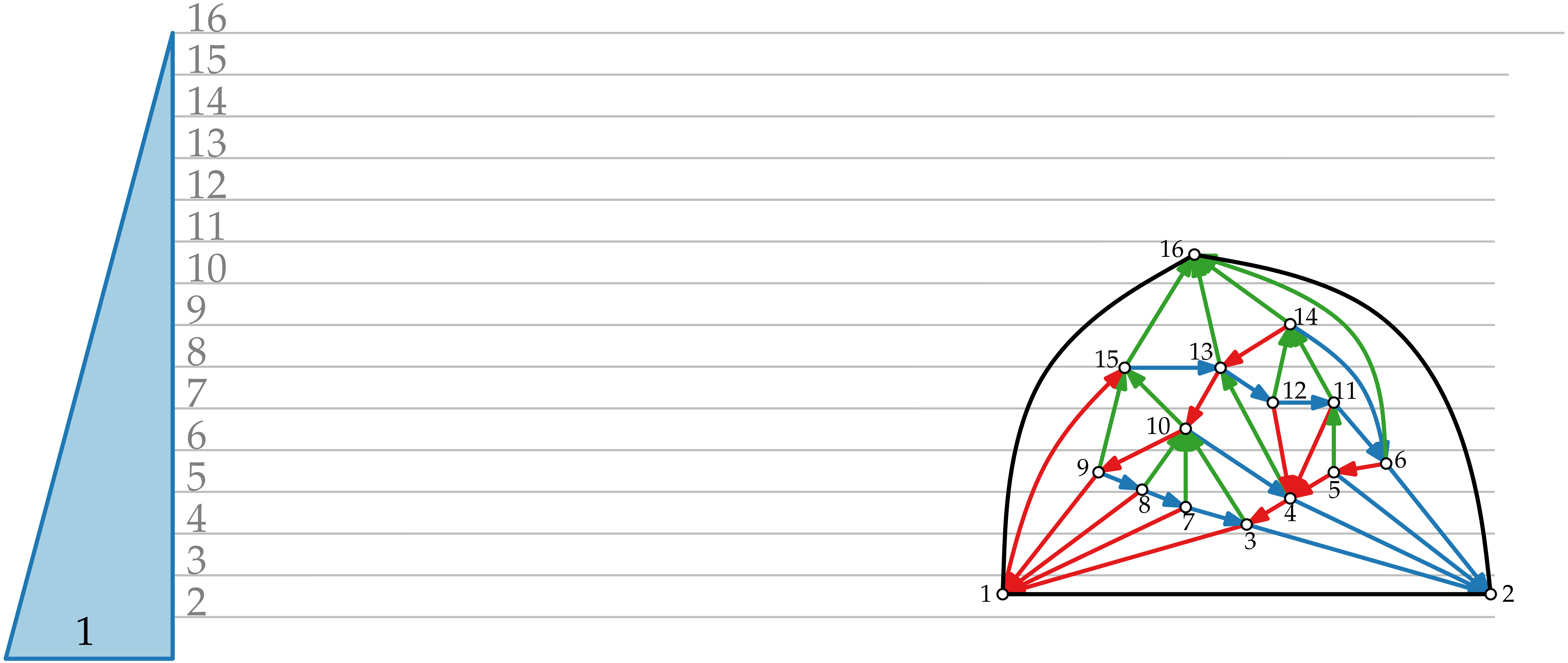
Triangle Contact Representation Example



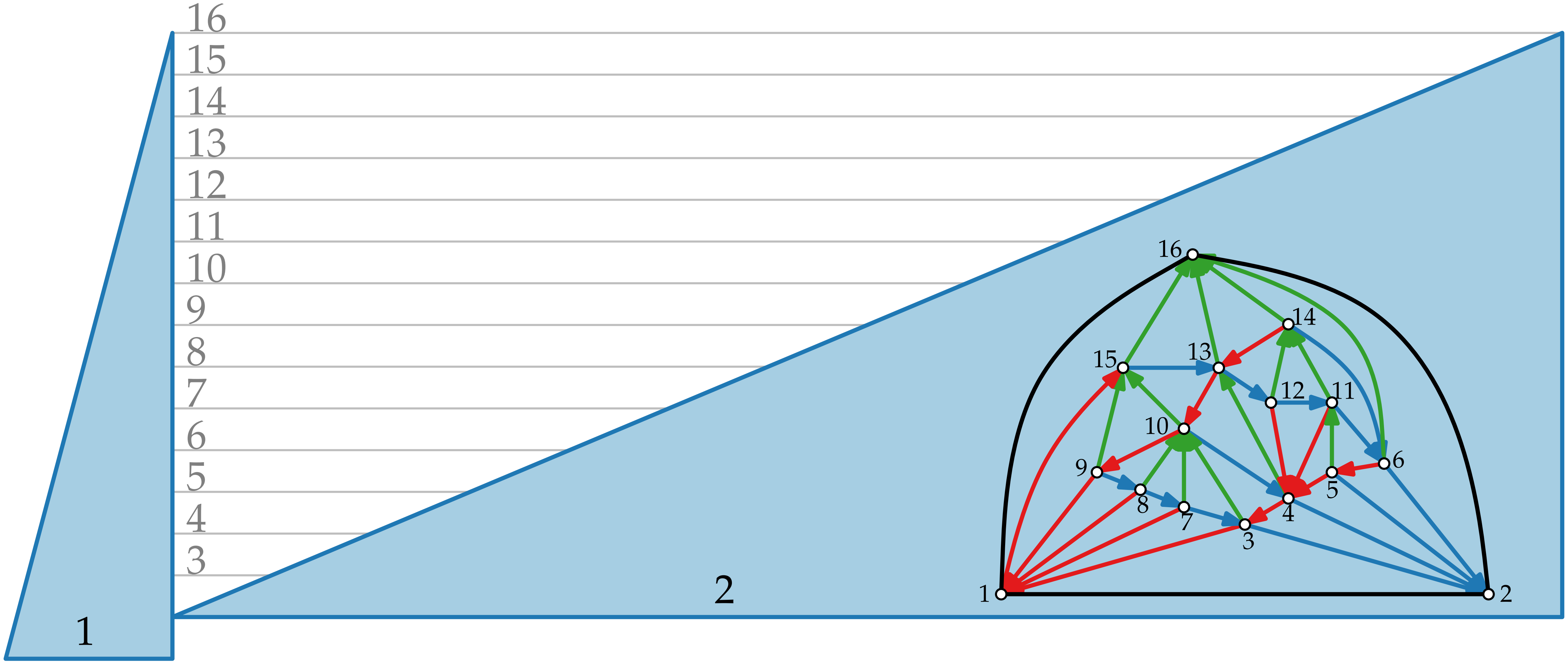
Triangle Contact Representation Example



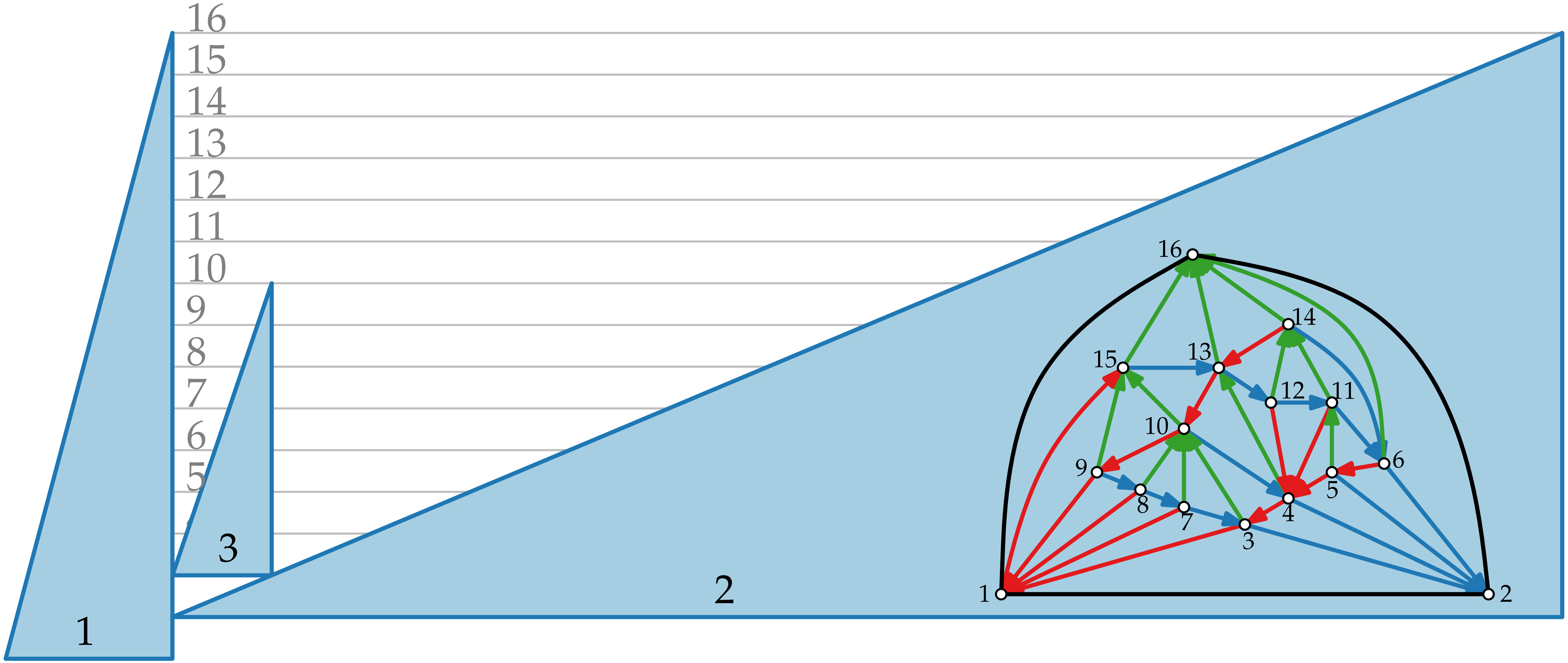
Triangle Contact Representation Example



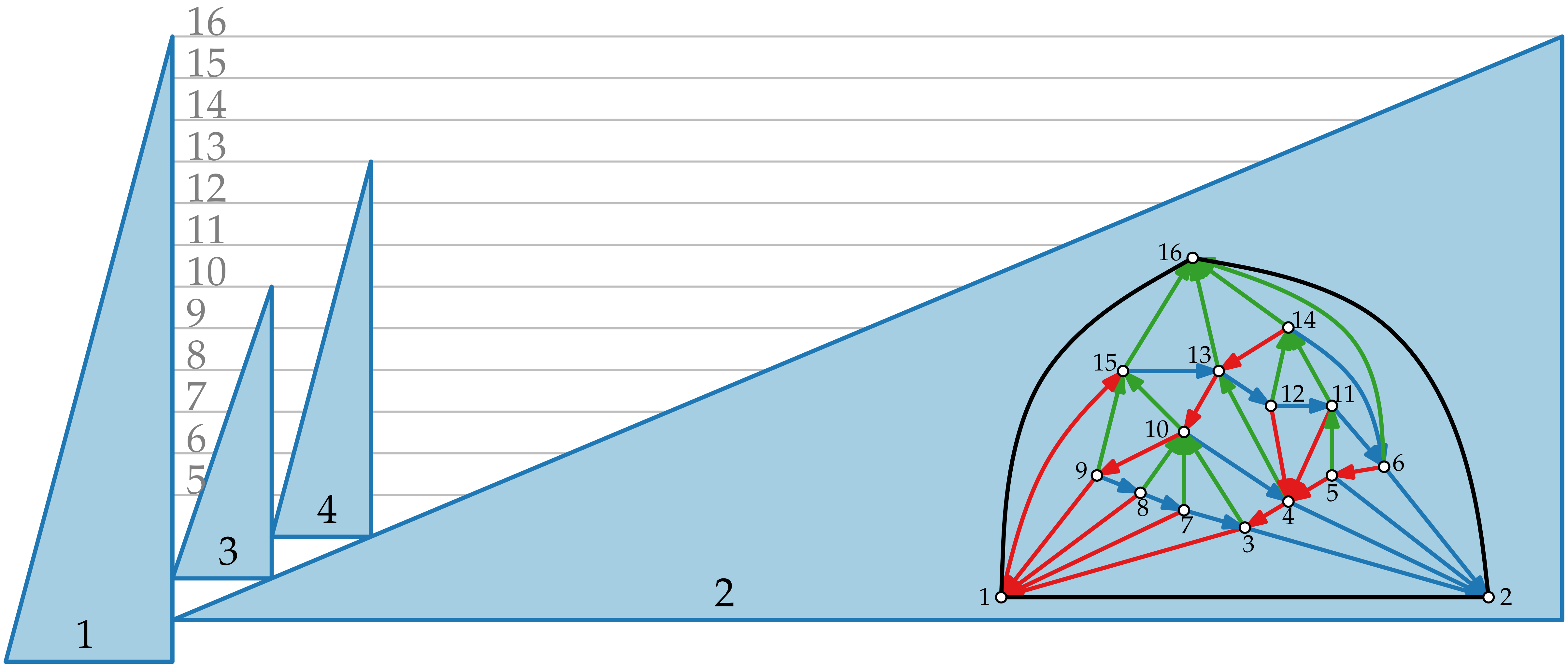
Triangle Contact Representation Example



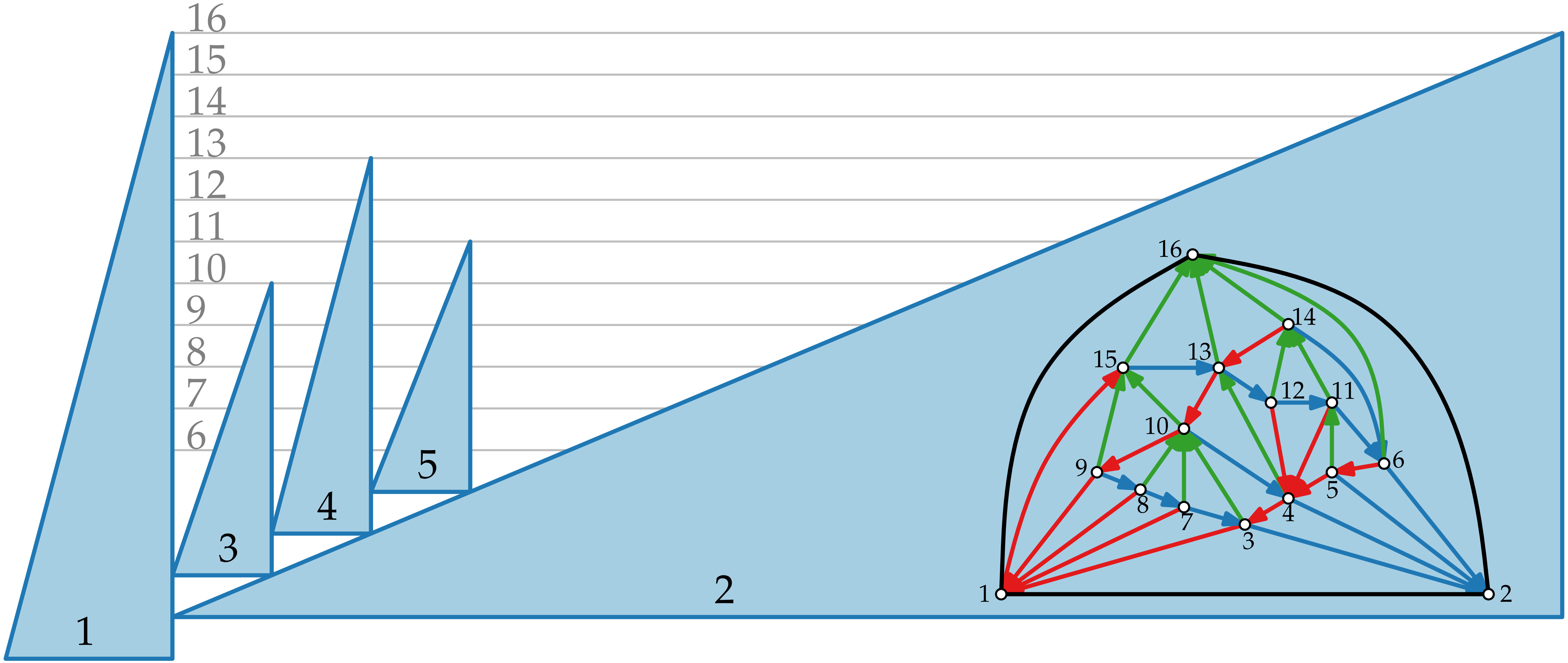
Triangle Contact Representation Example



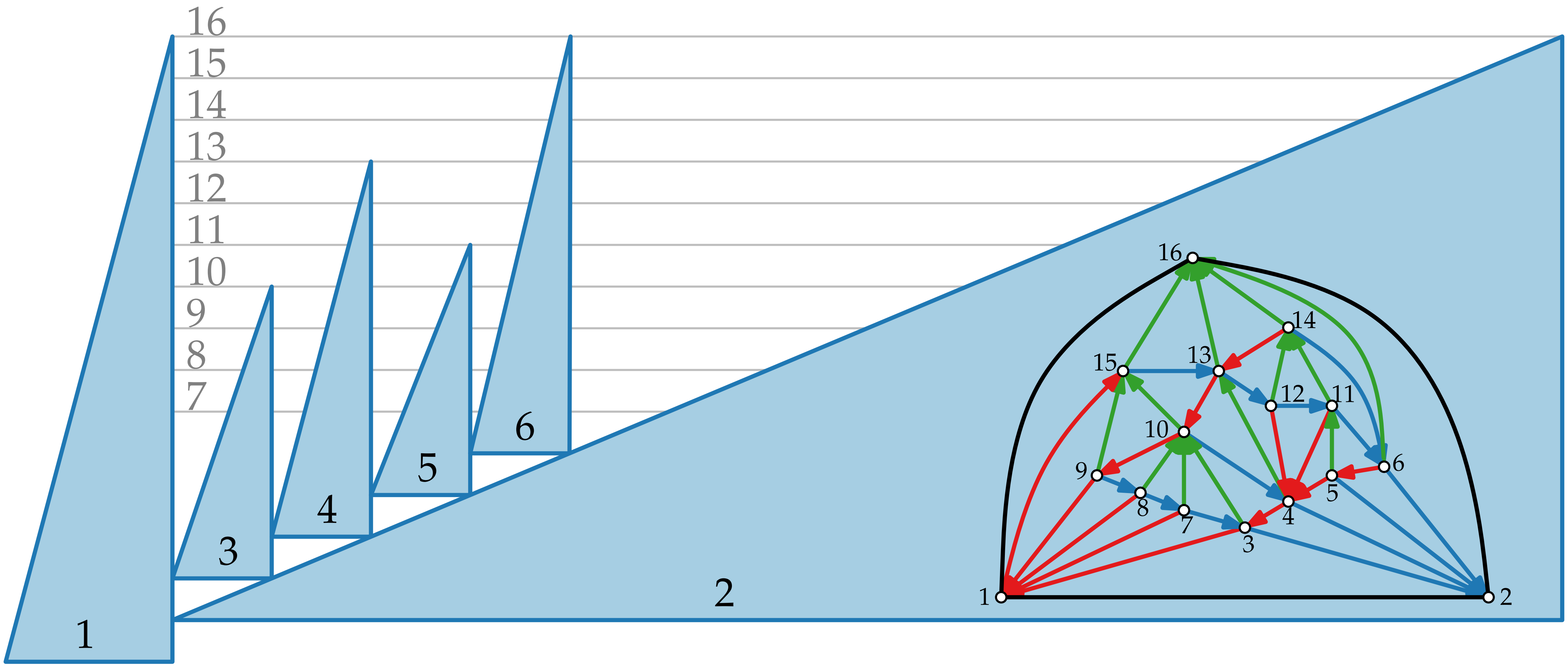
Triangle Contact Representation Example



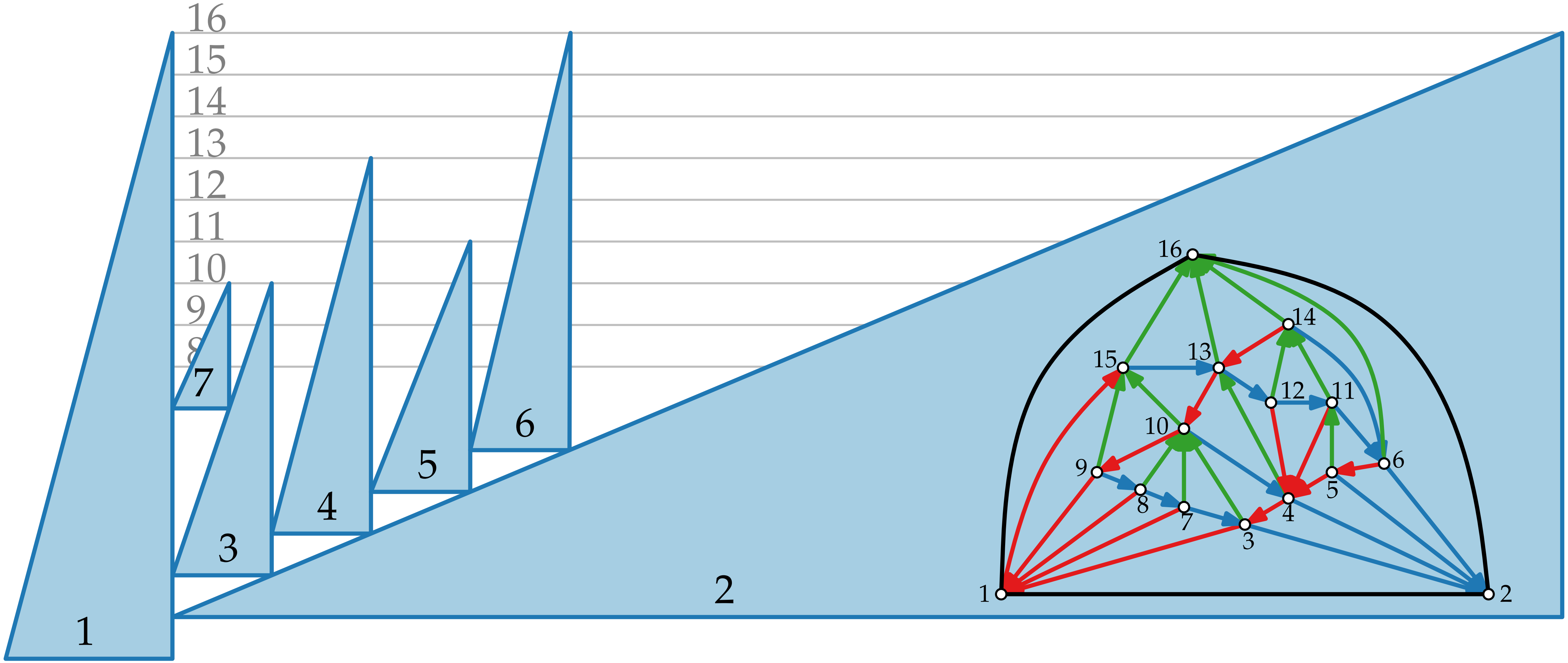
Triangle Contact Representation Example



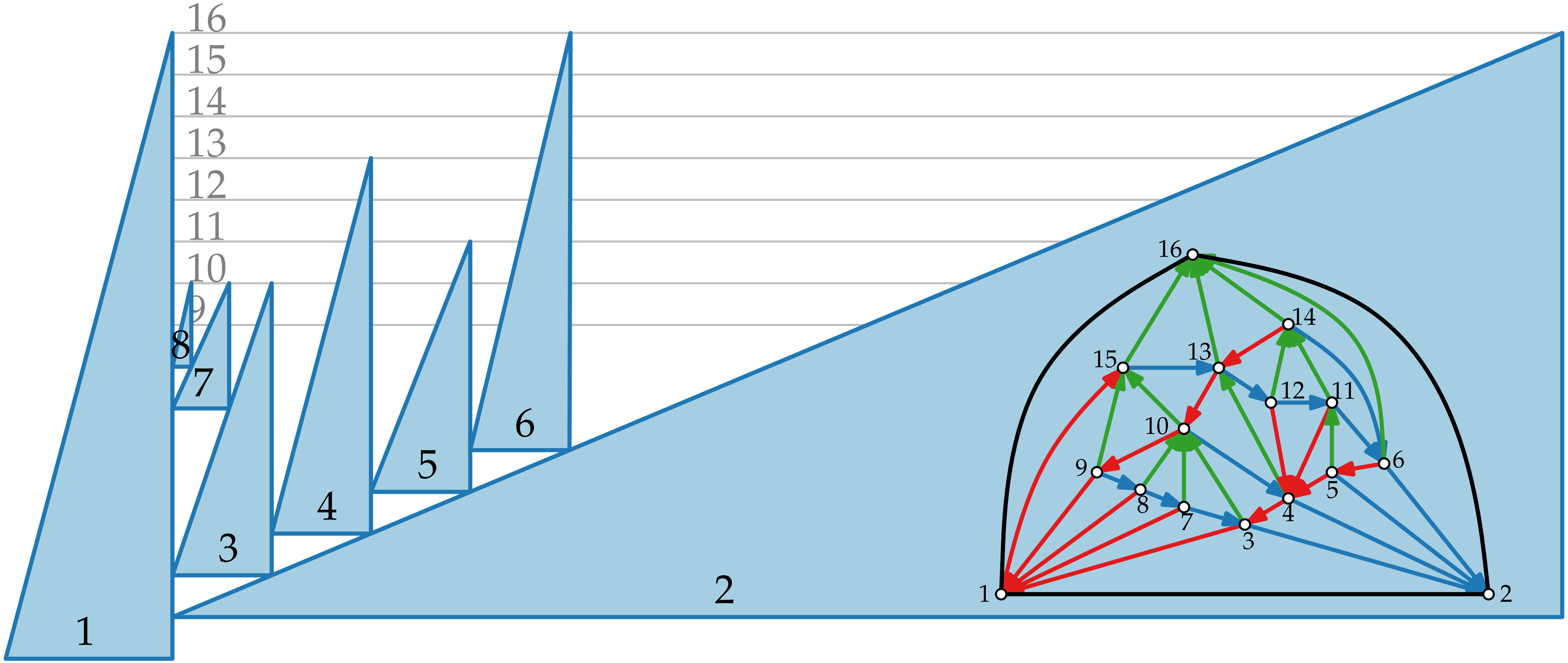
Triangle Contact Representation Example



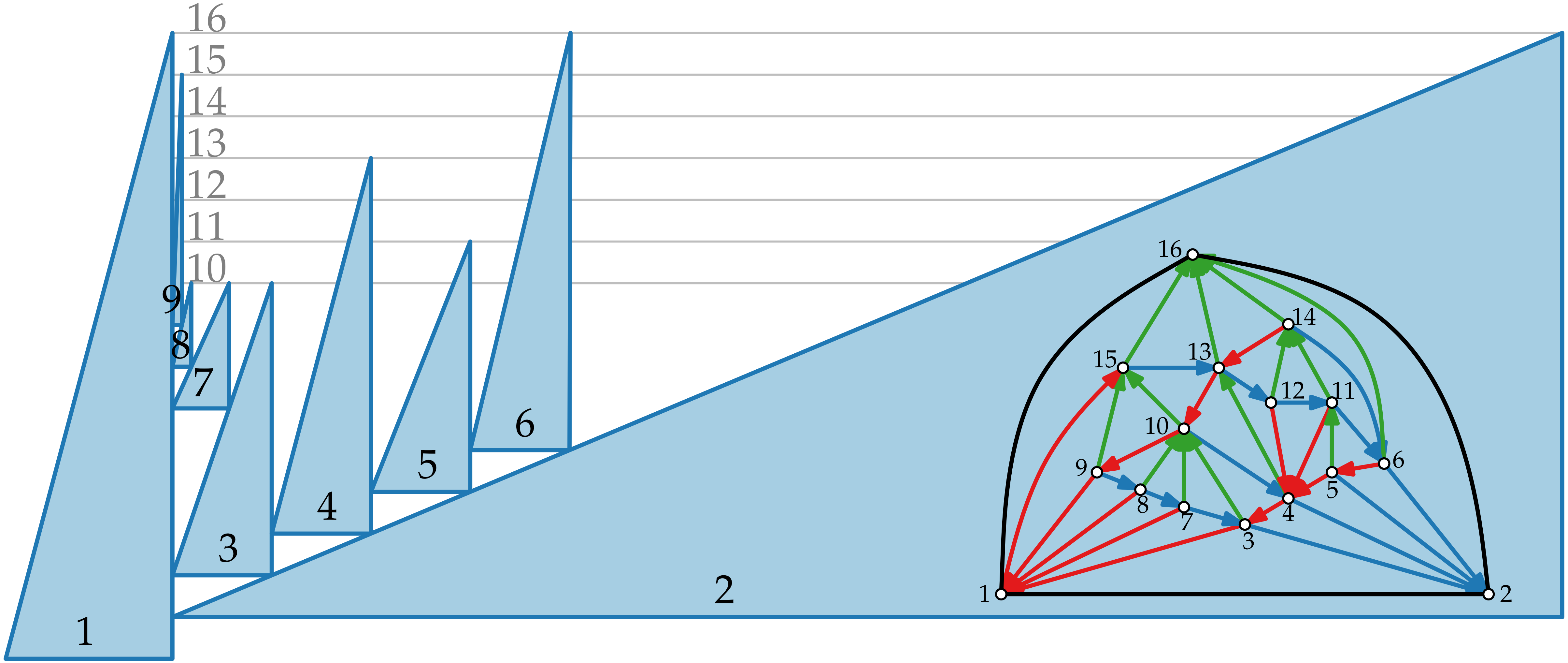
Triangle Contact Representation Example



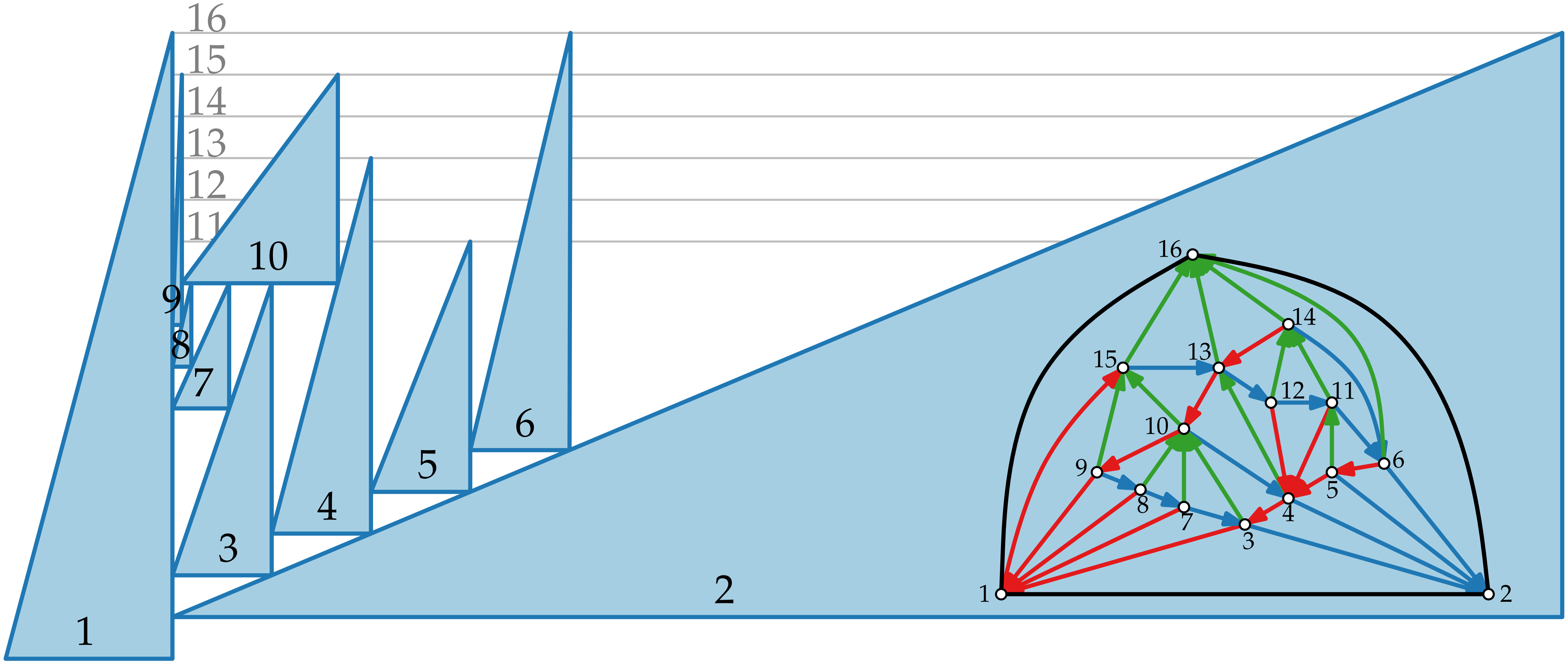
Triangle Contact Representation Example



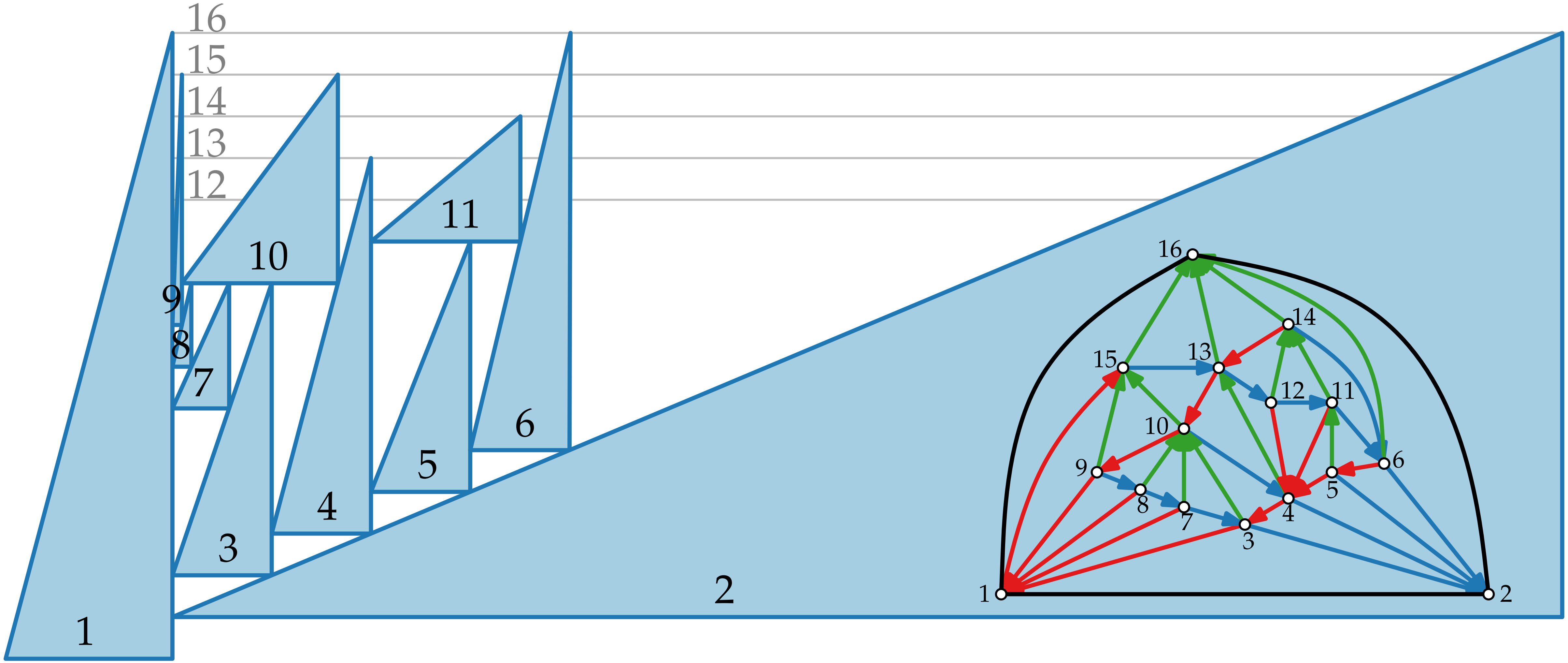
Triangle Contact Representation Example



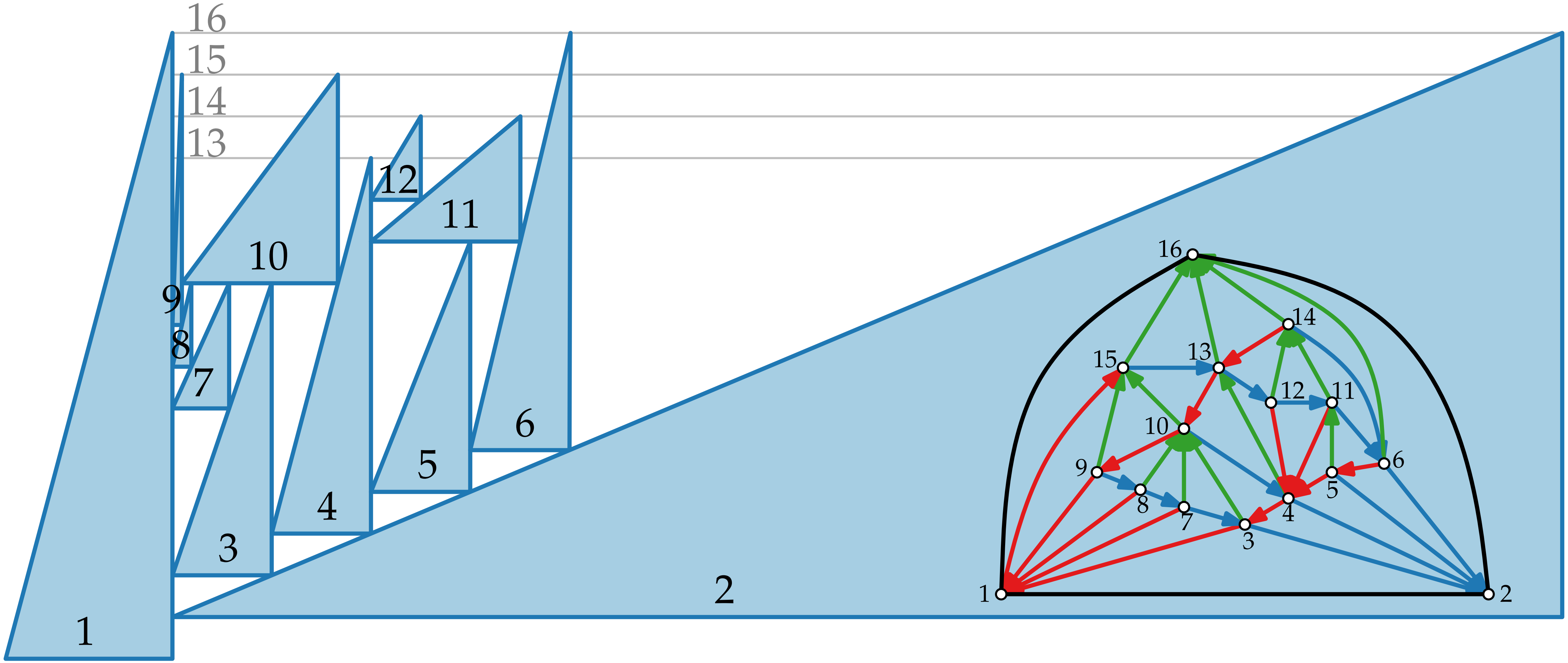
Triangle Contact Representation Example



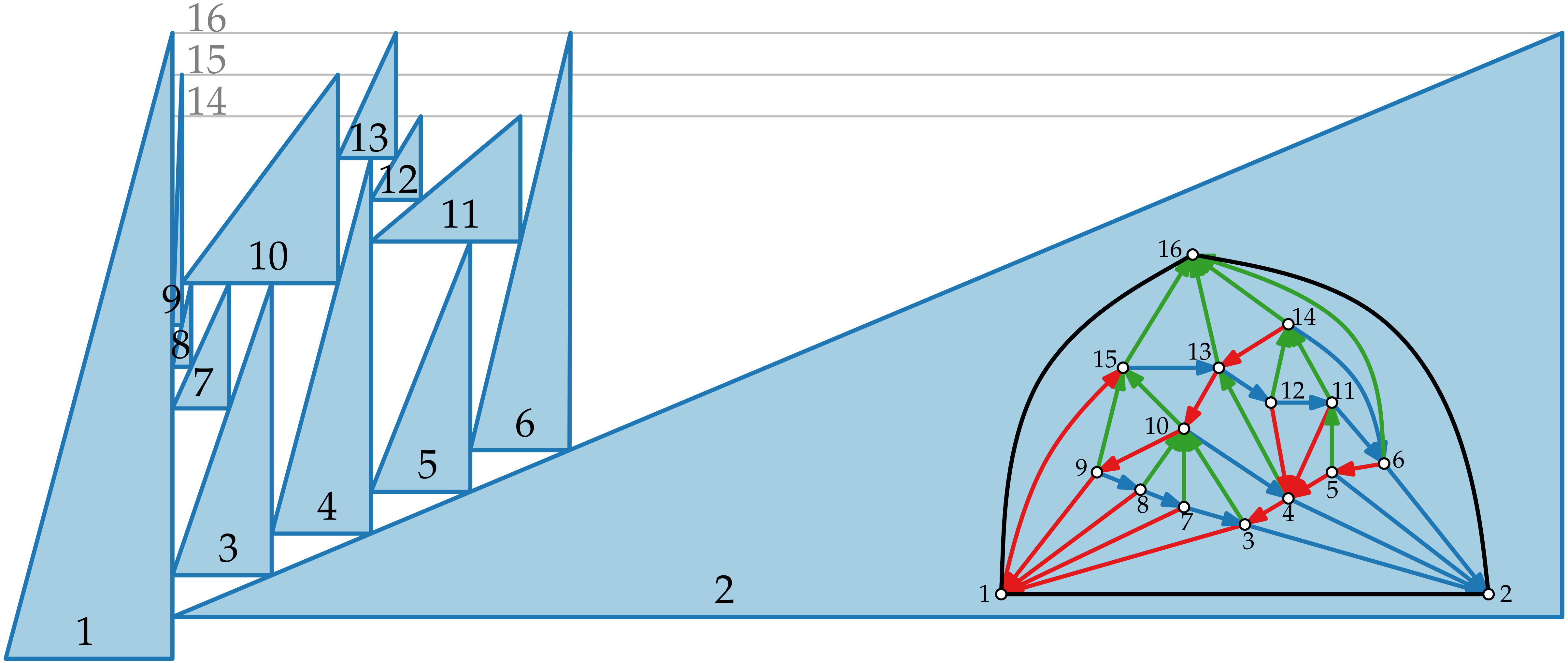
Triangle Contact Representation Example



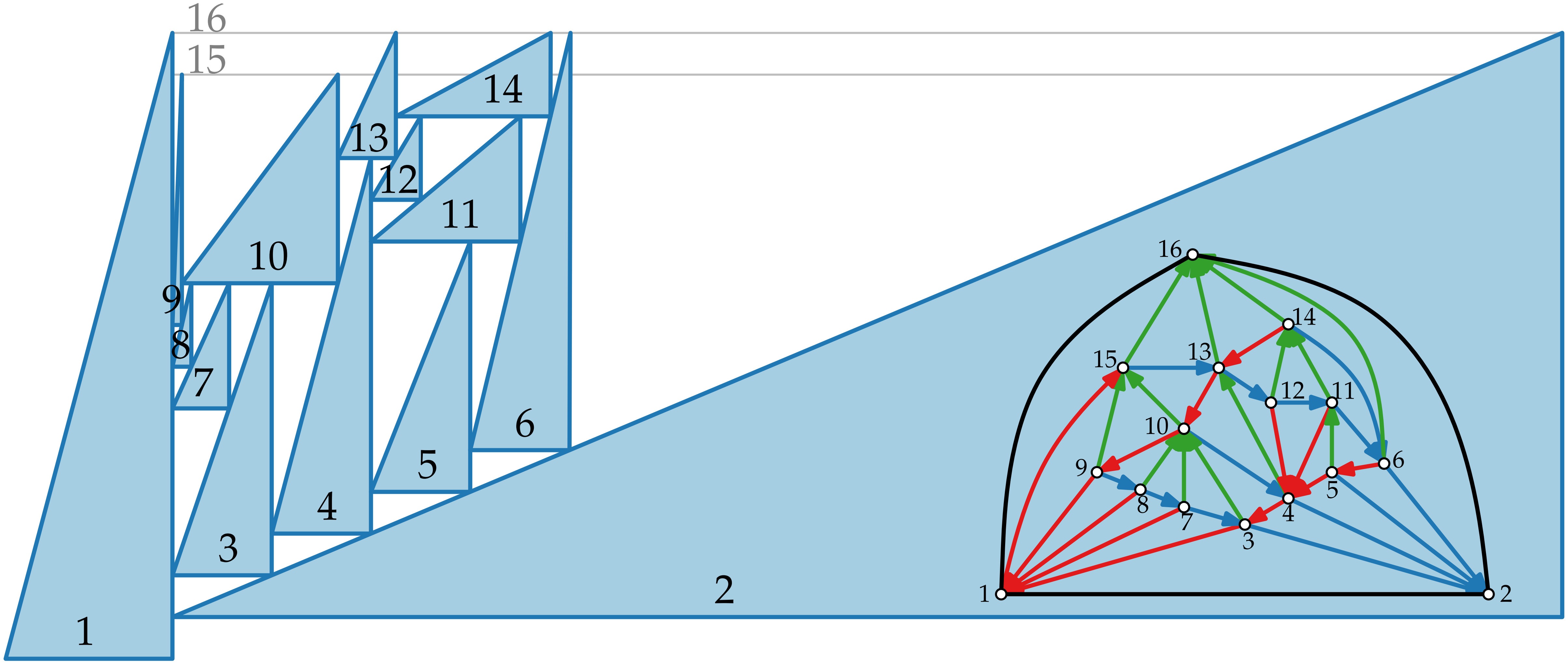
Triangle Contact Representation Example



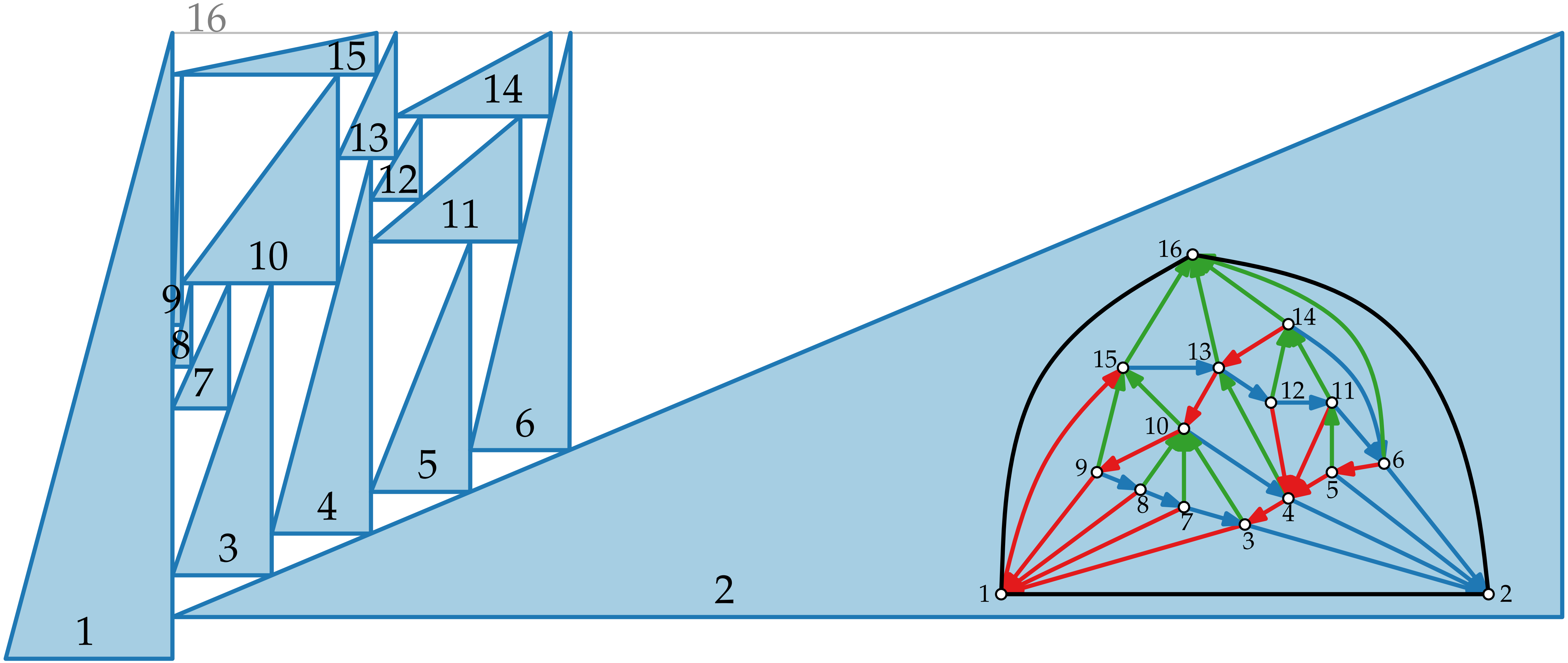
Triangle Contact Representation Example



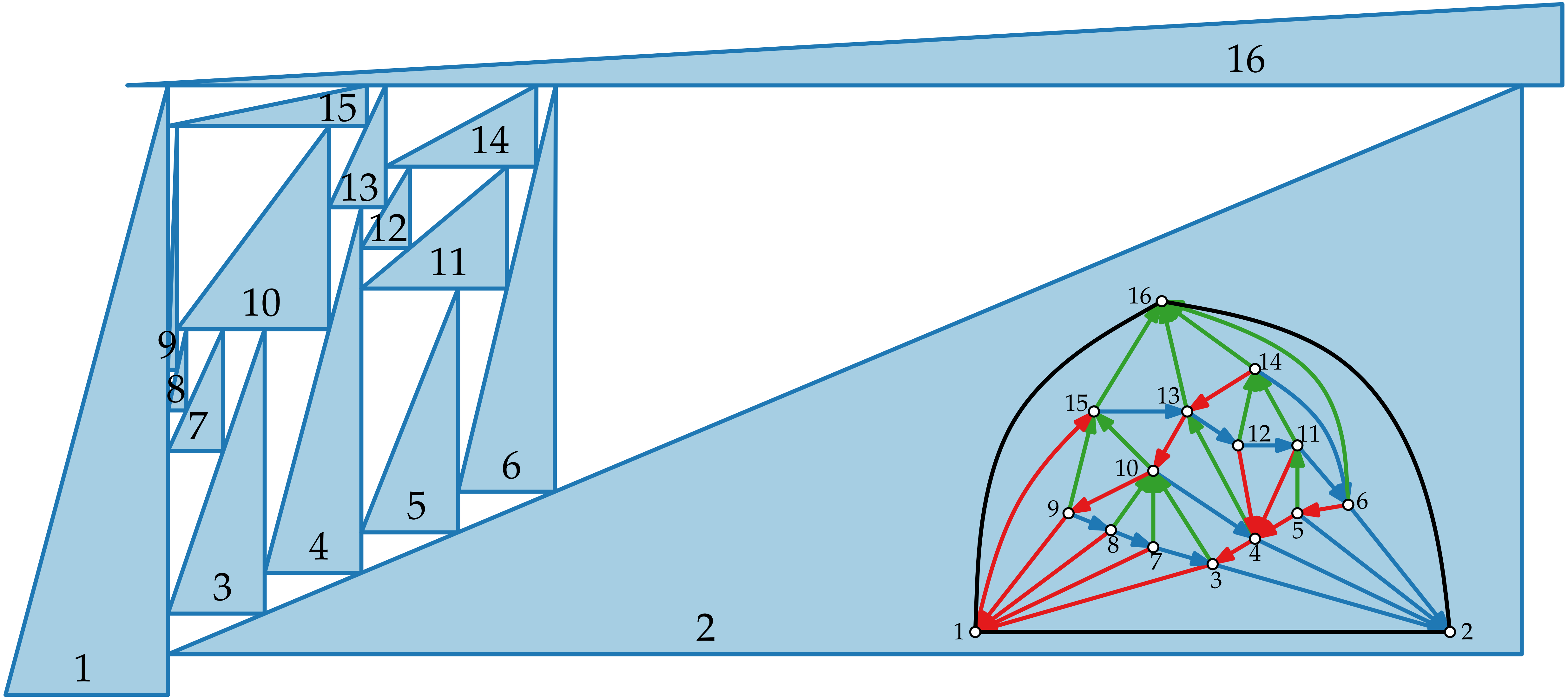
Triangle Contact Representation Example



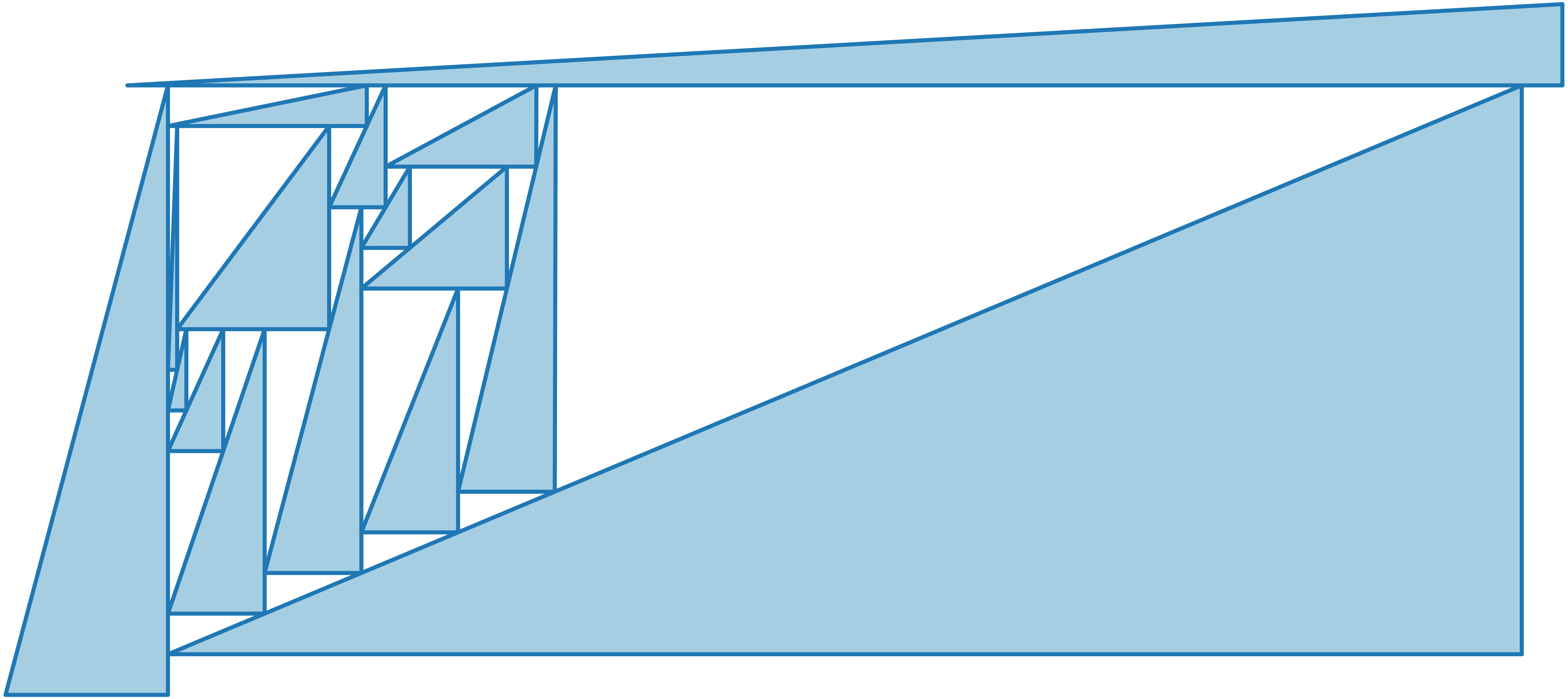
Triangle Contact Representation Example



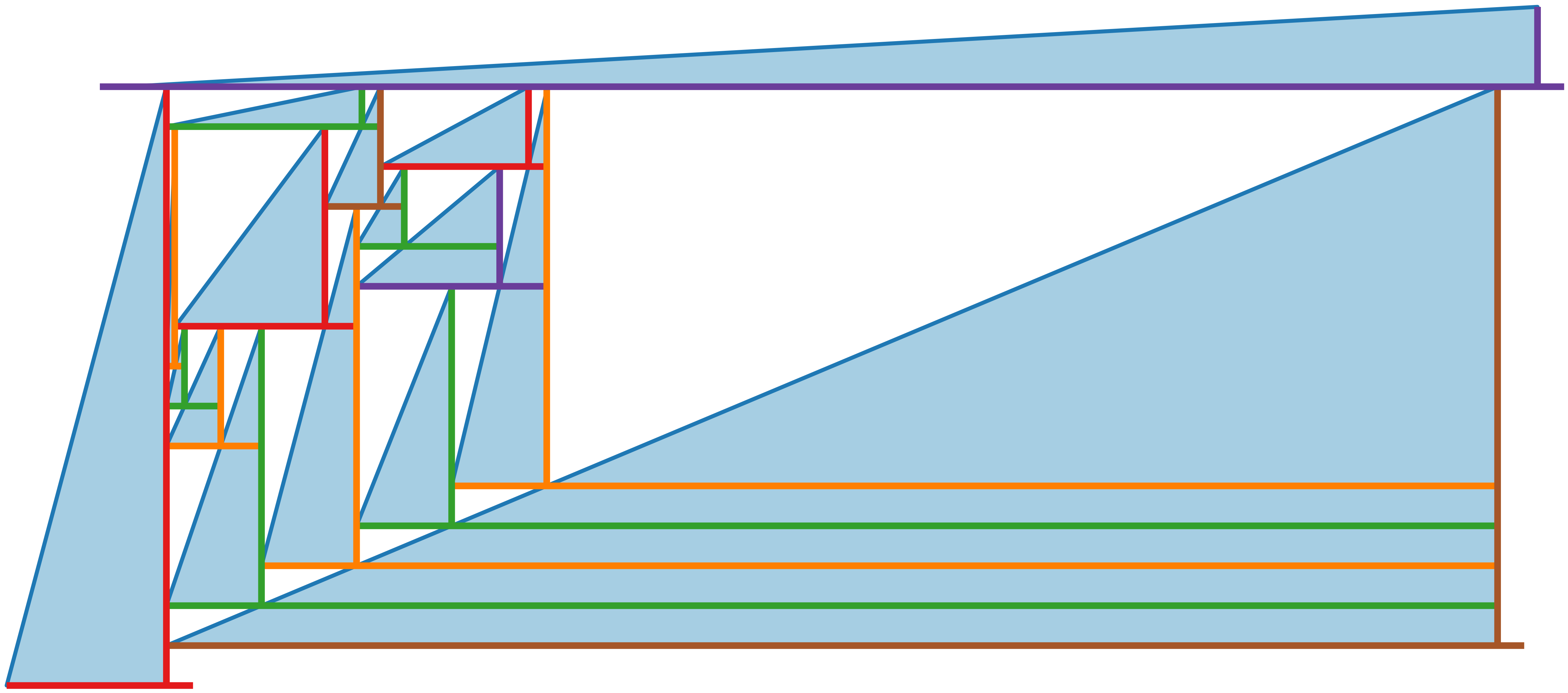
Triangle Contact Representation Example



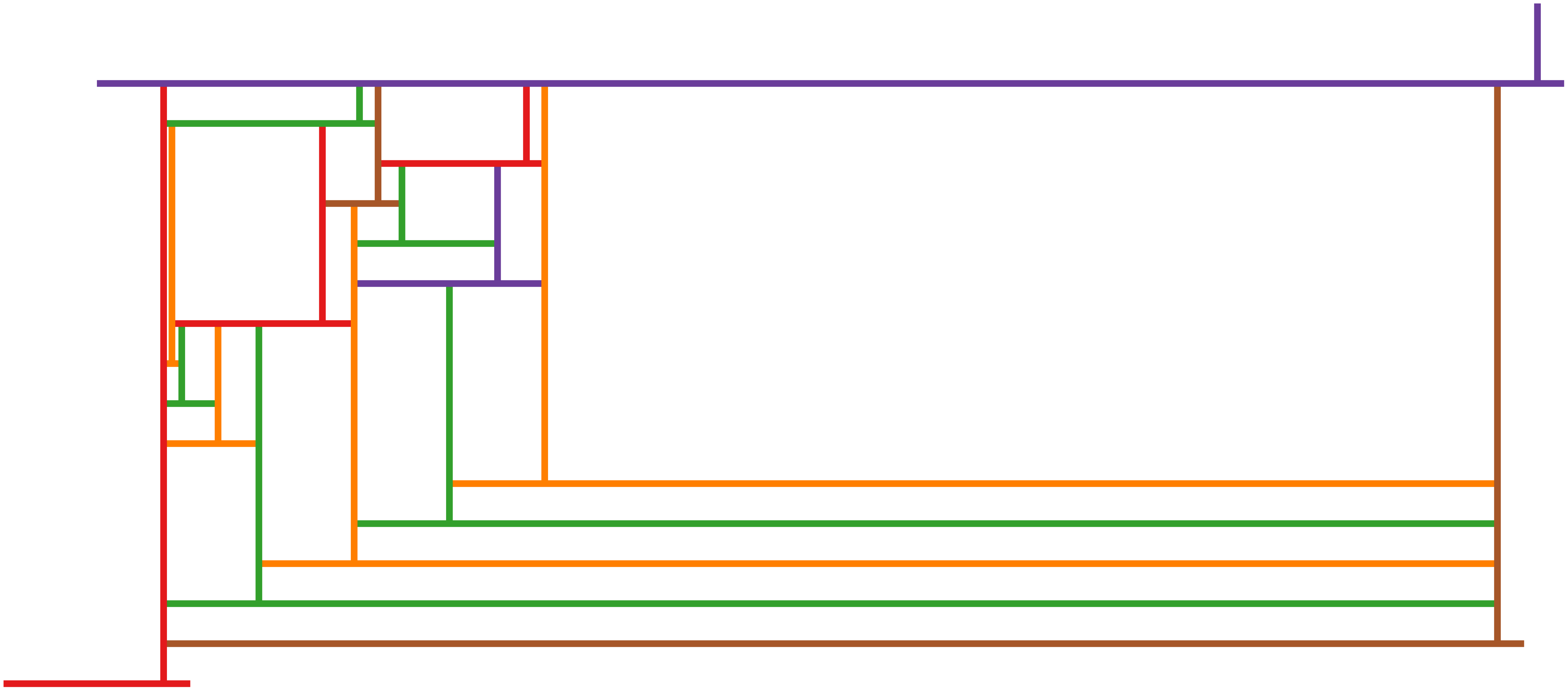
T-shape Contact Representation



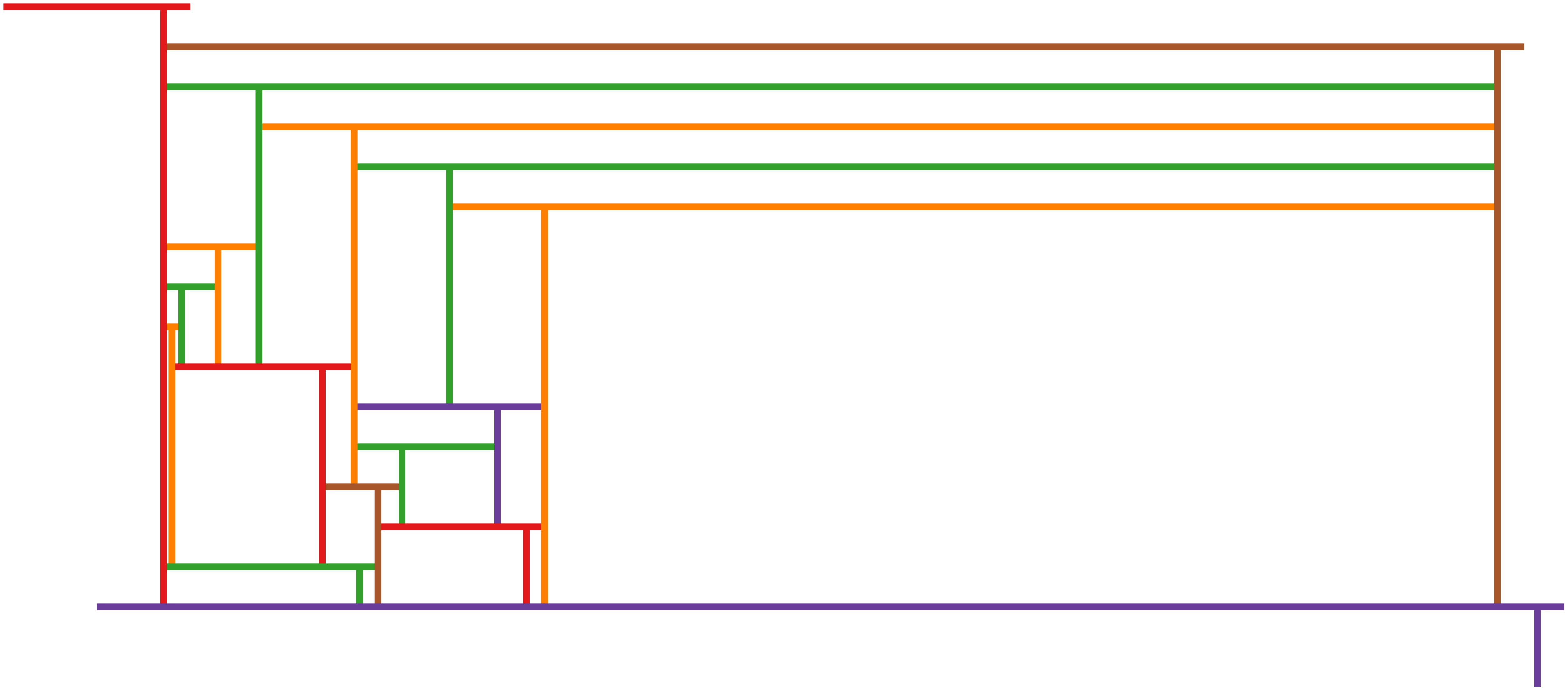
T-shape Contact Representation



T-shape Contact Representation



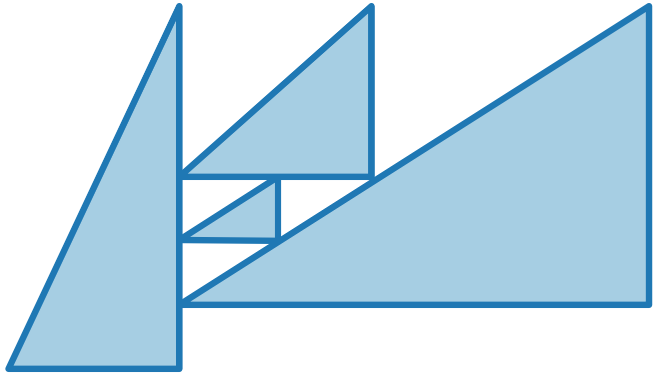
T-shape Contact Representation



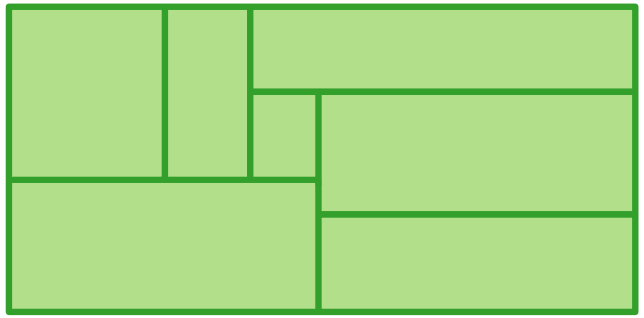
Universität Trier

Visualization of Graphs

Lecture 9:
Contact Representations of Planar Graphs: Triangle Contacts and Rectangular Duals



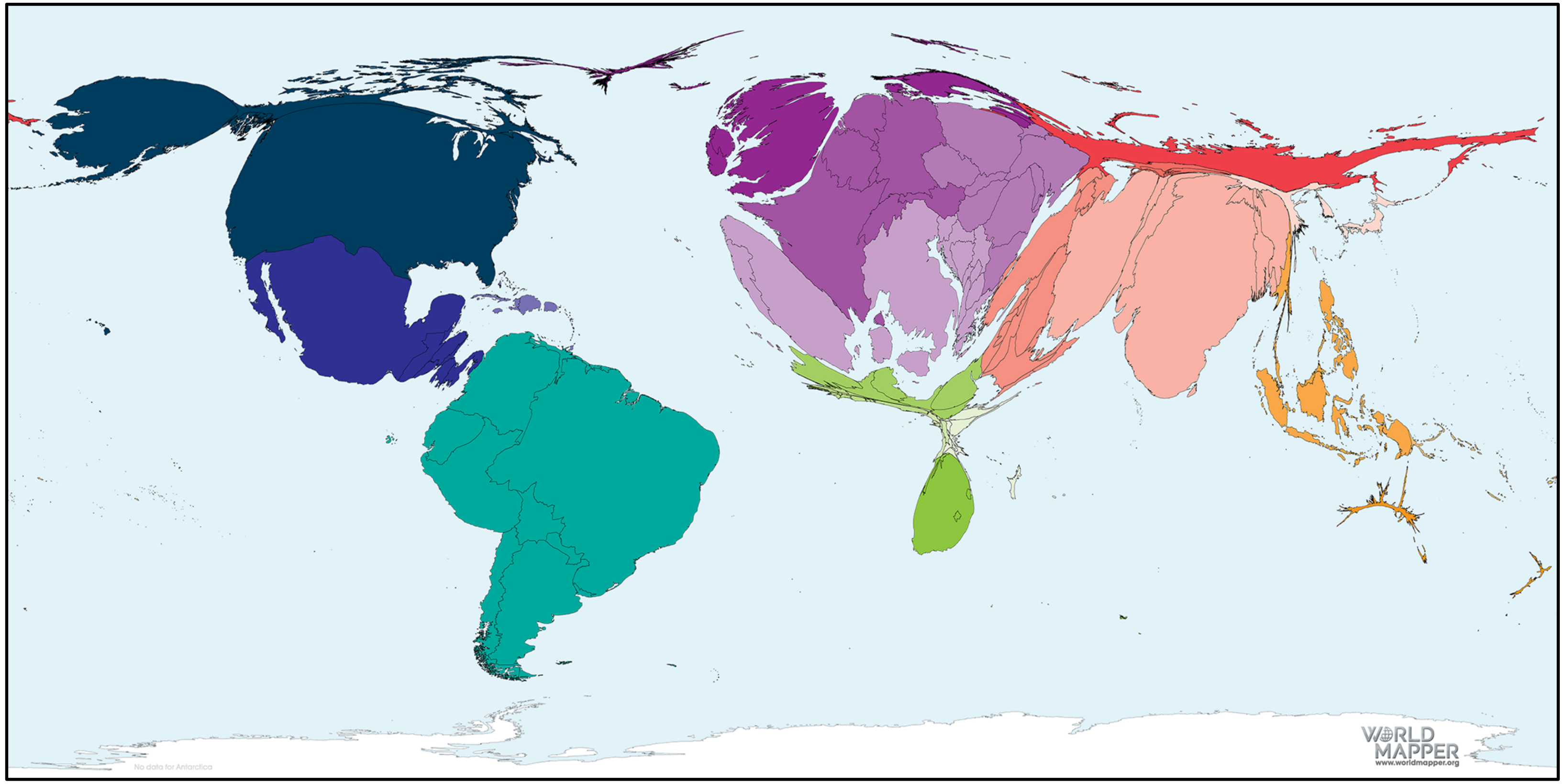
Part III:
Rectangular Duals



Philipp Kindermann

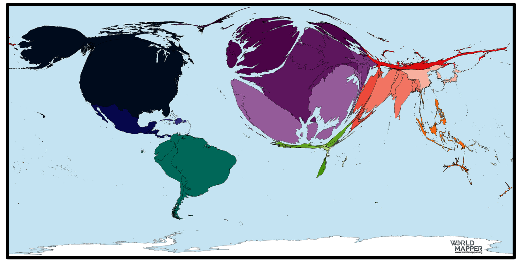
Cartograms

Cartograms



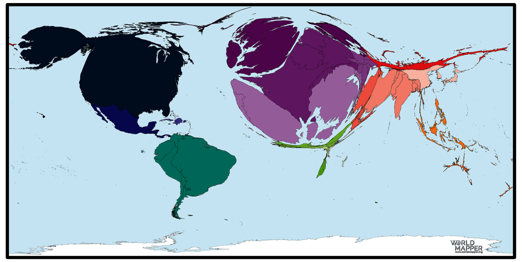
COVID19 reported deaths (January 1, 2021)

Cartograms

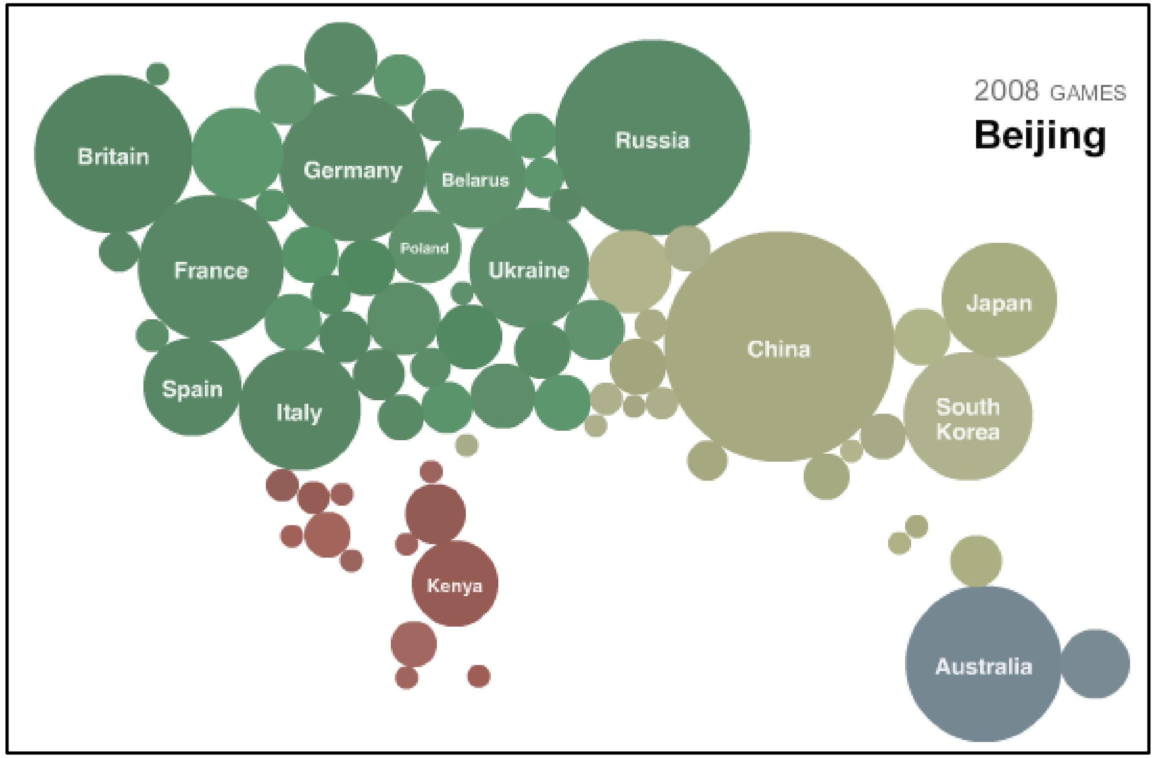


©worldmapper.org

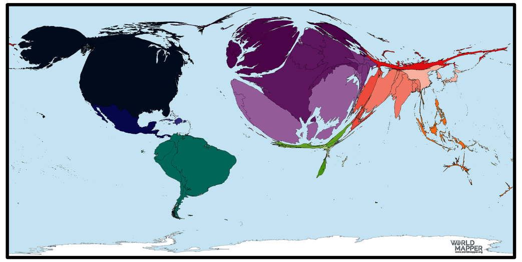
Cartograms



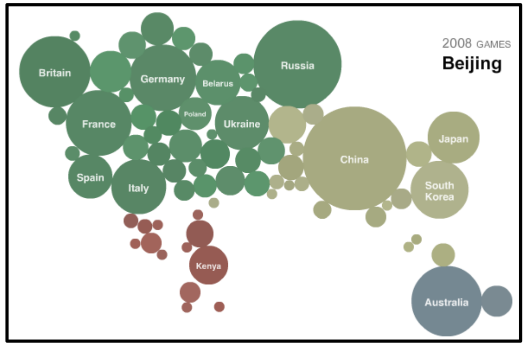
©worldmapper.org



Cartograms

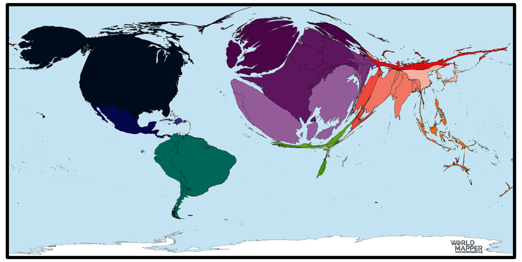


©worldmapper.org

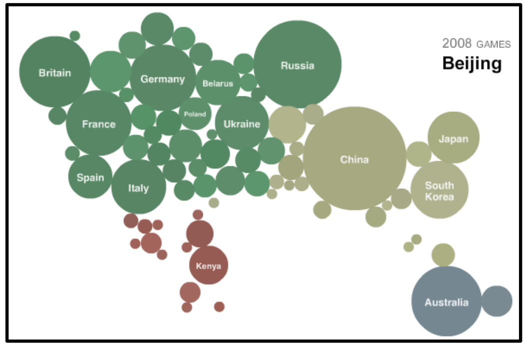


©New York Times

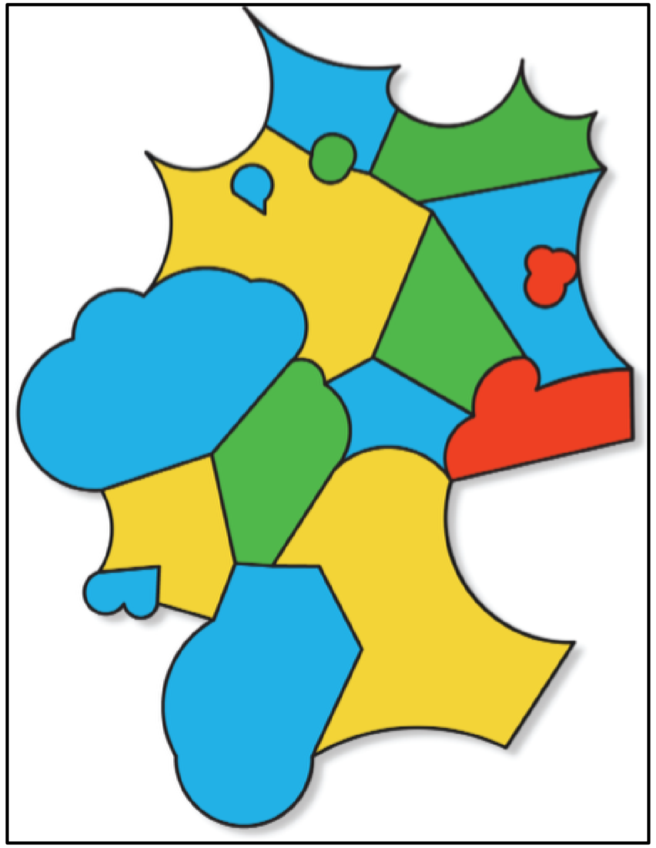
Cartograms



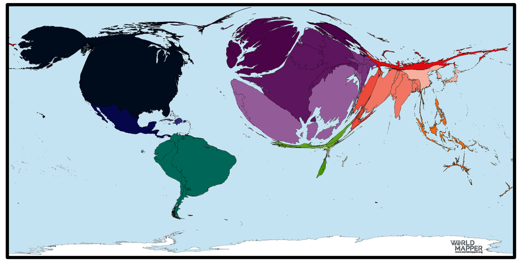
©worldmapper.org



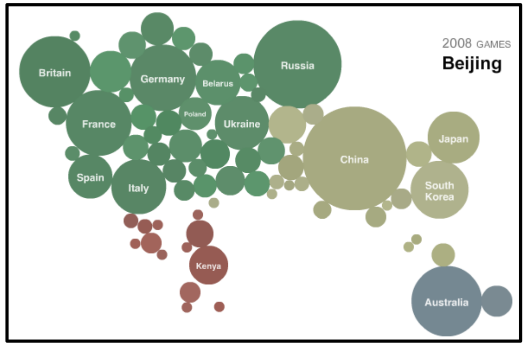
©New York Times



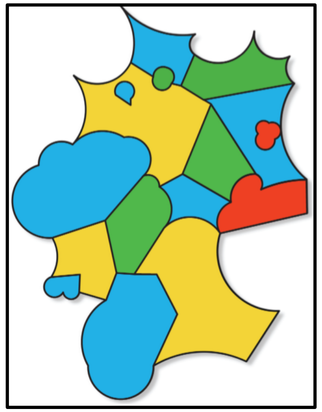
Cartograms



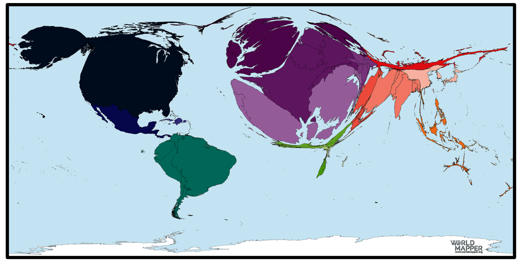
©worldmapper.org



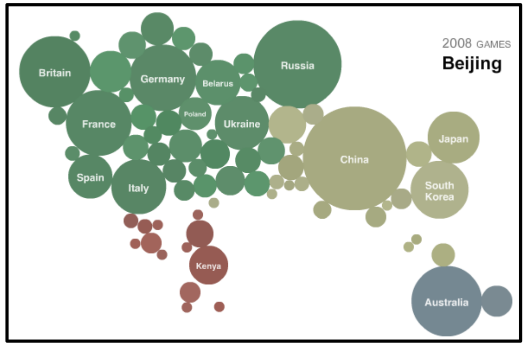
©New York Times



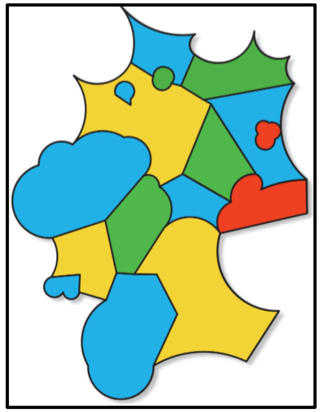
Cartograms

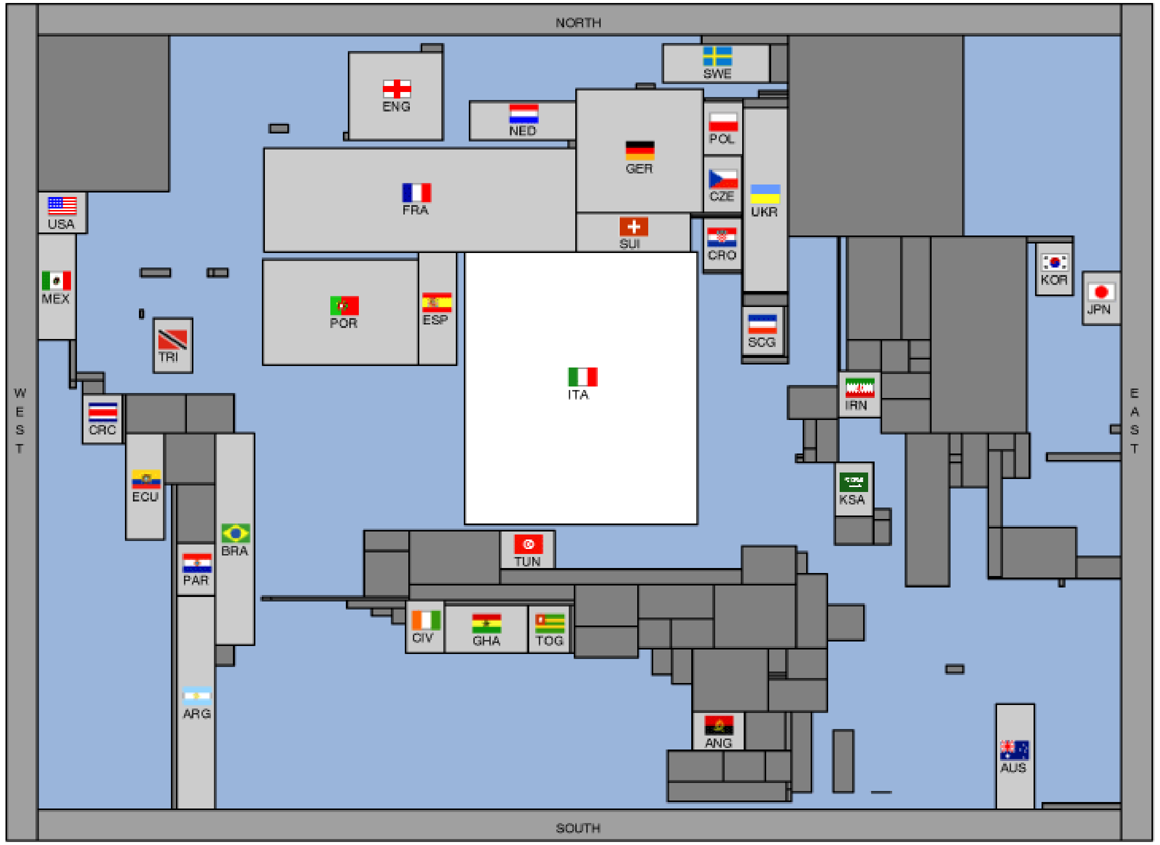


©worldmapper.org

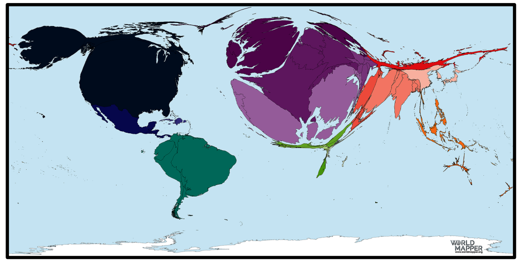


©New York Times

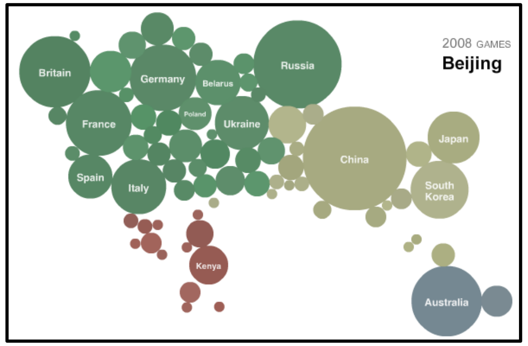




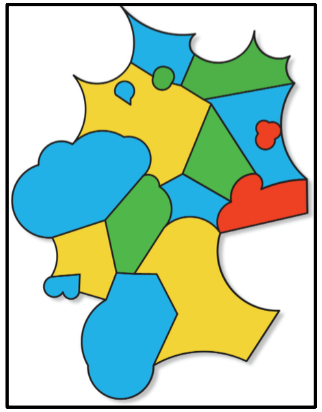
Cartograms

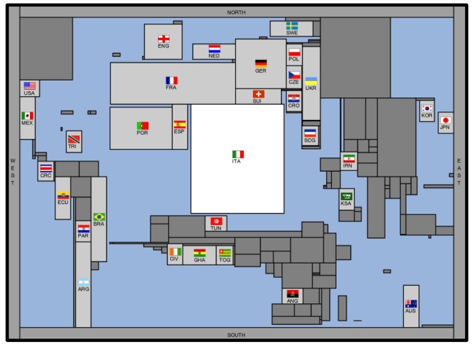


©worldmapper.org



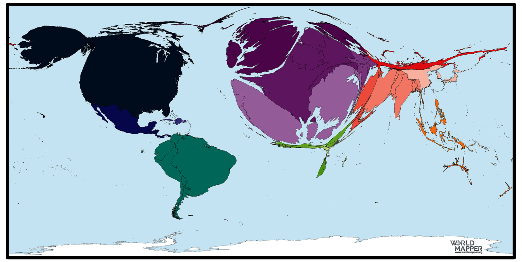
©New York Times



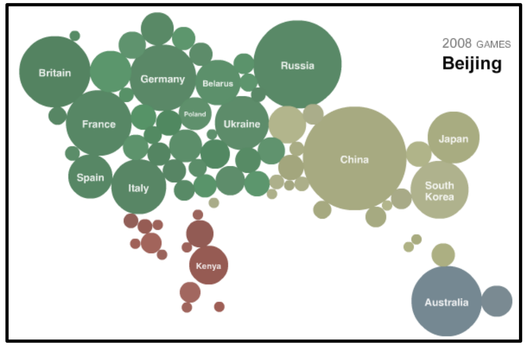


©Bettina Speckmann

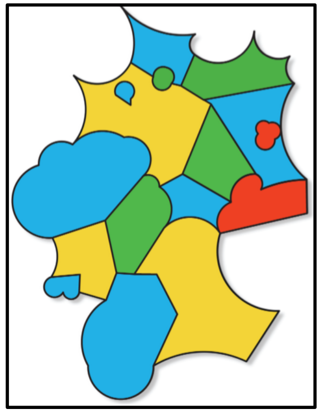
Cartograms

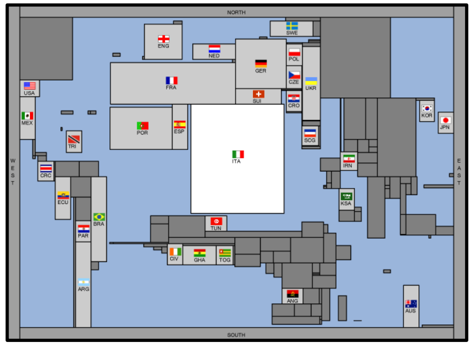


©worldmapper.org

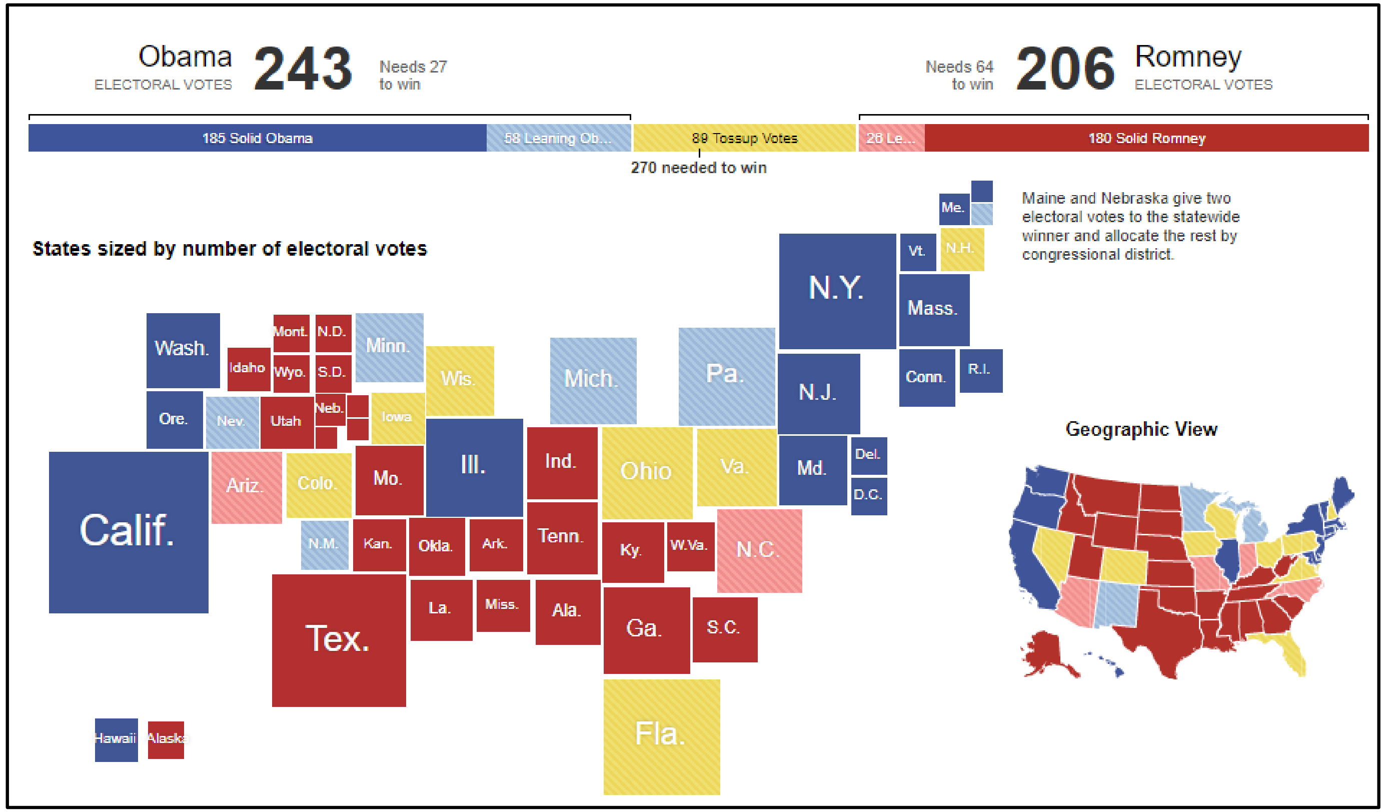


©New York Times

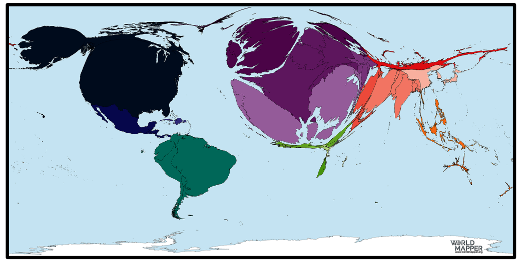




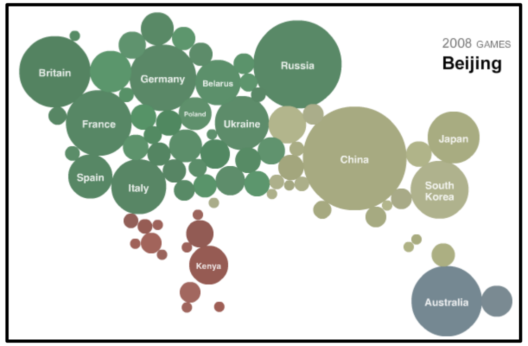
©Bettina Speckmann



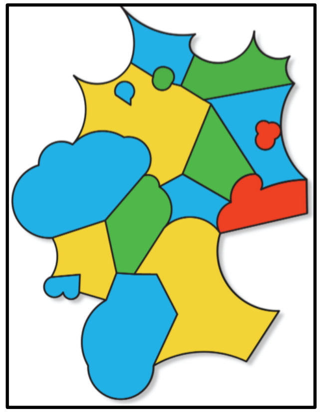
Cartograms

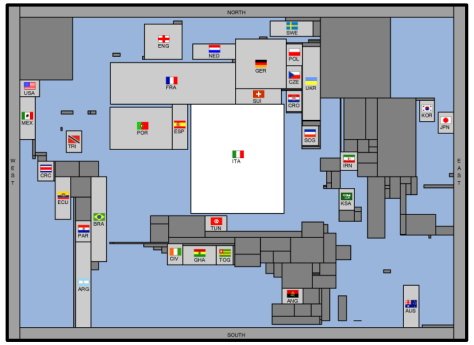


©worldmapper.org

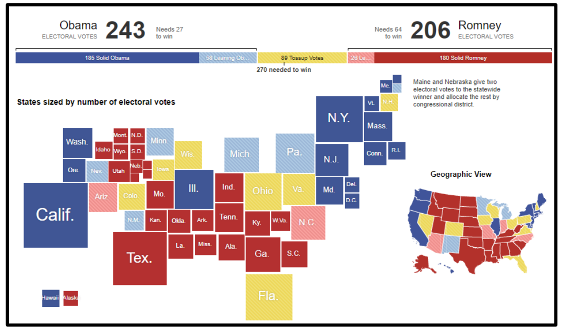


©New York Times



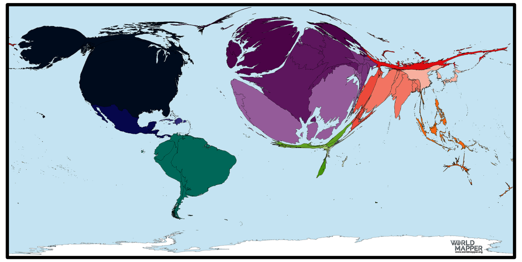


©Bettina Speckmann

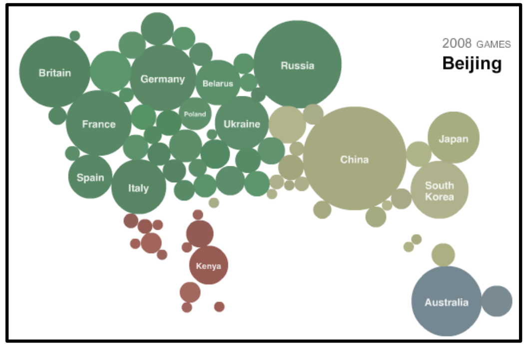


©New York Times

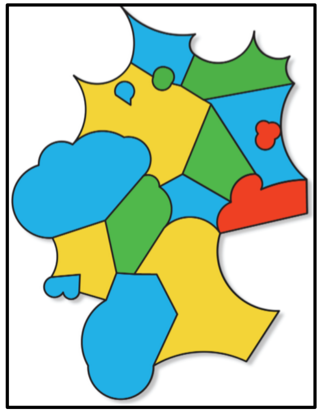
Cartograms

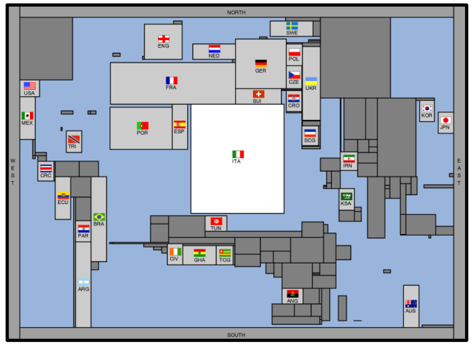


©worldmapper.org

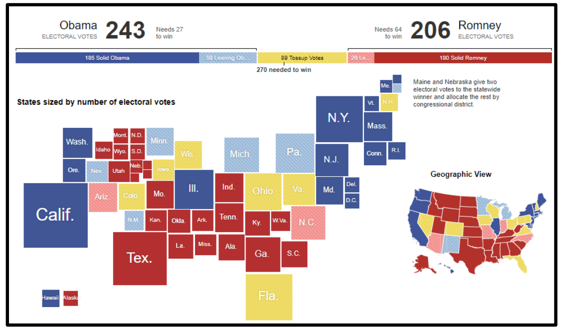


©New York Times

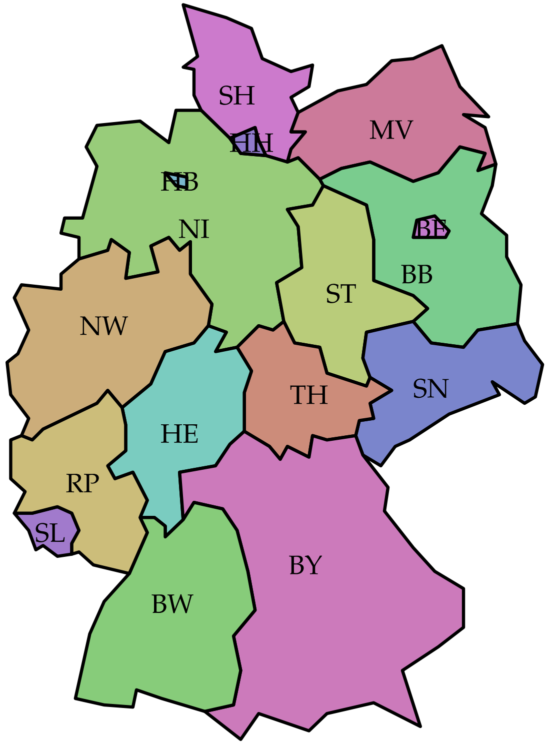




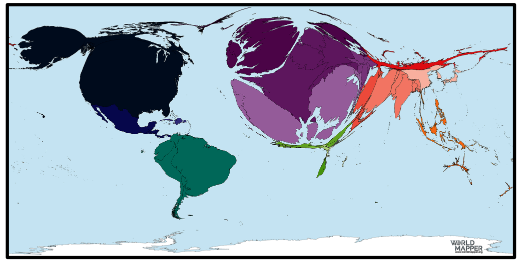
©Bettina Speckmann



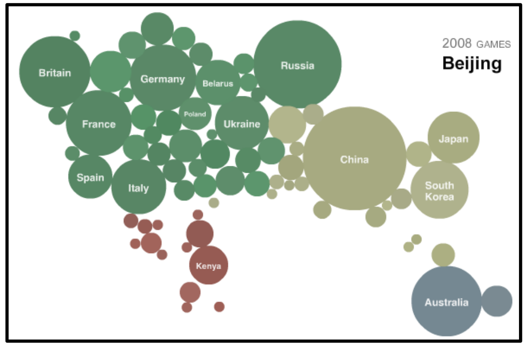
©New York Times



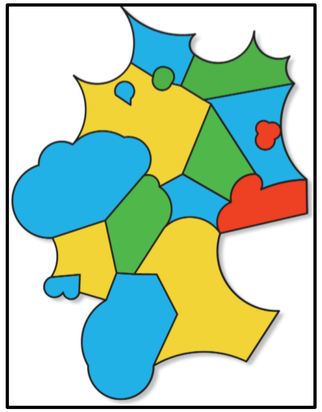
Cartograms

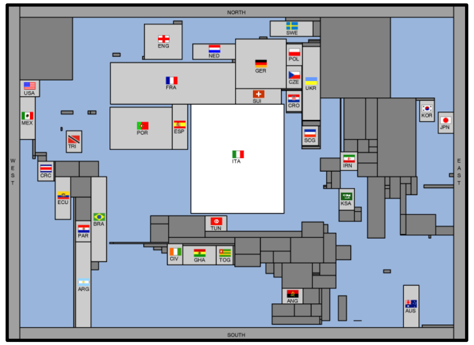


©worldmapper.org

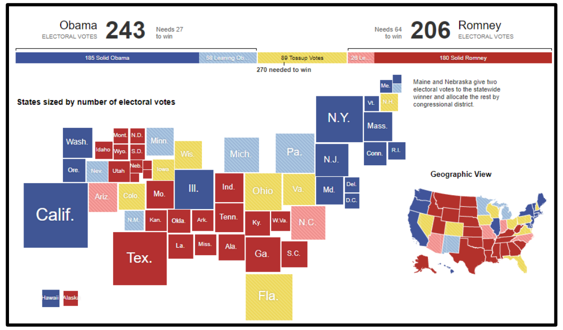


©New York Times

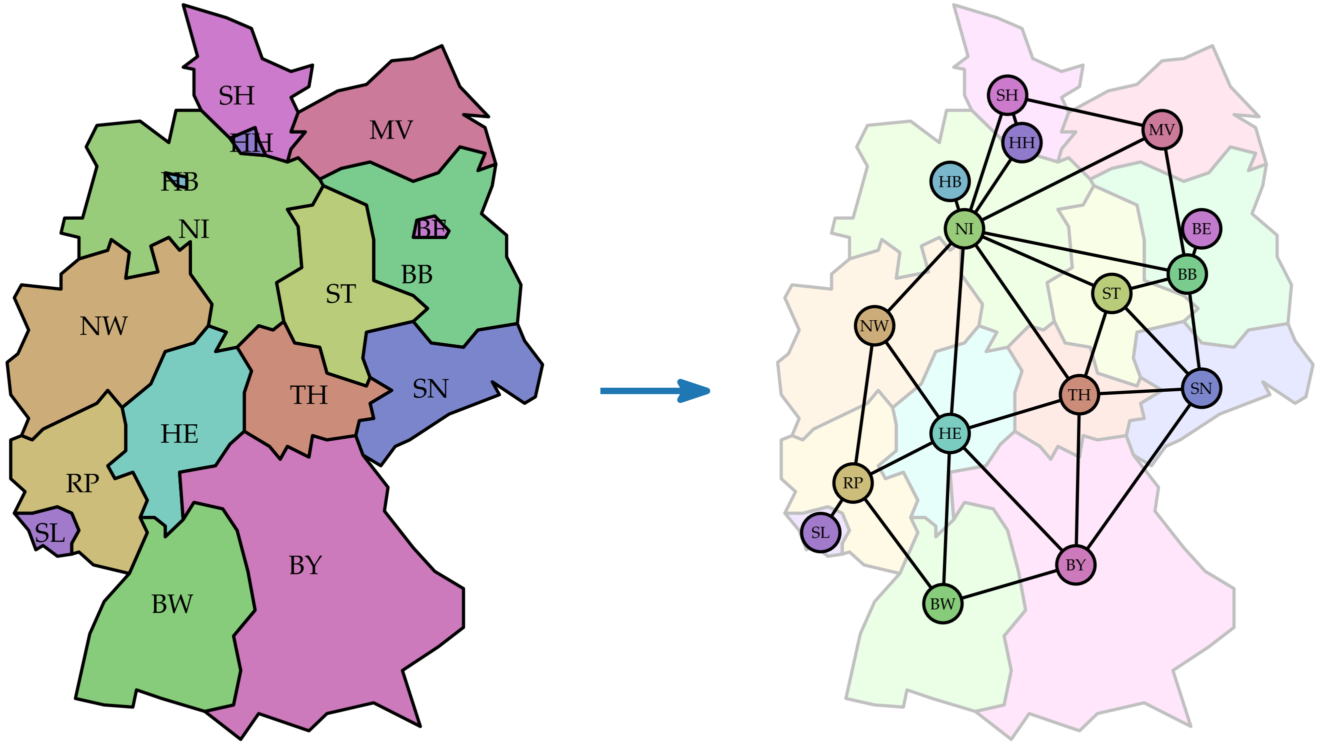




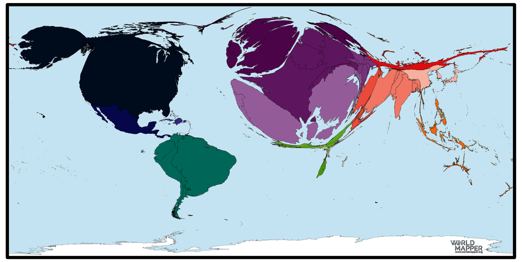
©Bettina Speckmann



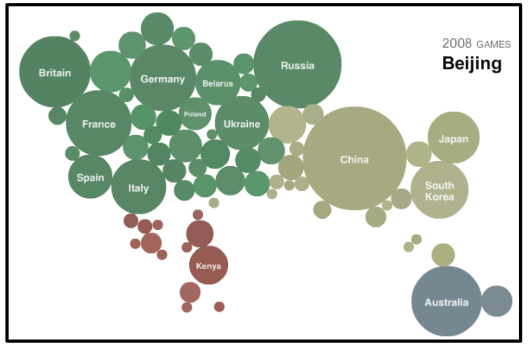
©New York Times



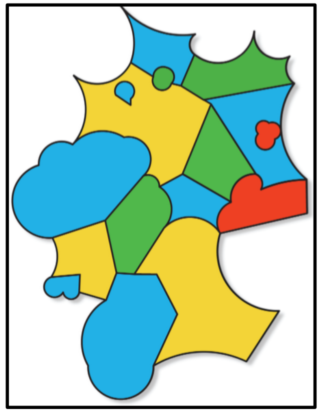
Cartograms

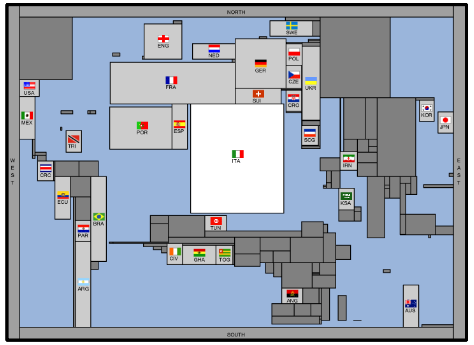


©worldmapper.org

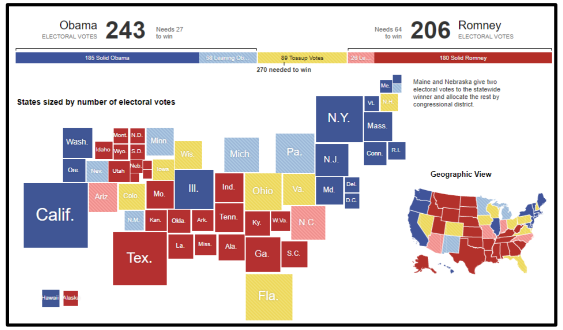


©New York Times

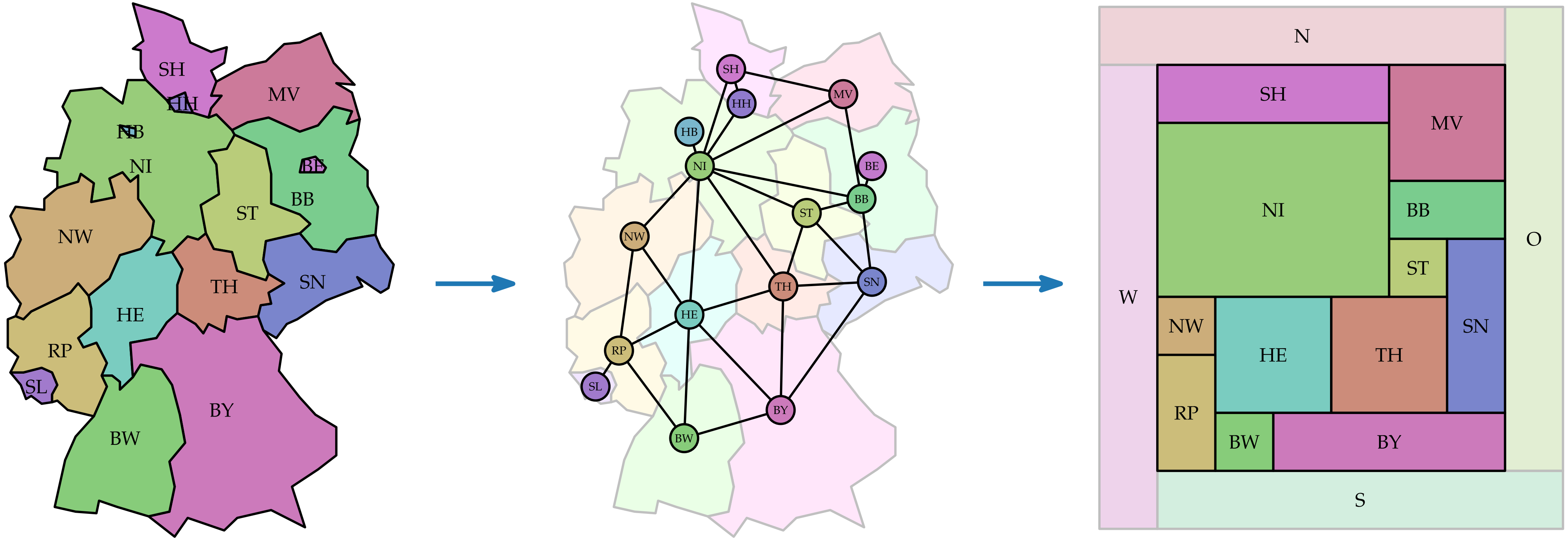




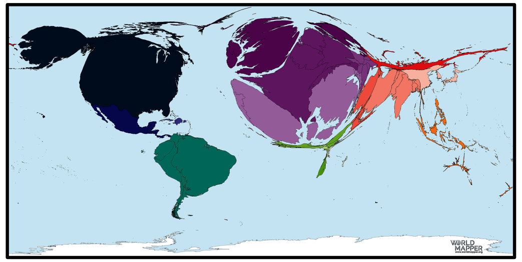
©Bettina Speckmann



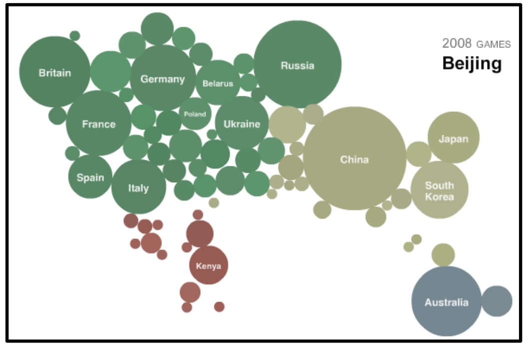
©New York Times



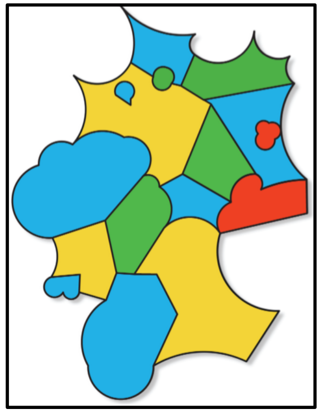
Cartograms

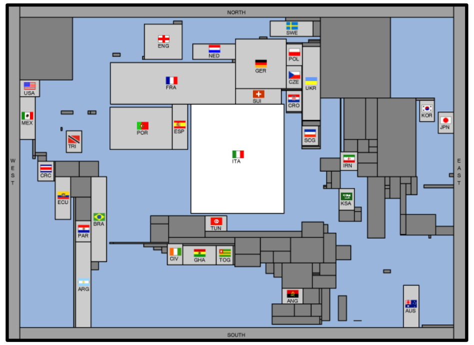


©worldmapper.org

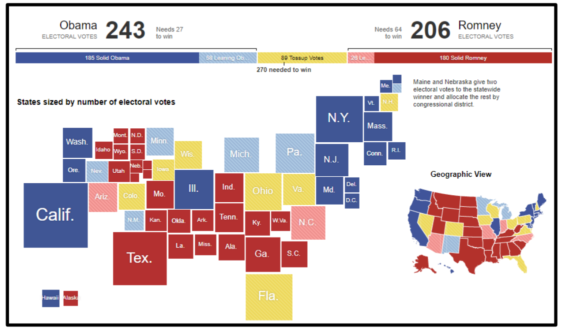


©New York Times

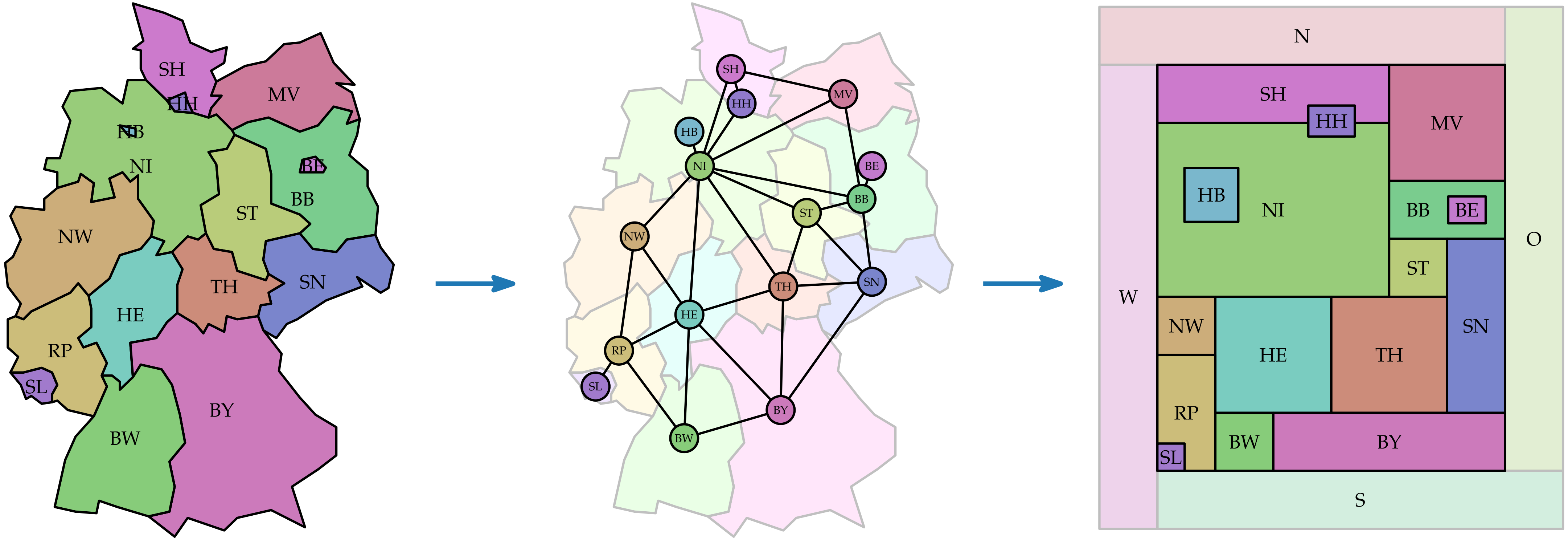




©Bettina Speckmann

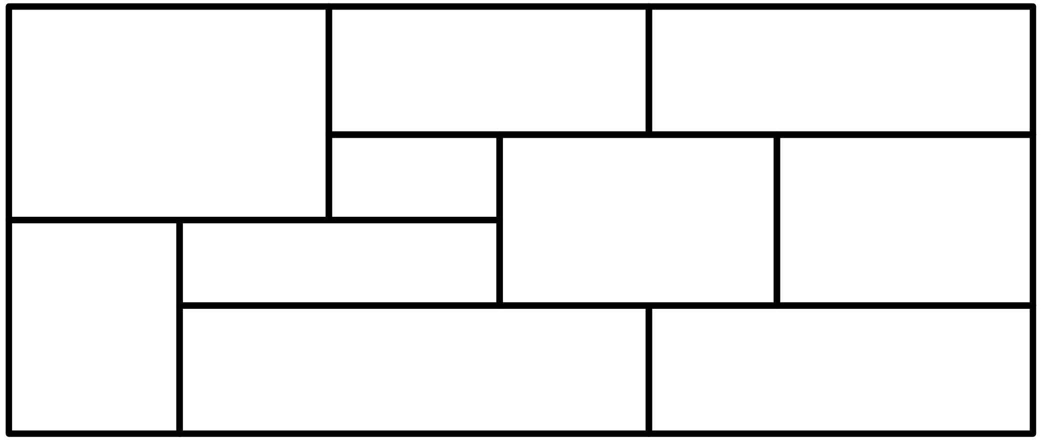


©New York Times

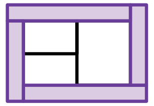


Rectangular Dual

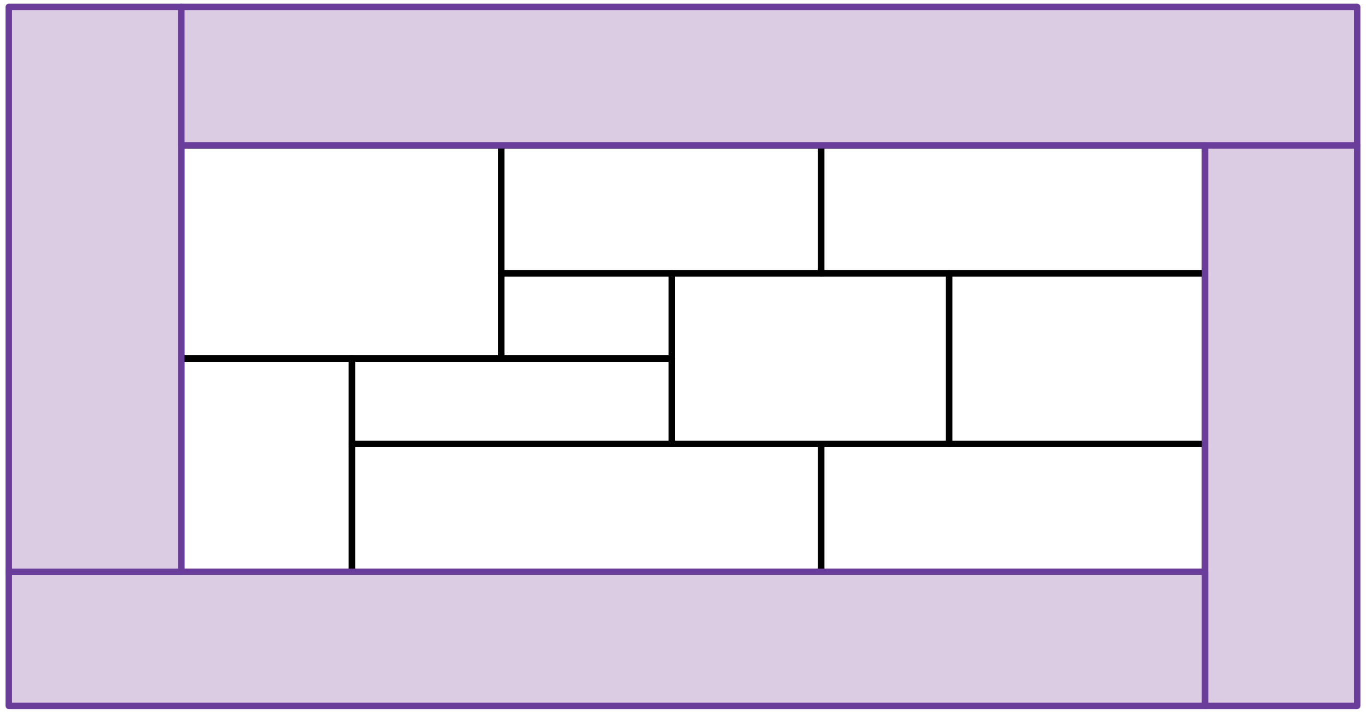
Rectangular Dual



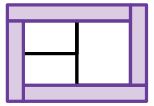
Rectangular Dual


RD

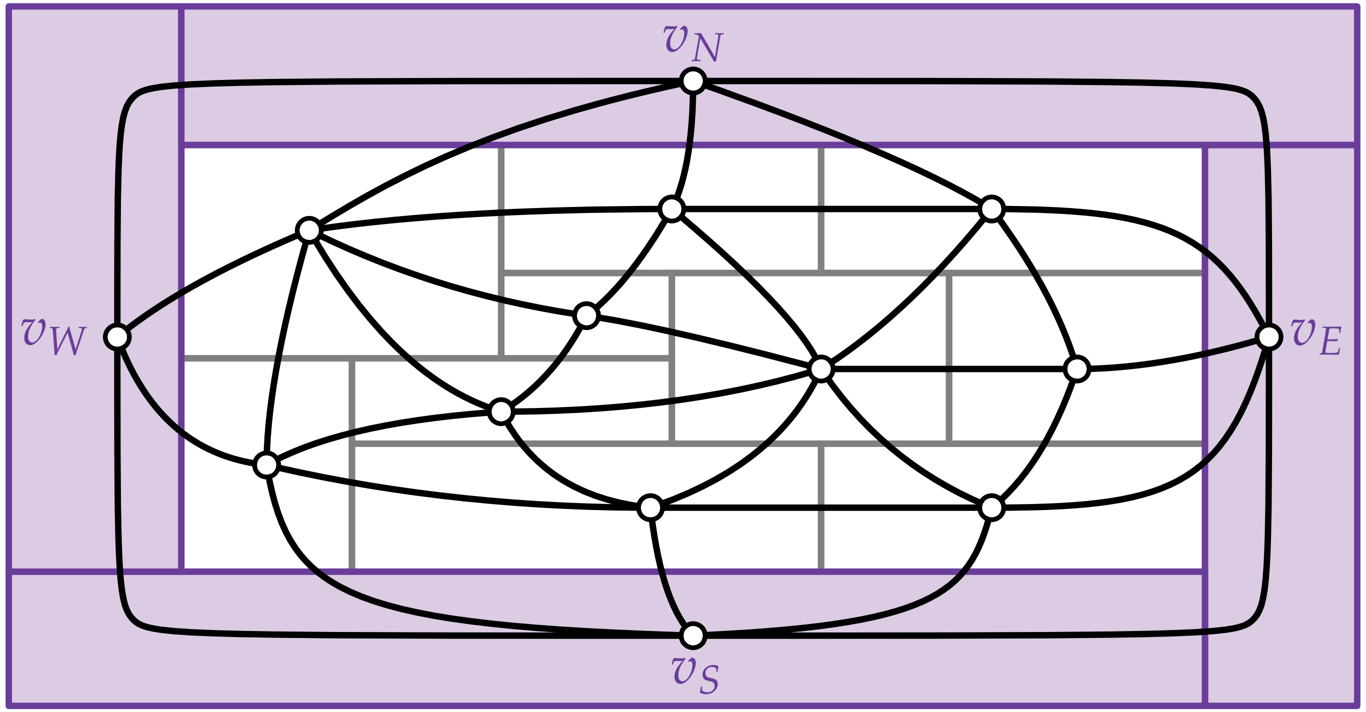
Rectangular Dual $\mathcal{R}$



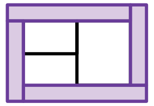
Rectangular Dual



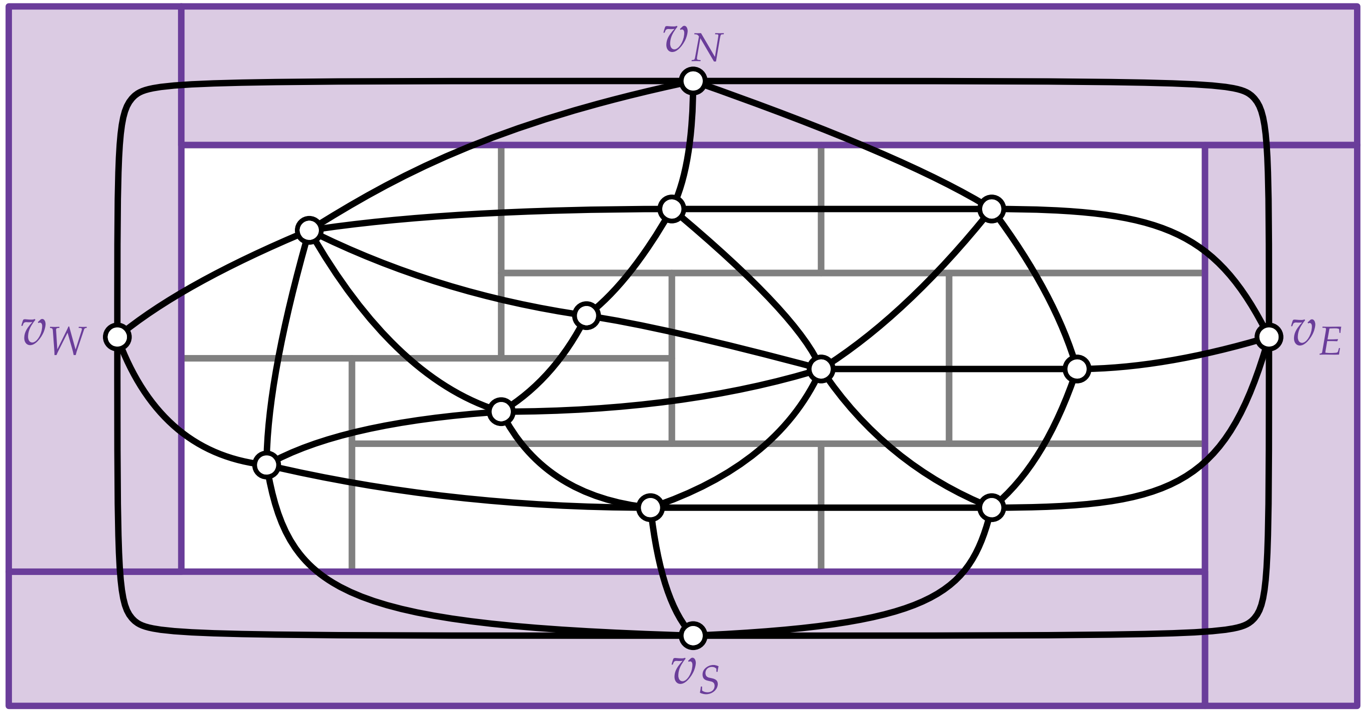
Rectangular Dual $\mathcal{R}$



Rectangular Dual

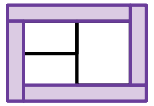


Rectangular Dual $\mathcal{R}$

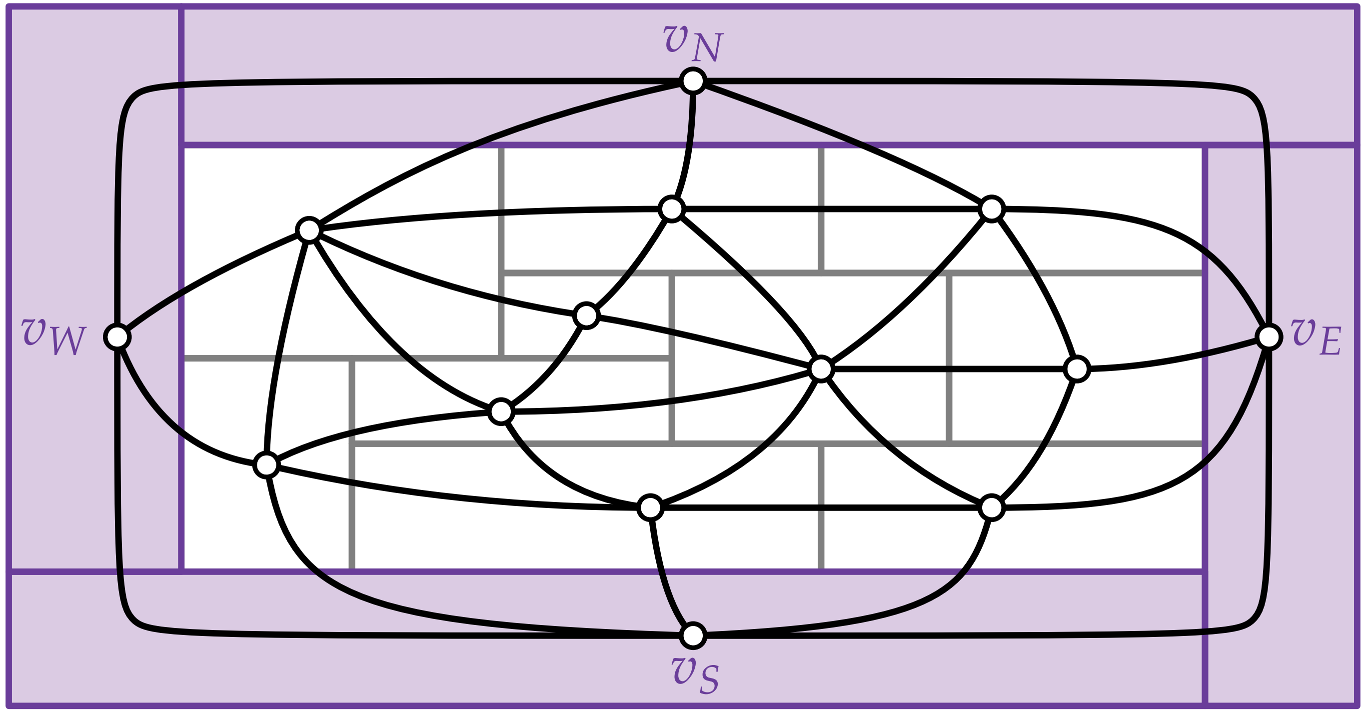


A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

Rectangular Dual

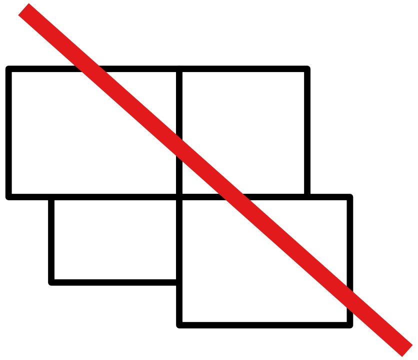

RD

Rectangular Dual $\mathcal{R}$

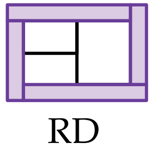


A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

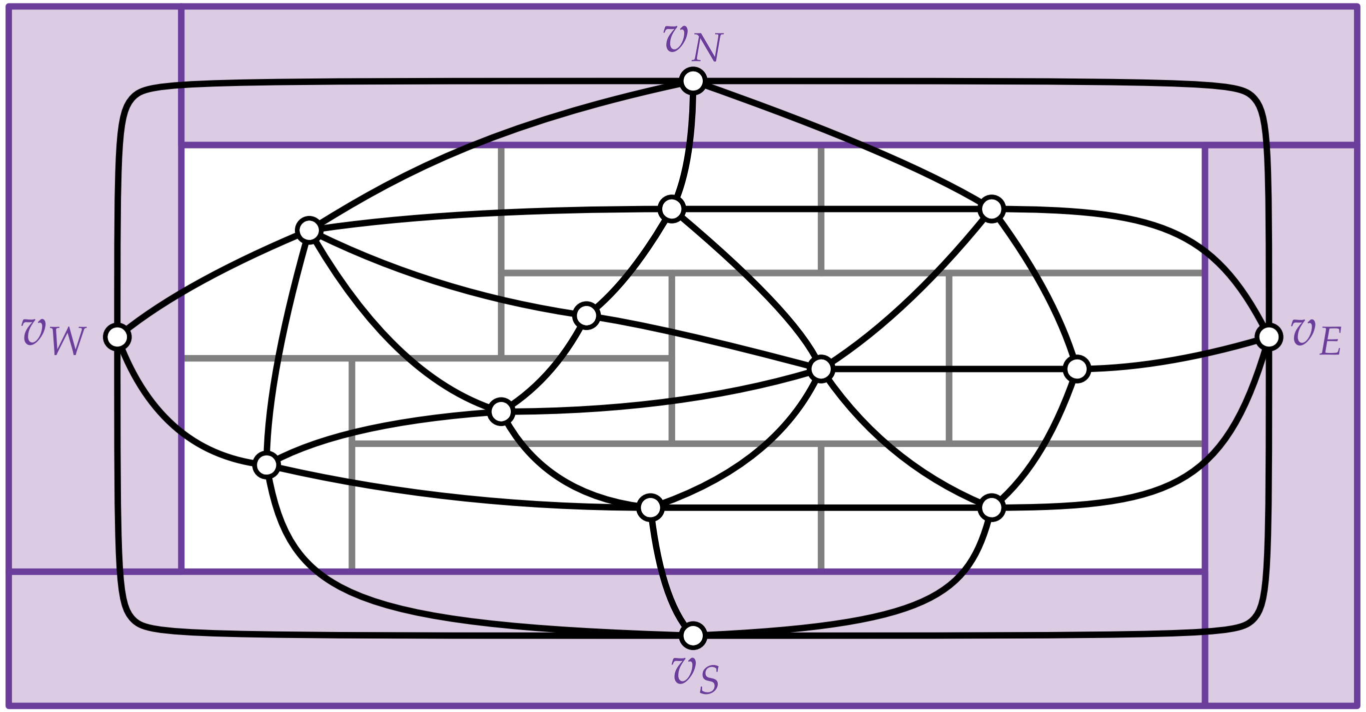
- no four rectangles share a point



Rectangular Dual

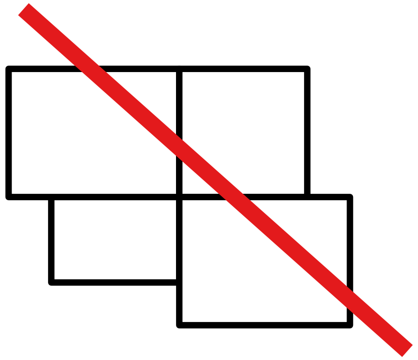


Rectangular Dual $\mathcal{R}$

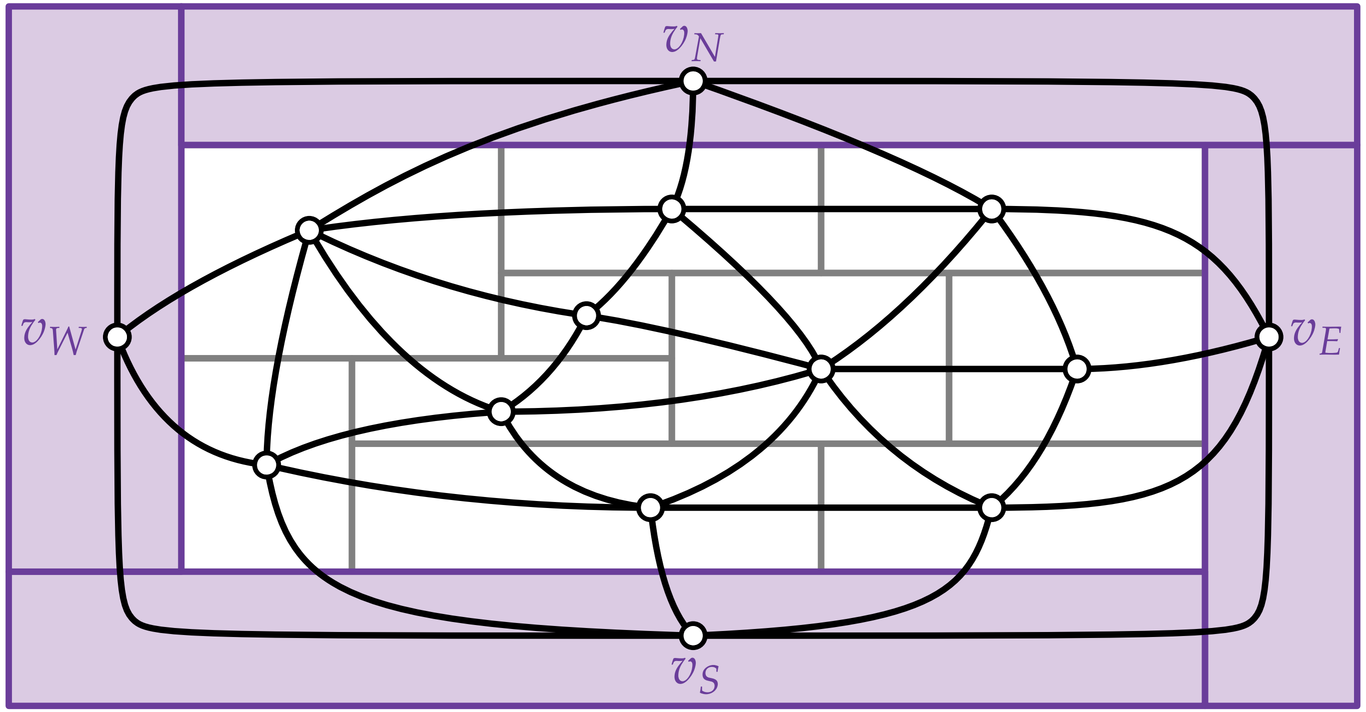


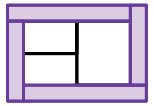
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle



Rectangular Dual



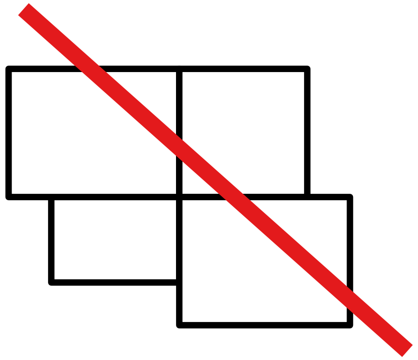


Rectangular Dual $\mathcal{R}$

RD

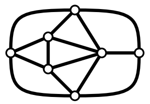
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

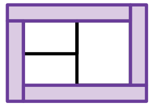


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

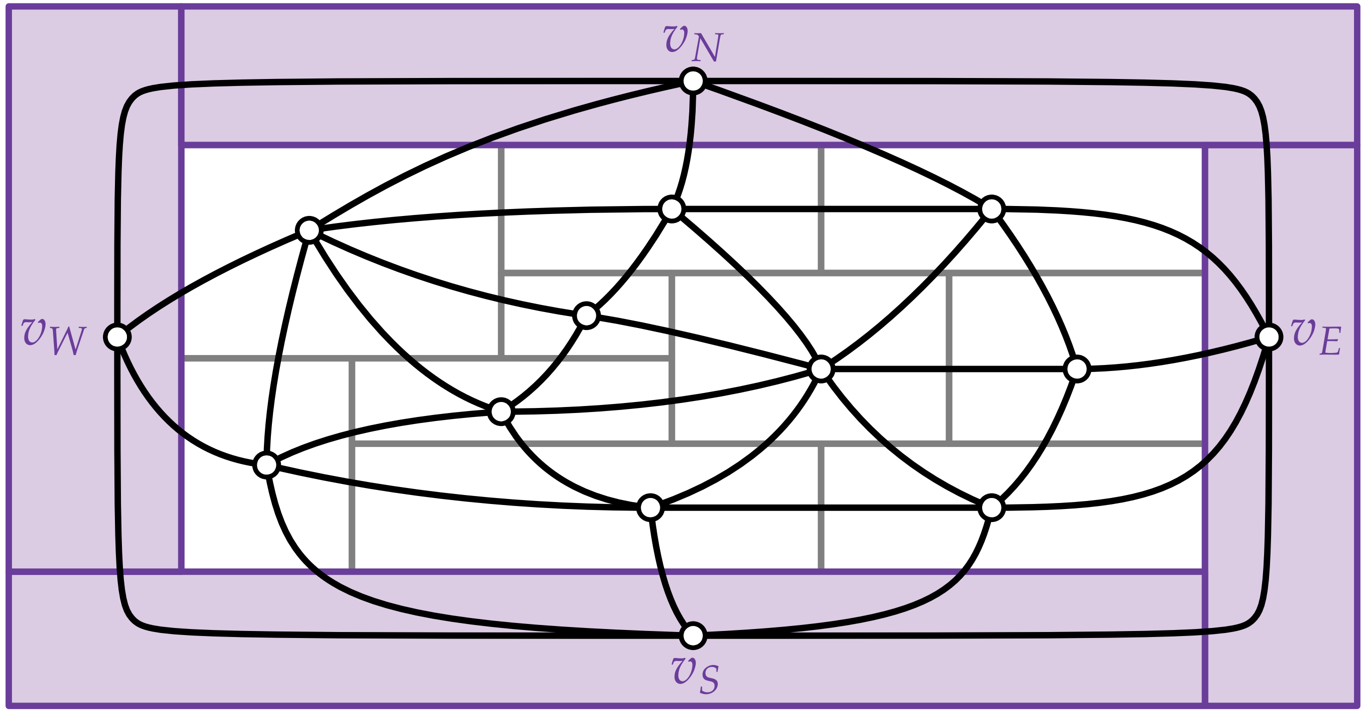
Rectangular Dual



Properly Triangulated Planar Graph G

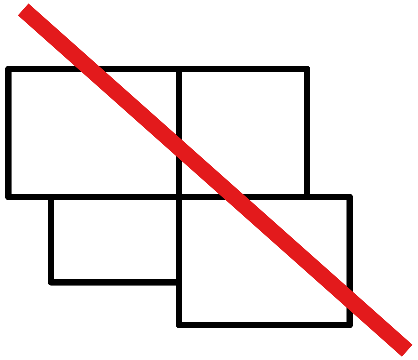


Rectangular Dual $\mathcal{R}$



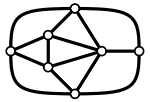
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle



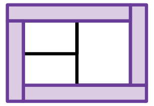
Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

Rectangular Dual



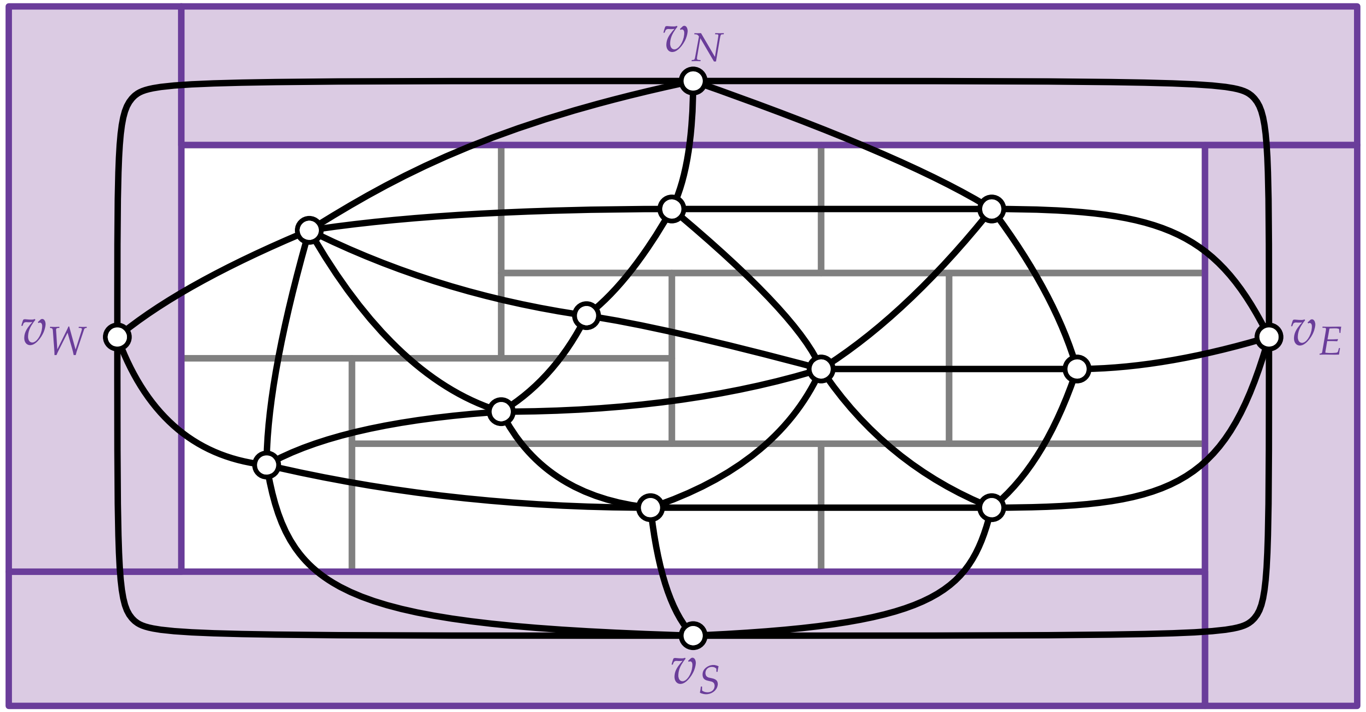
PTP

Properly Triangulated Planar Graph G



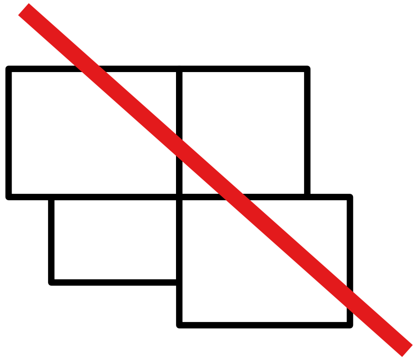
RD

Rectangular Dual $\mathcal{R}$



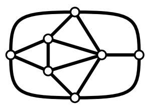
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle



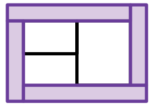
Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

Rectangular Dual



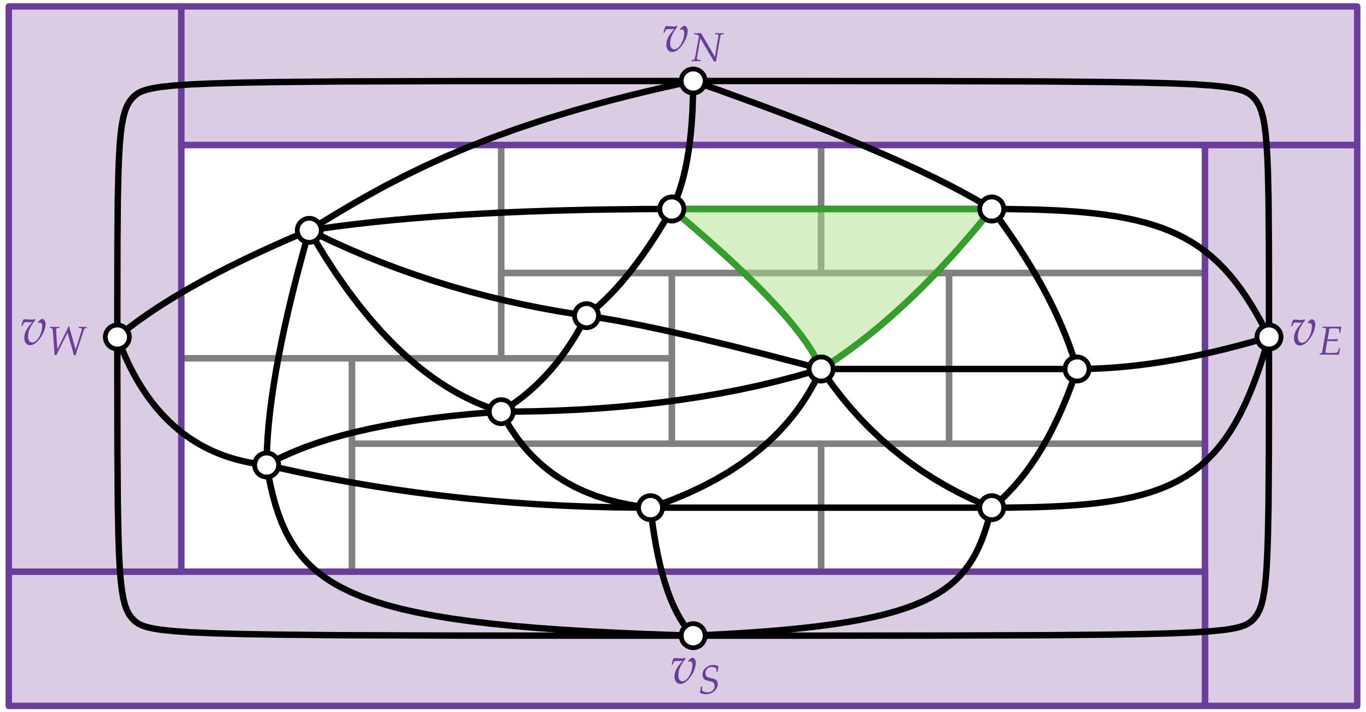
PTP

Properly Triangulated Planar Graph G



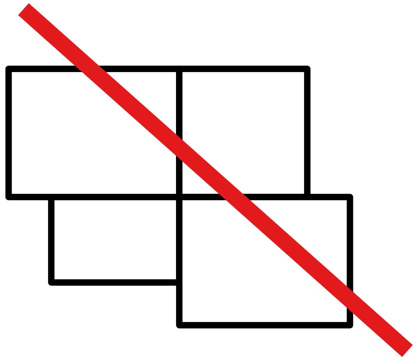
RD

Rectangular Dual $\mathcal{R}$



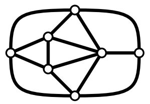
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

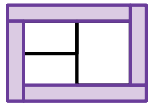


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

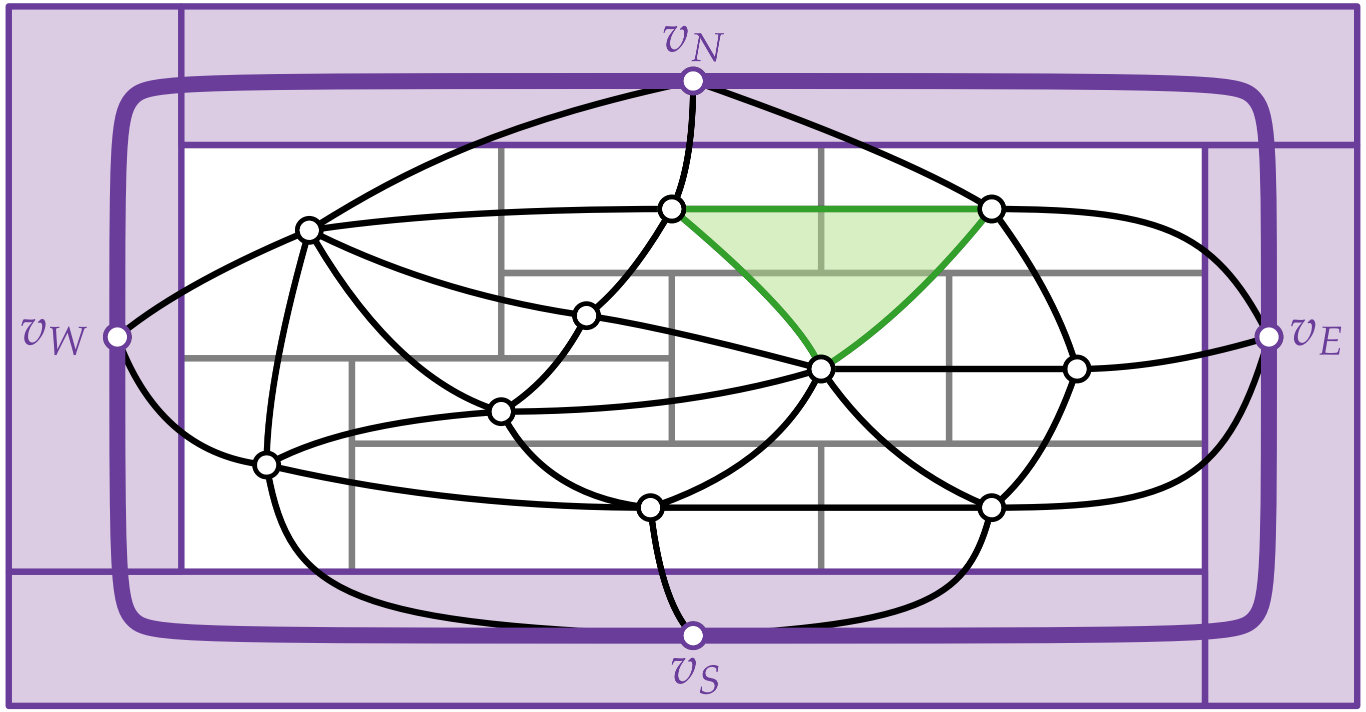
Rectangular Dual



Properly Triangulated Planar Graph G

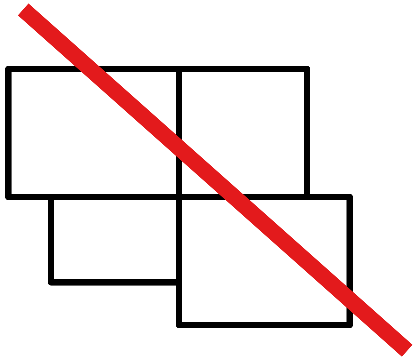


Rectangular Dual $\mathcal{R}$



A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

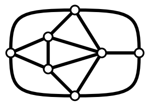
- no four rectangles share a point, and
- the union of all rectangles is a rectangle



Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

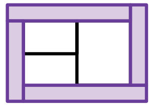
Rectangular Dual

Exactly 4 vertices on outer face



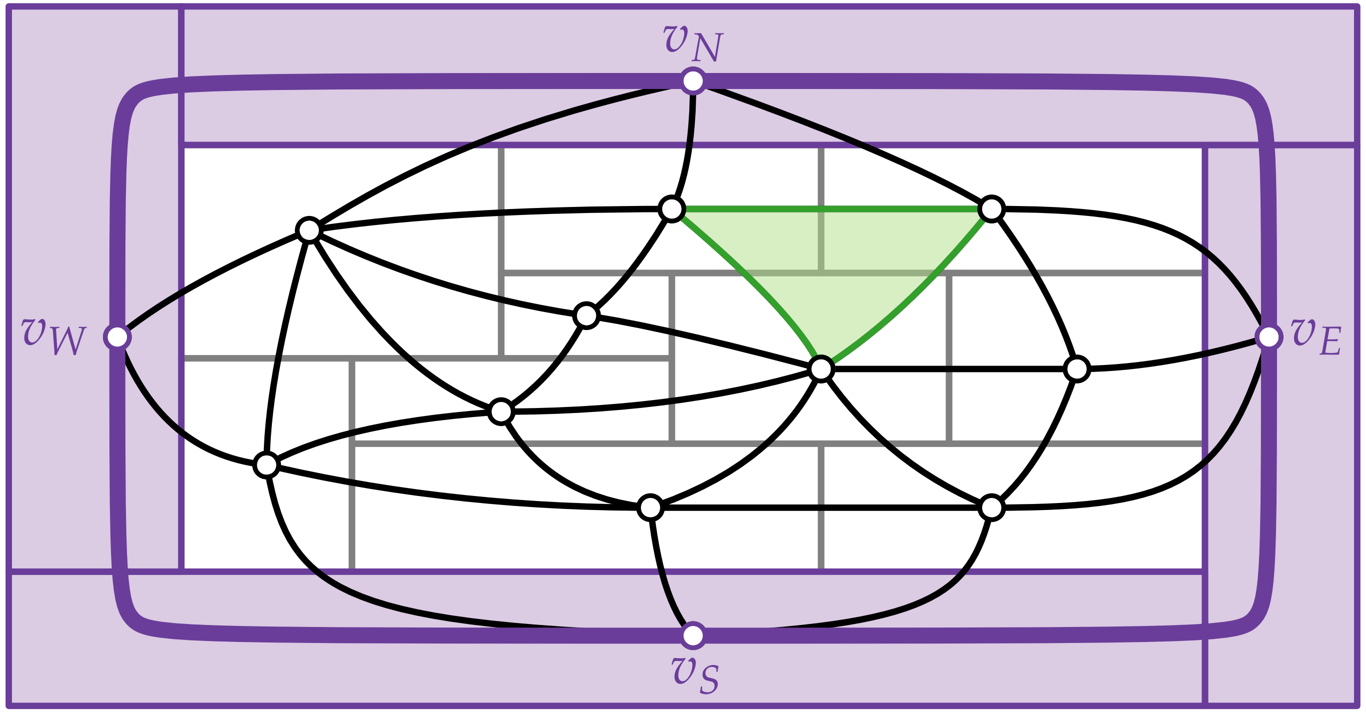
PTP

Properly Triangulated Planar Graph G



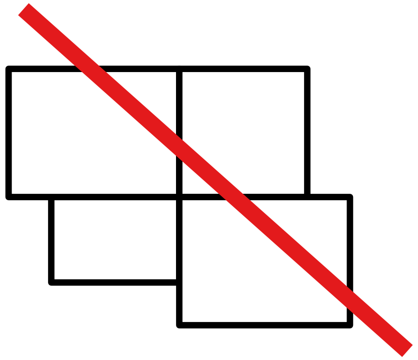
RD

Rectangular Dual $\mathcal{R}$



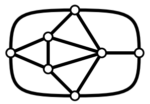
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

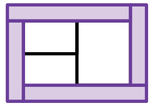


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

Rectangular Dual

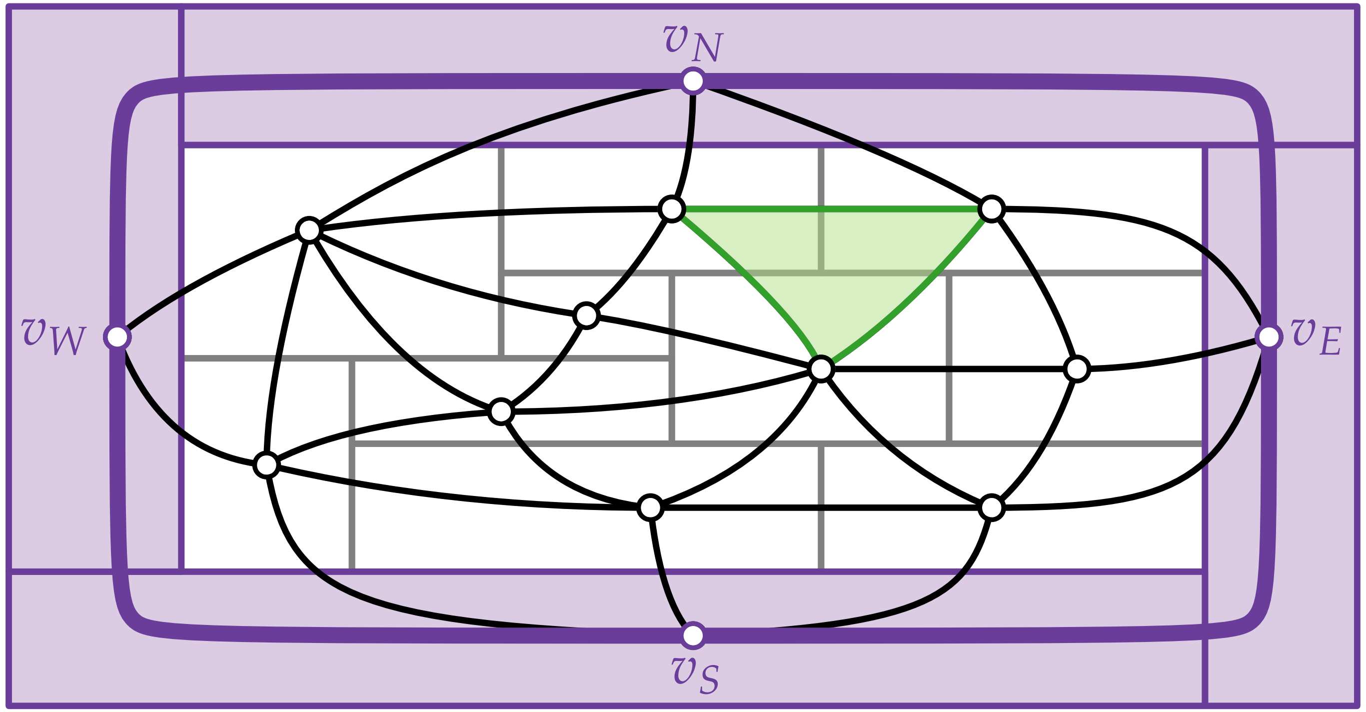


Properly Triangulated Planar Graph G



Rectangular Dual $\mathcal{R}$

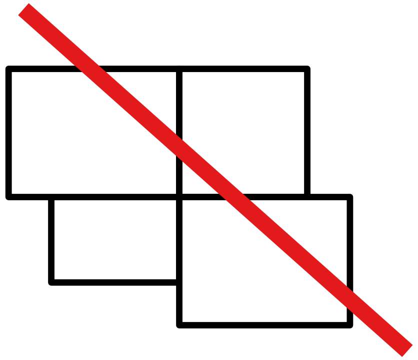
Exactly 4 vertices on outer face



no separating triangle

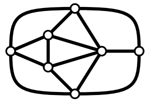
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

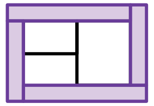


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

Rectangular Dual

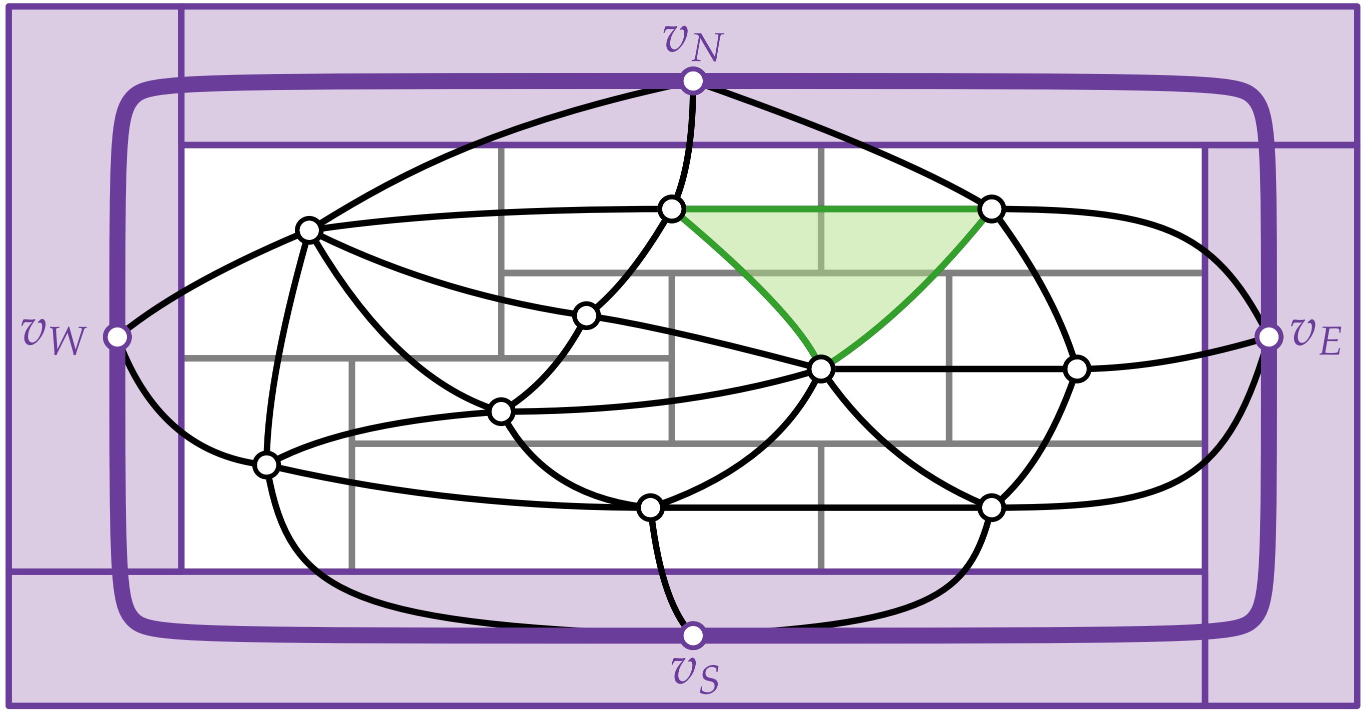


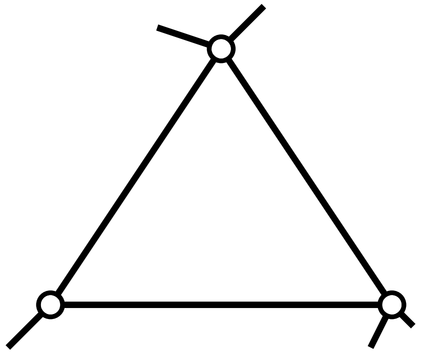
Properly Triangulated Planar Graph G



Rectangular Dual $\mathcal{R}$

Exactly 4 vertices on outer face

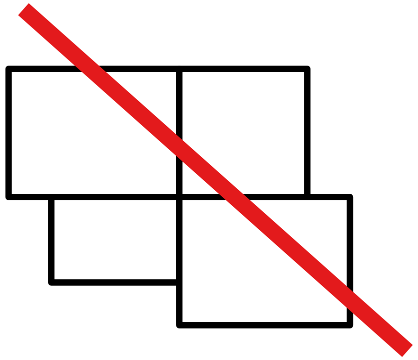




no separating triangle

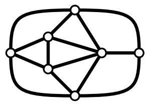
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

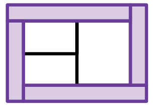
- no four rectangles share a point, and
- the union of all rectangles is a rectangle



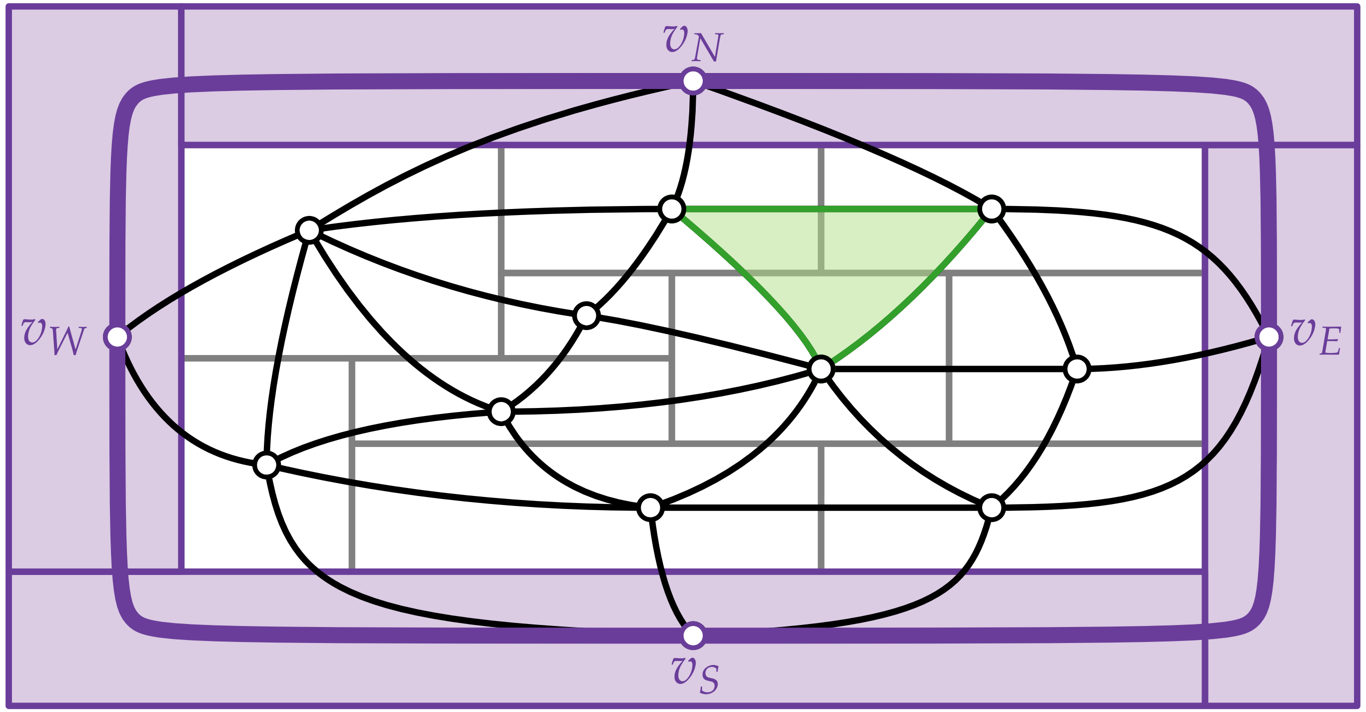
Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

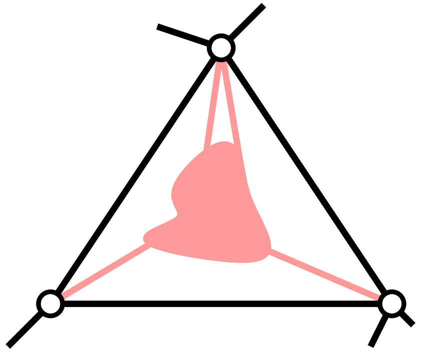
Rectangular Dual





Exactly 4 vertices on outer face

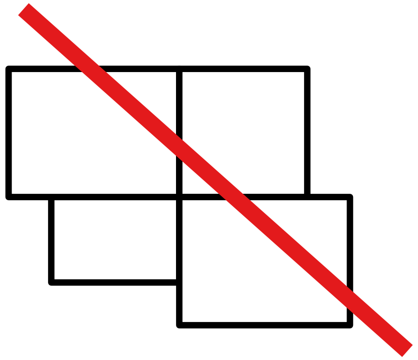




no separating triangle

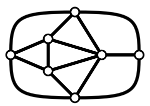
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

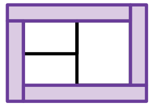


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

Rectangular Dual

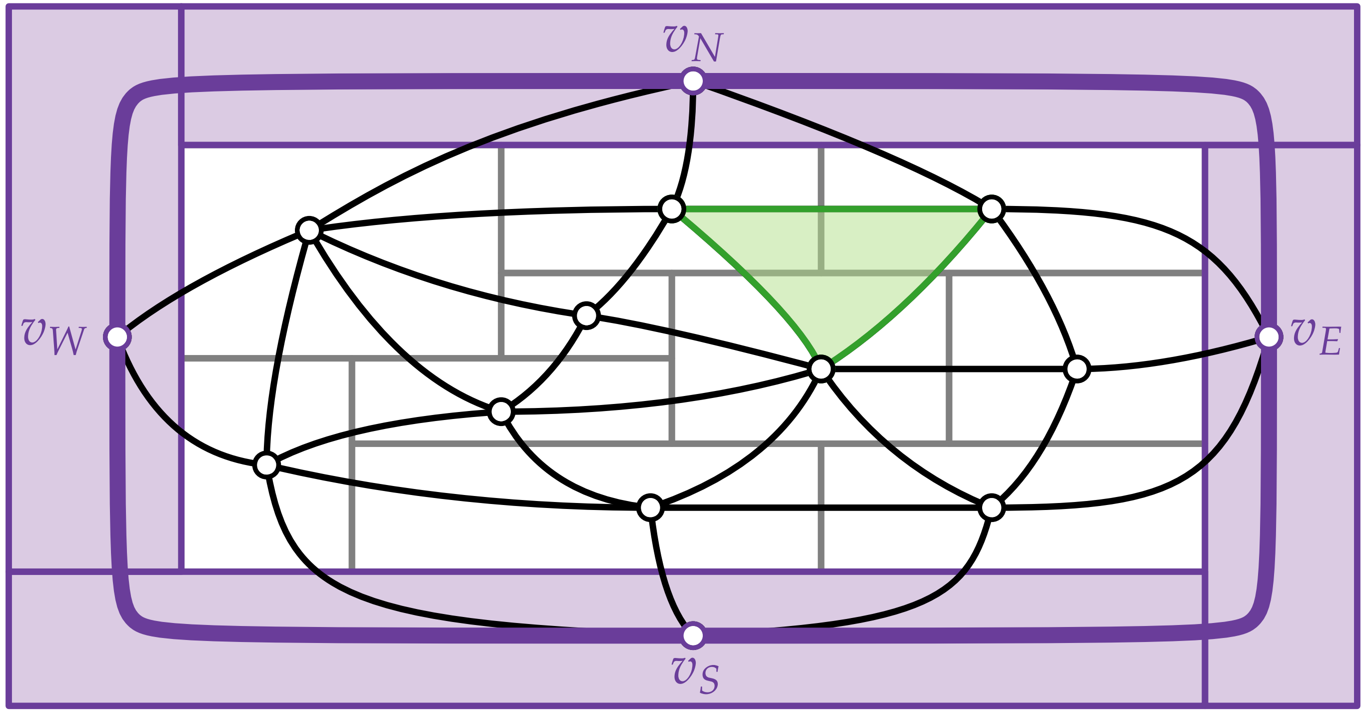


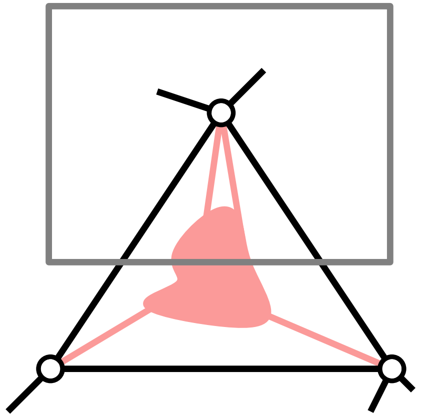
Properly Triangulated Planar Graph G



Rectangular Dual $\mathcal{R}$

Exactly 4 vertices on outer face

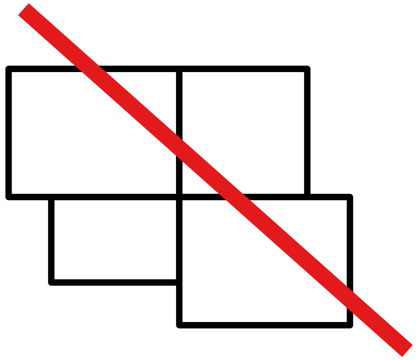




no separating triangle

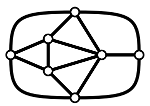
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

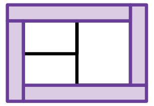


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

Rectangular Dual

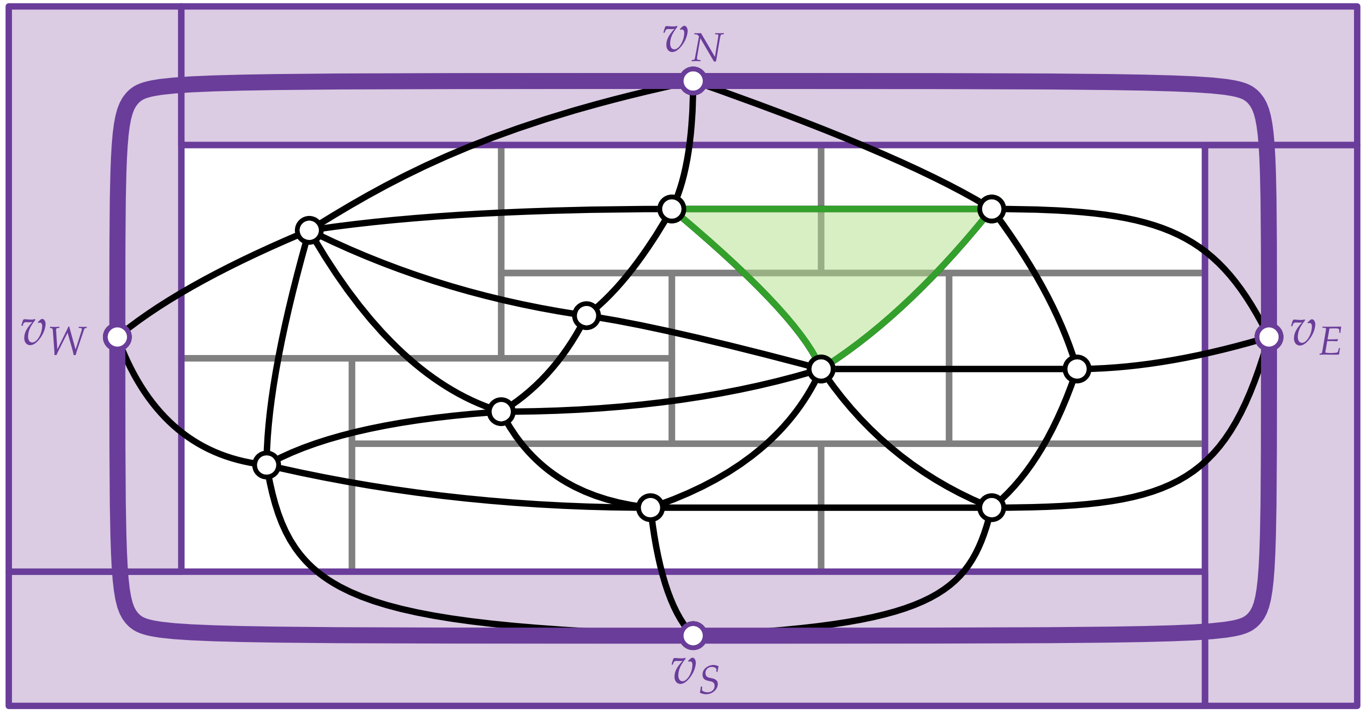

PTP

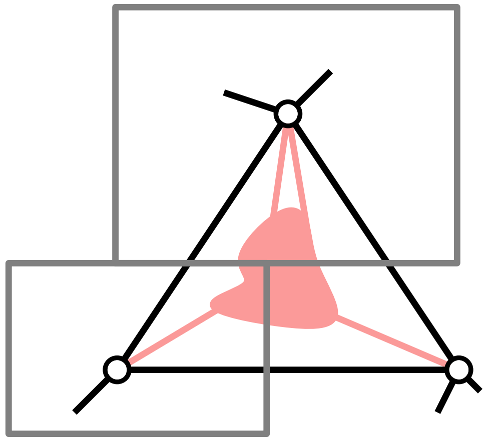
Properly Triangulated Planar Graph G


RD

Rectangular Dual $\mathcal{R}$

Exactly 4 vertices on outer face

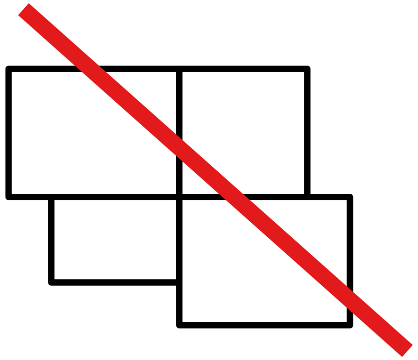




no separating triangle

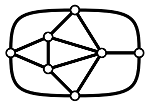
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle



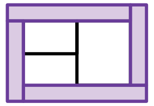
Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

Rectangular Dual



PTP

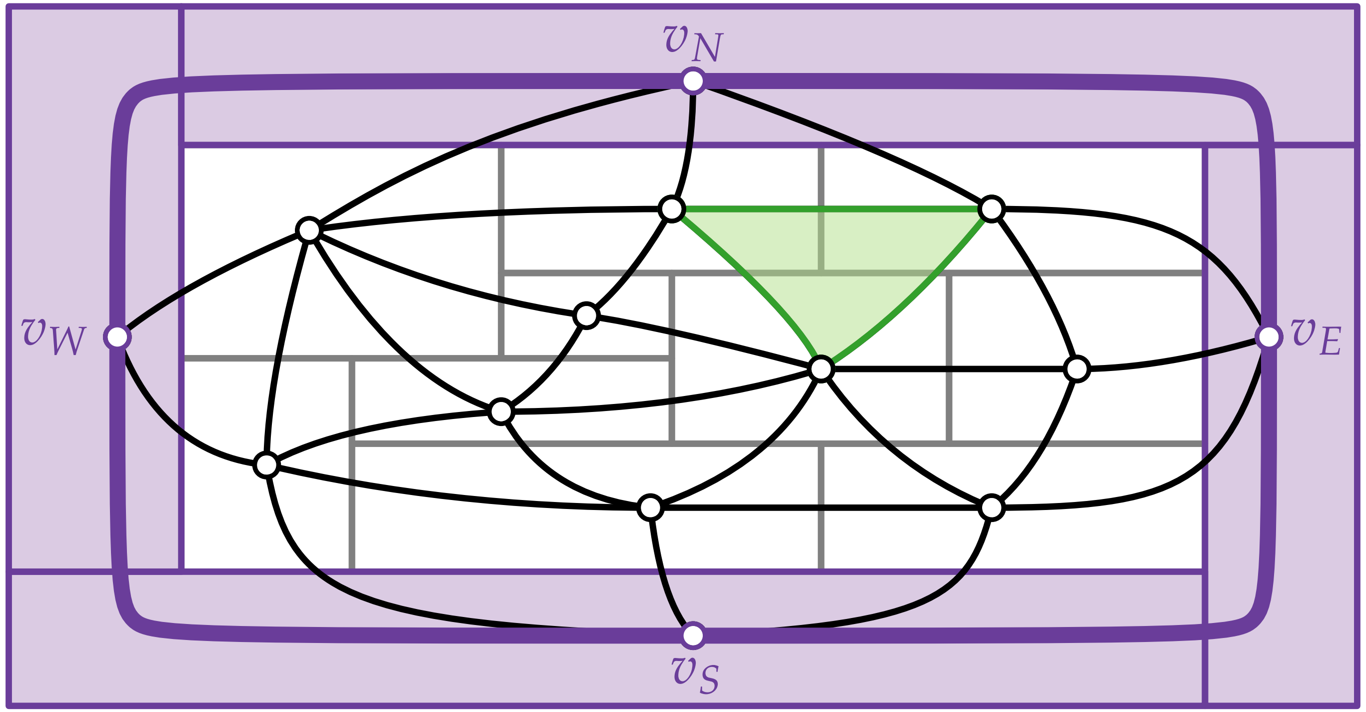
Properly Triangulated Planar Graph G

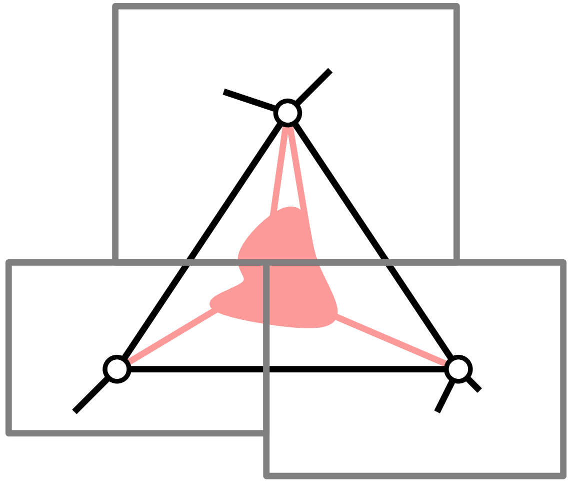


RD

Rectangular Dual $\mathcal{R}$

Exactly 4 vertices on outer face

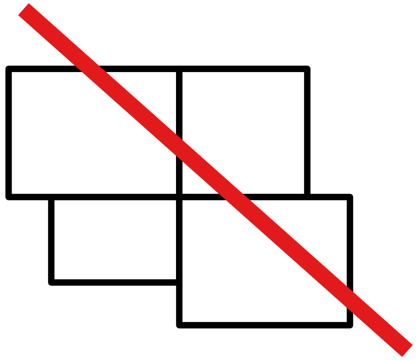




no separating triangle

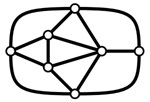
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

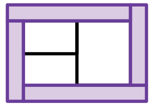


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

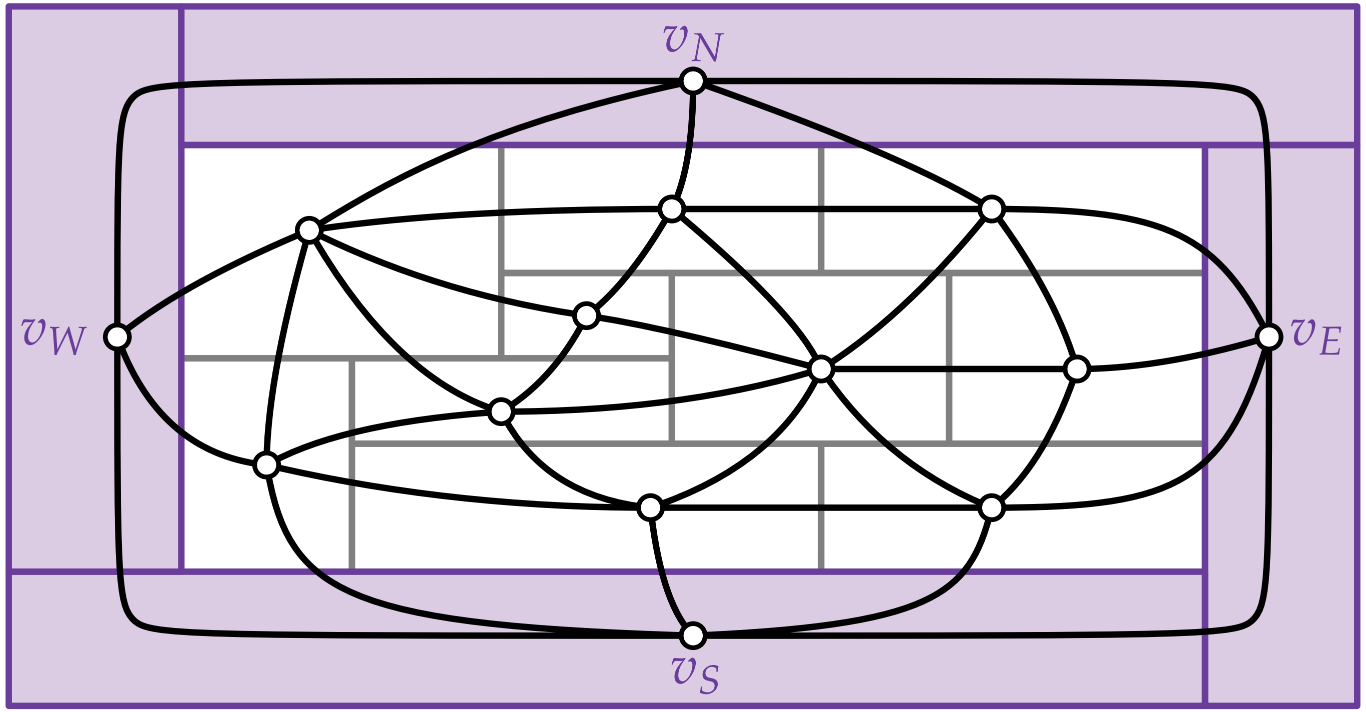
Regular Edge Labeling



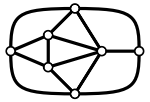
Properly Triangulated Planar Graph G



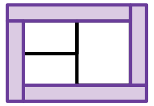
Rectangular Dual $\mathcal{R}$



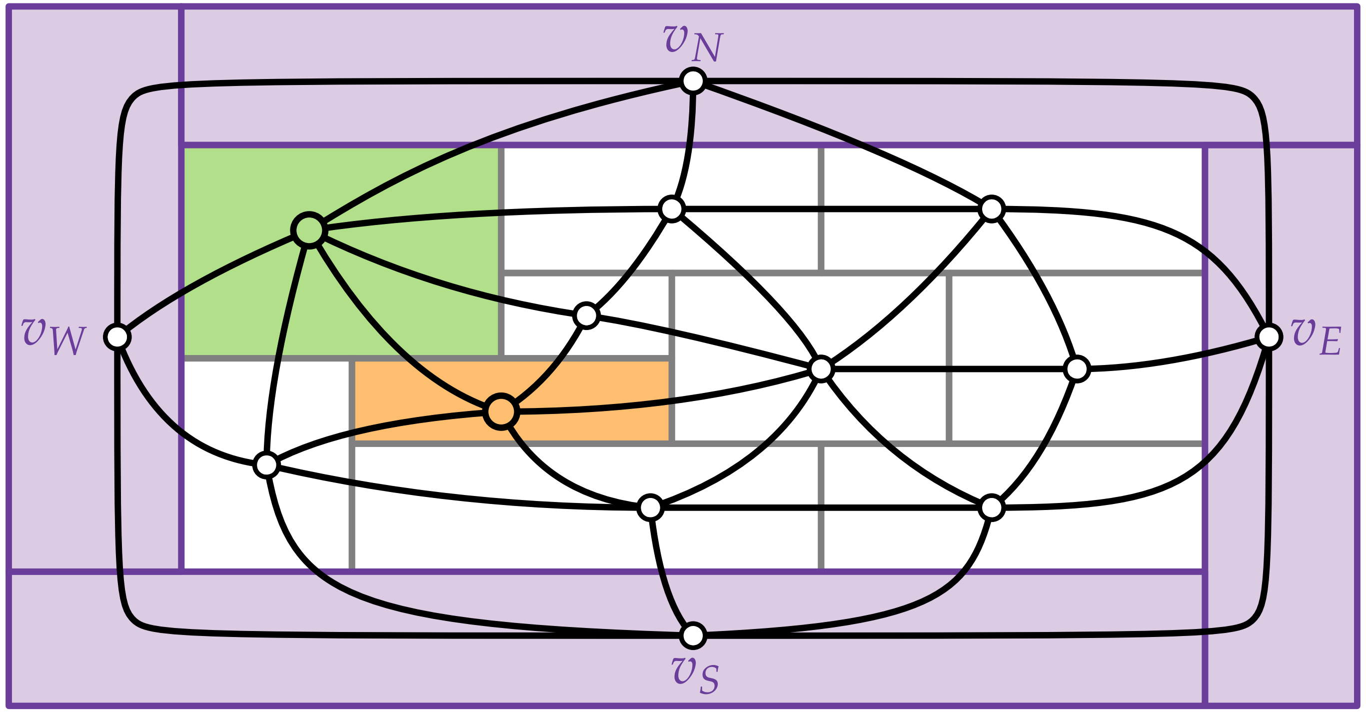
Regular Edge Labeling



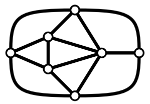
Properly Triangulated Planar Graph G



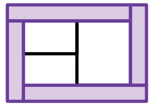
Rectangular Dual $\mathcal{R}$



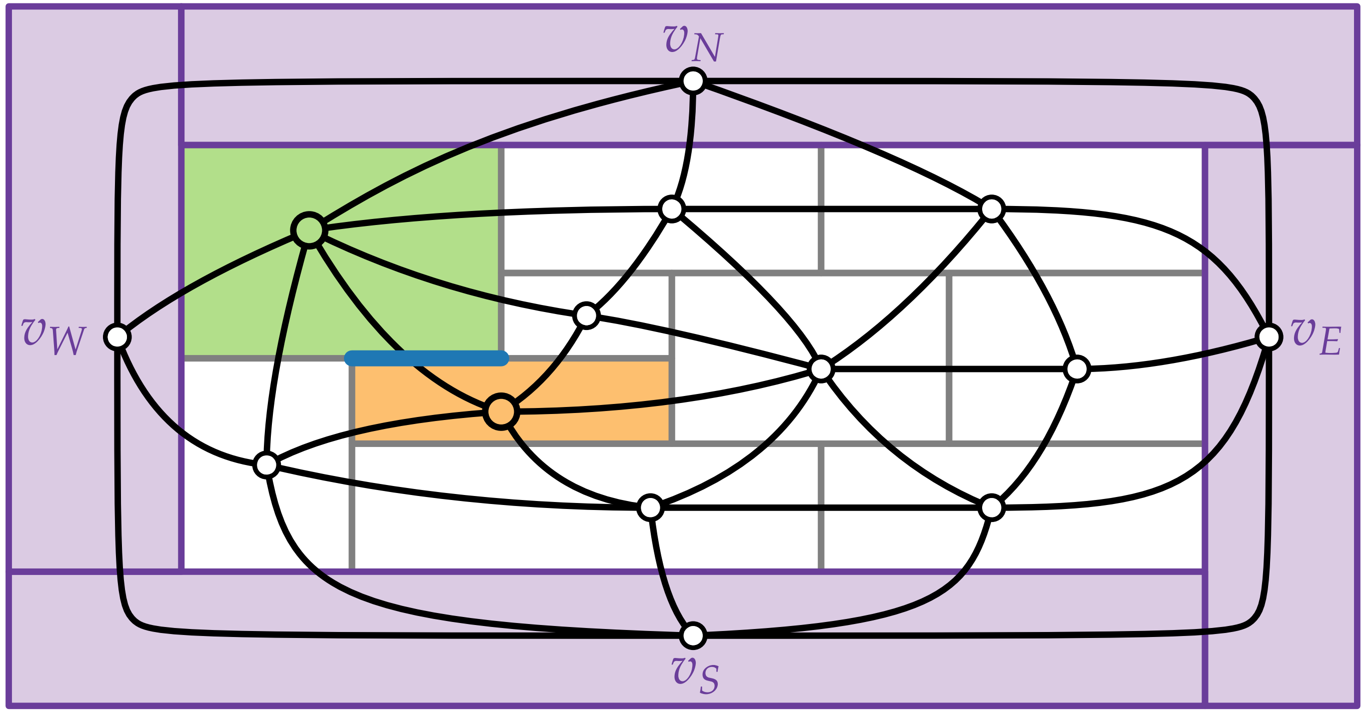
Regular Edge Labeling



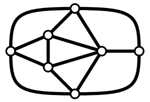
Properly Triangulated Planar Graph G



Rectangular Dual $\mathcal{R}$

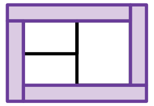


Regular Edge Labeling



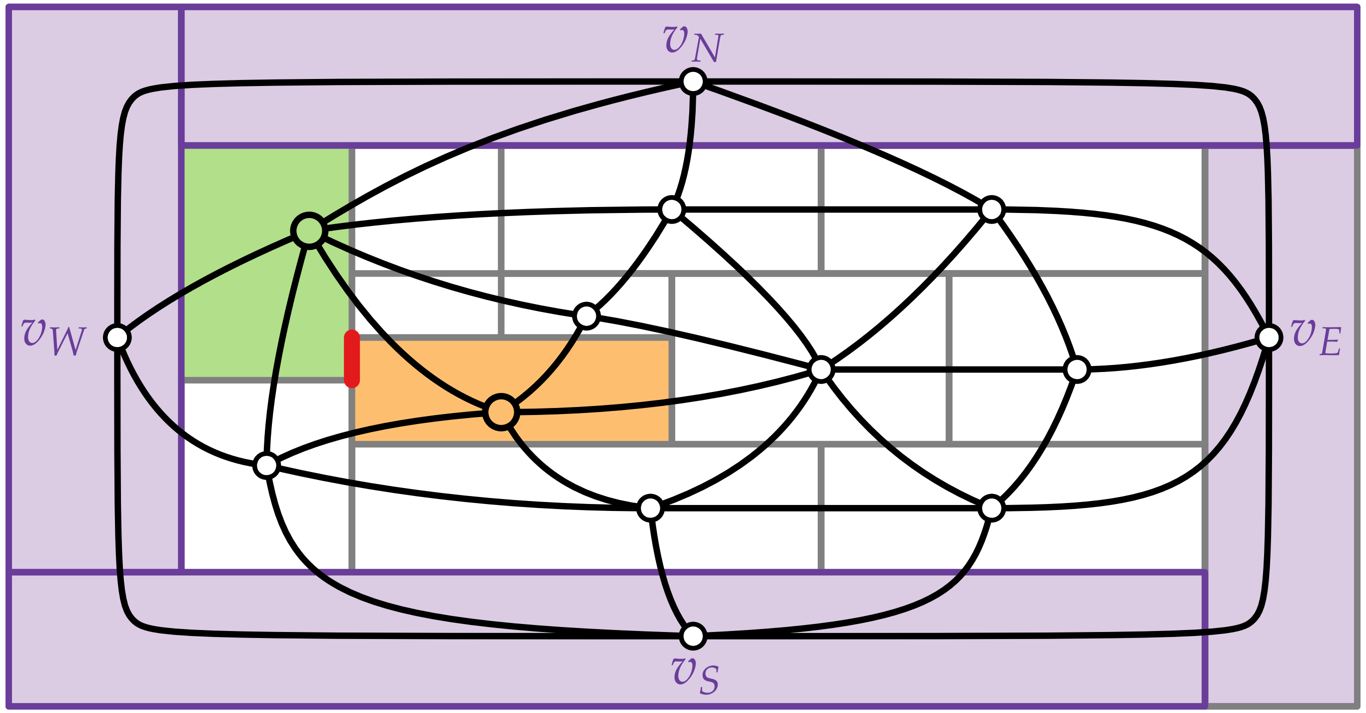
PTP

Properly Triangulated Planar Graph G

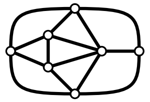


RD

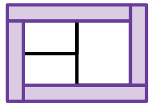
Rectangular Dual $\mathcal{R}$



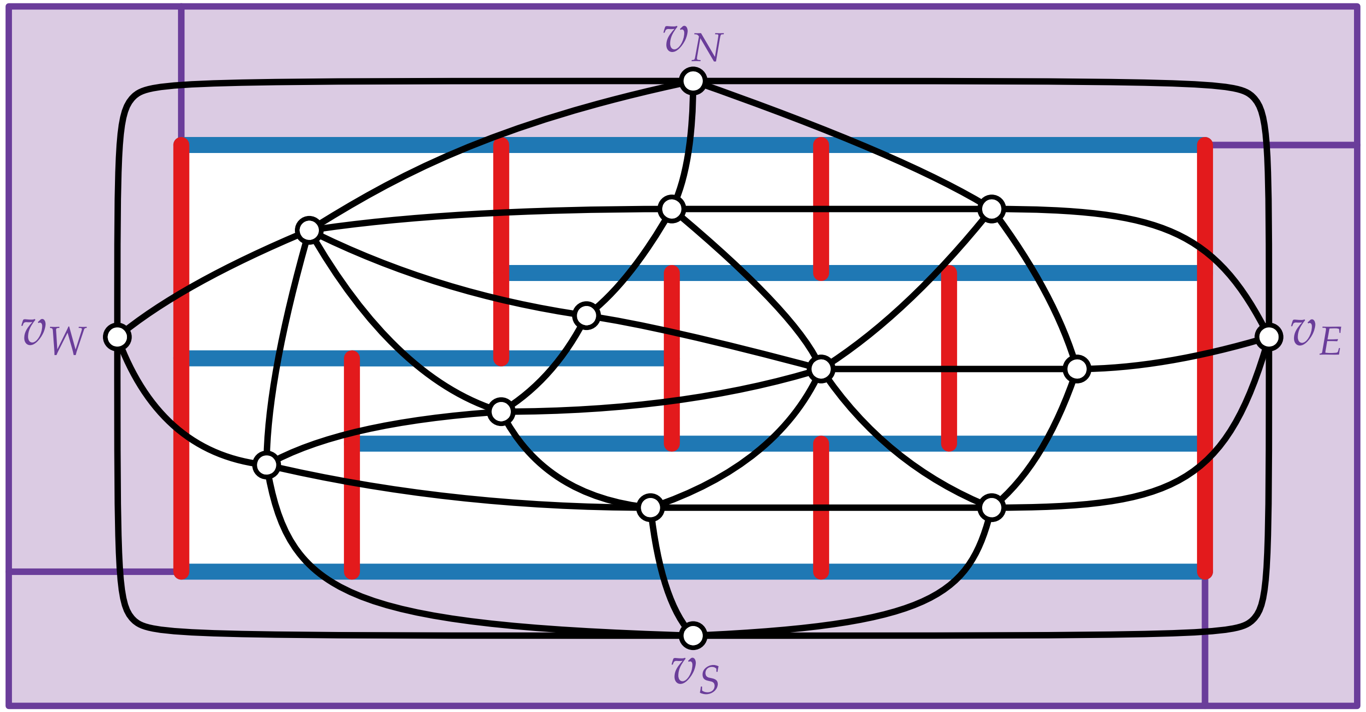
Regular Edge Labeling


PTP

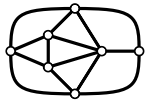
Properly Triangulated Planar Graph G


RD

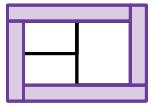
Rectangular Dual $\mathcal{R}$



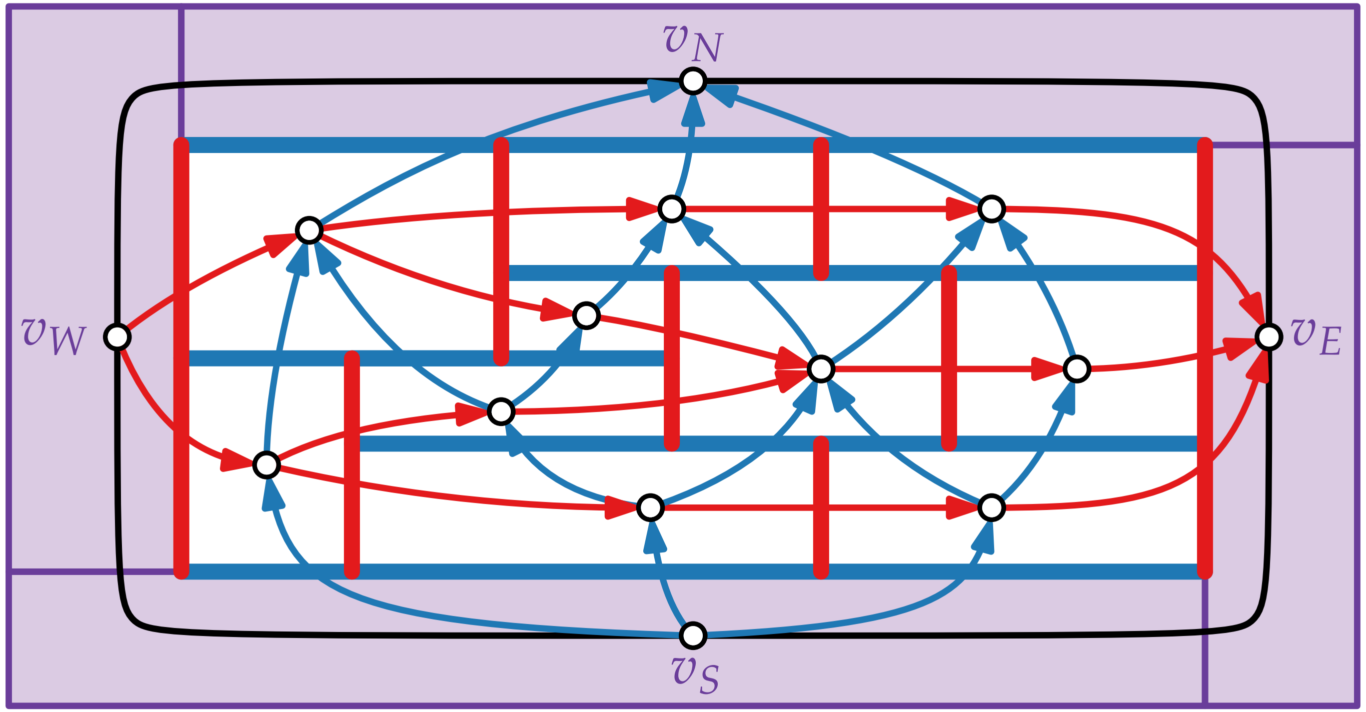
Regular Edge Labeling



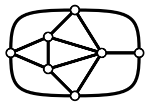
Properly Triangulated Planar Graph G



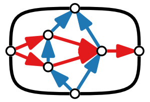
Rectangular Dual $\mathcal{R}$



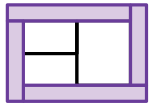
Regular Edge Labeling



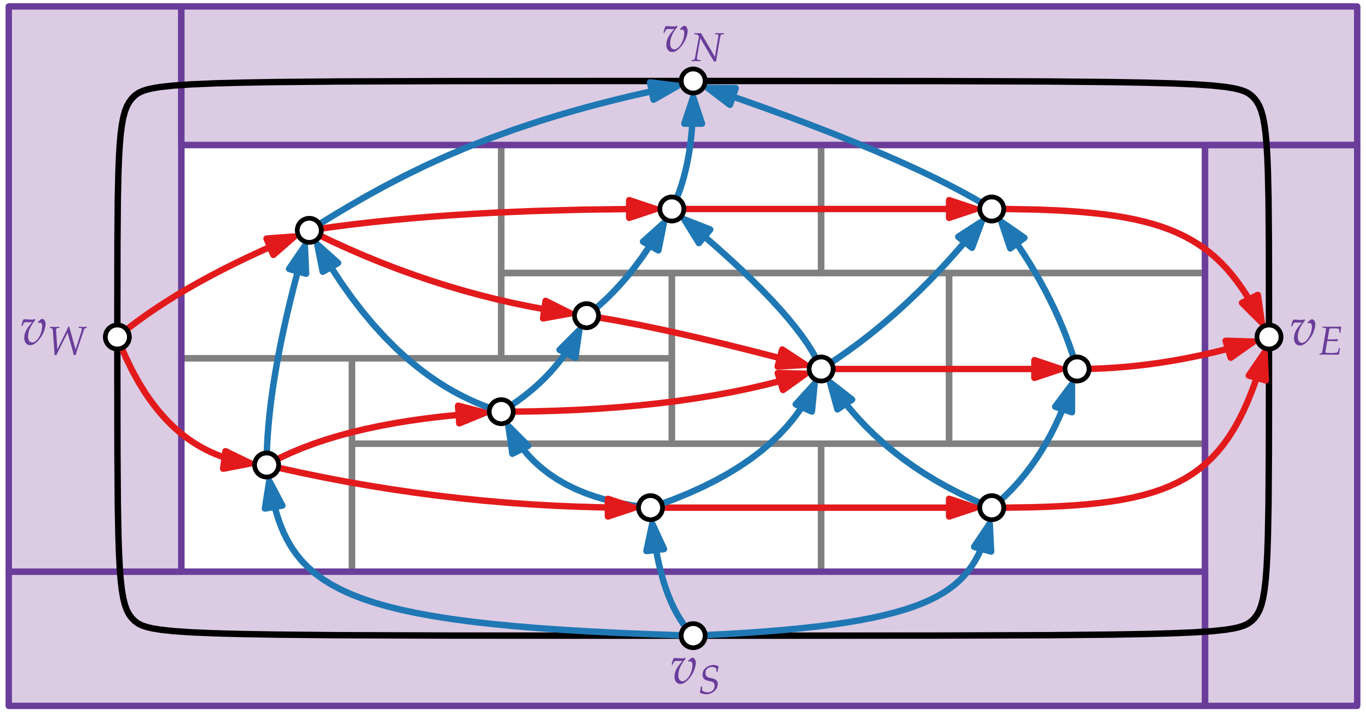
Properly Triangulated Planar Graph G



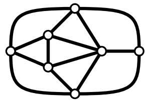
Regular Edge Labeling



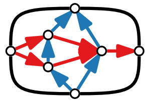
Rectangular Dual $\mathcal{R}$



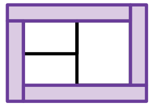
Regular Edge Labeling



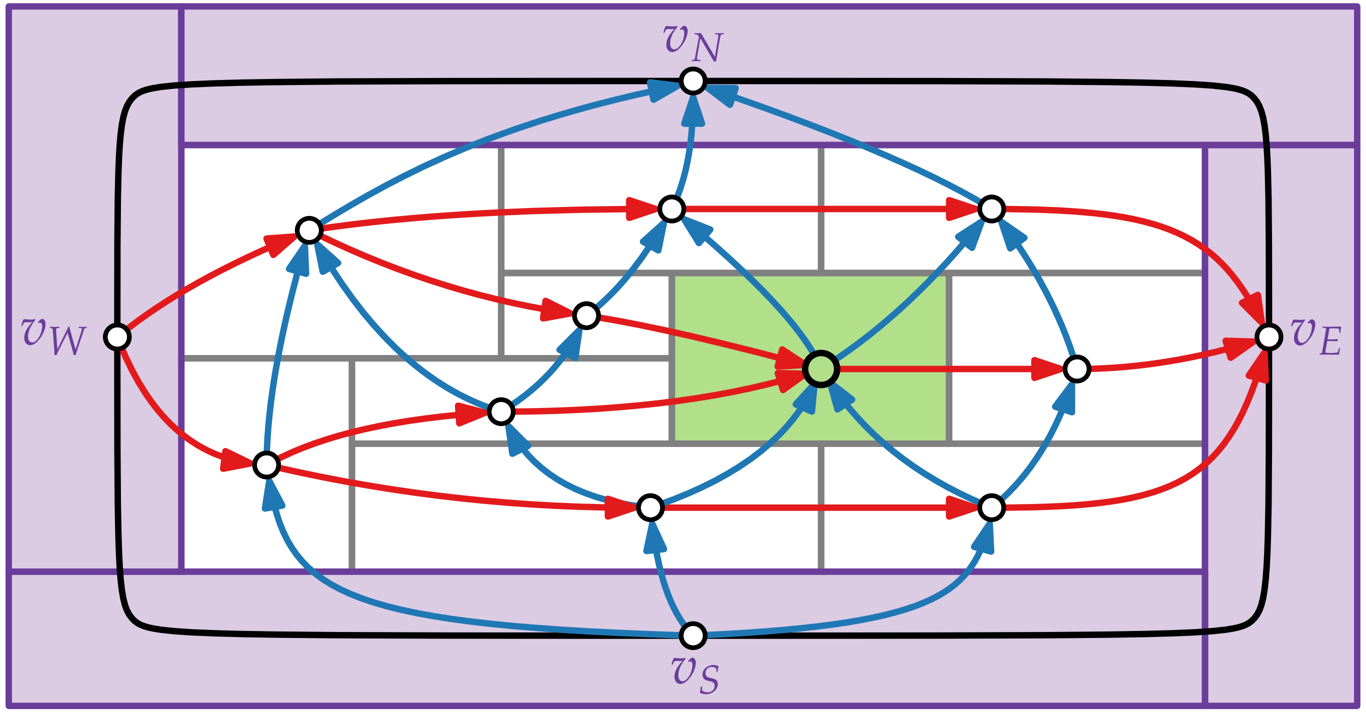
Properly Triangulated Planar Graph G



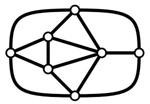
Regular Edge Labeling



Rectangular Dual $\mathcal{R}$

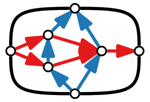


Regular Edge Labeling



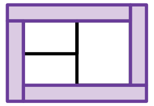
PTP

Properly Triangulated Planar Graph G



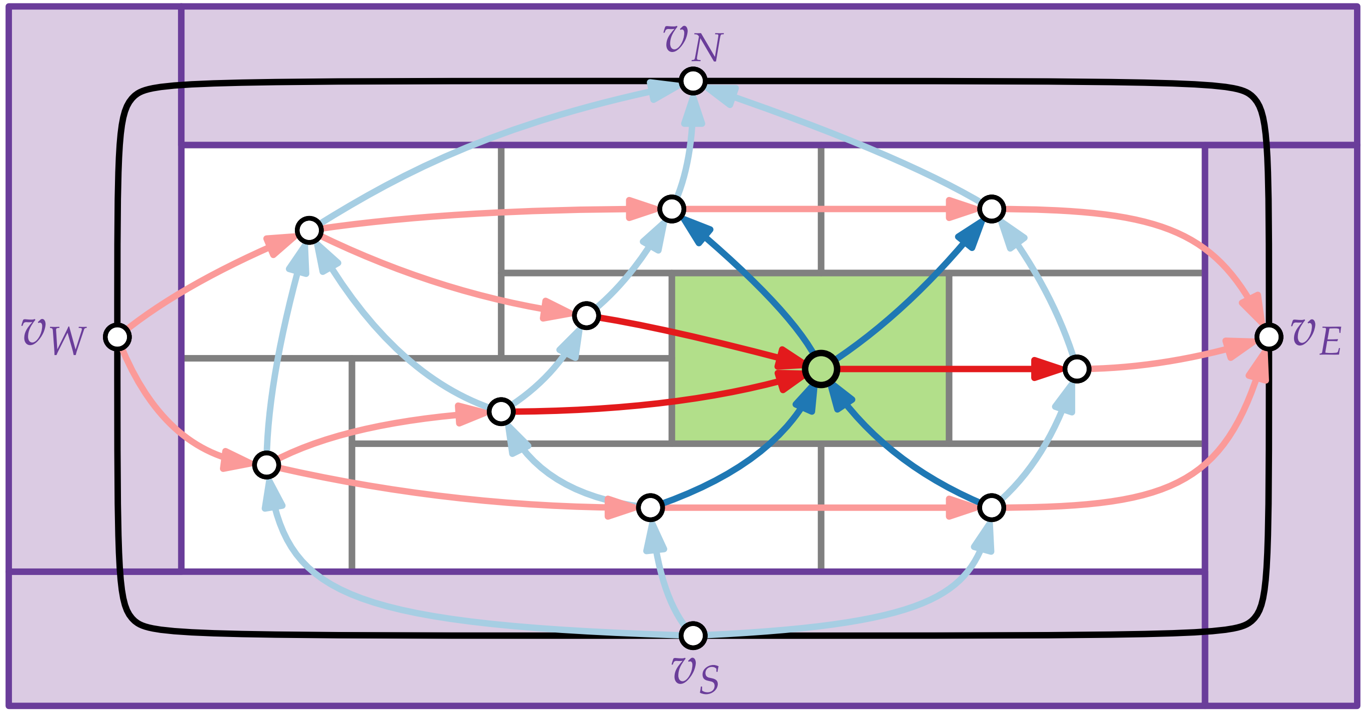
REL

Regular Edge Labeling

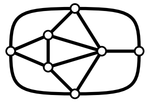


RD

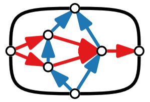
Rectangular Dual $\mathcal{R}$



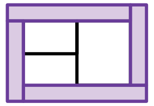
Regular Edge Labeling



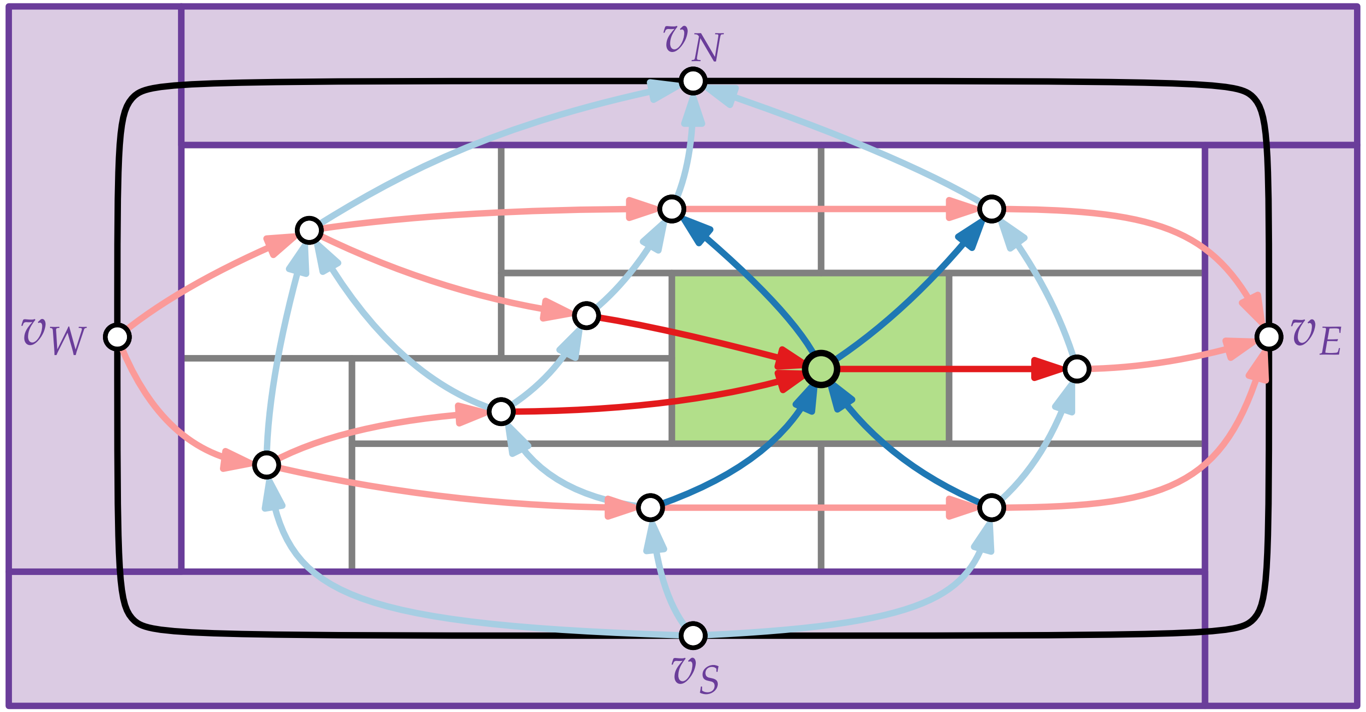
Properly Triangulated Planar Graph G

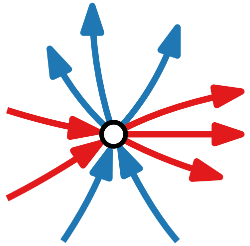


Regular Edge Labeling



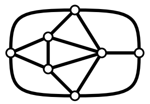
Rectangular Dual $\mathcal{R}$



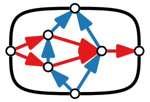


for every inner vertex

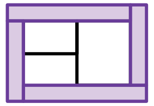
Regular Edge Labeling


PTP

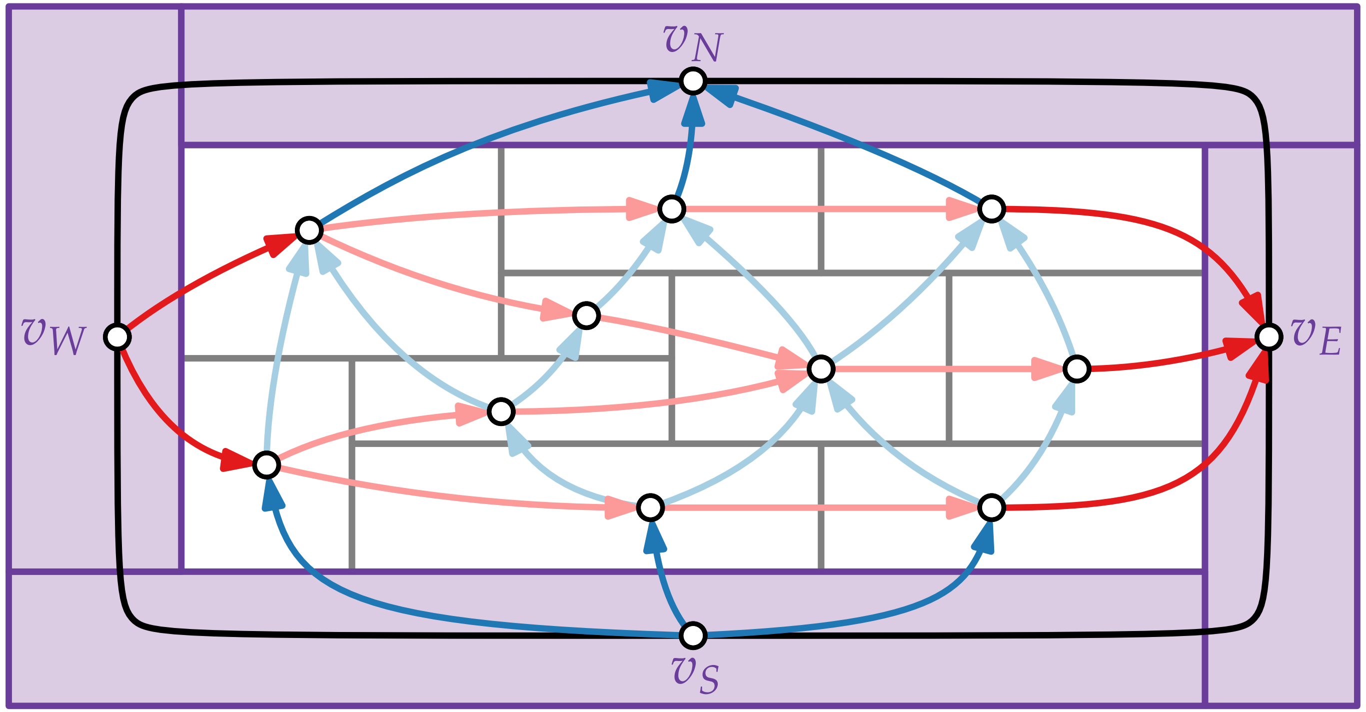
Properly Triangulated Planar Graph G

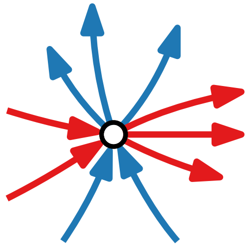

REL

Regular Edge Labeling


RD

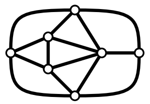
Rectangular Dual $\mathcal{R}$



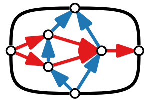


for every inner vertex

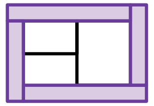
Regular Edge Labeling



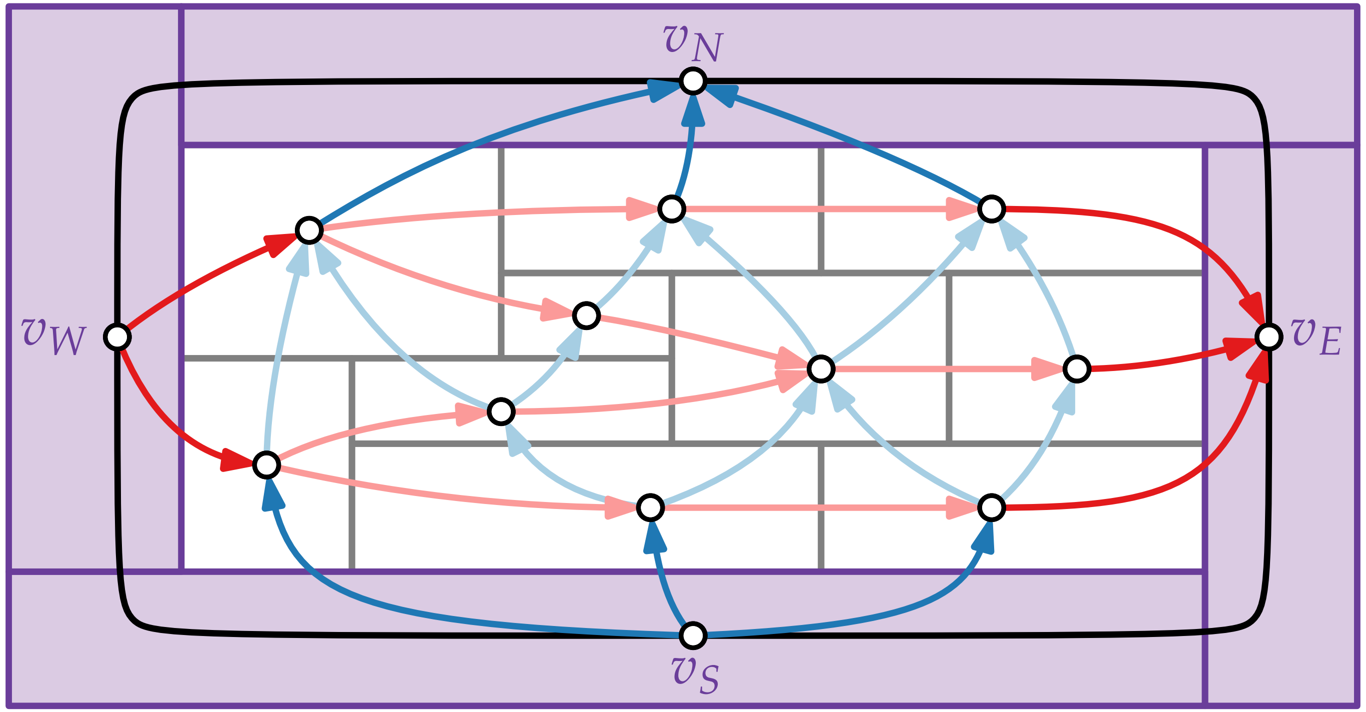
Properly Triangulated Planar Graph G

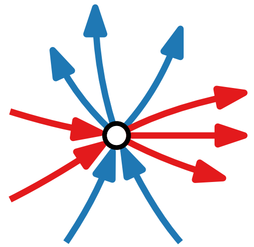


Regular Edge Labeling

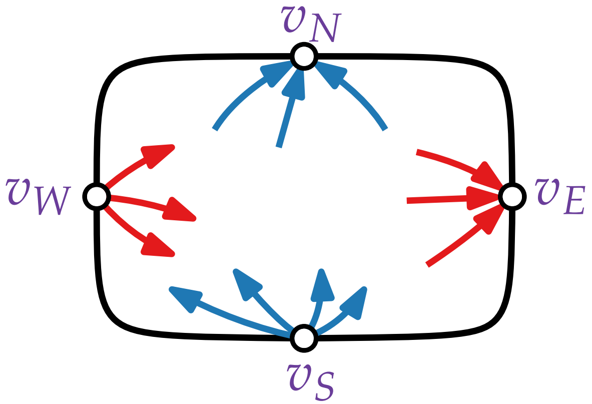


Rectangular Dual $\mathcal{R}$



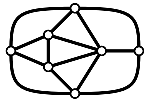


for every inner vertex

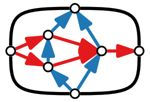


for four outer vertices

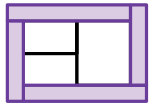
Regular Edge Labeling


PTP

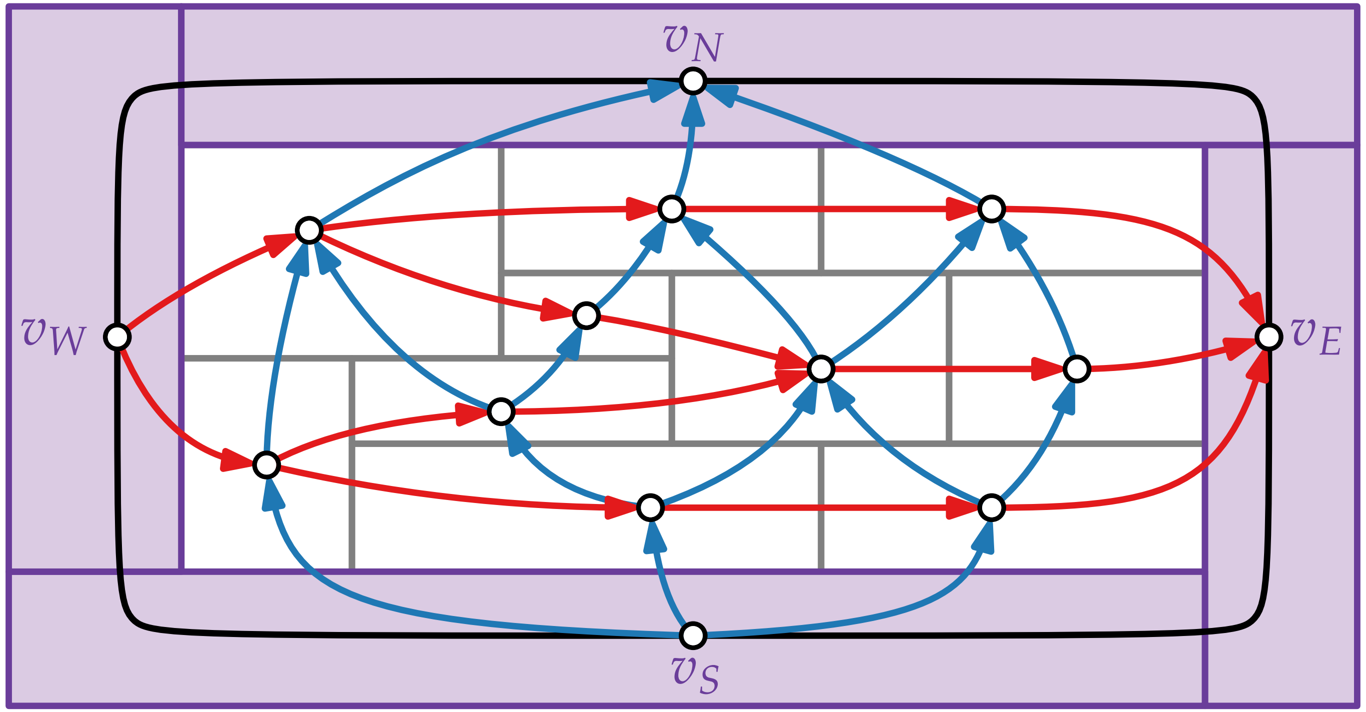
Properly Triangulated Planar Graph G

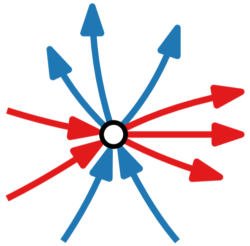

REL

Regular Edge Labeling

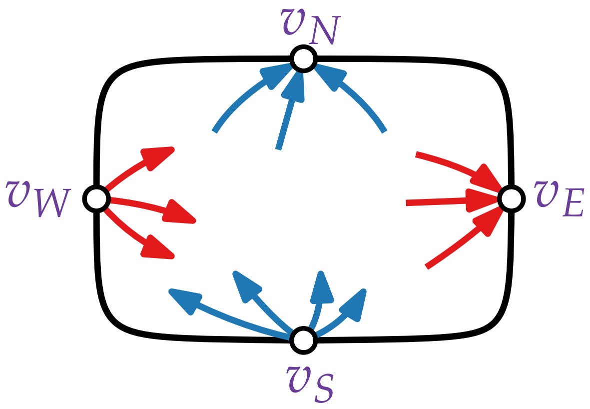

RD

Rectangular Dual $\mathcal{R}$



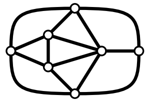


for every inner vertex

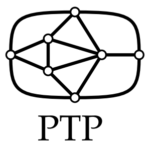


for four outer vertices

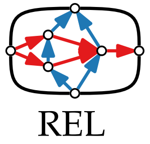
[Kant, He '94]: In linear time


PTP

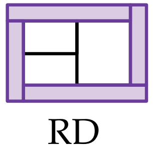
Regular Edge Labeling



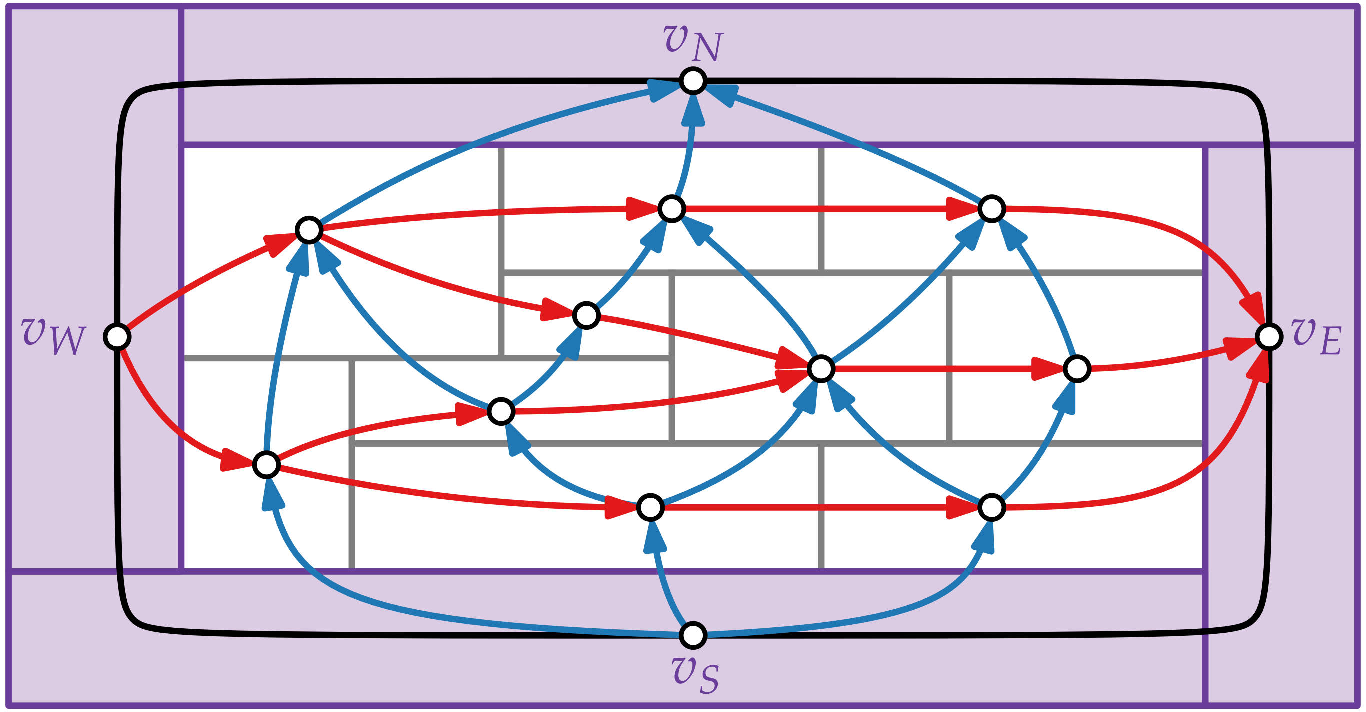
Properly Triangulated Planar Graph G

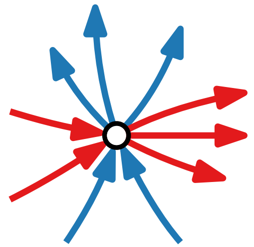


Regular Edge Labeling

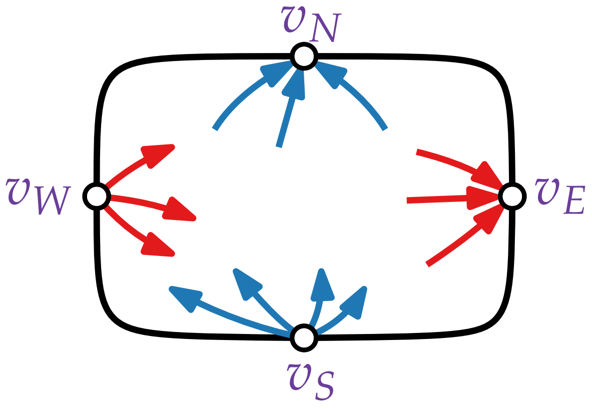


Rectangular Dual $\mathcal{R}$



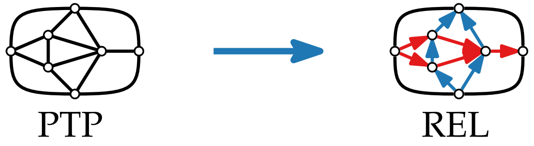


for every inner vertex

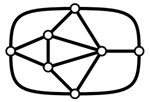


for four outer vertices

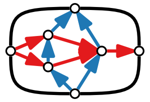
[Kant, He '94]: In linear time



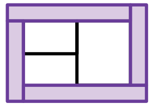
Regular Edge Labeling


PTP

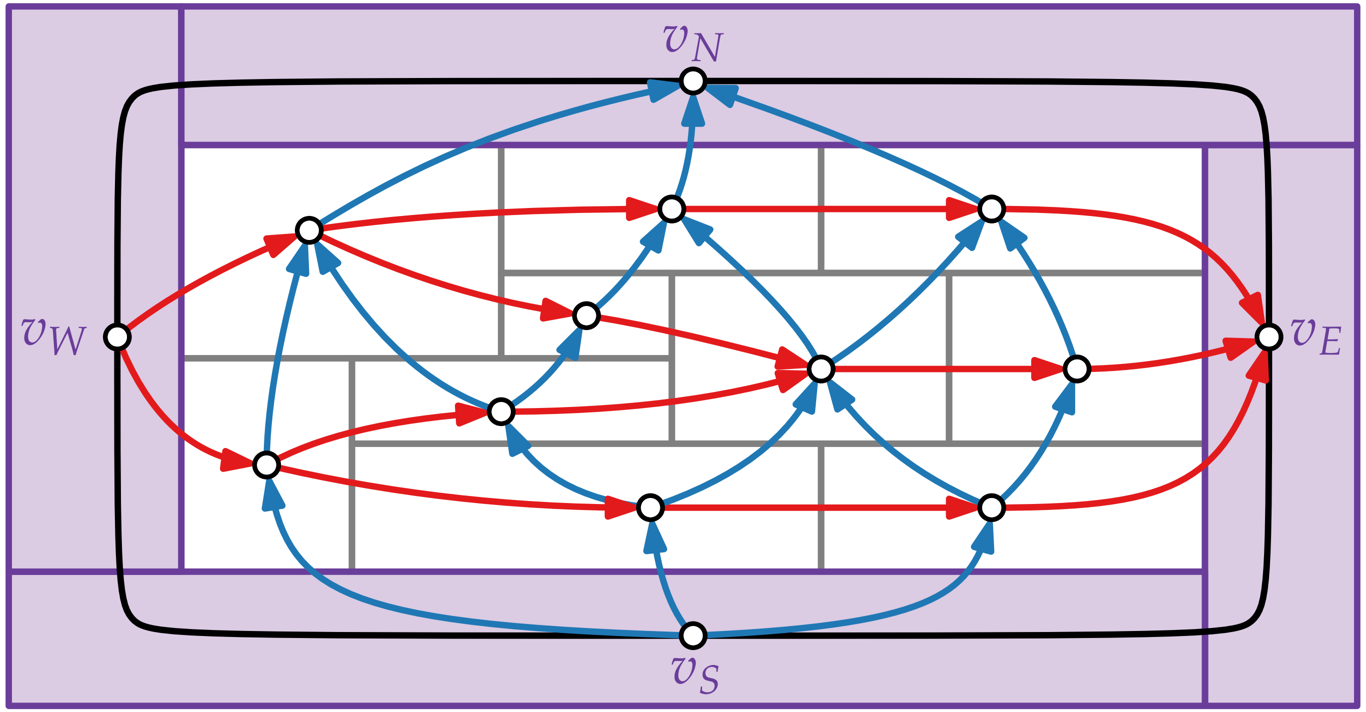
Properly Triangulated Planar Graph G

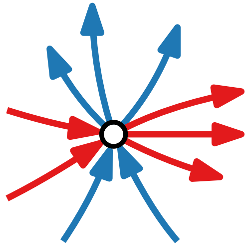

REL

Regular Edge Labeling

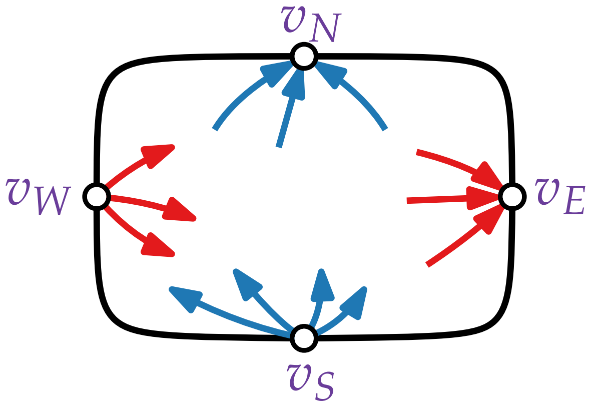

RD

Rectangular Dual $\mathcal{R}$



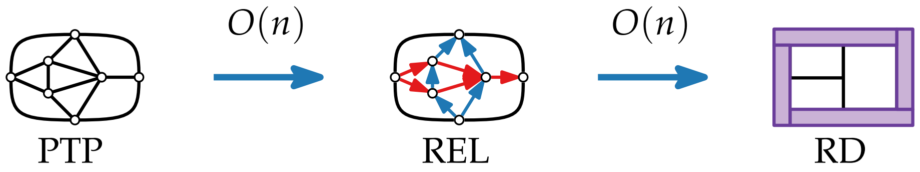


for every inner vertex



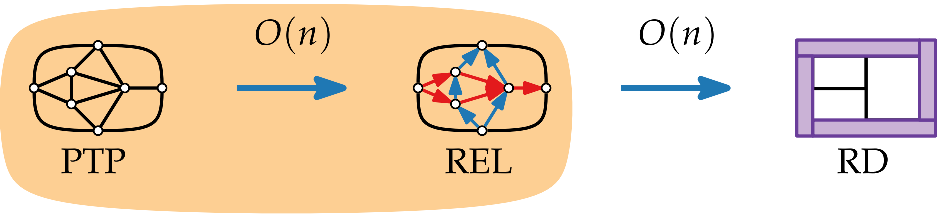
for four outer vertices

[Kant, He ’94]: In linear time



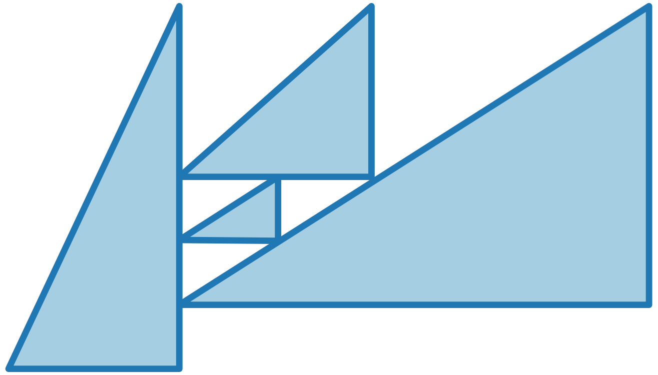
Universität Trier

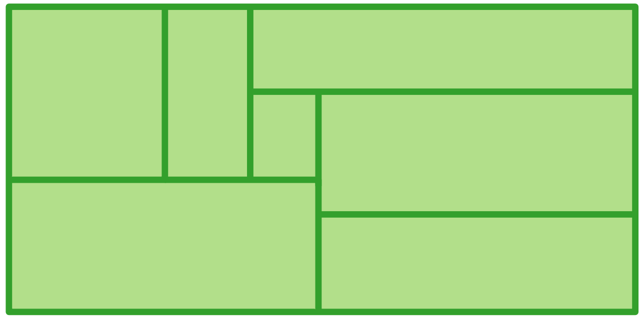
[Kant, He '94]:



Visualization of Graphs

Lecture 9:
Contact Representations of Planar Graphs: Triangle Contacts and Rectangular Duals





Part IV:
Computing a REL

Philipp Kindermann

Refined Canonical Order

Theorem.
Let G be a PTP graph.

Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G

Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

Refined Canonical Order

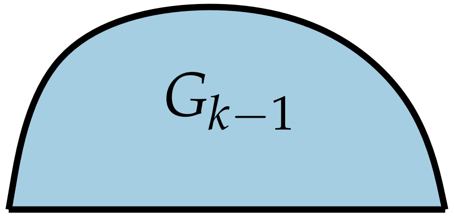
Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected

Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

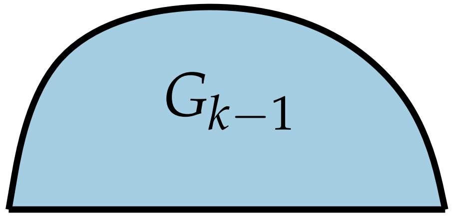
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected



Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

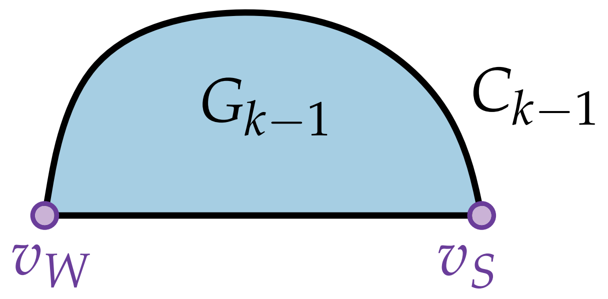
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).



Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

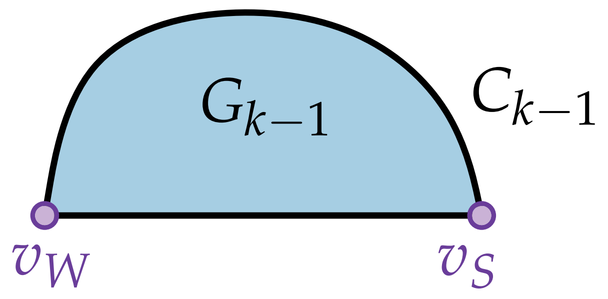
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).



Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

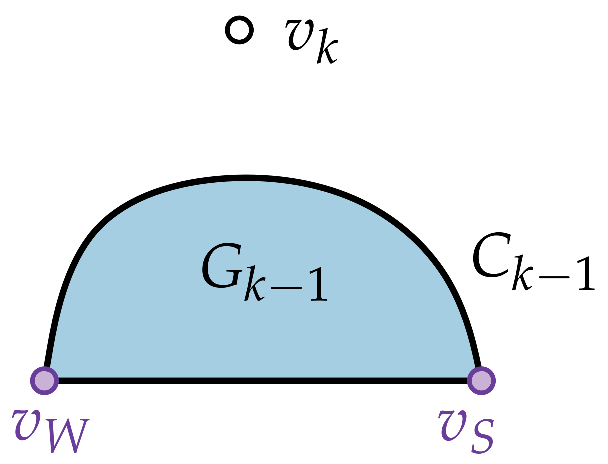
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).
- v_k is in exterior face of G_{k-1}



Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

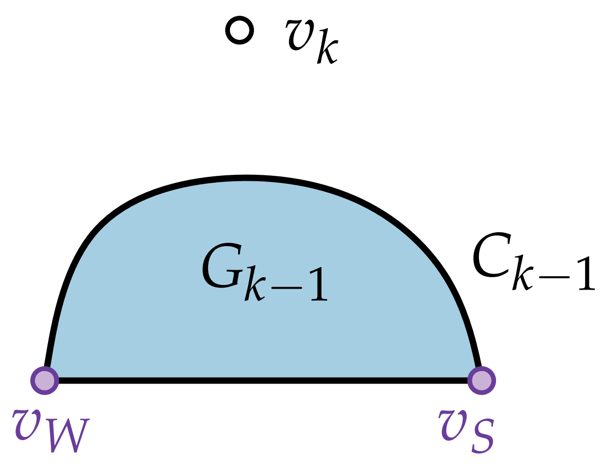
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).
- v_k is in exterior face of G_{k-1}



Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

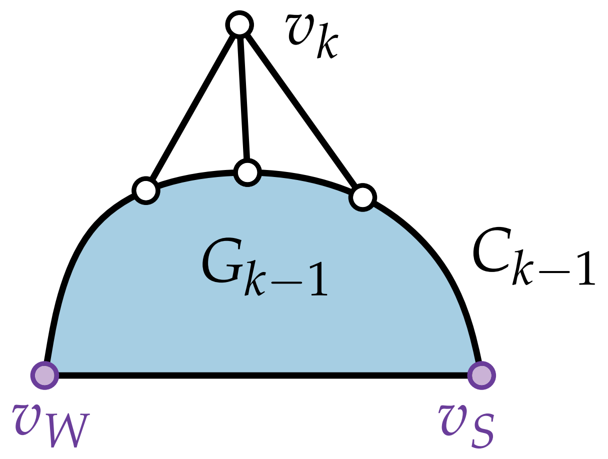
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).
- v_k is in exterior face of G_{k-1}, and its neighbors in G_{k-1} form a (at least 2-element) subinterval of the path $C_{k-1} \setminus (v_S, v_W)$.



Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

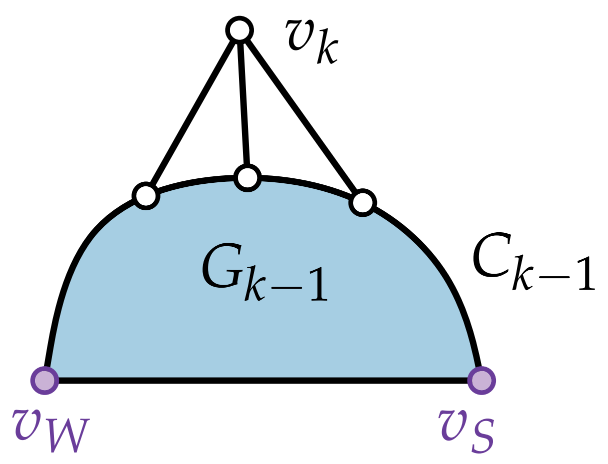
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).
- v_k is in exterior face of G_{k-1}, and its neighbors in G_{k-1} form a (at least 2-element) subinterval of the path $C_{k-1} \setminus (v_S, v_W)$.



Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

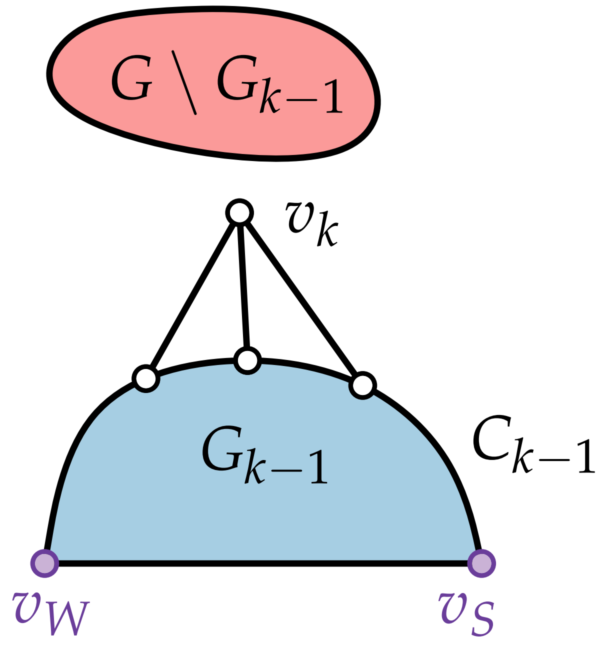
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).
- v_k is in exterior face of G_{k-1}, and its neighbors in G_{k-1} form a (at least 2-element) subinterval of the path $C_{k-1} \setminus (v_S, v_W)$.
- If $k \leq k-2$, then v_k has at least 2 neighbors in $G \setminus G_{k-1}$.



Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

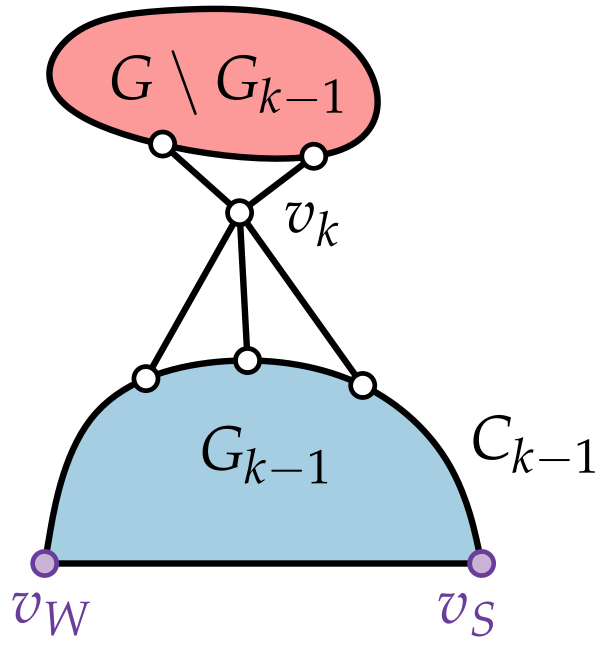
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).
- v_k is in exterior face of G_{k-1}, and its neighbors in G_{k-1} form a (at least 2-element) subinterval of the path $C_{k-1} \setminus (v_S, v_W)$.
- If $k \leq k-2$, then v_k has at least 2 neighbors in $G \setminus G_{k-1}$.



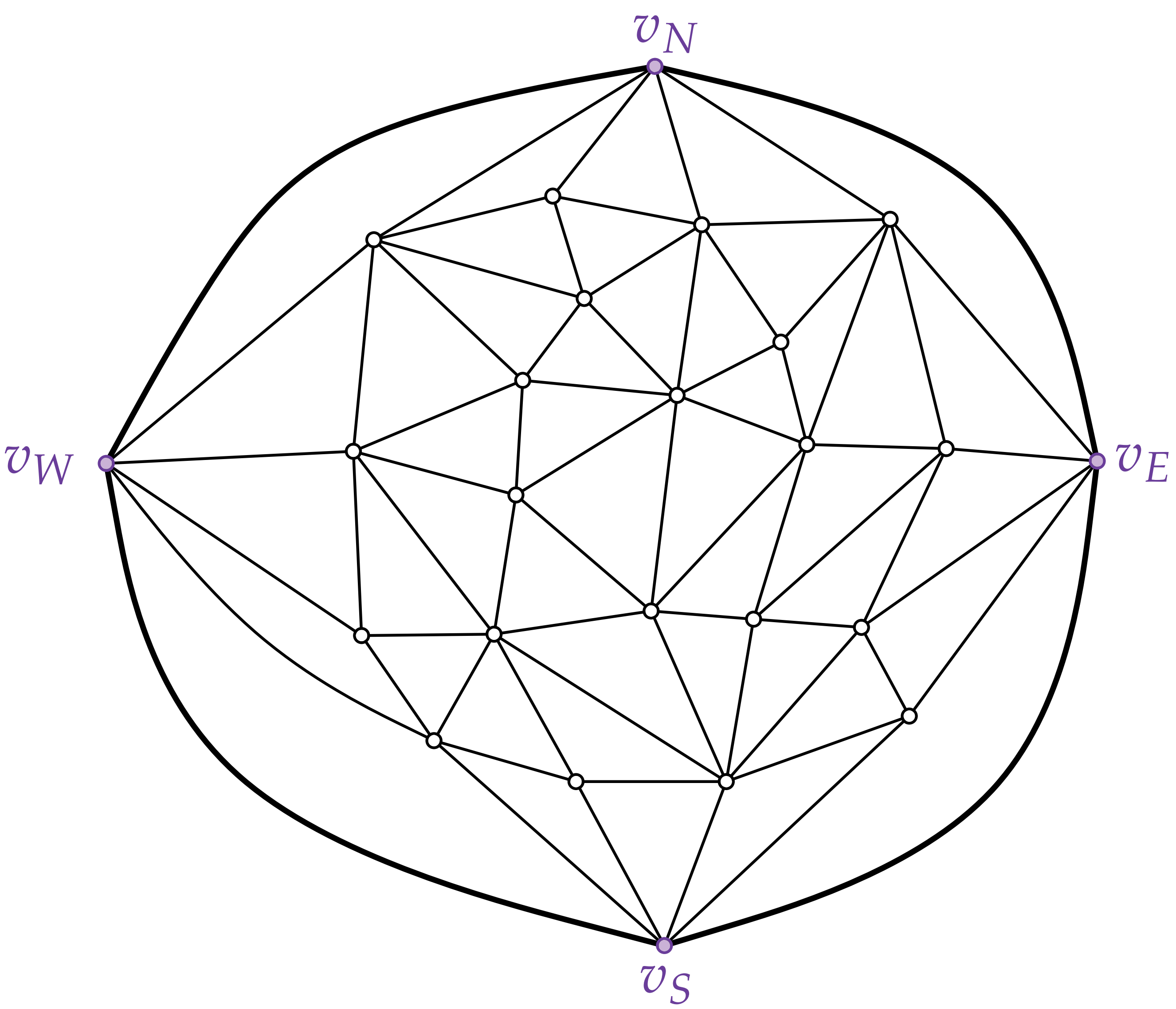
Refined Canonical Order

Theorem.
Let G be a PTP graph. There exists a labeling $v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G such that for every $4 \leq k \leq n$:

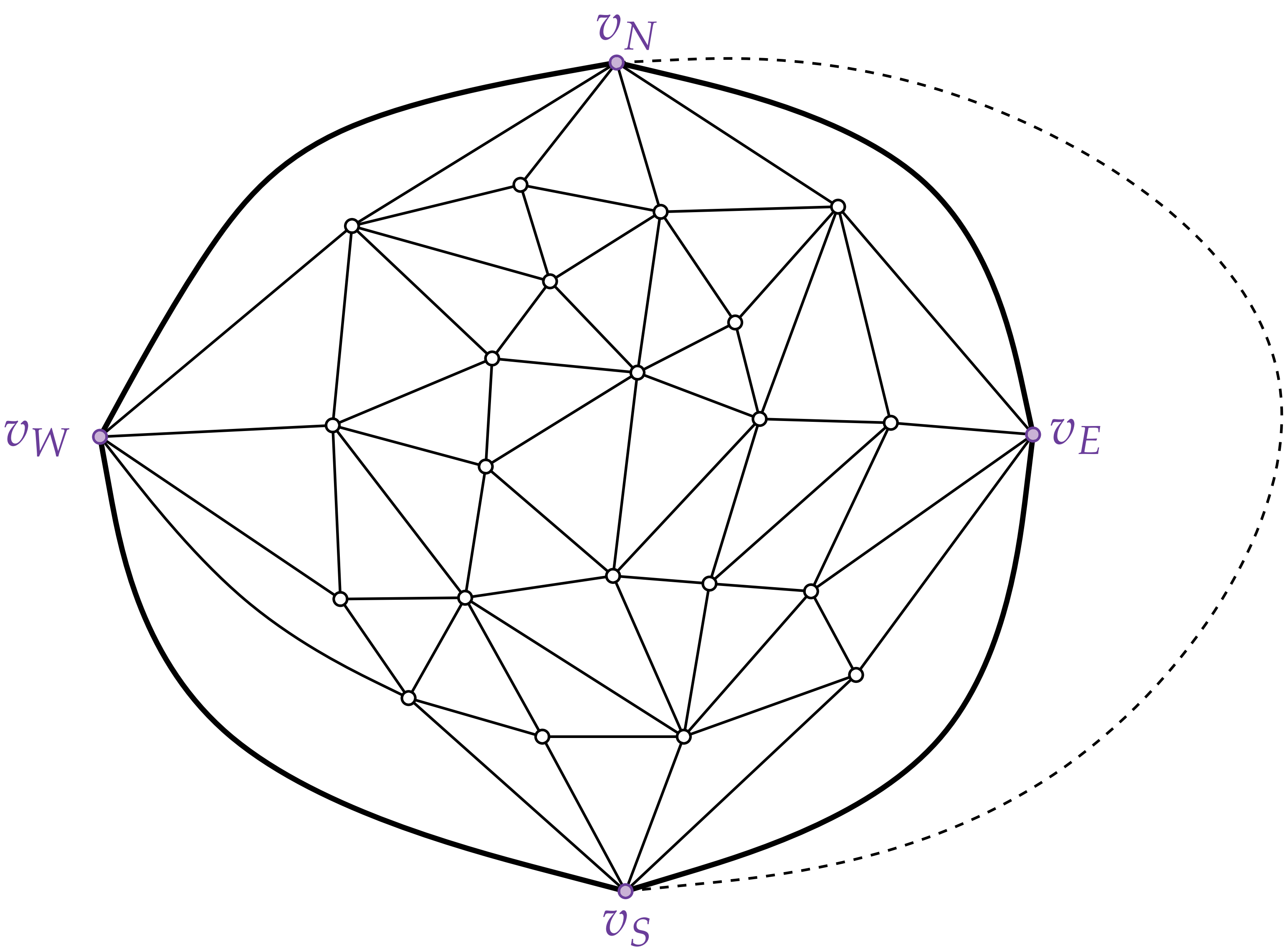
- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and boundary C_{k-1} of G_{k-1} contains the edge (v_S, v_W).
- v_k is in exterior face of G_{k-1}, and its neighbors in G_{k-1} form a (at least 2-element) subinterval of the path $C_{k-1} \setminus (v_S, v_W)$.
- If $k \leq k-2$, then v_k has at least 2 neighbors in $G \setminus G_{k-1}$.



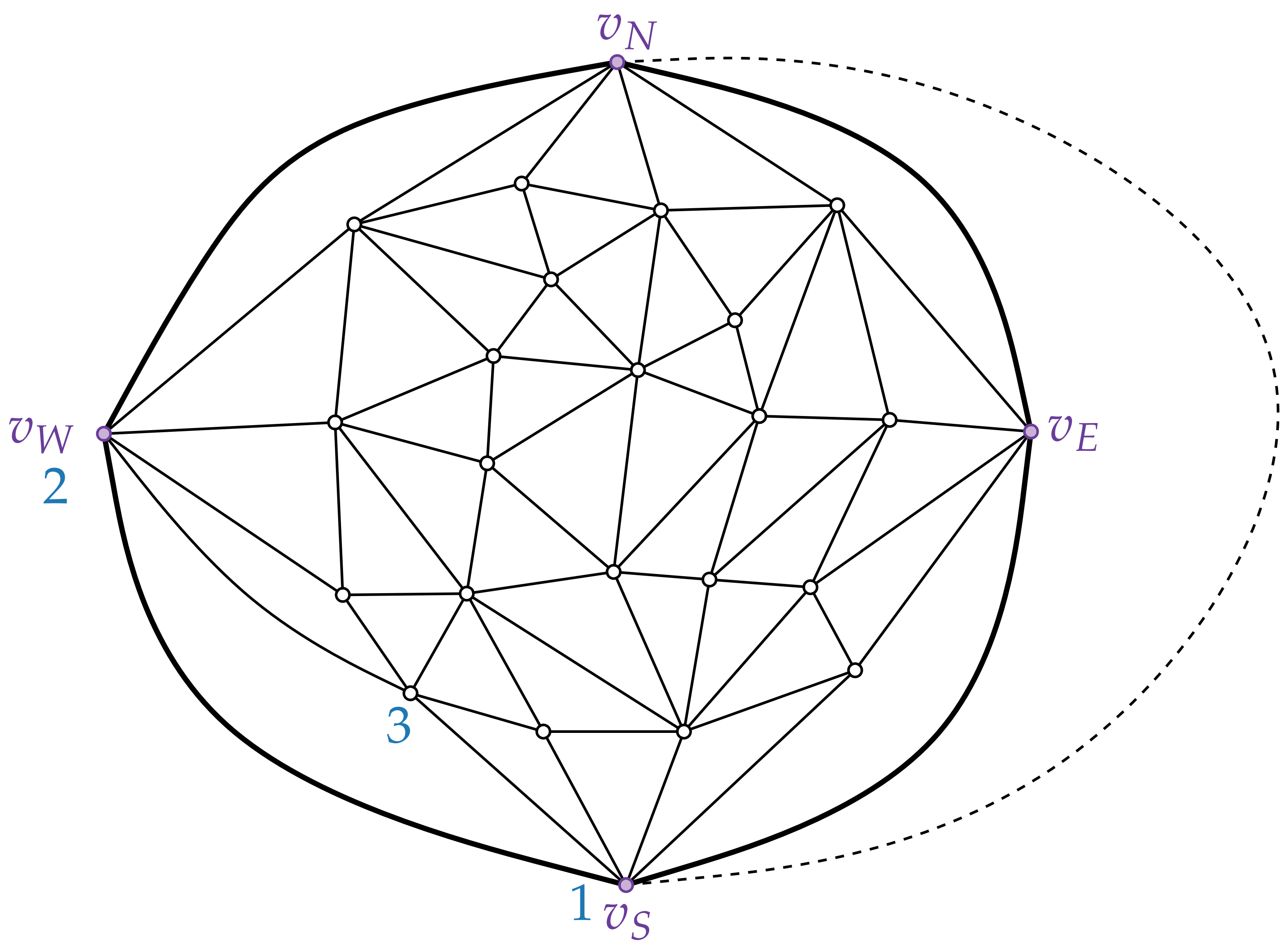
Refined Canonical Order Example



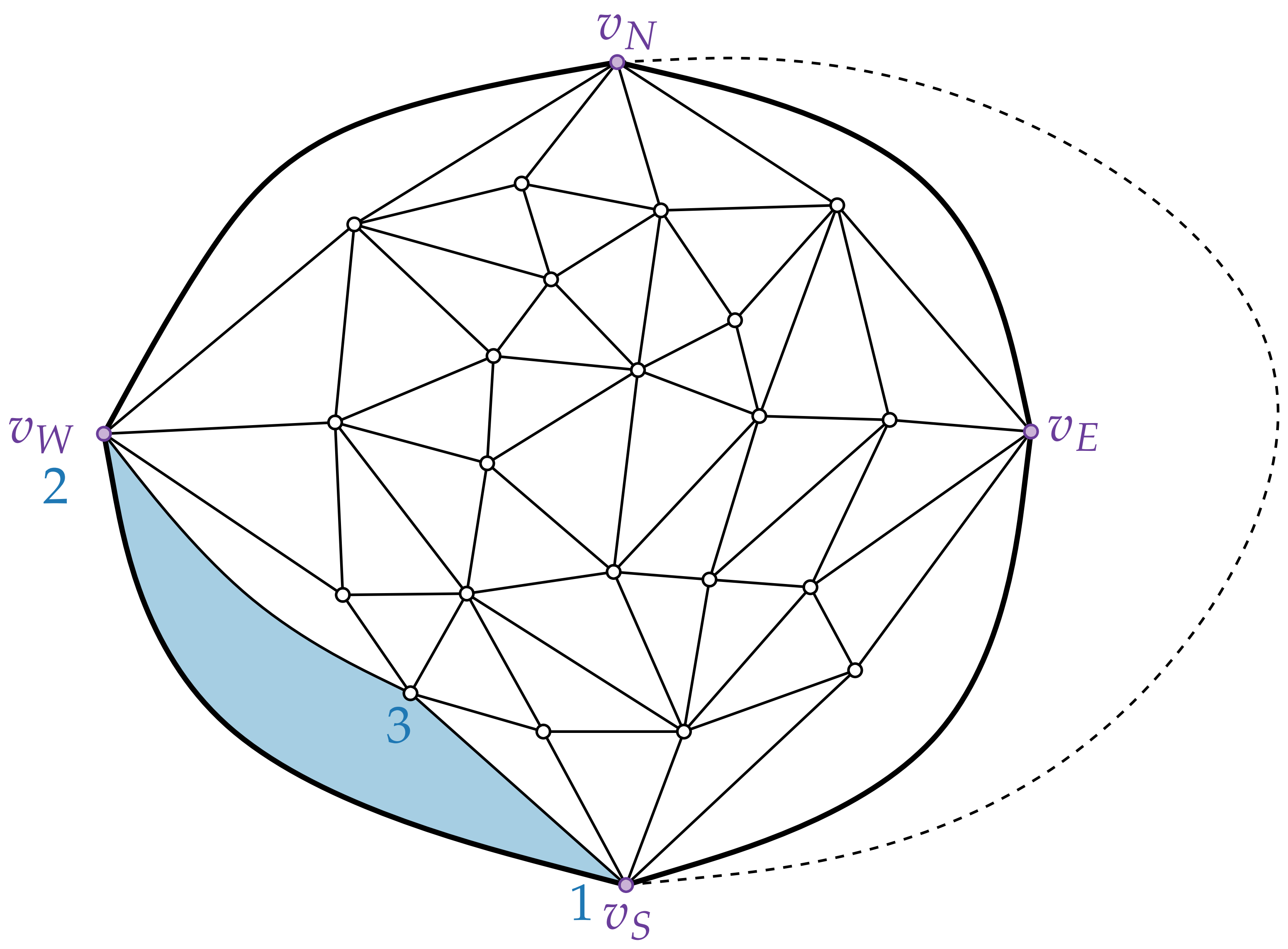
Refined Canonical Order Example



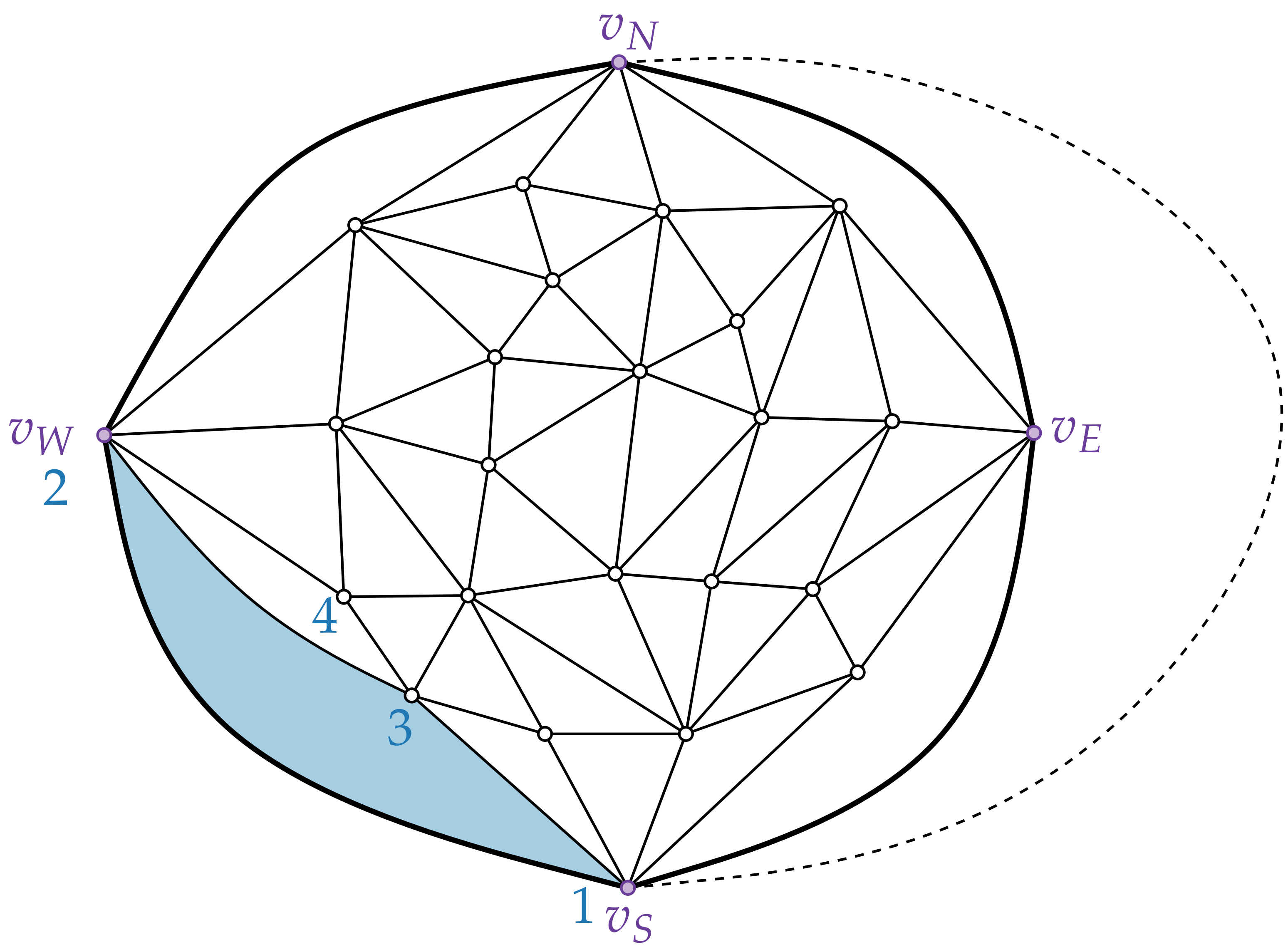
Refined Canonical Order Example



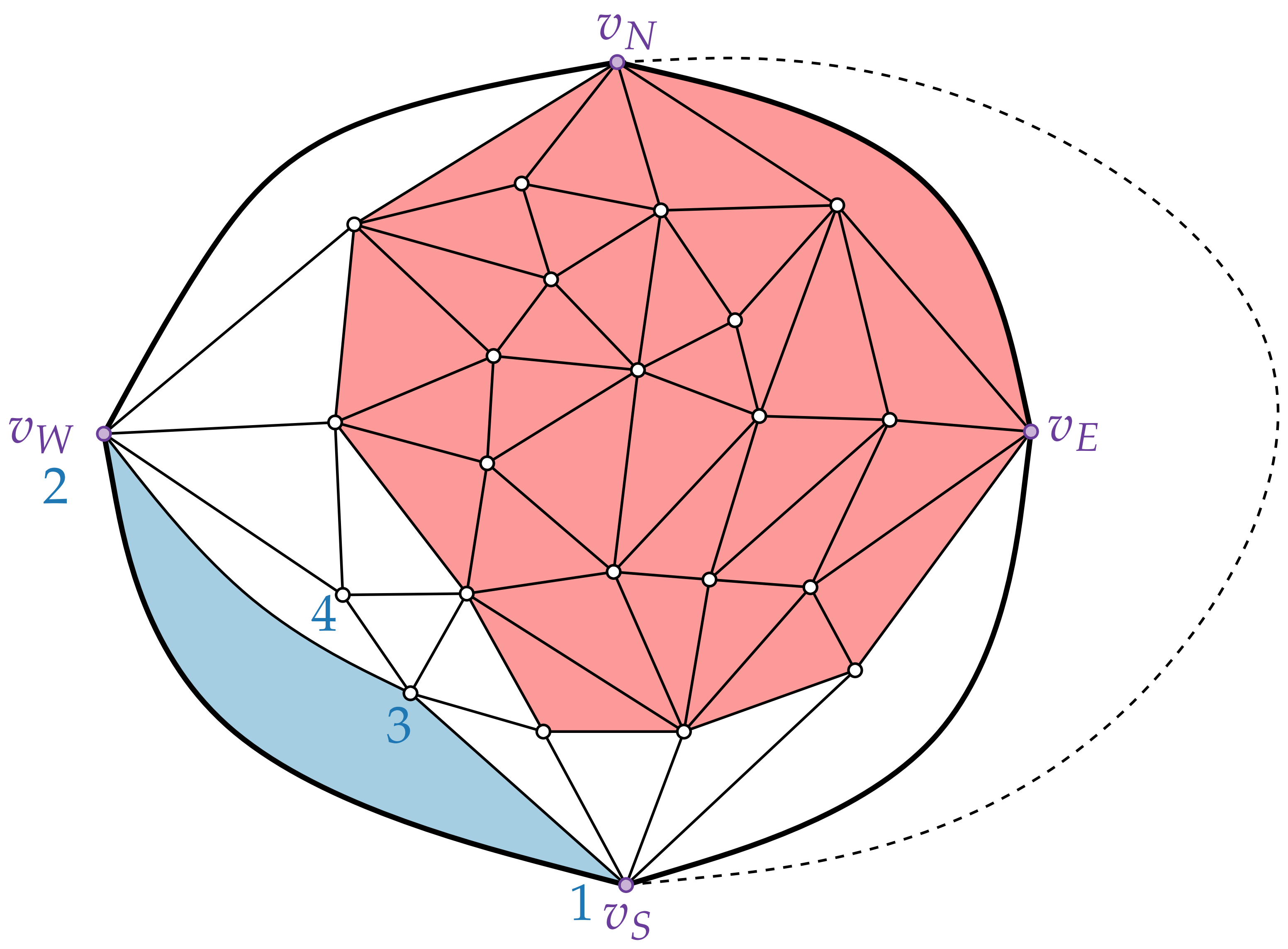
Refined Canonical Order Example



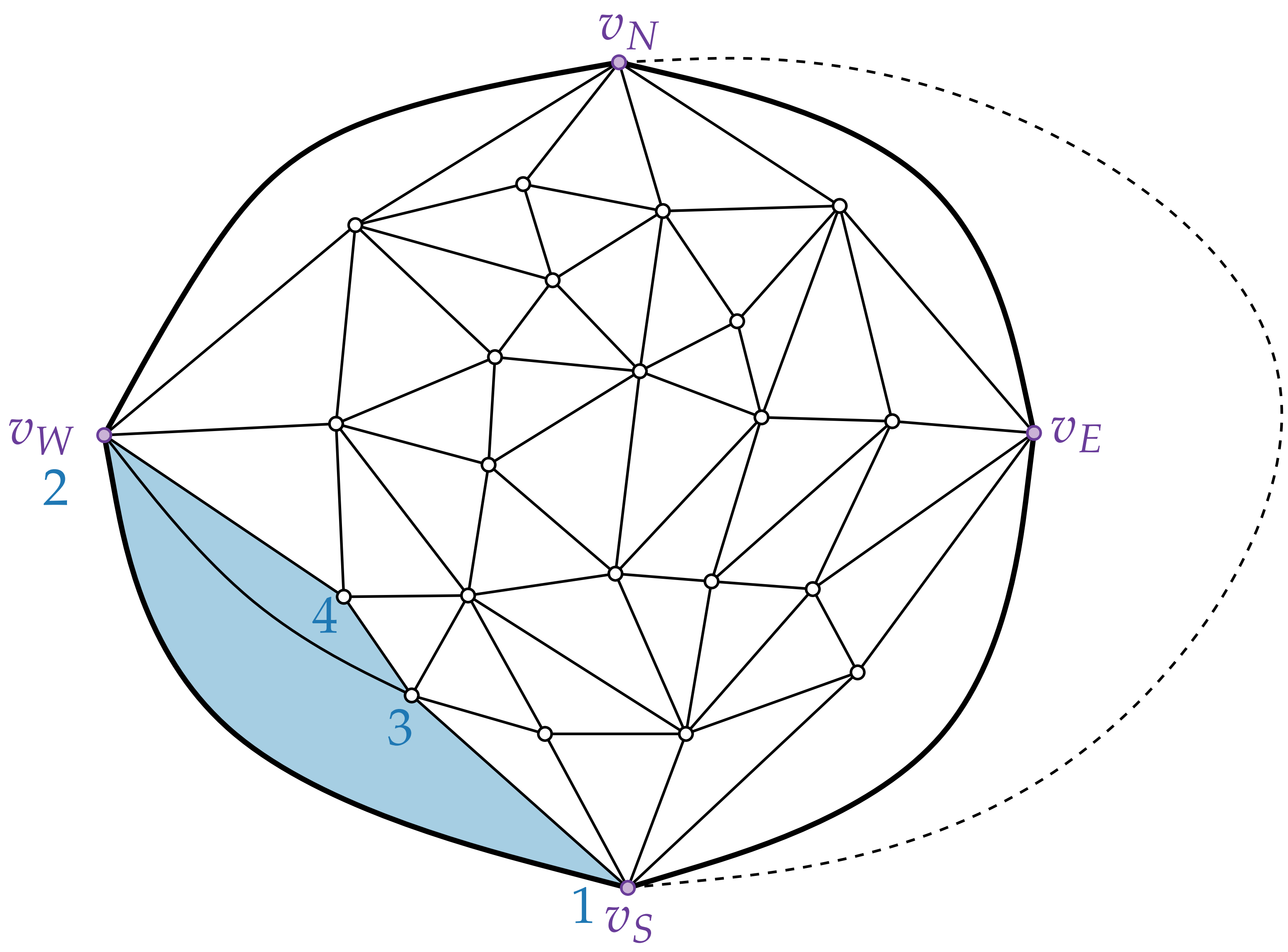
Refined Canonical Order Example



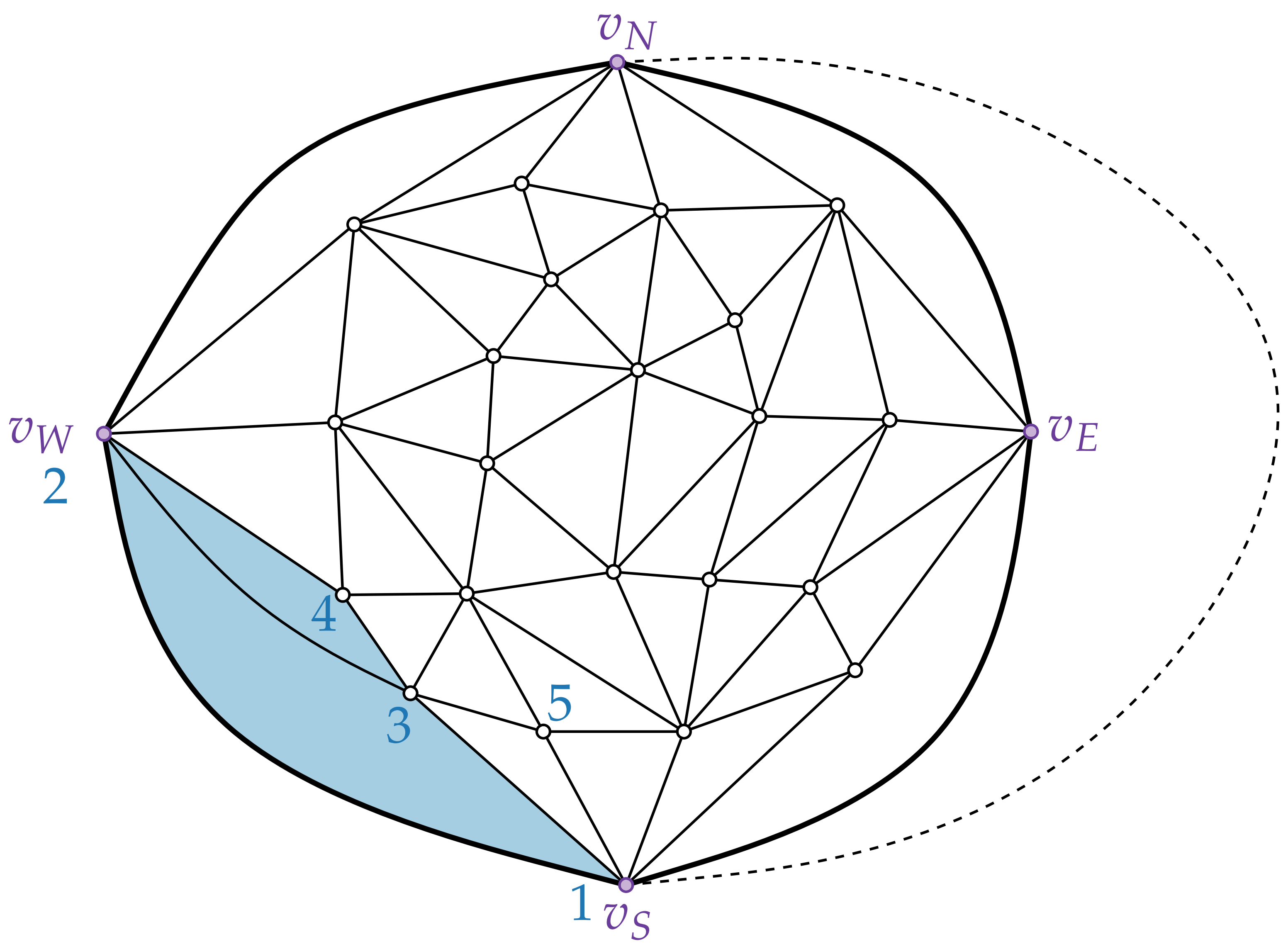
Refined Canonical Order Example



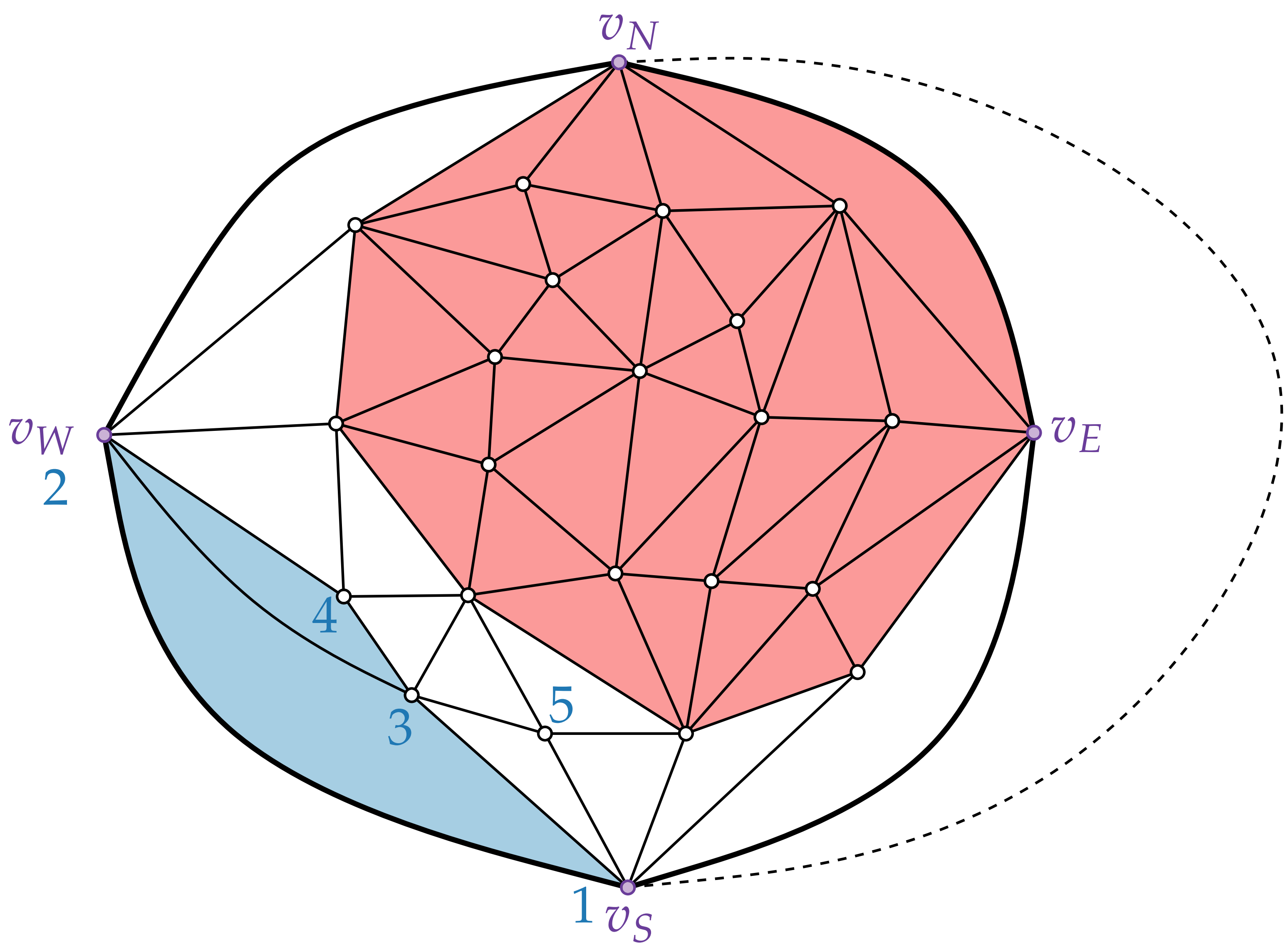
Refined Canonical Order Example



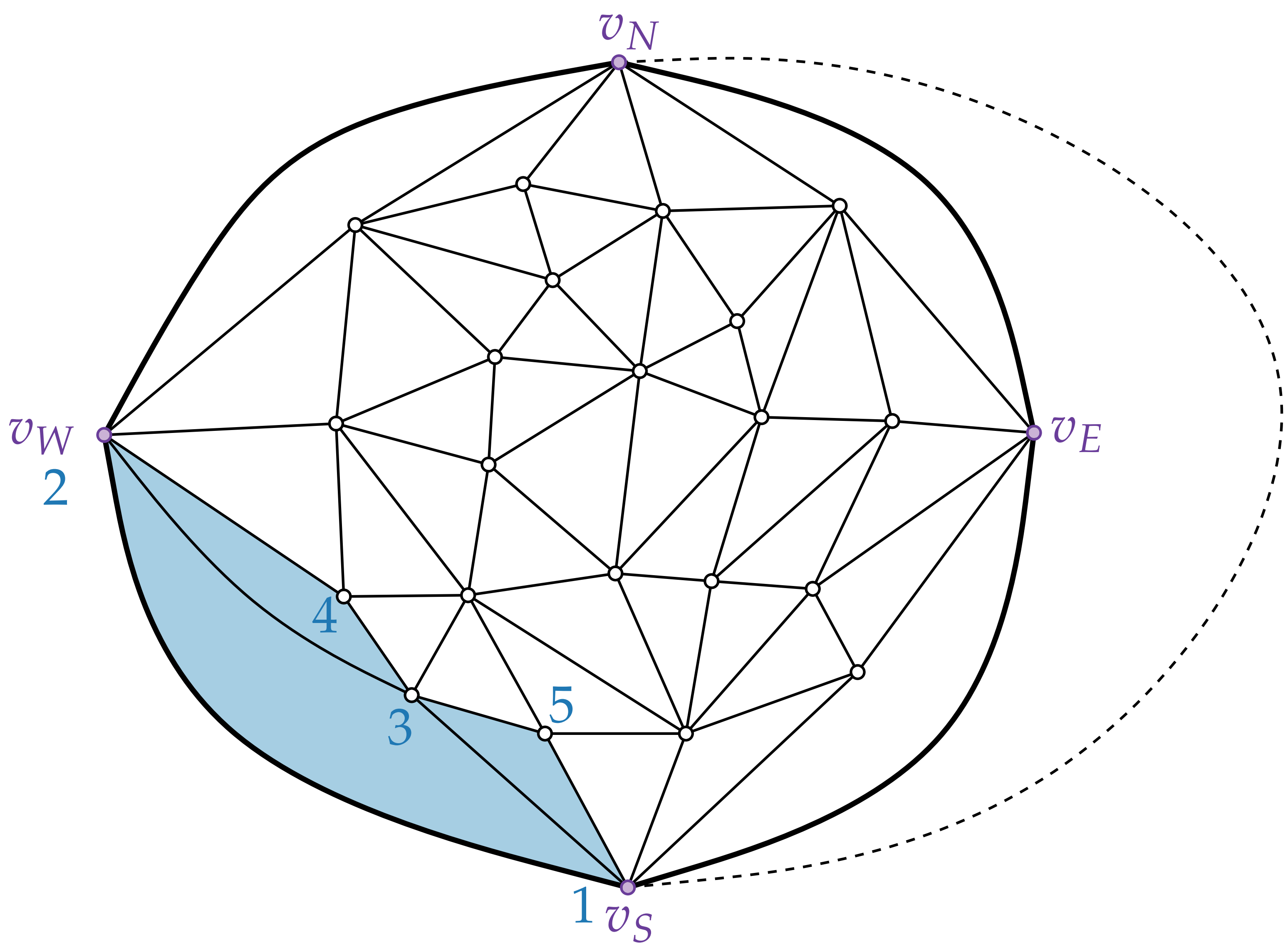
Refined Canonical Order Example



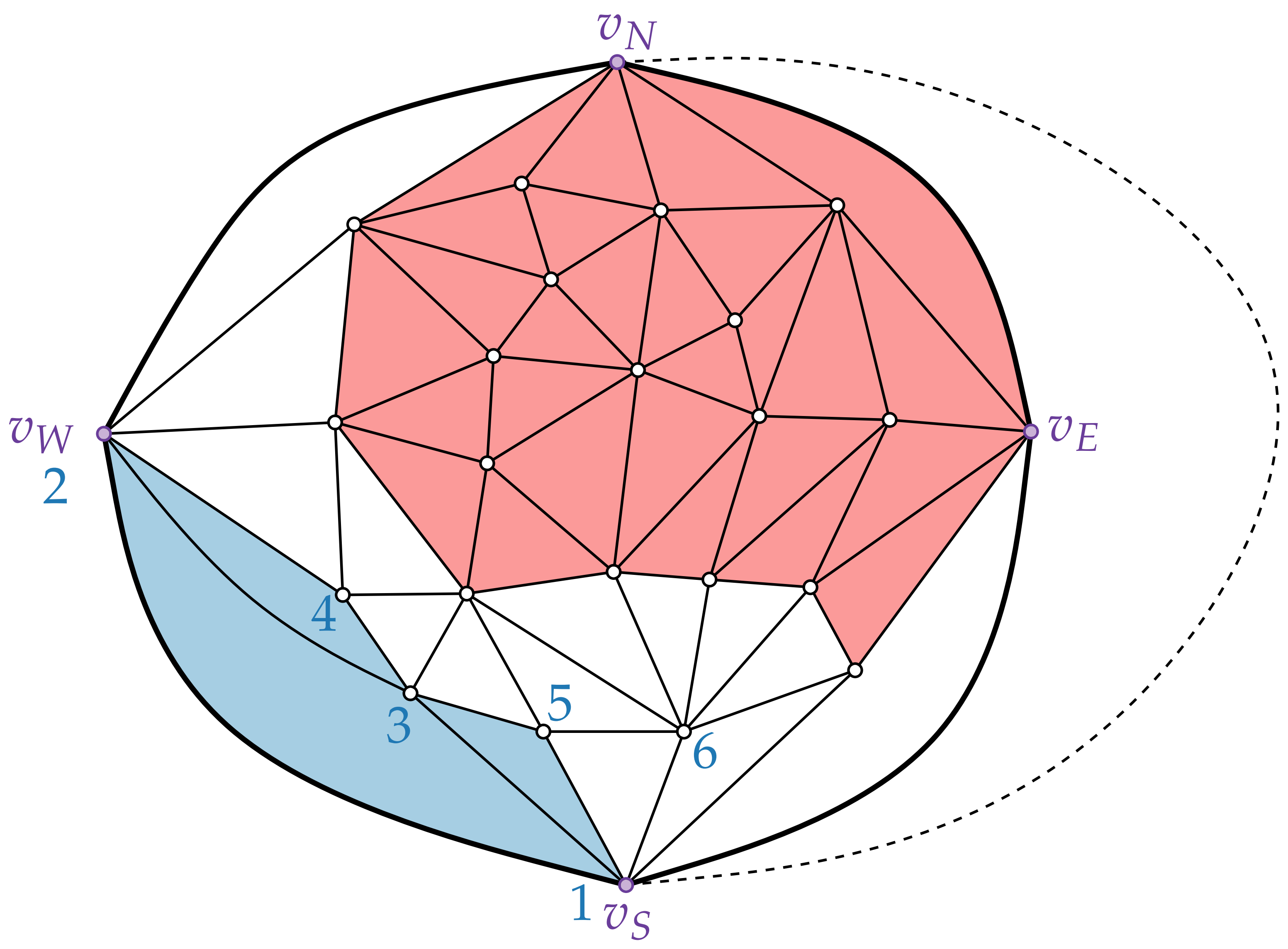
Refined Canonical Order Example



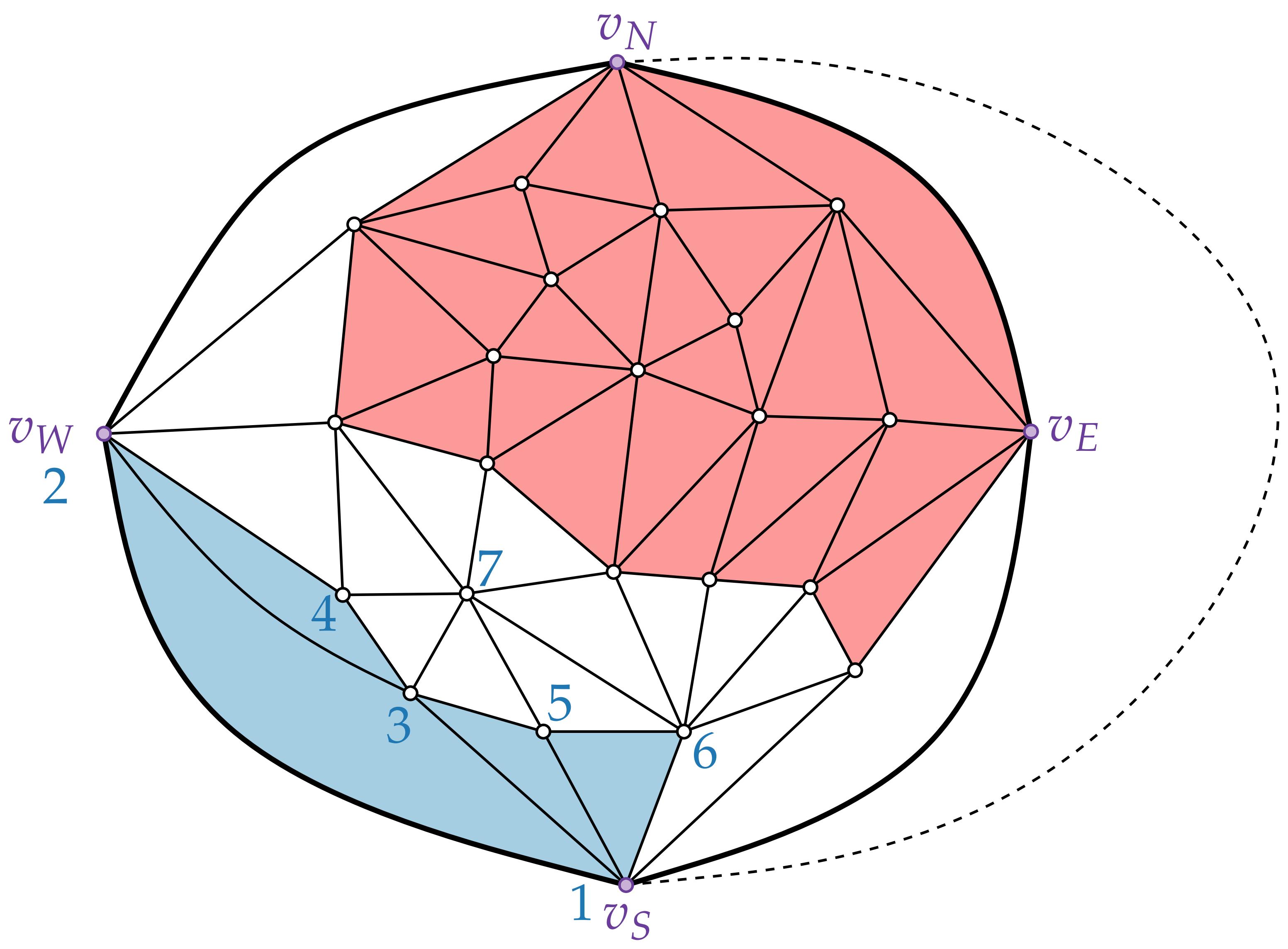
Refined Canonical Order Example



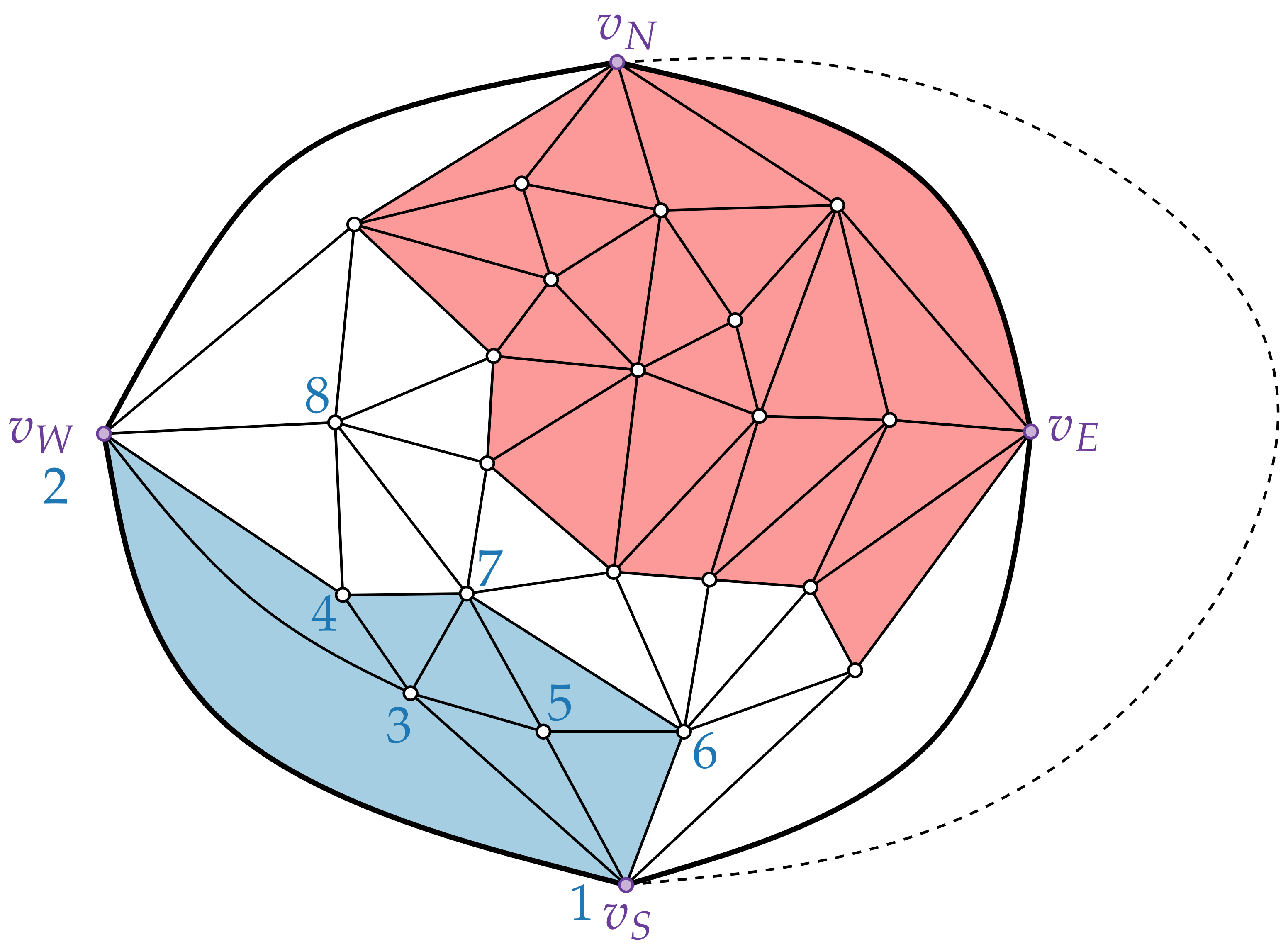
Refined Canonical Order Example



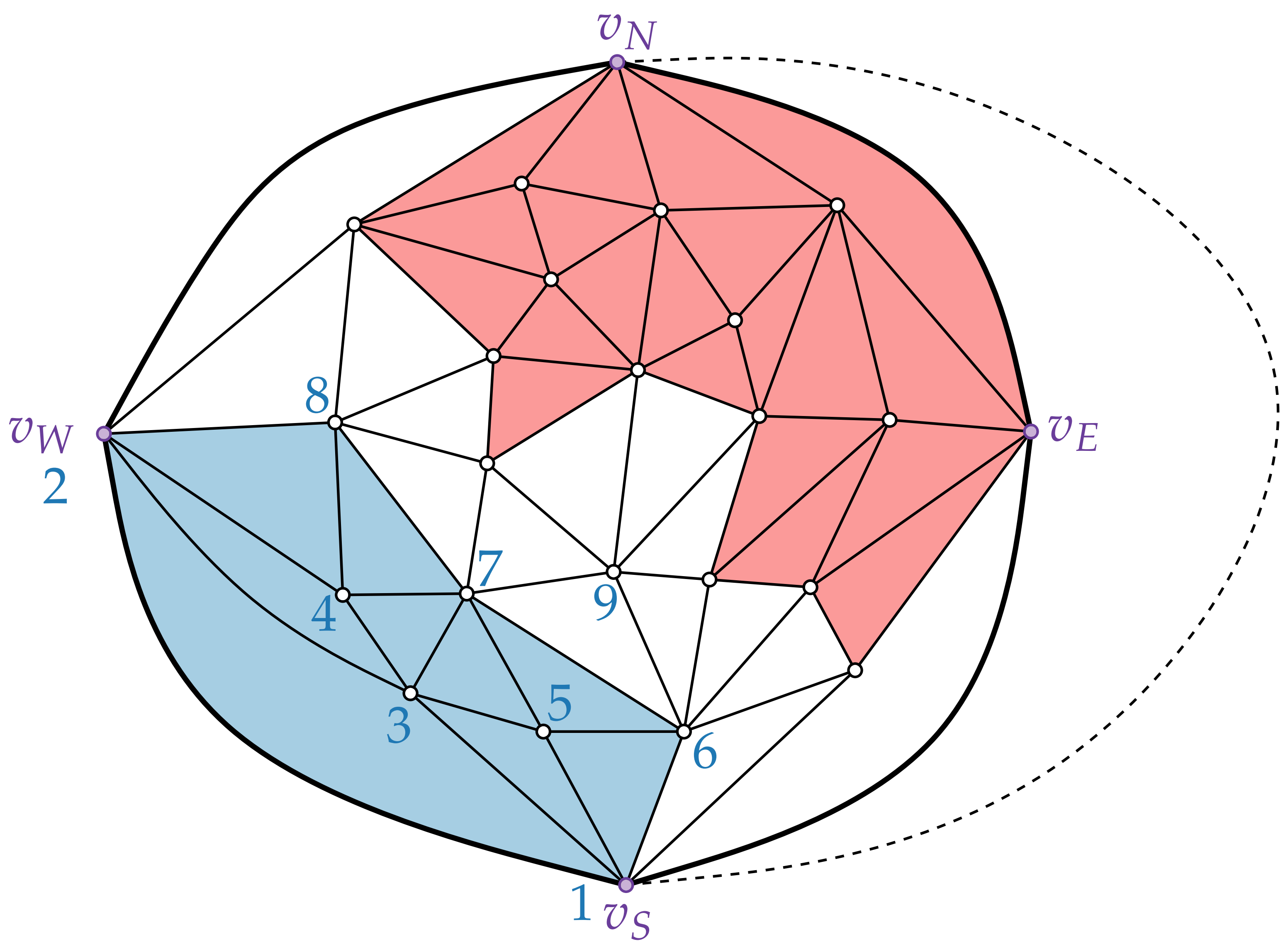
Refined Canonical Order Example



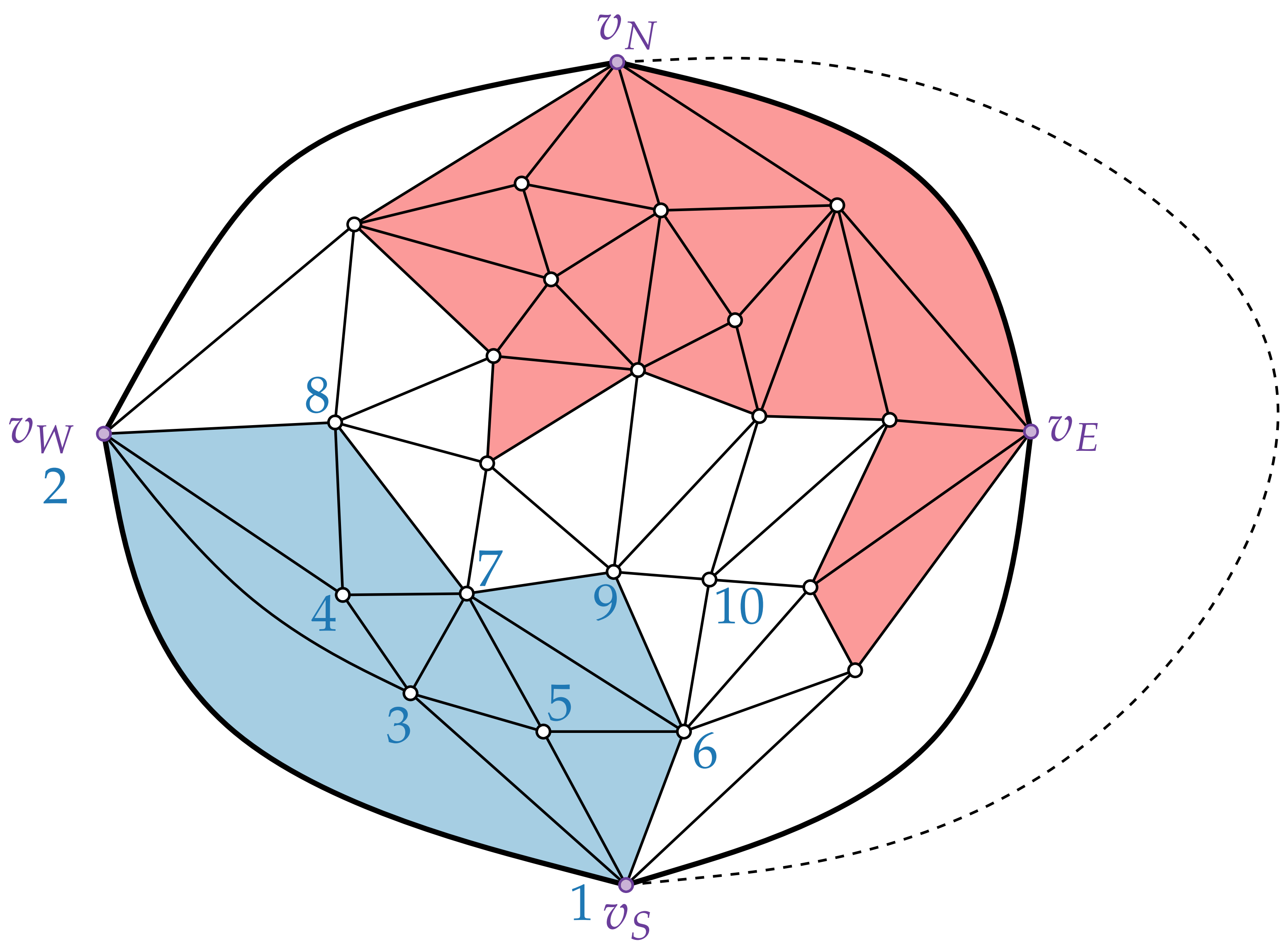
Refined Canonical Order Example



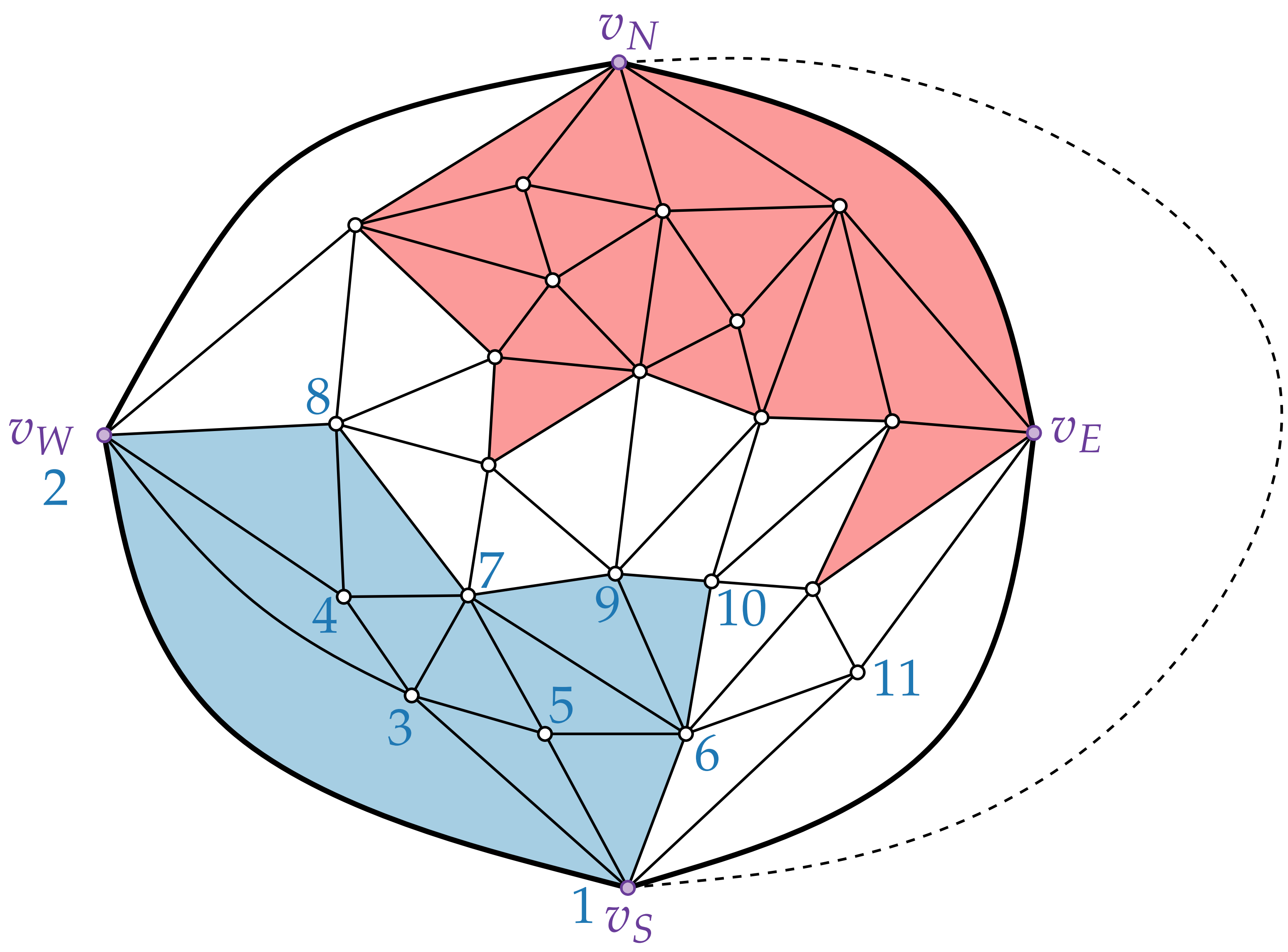
Refined Canonical Order Example



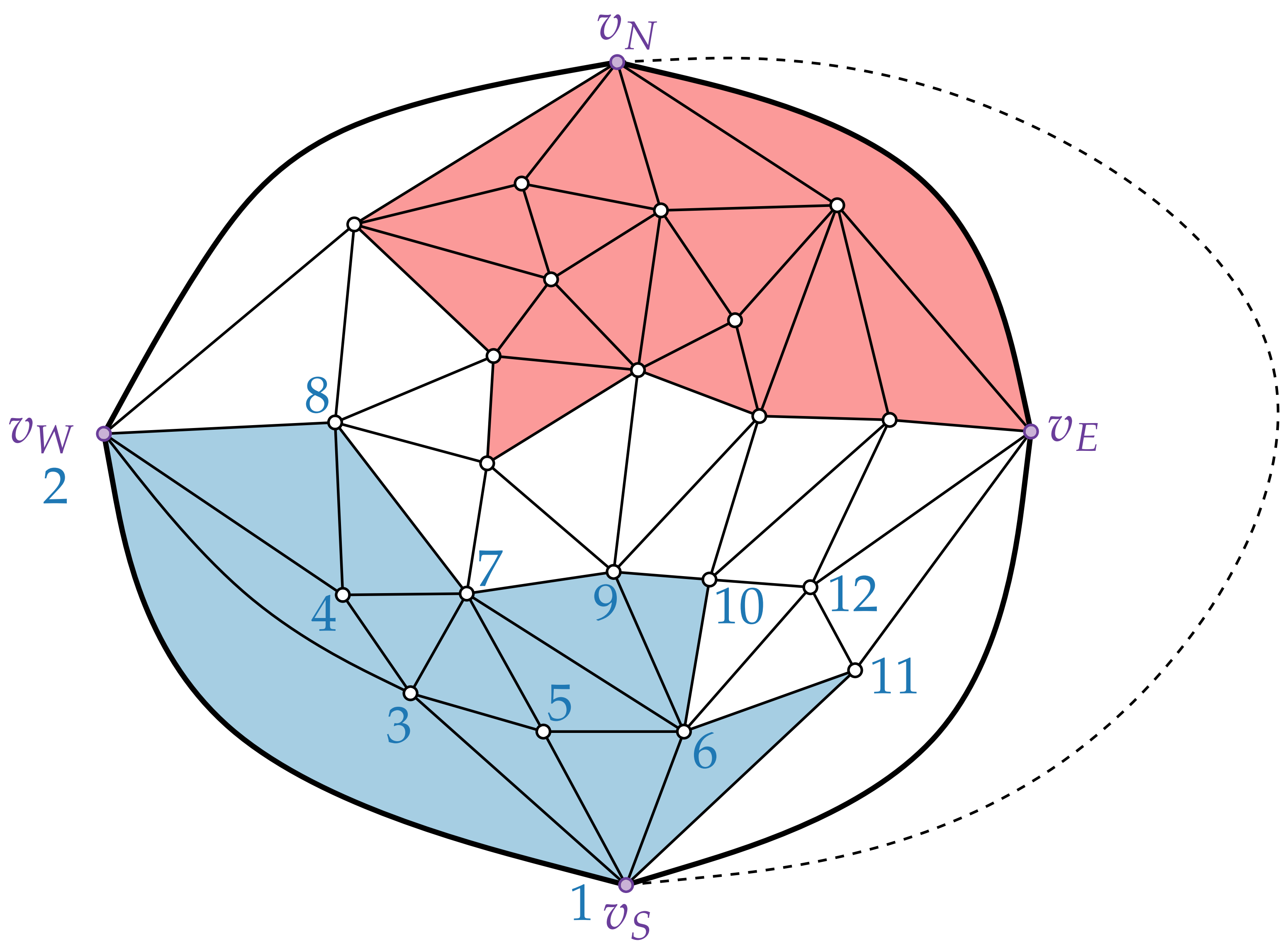
Refined Canonical Order Example



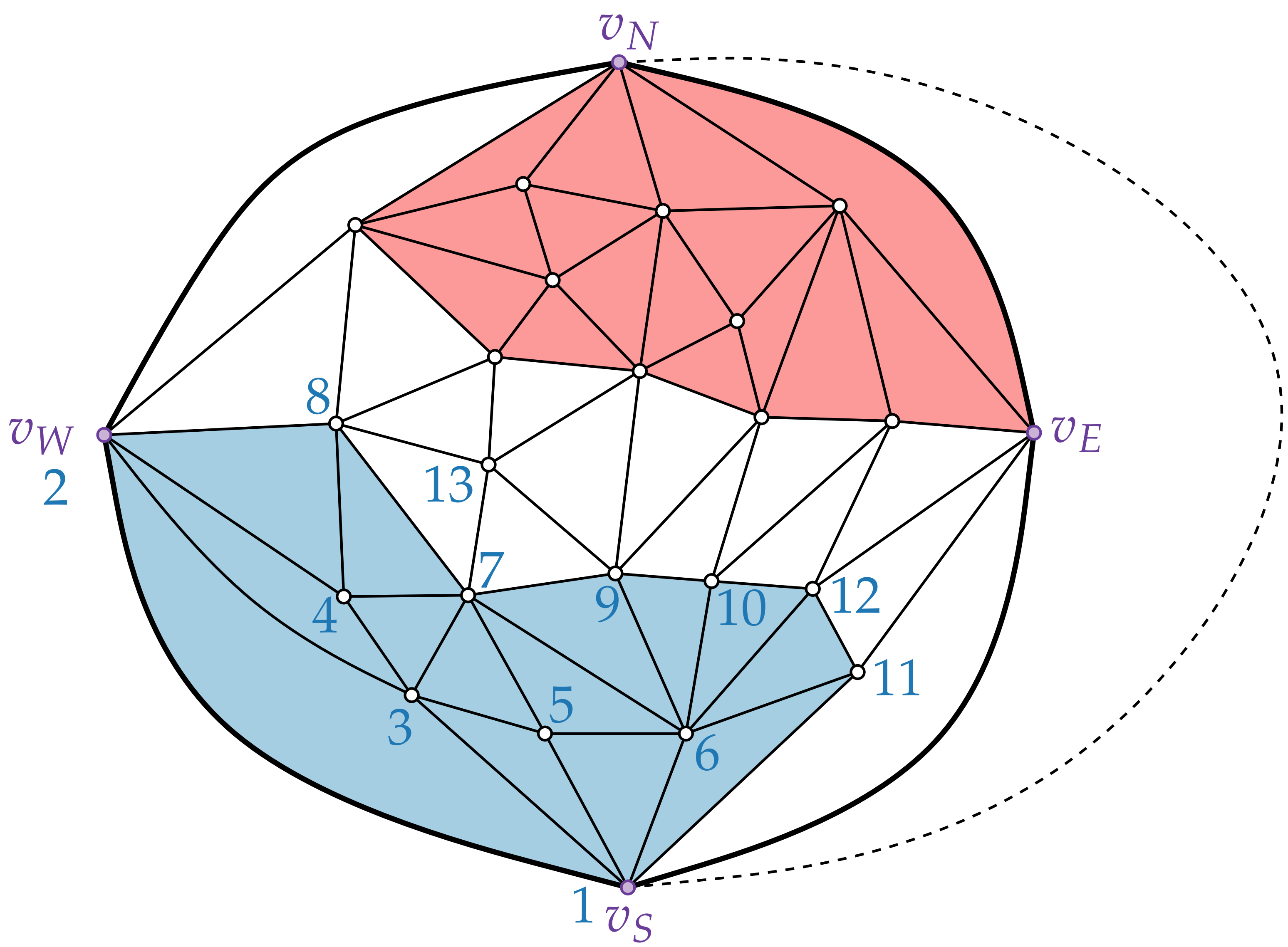
Refined Canonical Order Example



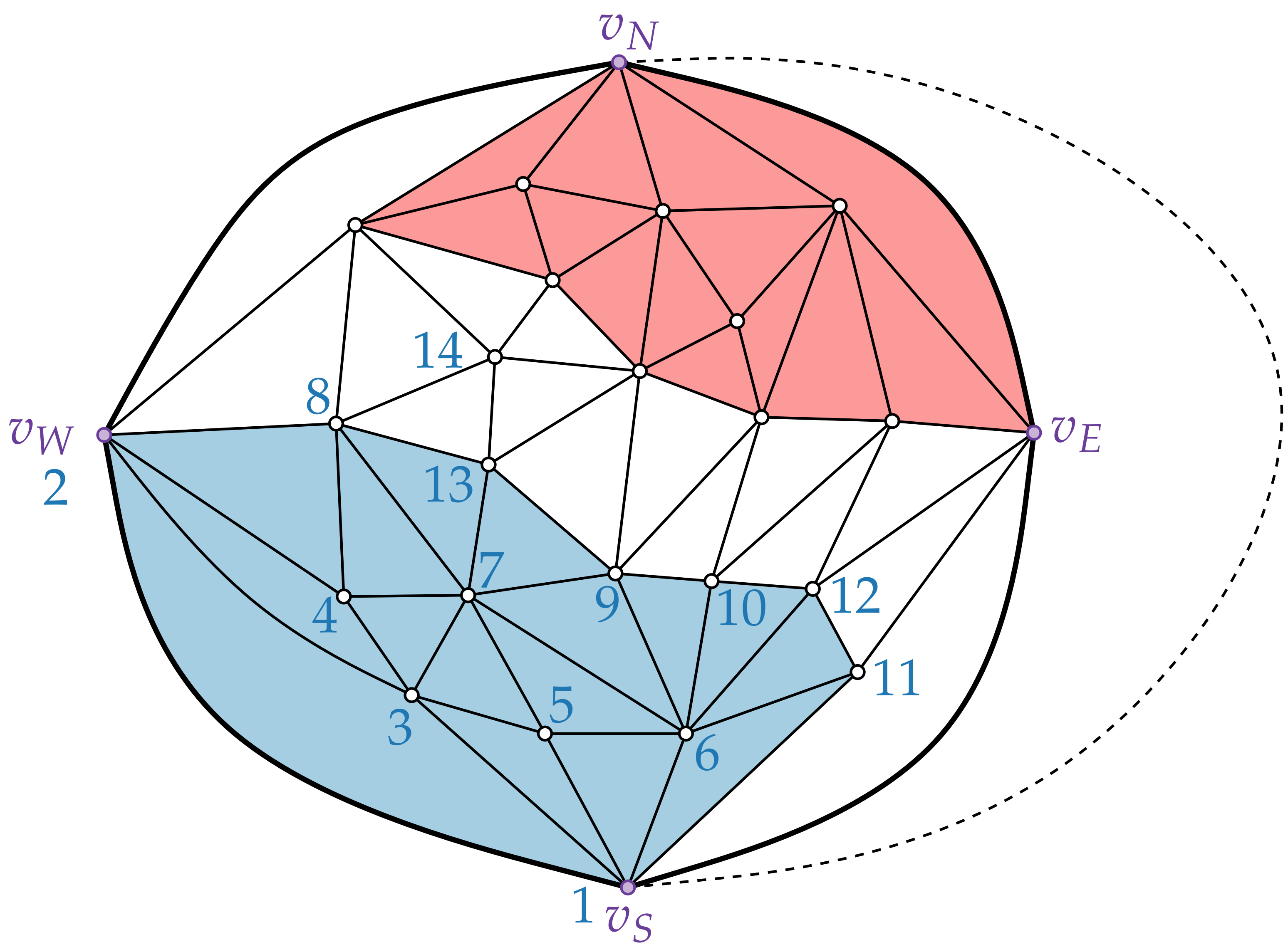
Refined Canonical Order Example



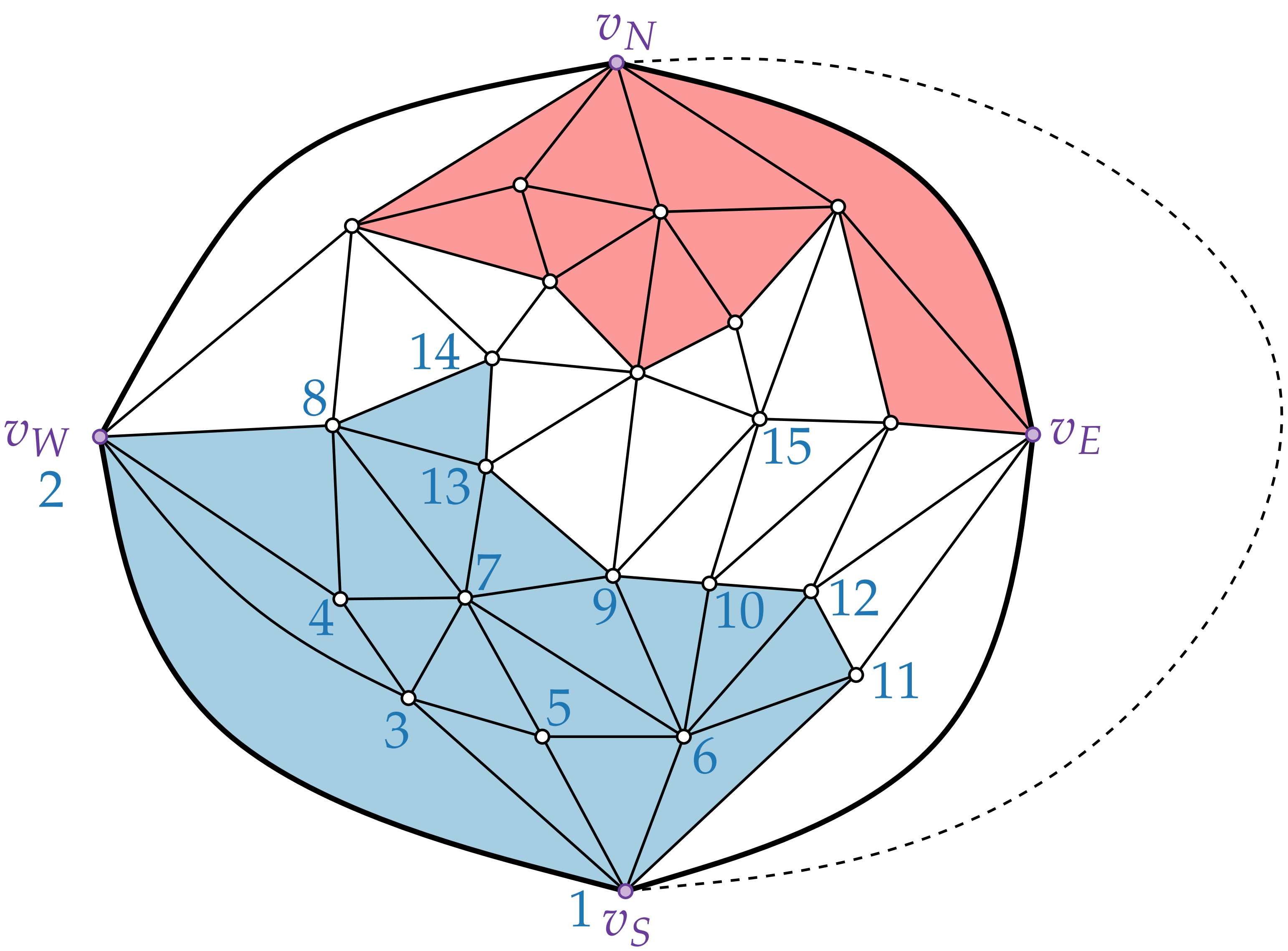
Refined Canonical Order Example



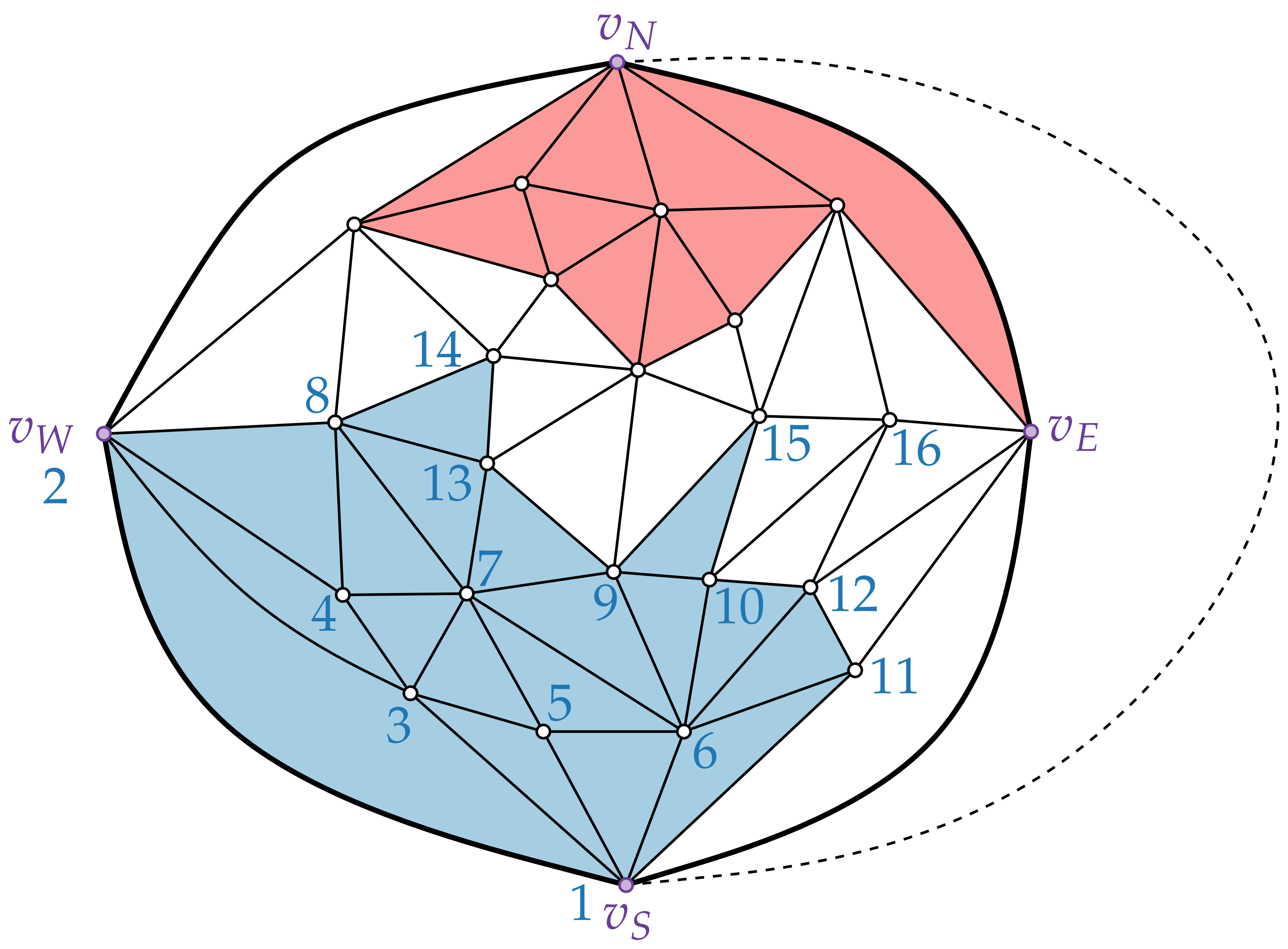
Refined Canonical Order Example



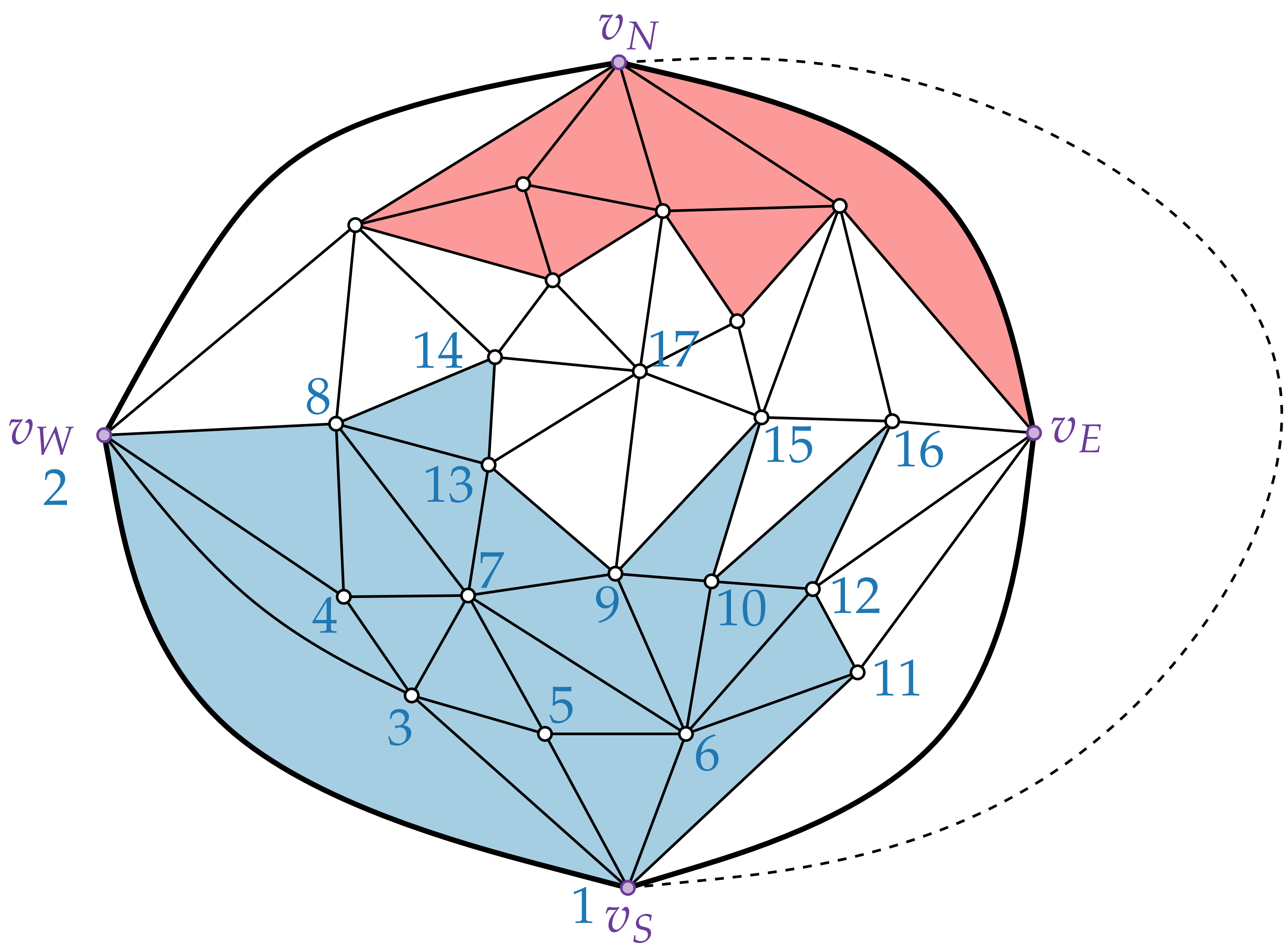
Refined Canonical Order Example



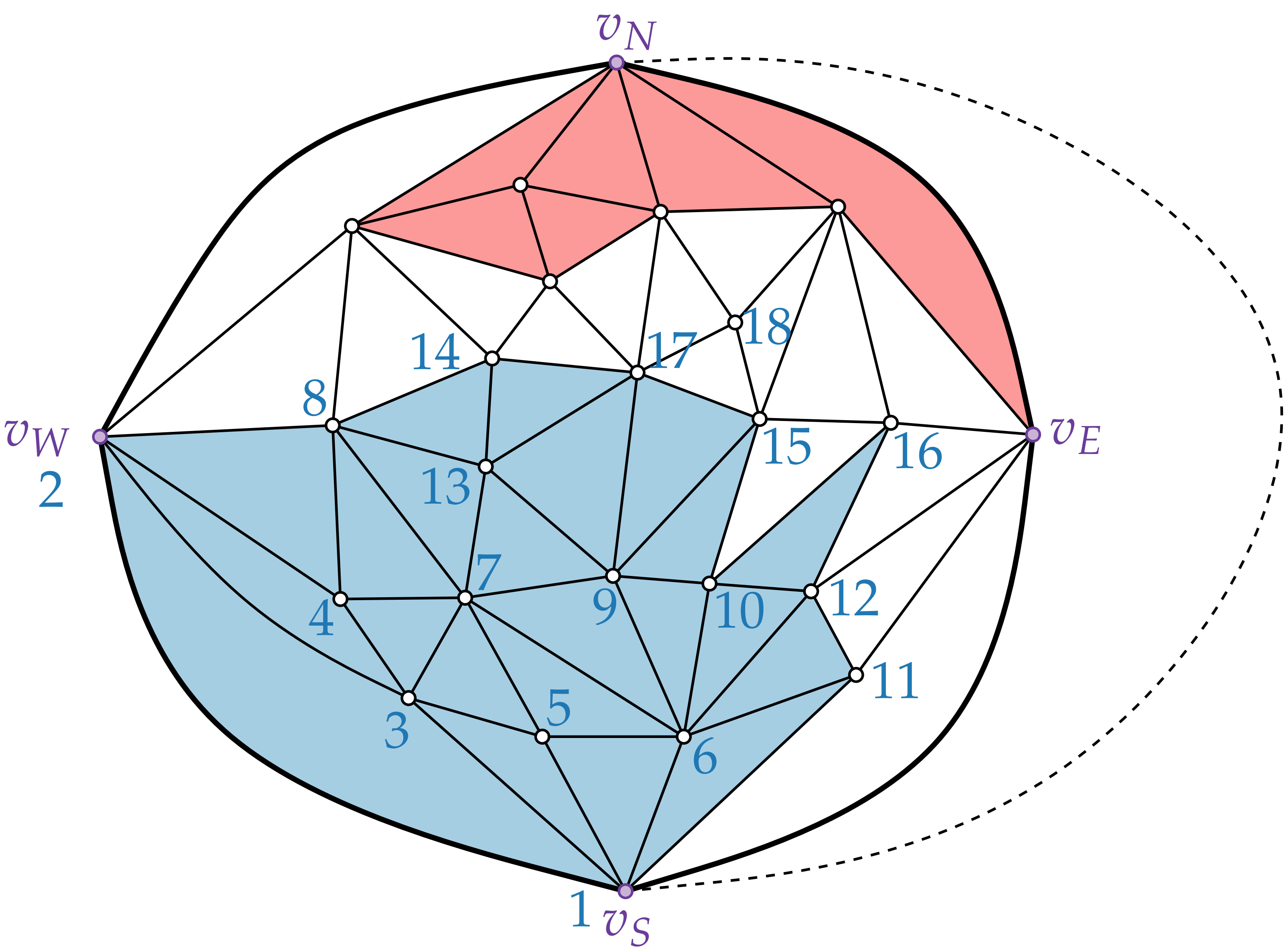
Refined Canonical Order Example



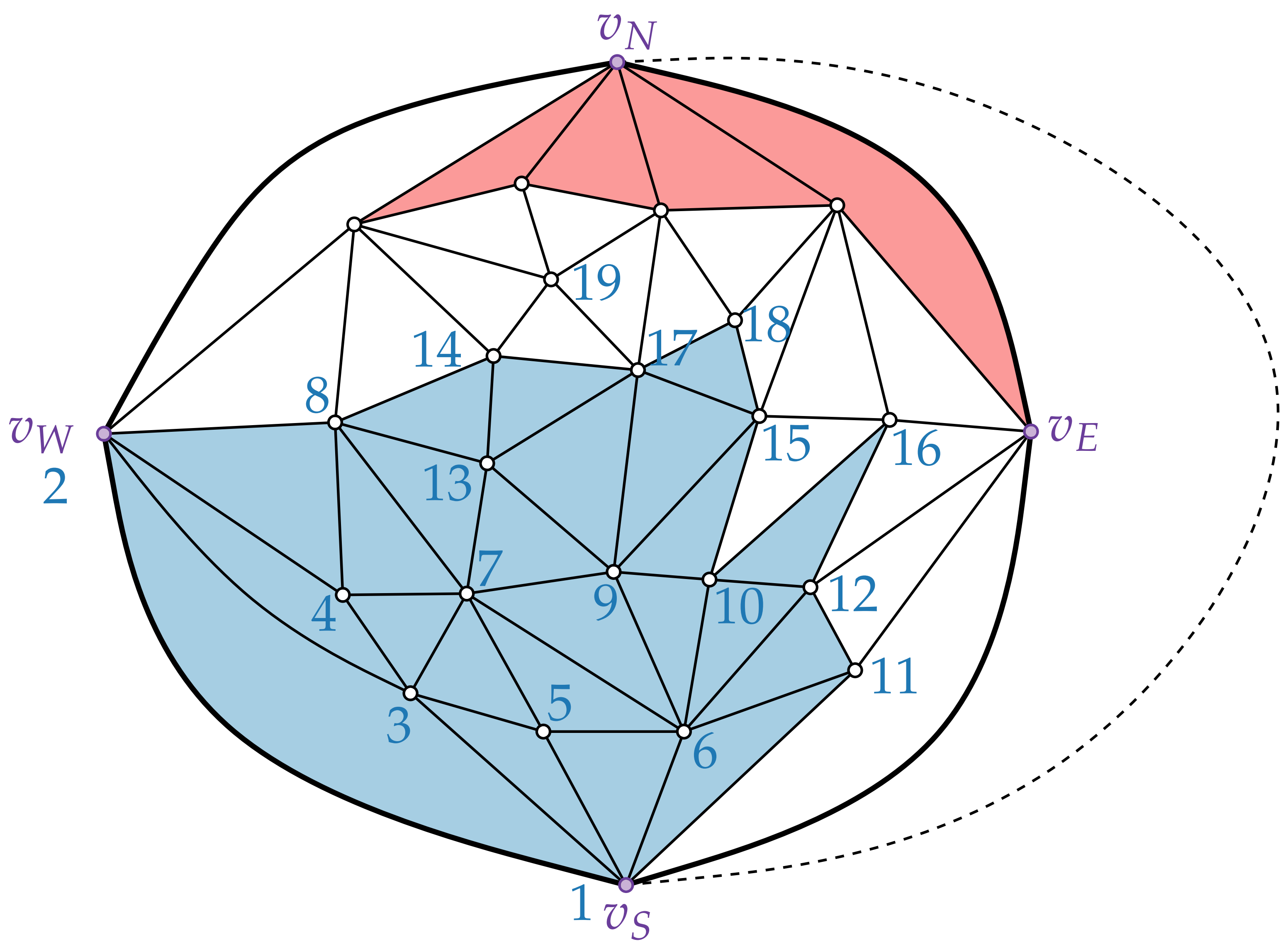
Refined Canonical Order Example



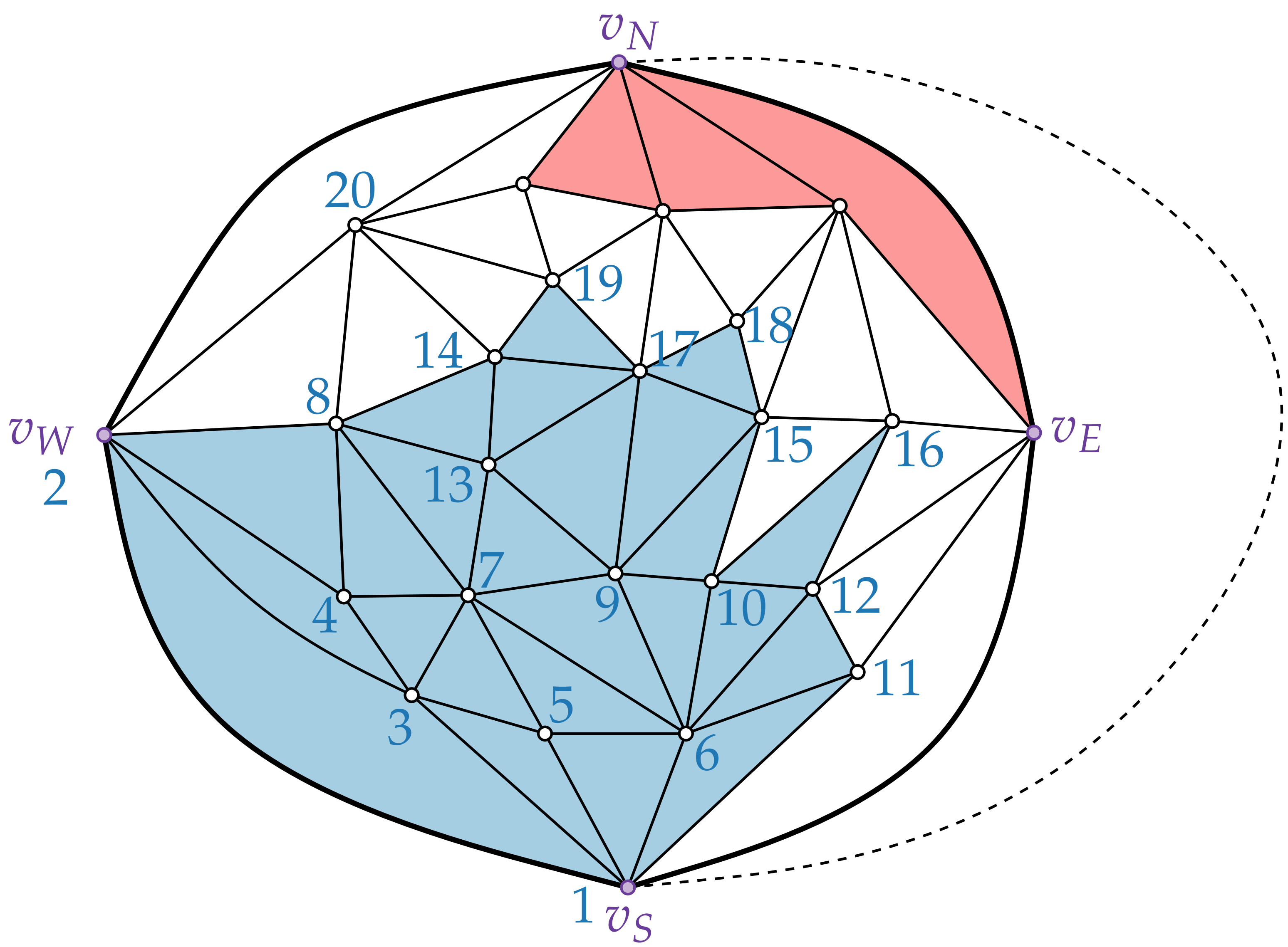
Refined Canonical Order Example



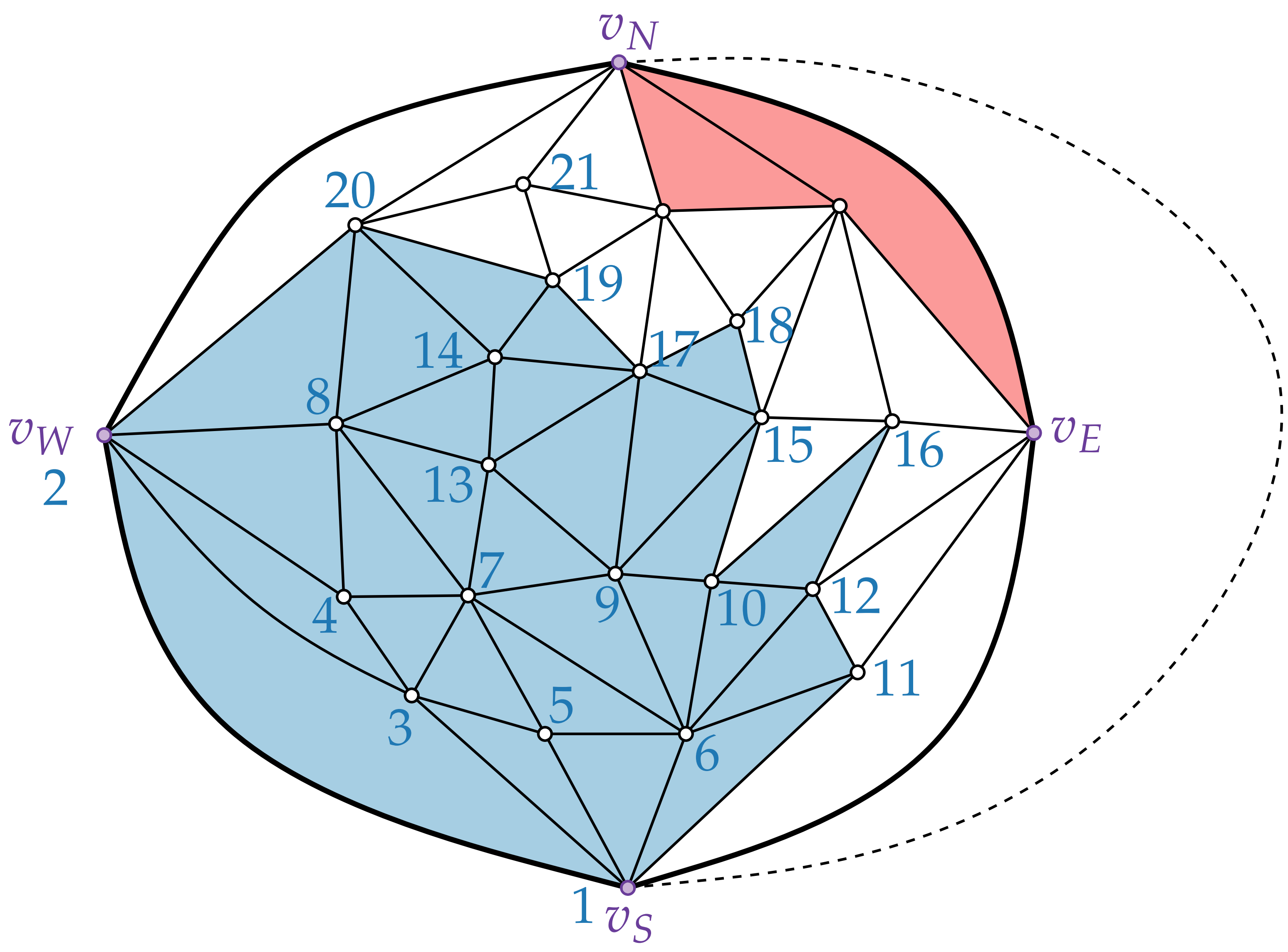
Refined Canonical Order Example



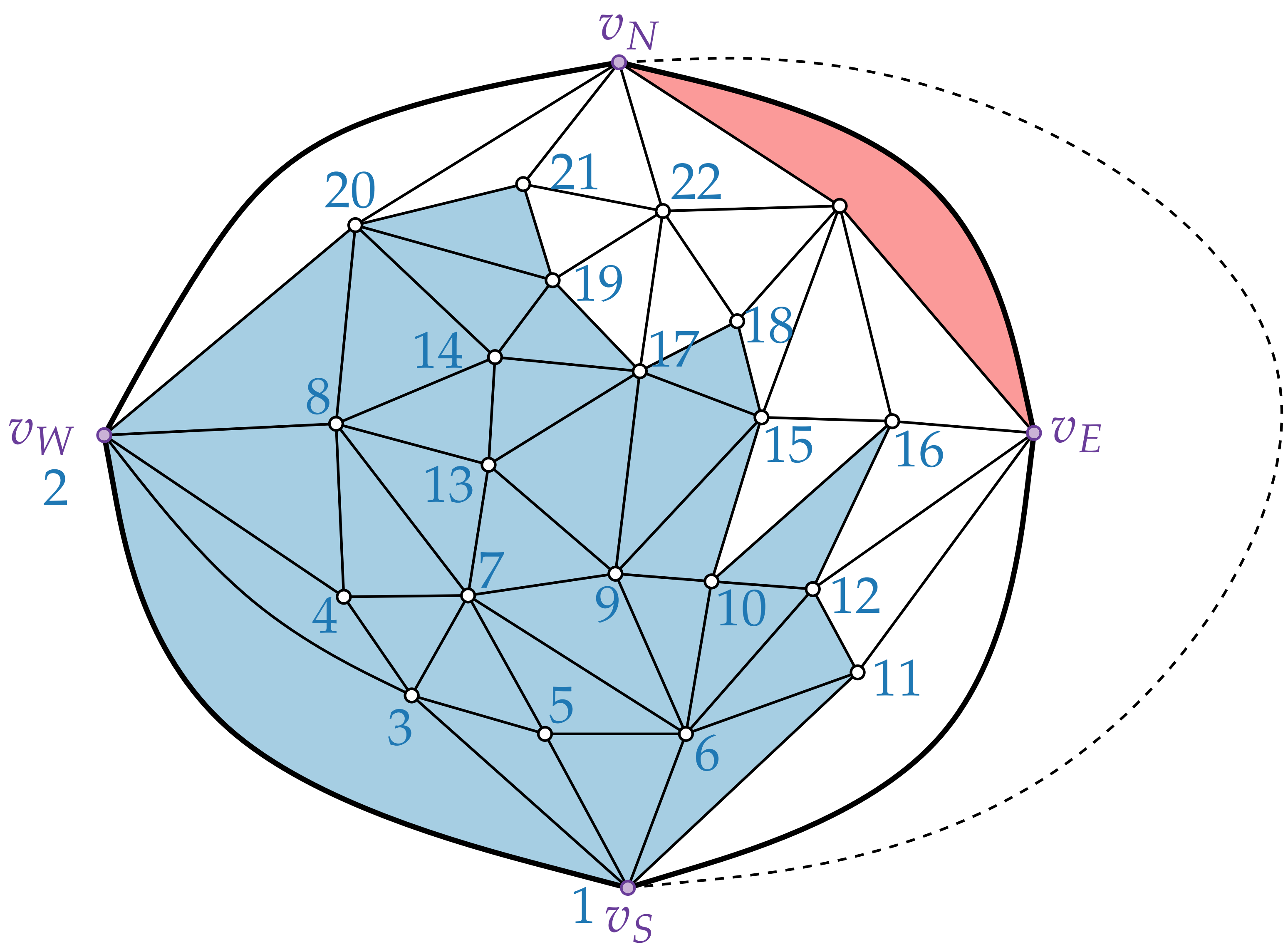
Refined Canonical Order Example



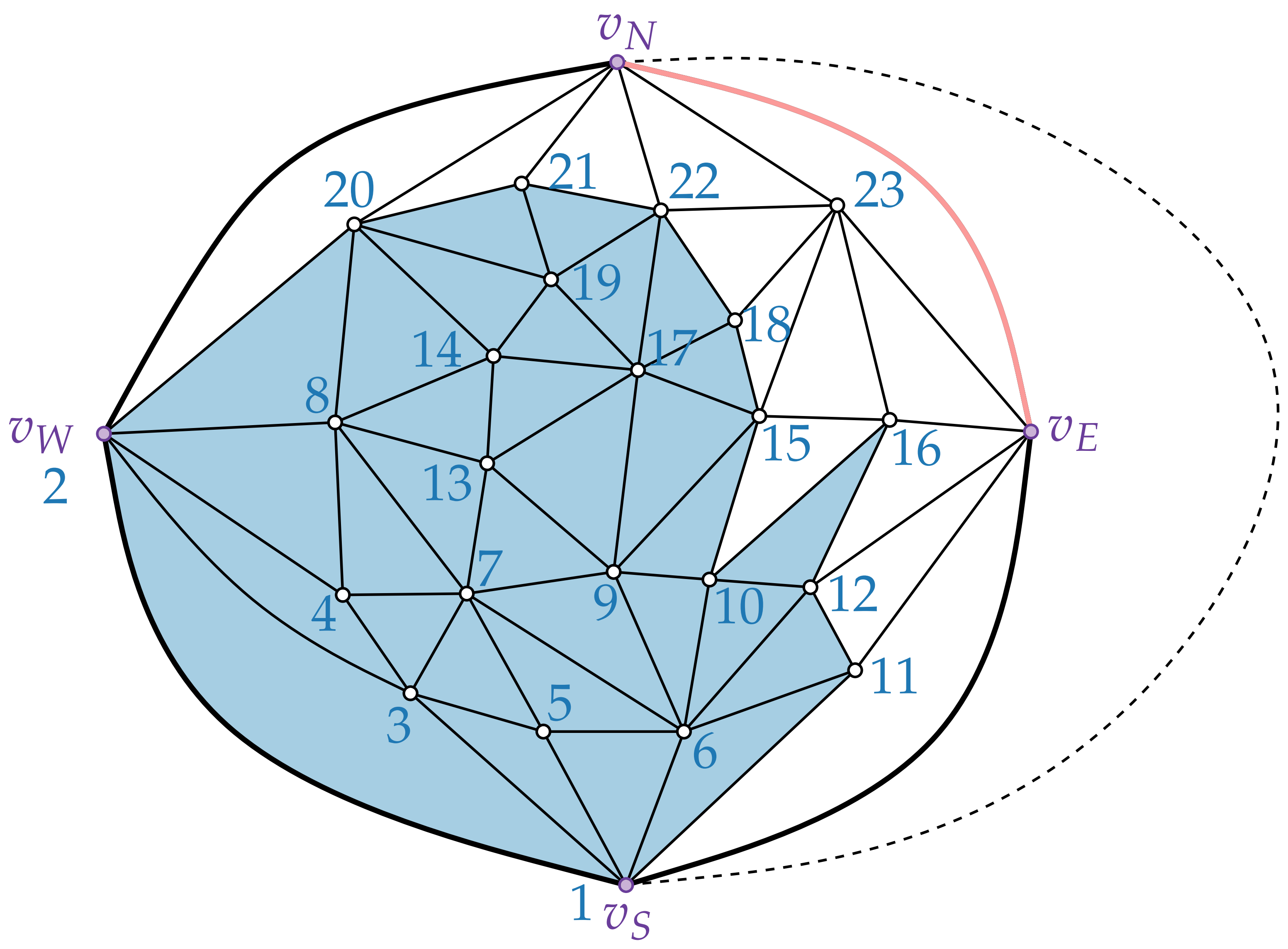
Refined Canonical Order Example



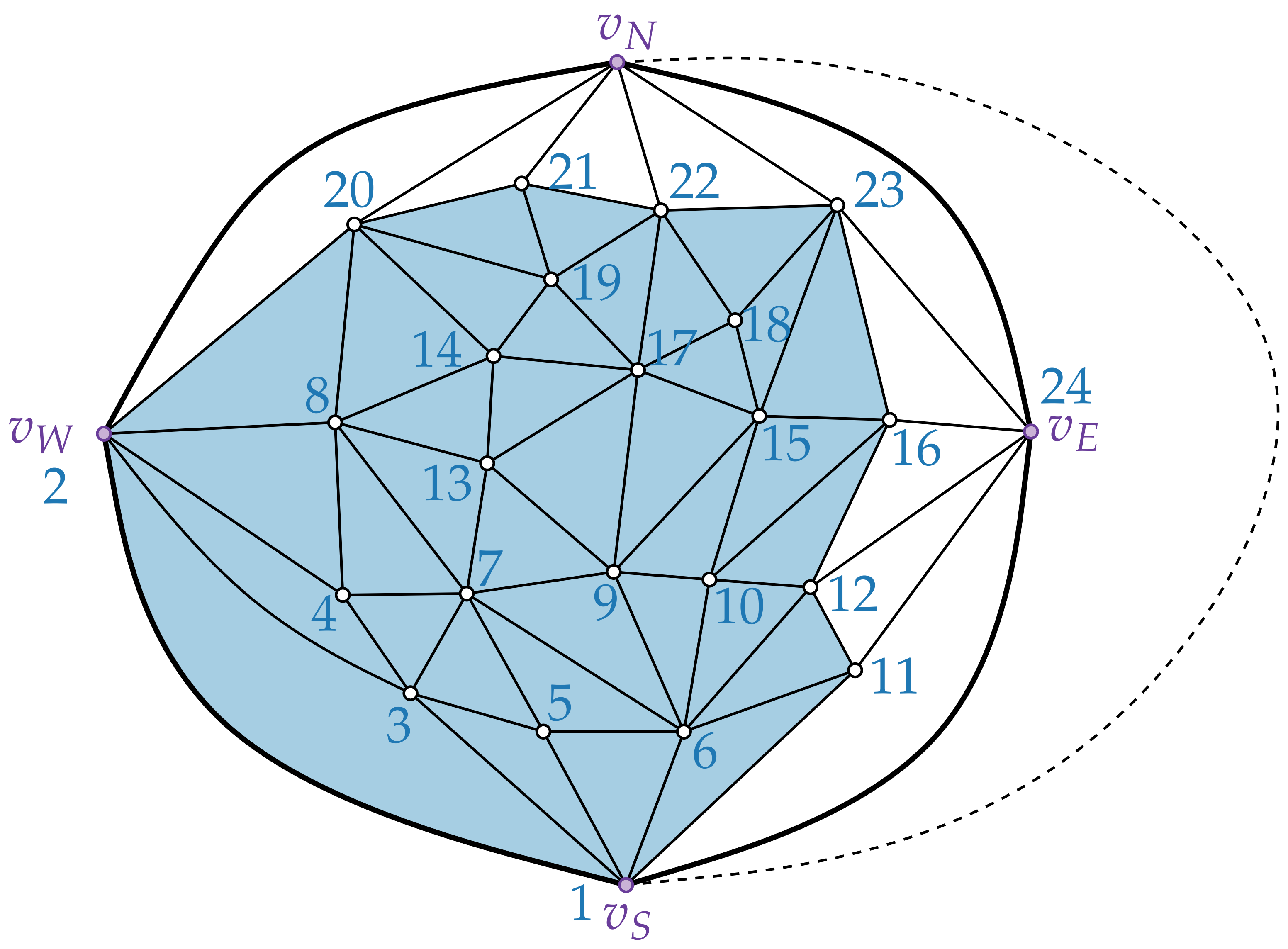
Refined Canonical Order Example



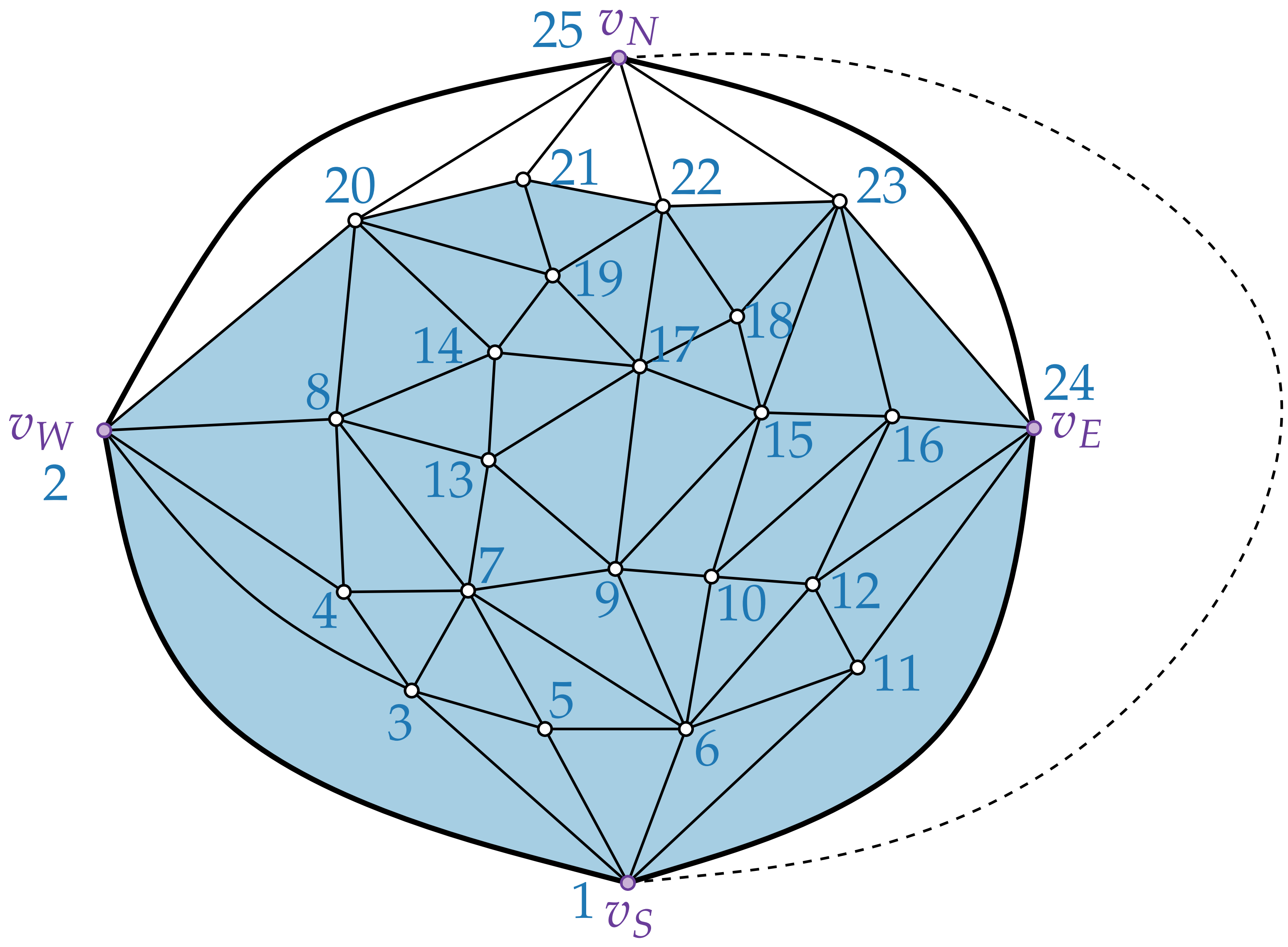
Refined Canonical Order Example



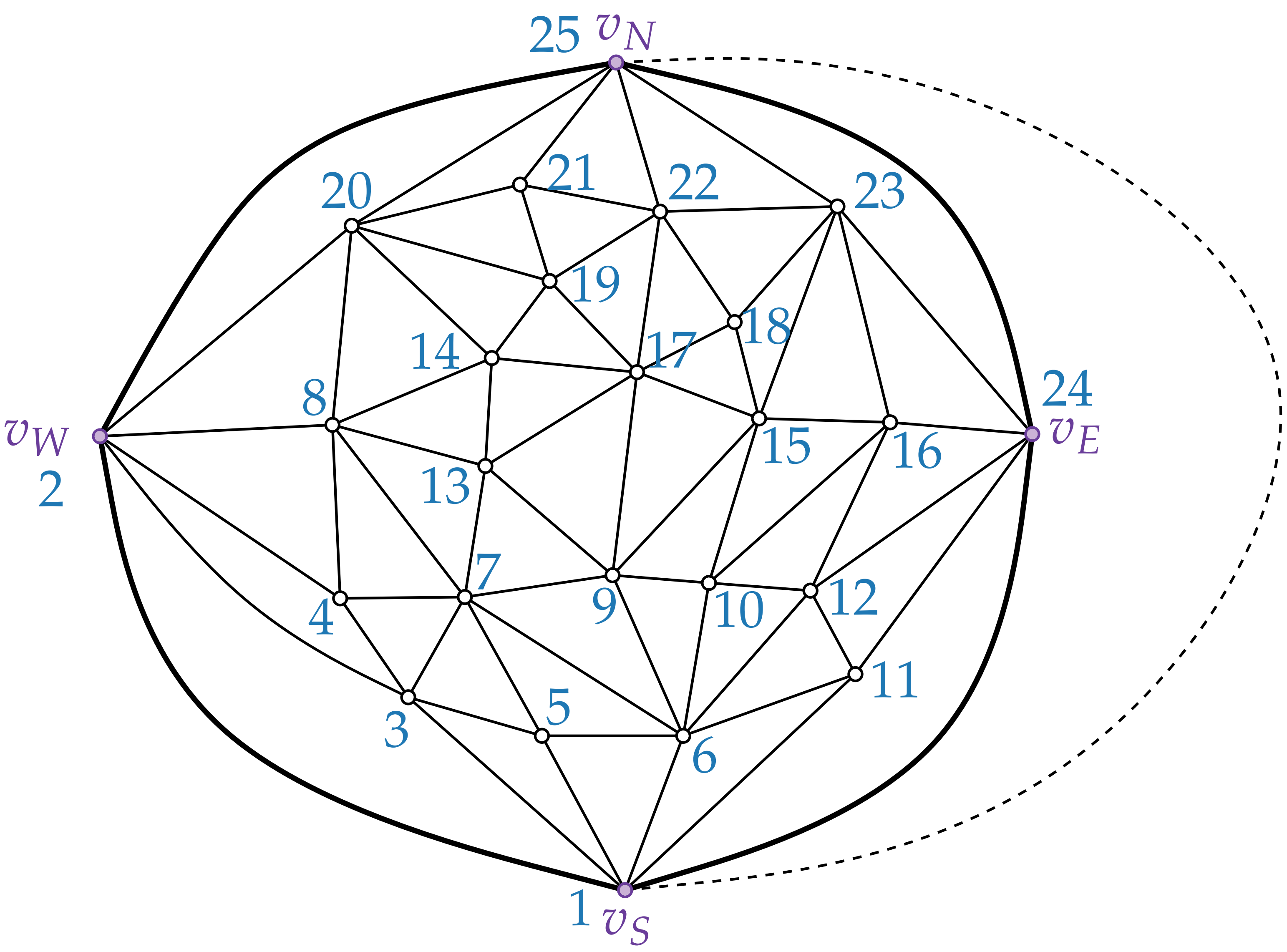
Refined Canonical Order Example



Refined Canonical Order Example

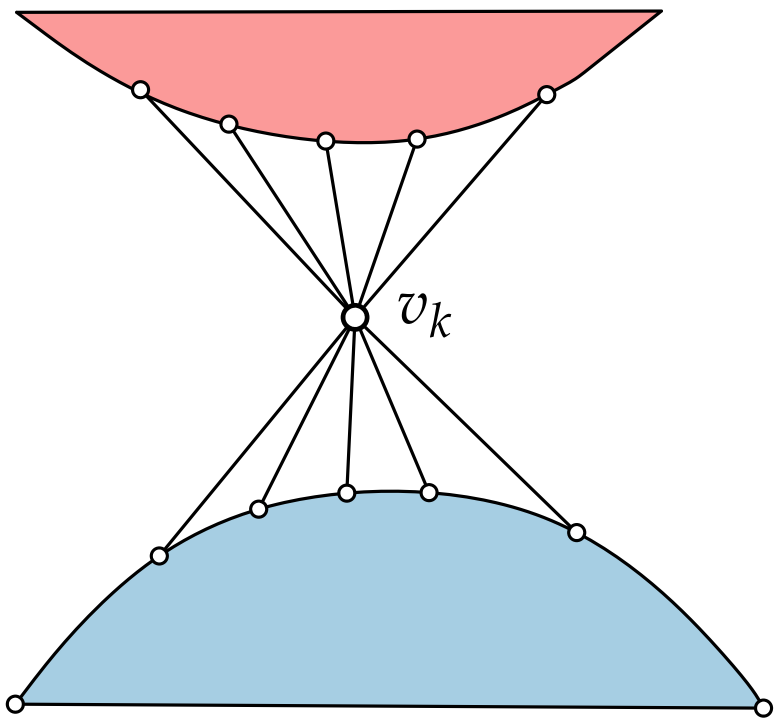


Refined Canonical Order Example



Refined Canonical Order → REL

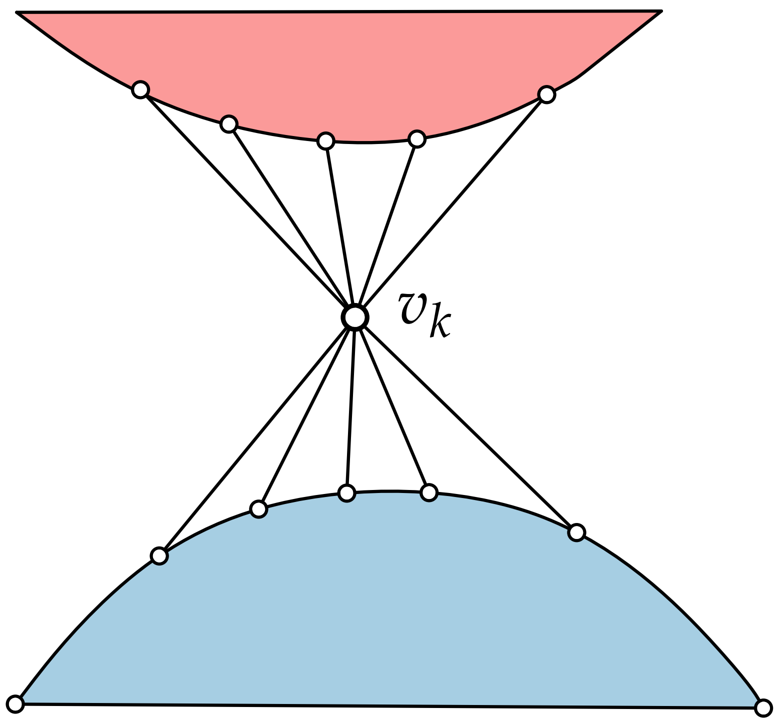
We construct a REL as follows:



Refined Canonical Order → REL

We construct a REL as follows:

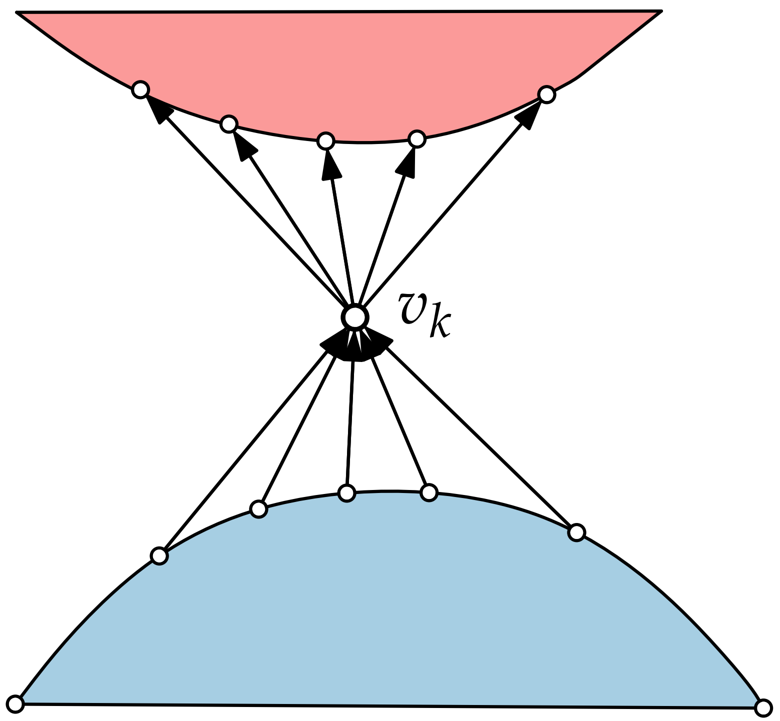
- For $i < j$, orient (v_i, v_j) from v_i to v_j;



Refined Canonical Order $\rightarrow$ REL

We construct a REL as follows:

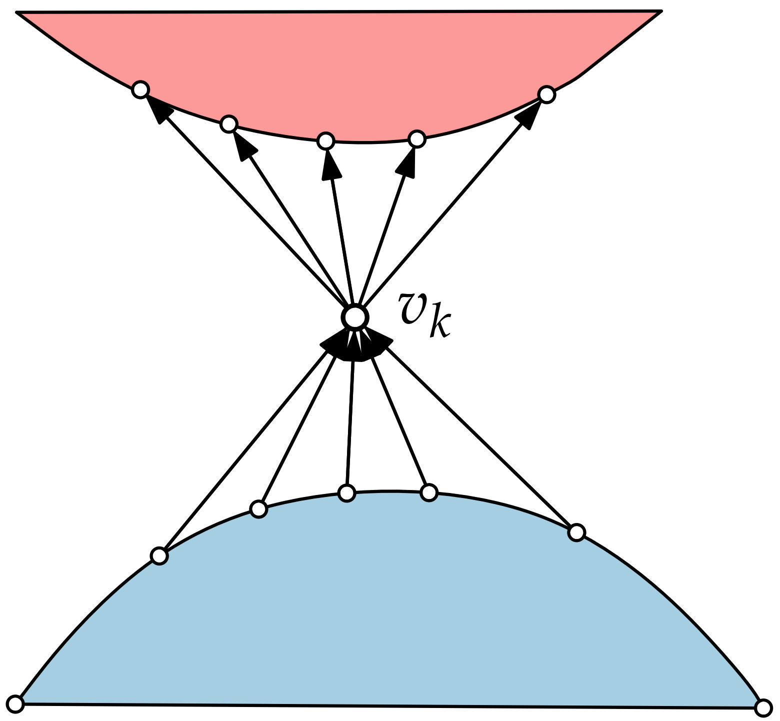
- For $i < j$, orient (v_i, v_j) from v_i to v_j;



Refined Canonical Order → REL

We construct a REL as follows:

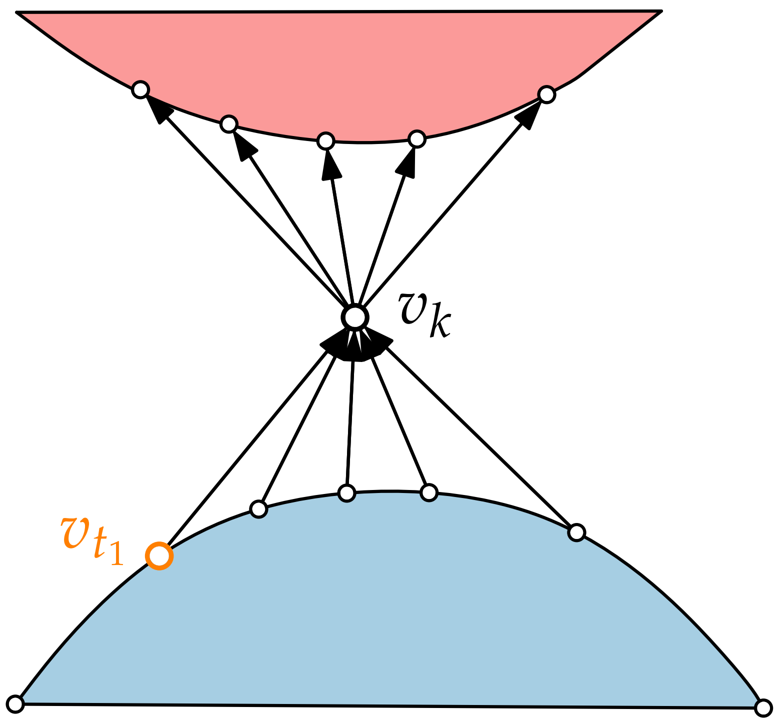
- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.



Refined Canonical Order → REL

We construct a REL as follows:

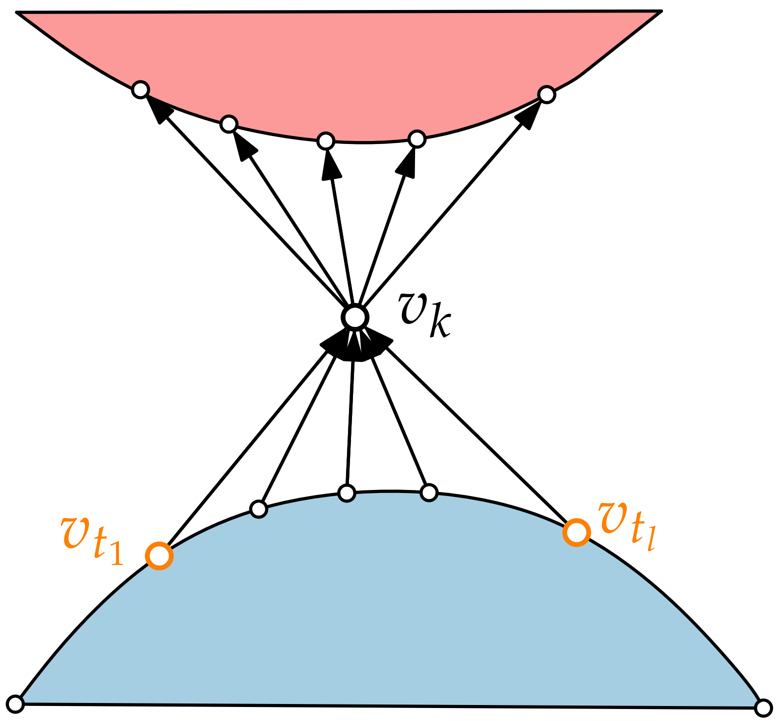
- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.



Refined Canonical Order → REL

We construct a REL as follows:

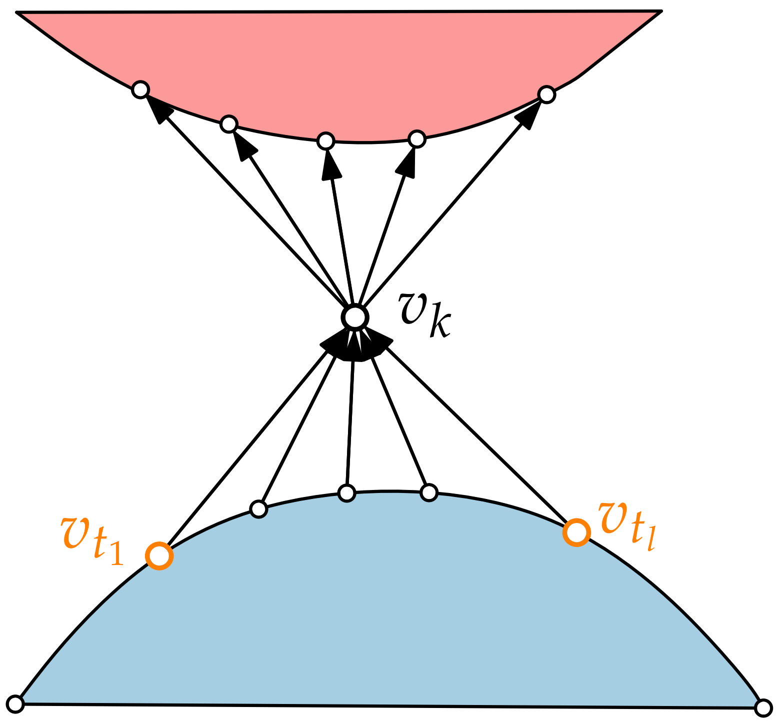
- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.



Refined Canonical Order → REL

We construct a REL as follows:

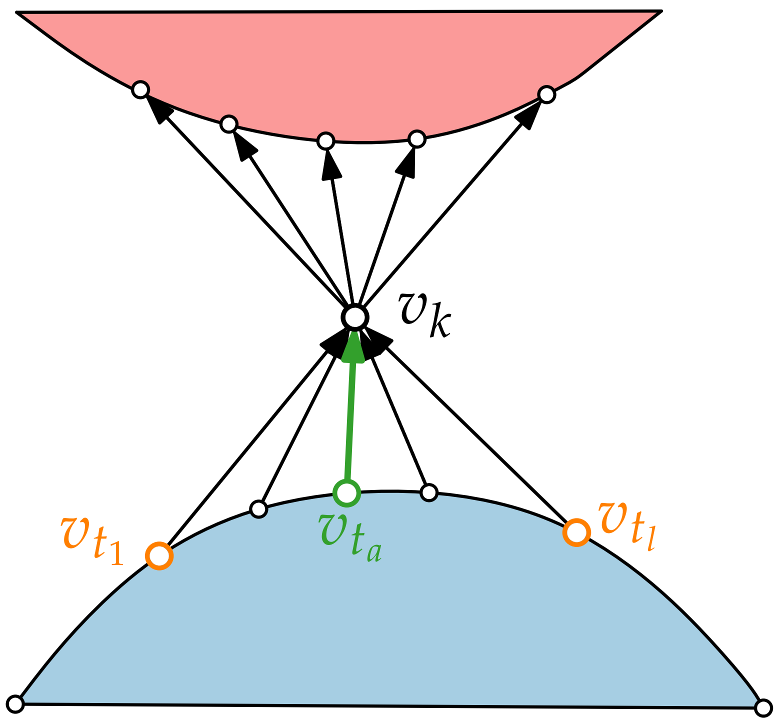
- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.



Refined Canonical Order → REL

We construct a REL as follows:

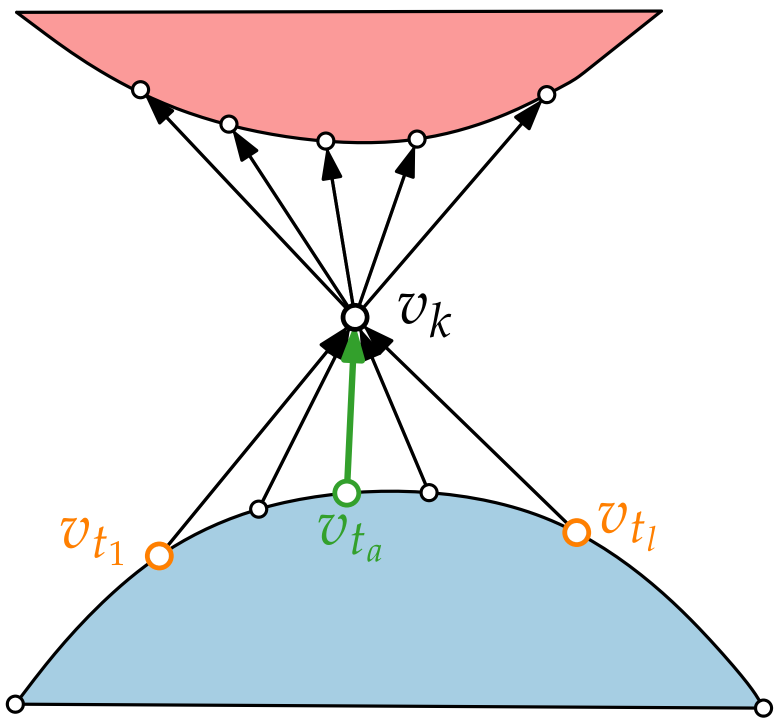
- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.



Refined Canonical Order → REL

We construct a REL as follows:

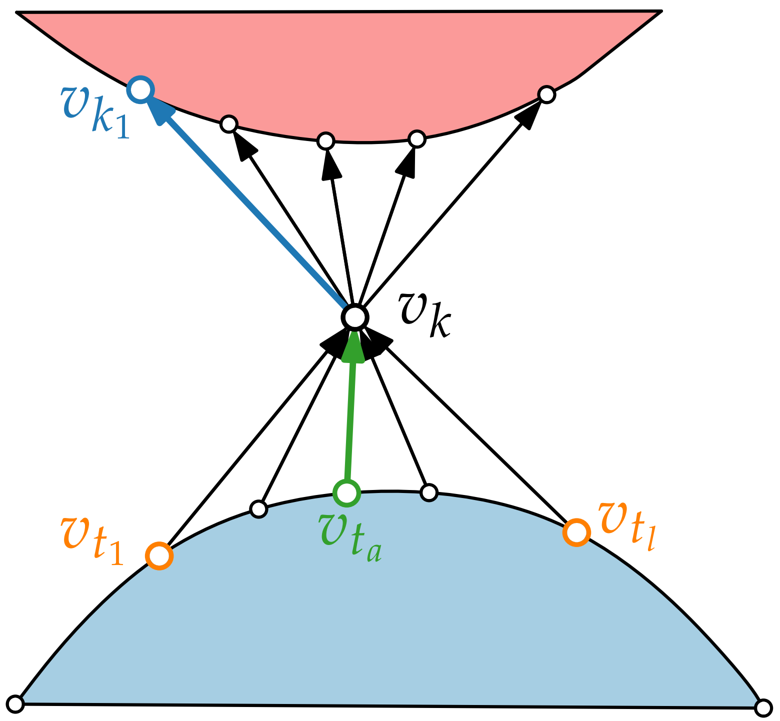
- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.
- If $v_{k_1}, \ldots, v_{k_o}$ are higher numbered neighbors of v_k, we call (v_k, v_{k_1}) **left edge** and (v_k, v_{k_o}) **right edge**.



Refined Canonical Order → REL

We construct a REL as follows:

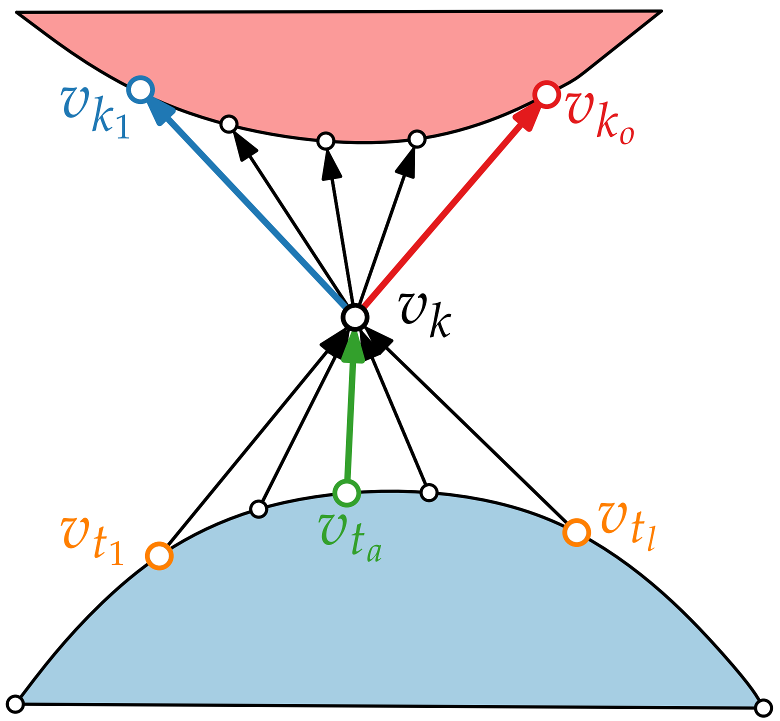
- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.
- If $v_{k_1}, \ldots, v_{k_o}$ are higher numbered neighbors of v_k, we call (v_k, v_{k_1}) **left edge** and (v_k, v_{k_o}) **right edge**.



Refined Canonical Order → REL

We construct a REL as follows:

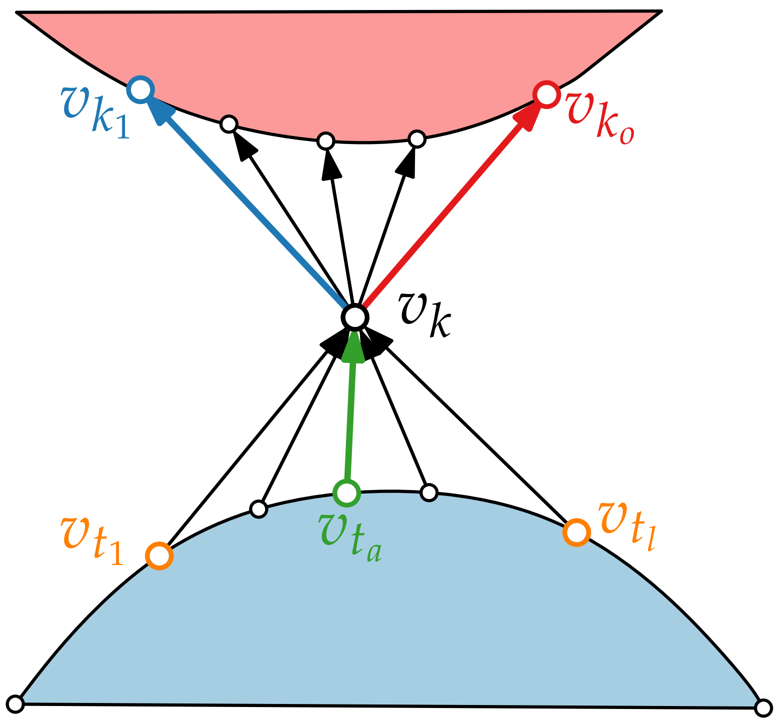
- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.
- If $v_{k_1}, \ldots, v_{k_o}$ are higher numbered neighbors of v_k, we call (v_k, v_{k_1}) **left edge** and (v_k, v_{k_o}) **right edge**.



Refined Canonical Order → REL

We construct a REL as follows:

- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.
- If $v_{k_1}, \ldots, v_{k_o}$ are higher numbered neighbors of v_k, we call (v_k, v_{k_1}) **left edge** and (v_k, v_{k_o}) **right edge**.

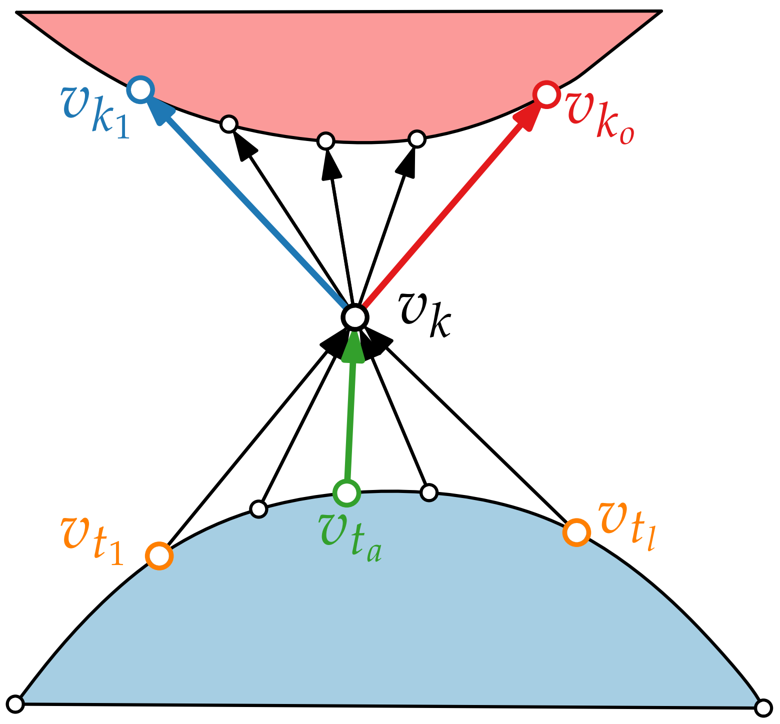


Lemma 1.
A left edge or right edge cannot be a base edge.

Refined Canonical Order → REL

We construct a REL as follows:

- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.
- If $v_{k_1}, \ldots, v_{k_o}$ are higher numbered neighbors of v_k, we call (v_k, v_{k_1}) **left edge** and (v_k, v_{k_o}) **right edge**.



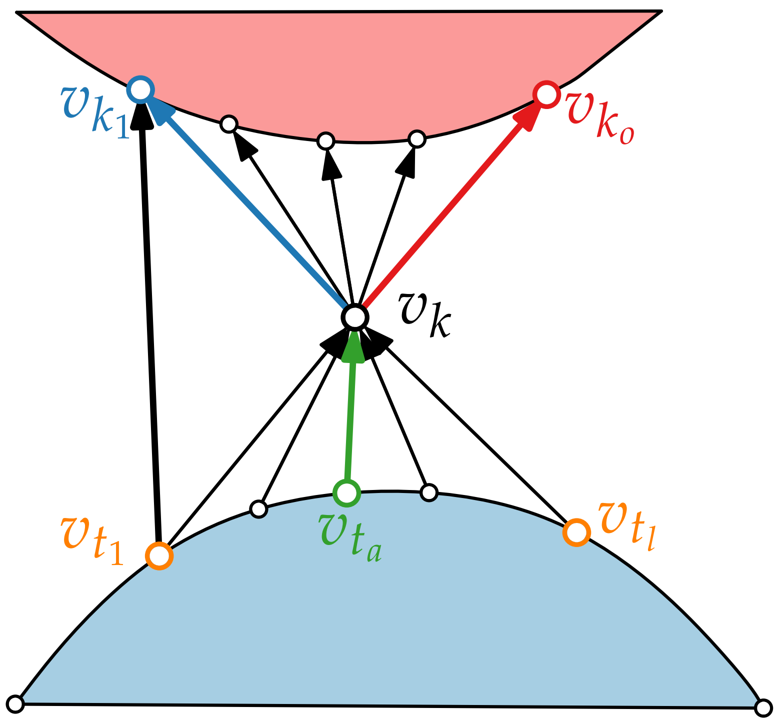
Lemma 1.
A left edge or right edge cannot be a base edge.

Proof. Suppose left edge (v_k, v_{k_1}) is base edge of v_{k_1}.

Refined Canonical Order → REL

We construct a REL as follows:

- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.
- If $v_{k_1}, \ldots, v_{k_o}$ are higher numbered neighbors of v_k, we call (v_k, v_{k_1}) **left edge** and (v_k, v_{k_o}) **right edge**.



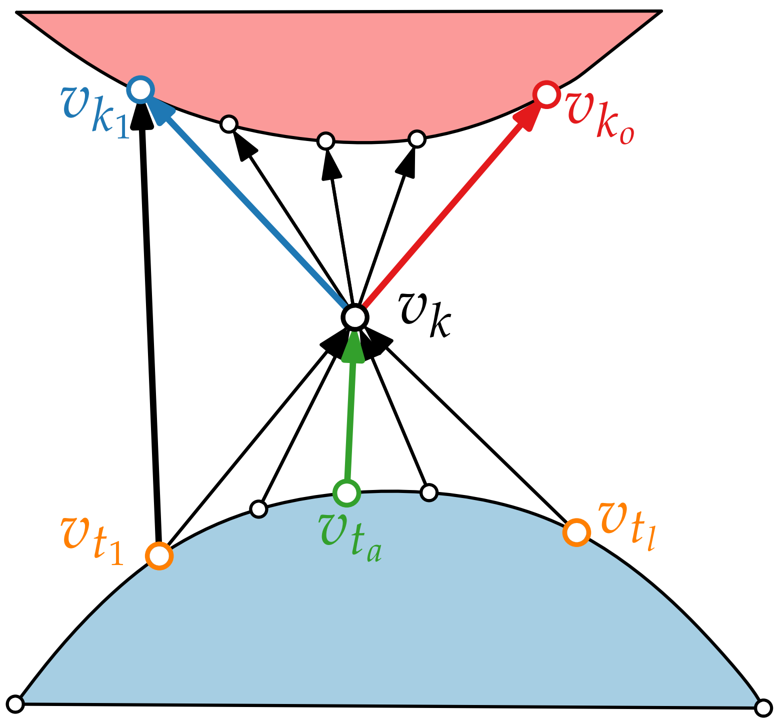
Lemma 1.
A left edge or right edge cannot be a base edge.

Proof. Suppose left edge (v_k, v_{k_1}) is base edge of v_{k_1}.
Since G triangulated, $(v_{t_1}, v_{k_1}) \in E(G)$.

Refined Canonical Order → REL

We construct a REL as follows:

- For $i < j$, orient (v_i, v_j) from v_i to v_j;
- v_k has incoming edges from $v_{t_1}, \ldots, v_{t_l}$, we say that v_{t_1} is **left point** of v_k and v_{t_l} is **right point** of v_k.
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.
- If $v_{k_1}, \ldots, v_{k_o}$ are higher numbered neighbors of v_k, we call (v_k, v_{k_1}) **left edge** and (v_k, v_{k_o}) **right edge**.

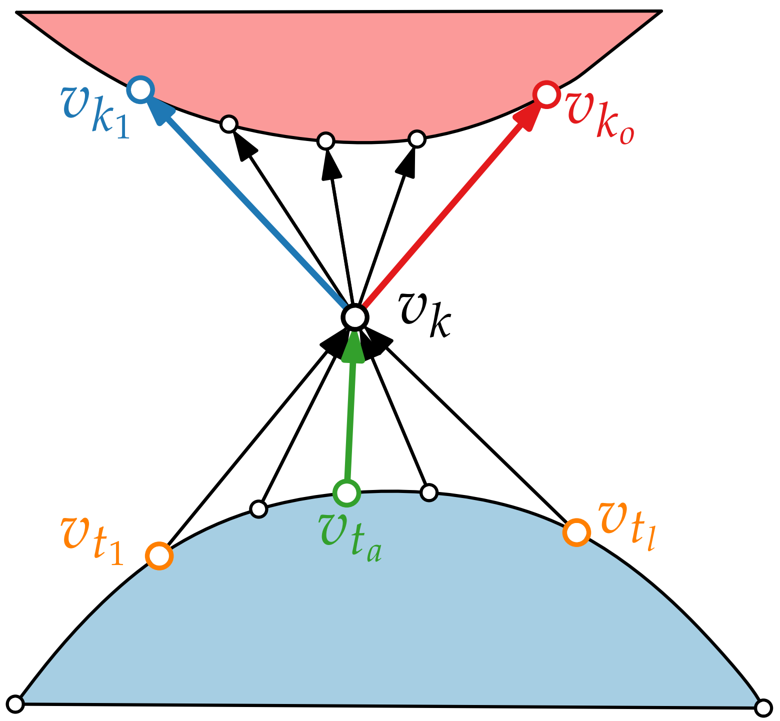


Lemma 1.
A left edge or right edge cannot be a base edge.

Proof. Suppose left edge (v_k, v_{k_1}) is base edge of v_{k_1}.
Since G triangulated, $(v_{t_1}, v_{k_1}) \in E(G)$.
Contradiction since $v_k > v_{t_1}$.

Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

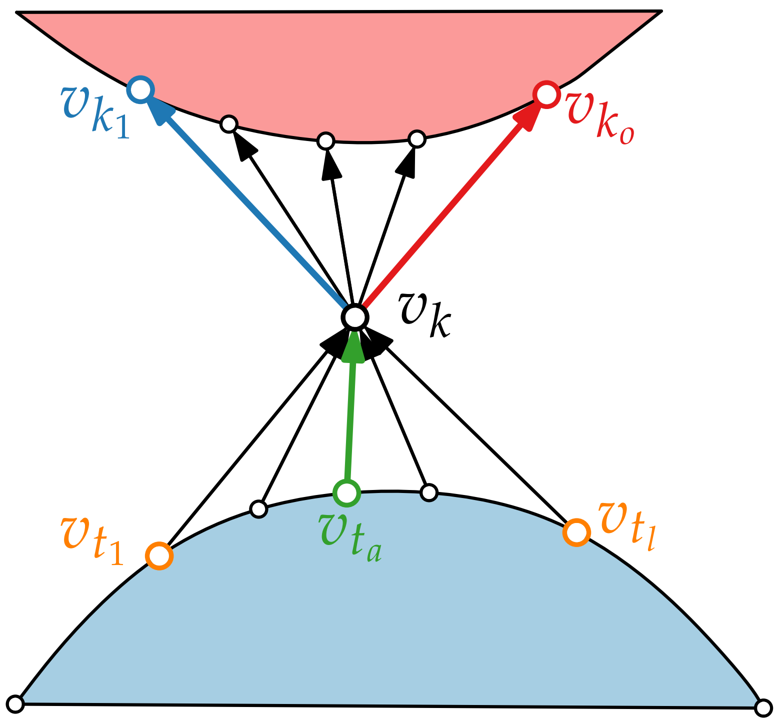


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.

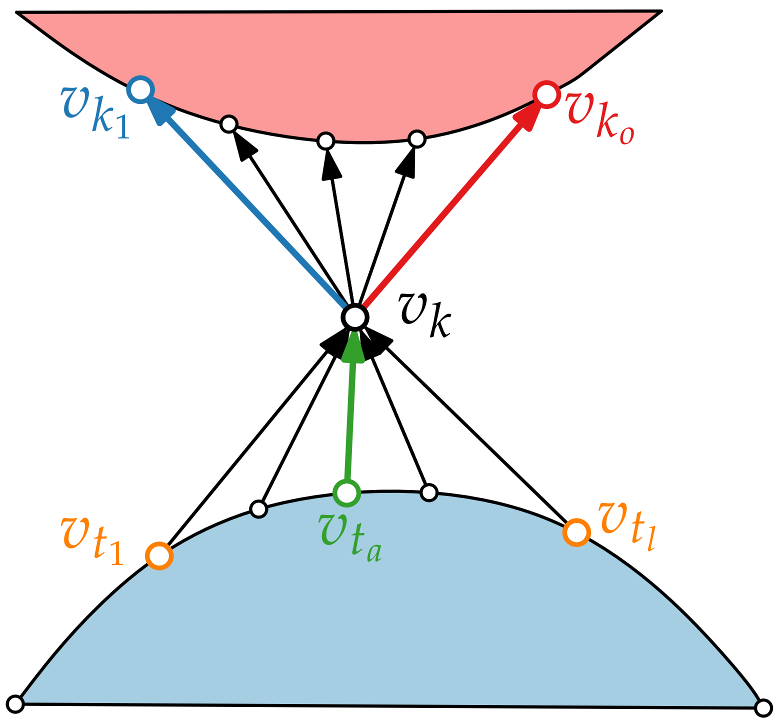


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.

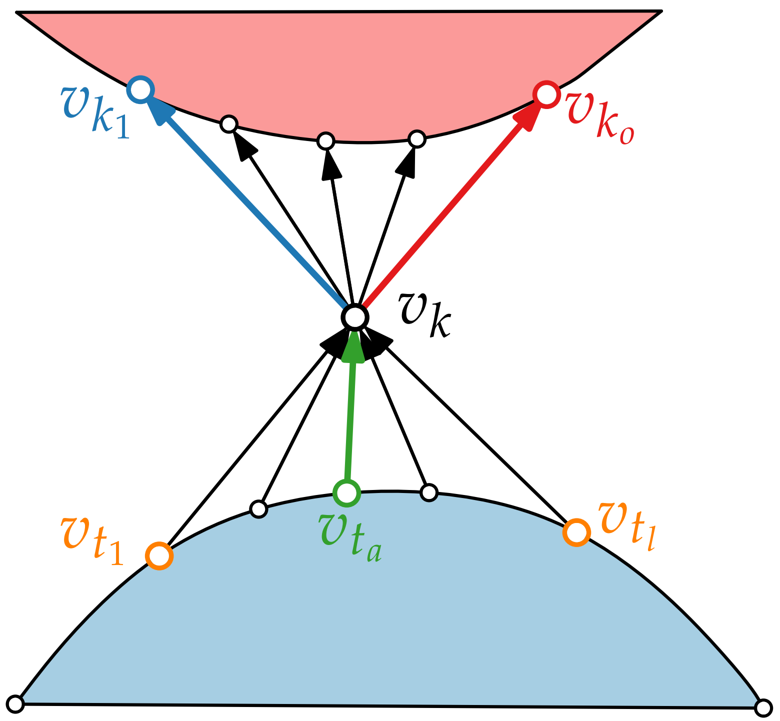


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive “or” follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$

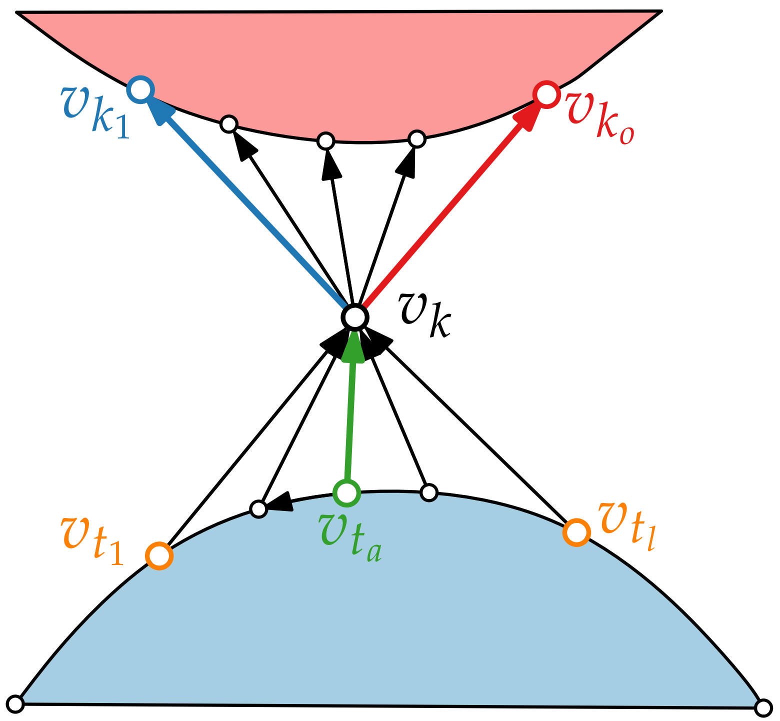


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$

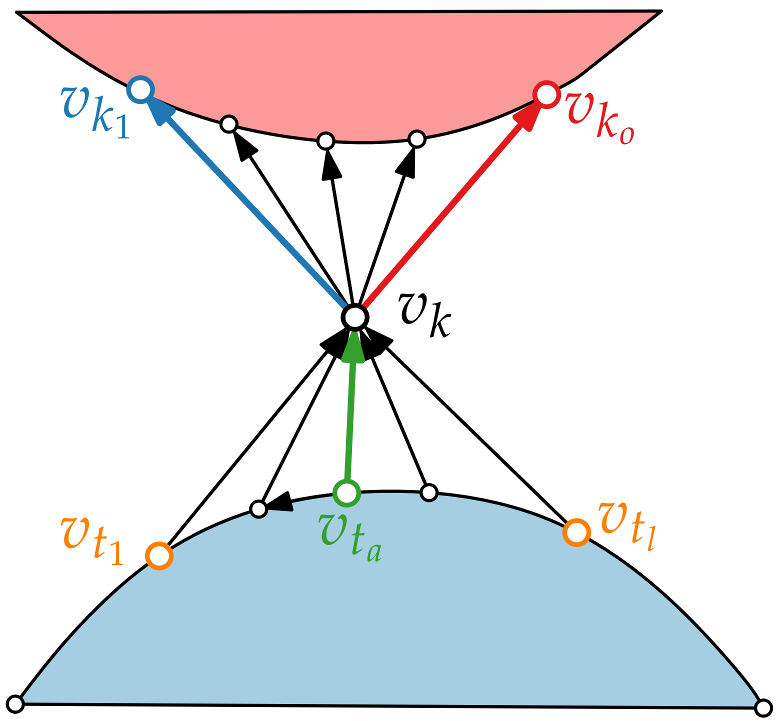


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$; $v_{t_{i<a}}$ is right point of $v_{t_{i-1}}$:

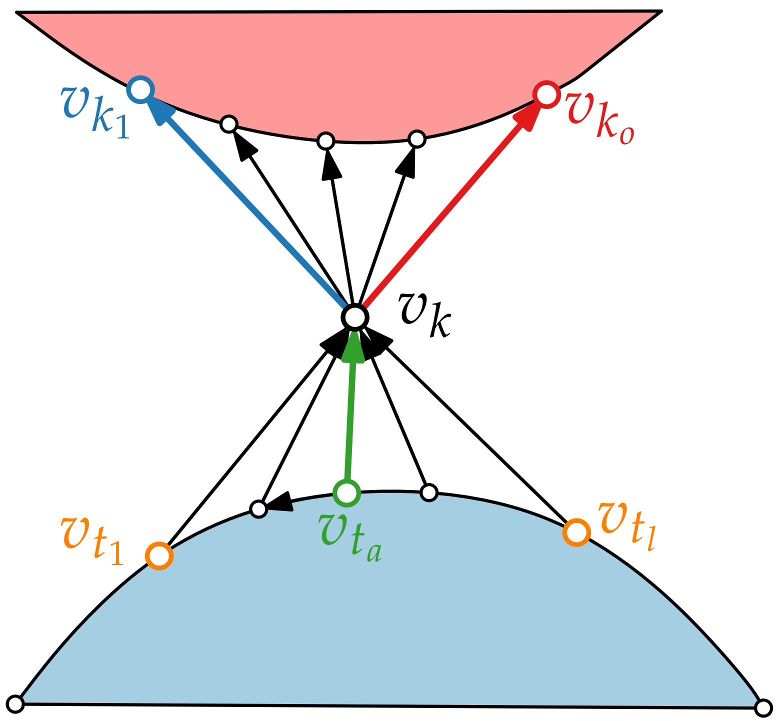


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$; $v_{t_{i<a}}$ is right point of $v_{t_{i-1}}$:
  - v_{t_i} has at least two higher-numbered neighbors.

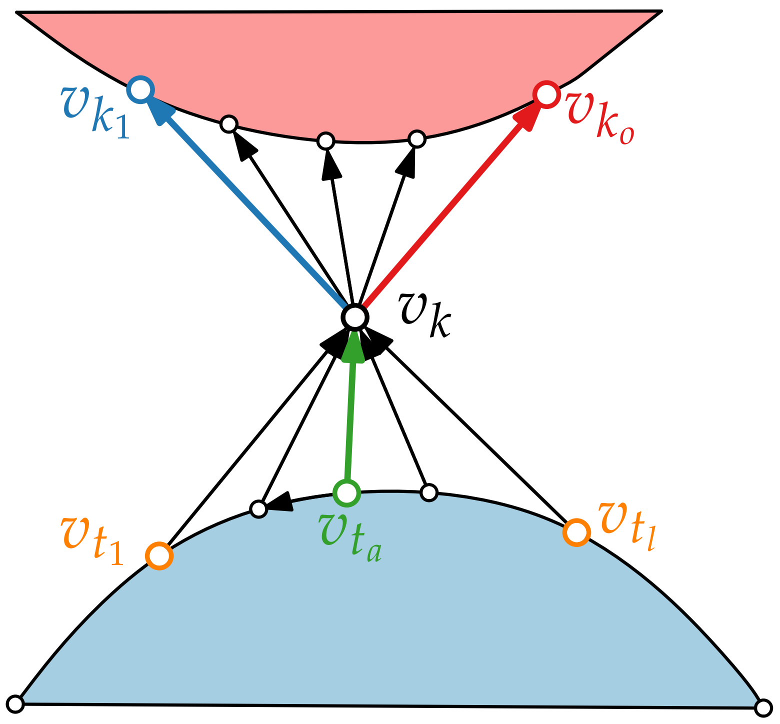


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$; $v_{t_{i<a}}$ is right point of $v_{t_{i-1}}$:
  - v_{t_i} has at least two higher-numbered neighbors.
  - One of them is v_k; the other one is either $v_{t_{i-1}}$ or $v_{t_{i+1}}$.

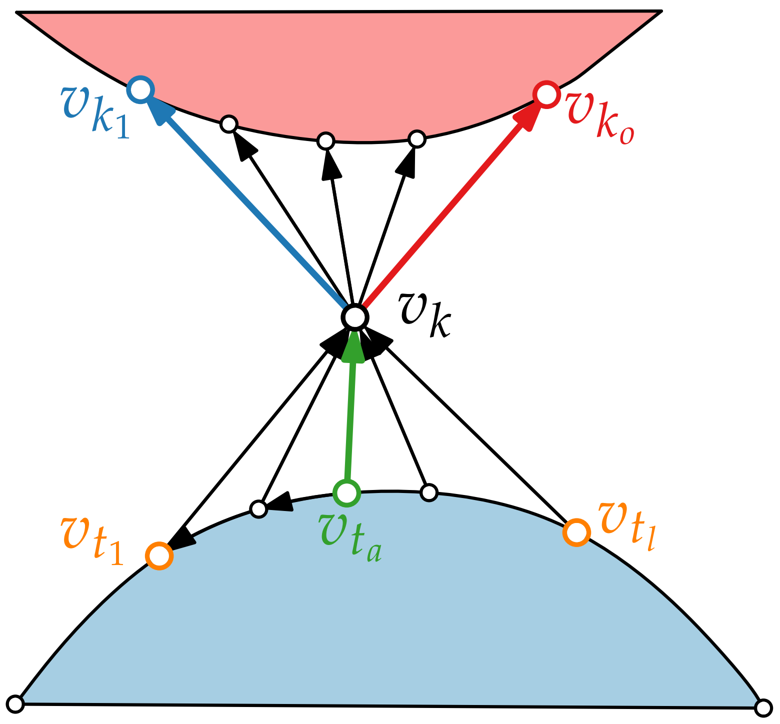


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$; $v_{t_{i<a}}$ is right point of $v_{t_{i-1}}$:
  - v_{t_i} has at least two higher-numbered neighbors.
  - One of them is v_k; the other one is either $v_{t_{i-1}}$ or $v_{t_{i+1}}$.
  - For $1 \leq i < a - 1$, it is $v_{t_{i-1}}$.

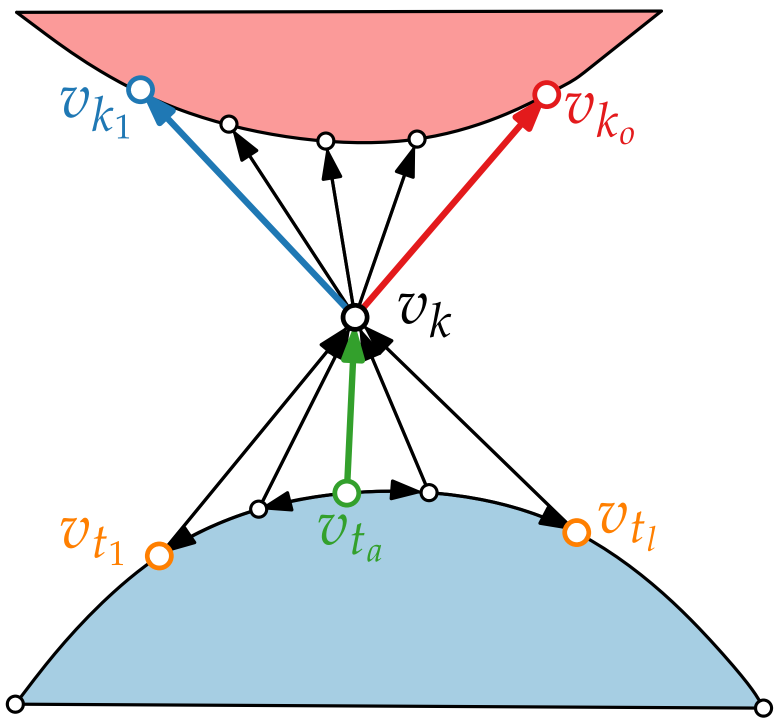


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$; $v_{t_{i<a}}$ is right point of $v_{t_{i-1}}$:
  - v_{t_i} has at least two higher-numbered neighbors.
  - One of them is v_k; the other one is either $v_{t_{i-1}}$ or $v_{t_{i+1}}$.
  - For $1 \leq i < a-1$, it is $v_{t_{i-1}}$.
- Analogously, $v_{t_{i \geq a}}$ is left point of $v_{t_{i+1}}$

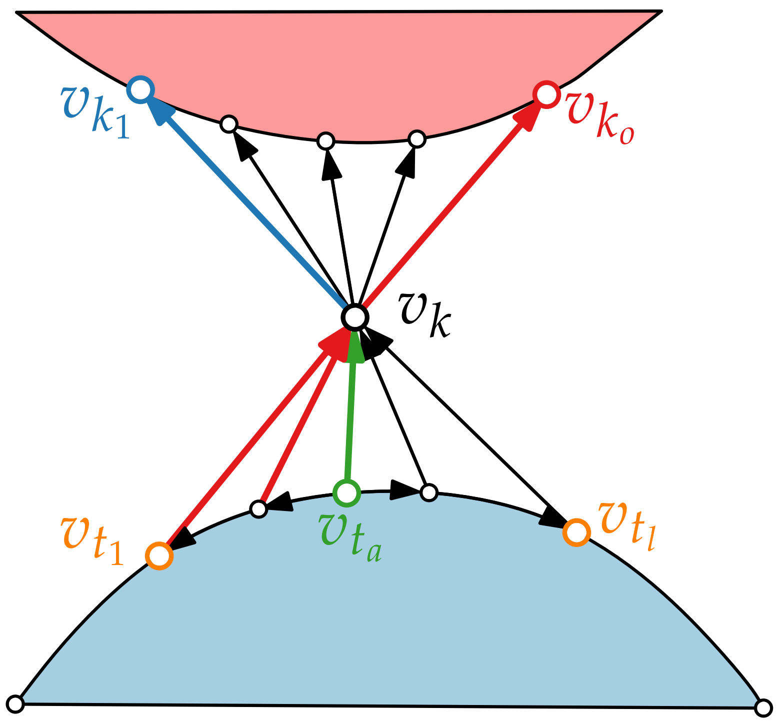


Refined Canonical Order → REL

Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive "or" follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$; $v_{t_{i<a}}$ is right point of $v_{t_{i-1}}$:
  - v_{t_i} has at least two higher-numbered neighbors.
  - One of them is v_k; the other one is either $v_{t_{i-1}}$ or $v_{t_{i+1}}$.
  - For $1 \leq i < a-1$, it is $v_{t_{i-1}}$.
- Analogously, $v_{t_{i \geq a}}$ is left point of $v_{t_{i+1}}$
- Edges (v_{t_i}, v_k), $1 \leq i < a-1$, are right edges.

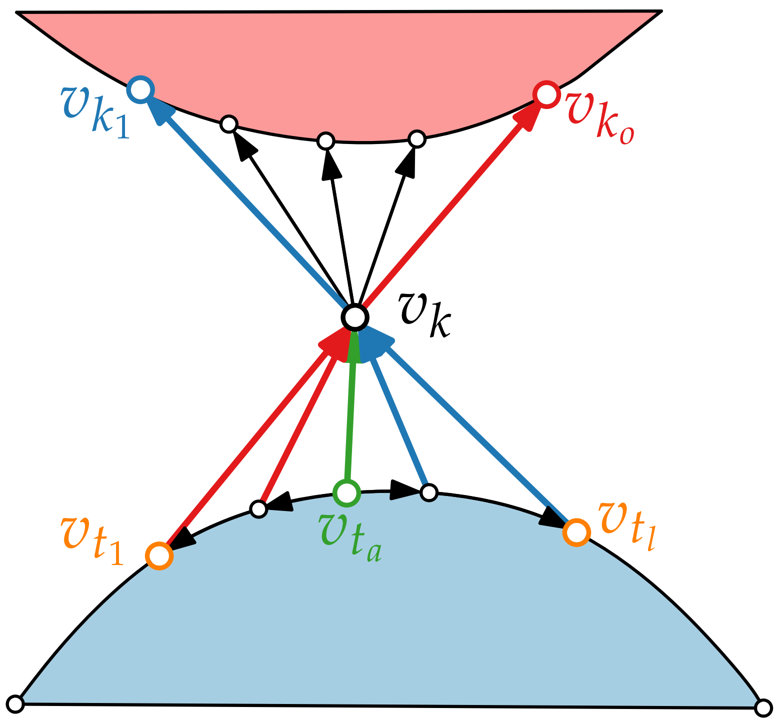


Refined Canonical Order → REL

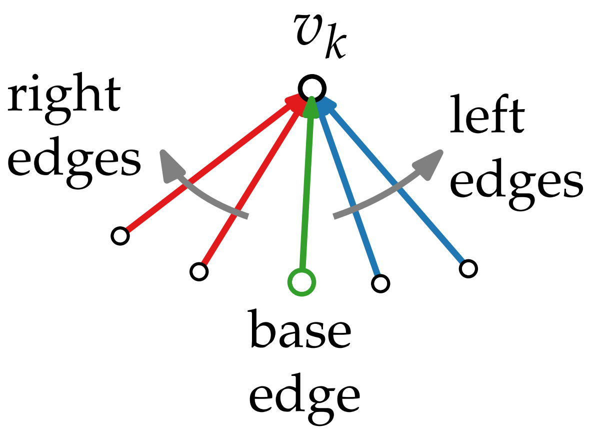
Lemma 2.
An edge is either a left edge, a right edge or a base edge.

Proof.

- Exclusive “or” follows from Lemma 1.
- Let (v_{t_a}, v_k) be base edge of v_k.
- v_{t_a} is right point of $v_{t_{a-1}}$; $v_{t_{i<a}}$ is right point of $v_{t_{i-1}}$:
  - v_{t_i} has at least two higher-numbered neighbors.
  - One of them is v_k; the other one is either $v_{t_{i-1}}$ or $v_{t_{i+1}}$.
  - For $1 \leq i < a-1$, it is $v_{t_{i-1}}$.
- Analogously, $v_{t_{i \geq a}}$ is left point of $v_{t_{i+1}}$
- Edges (v_{t_i}, v_k), $1 \leq i < a-1$, are right edges.
- Similarly, (v_{t_i}, v_k), for $a+1 \leq i \leq l$, are left edges.



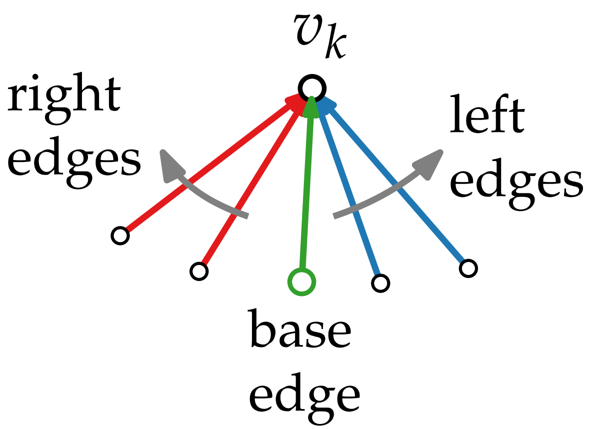
Refined Canonical Order → REL



Refined Canonical Order → REL

Coloring.

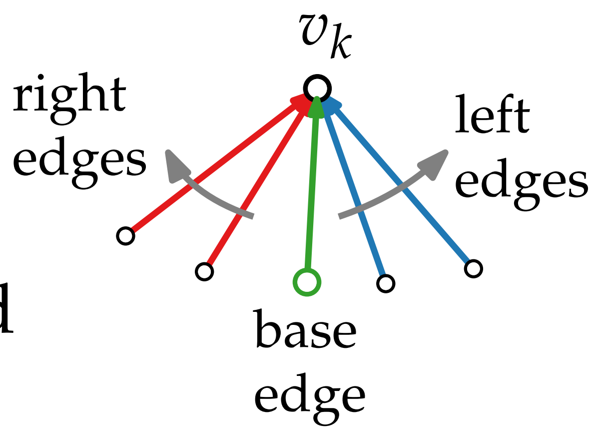
- Color right (left) edges in red (blue).



Refined Canonical Order → REL

Coloring.

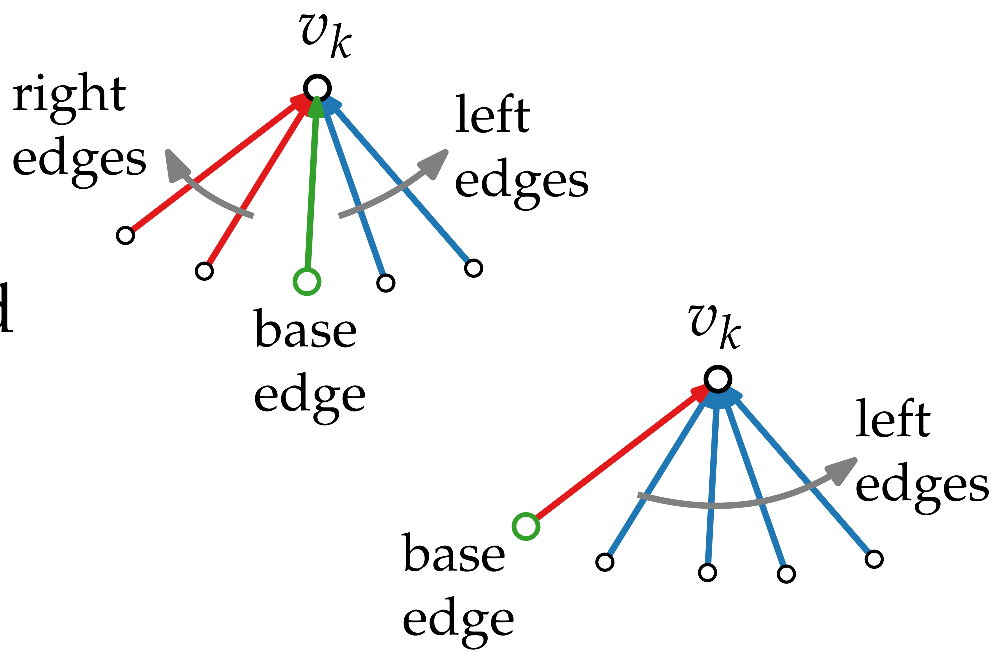
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.



Refined Canonical Order → REL

Coloring.

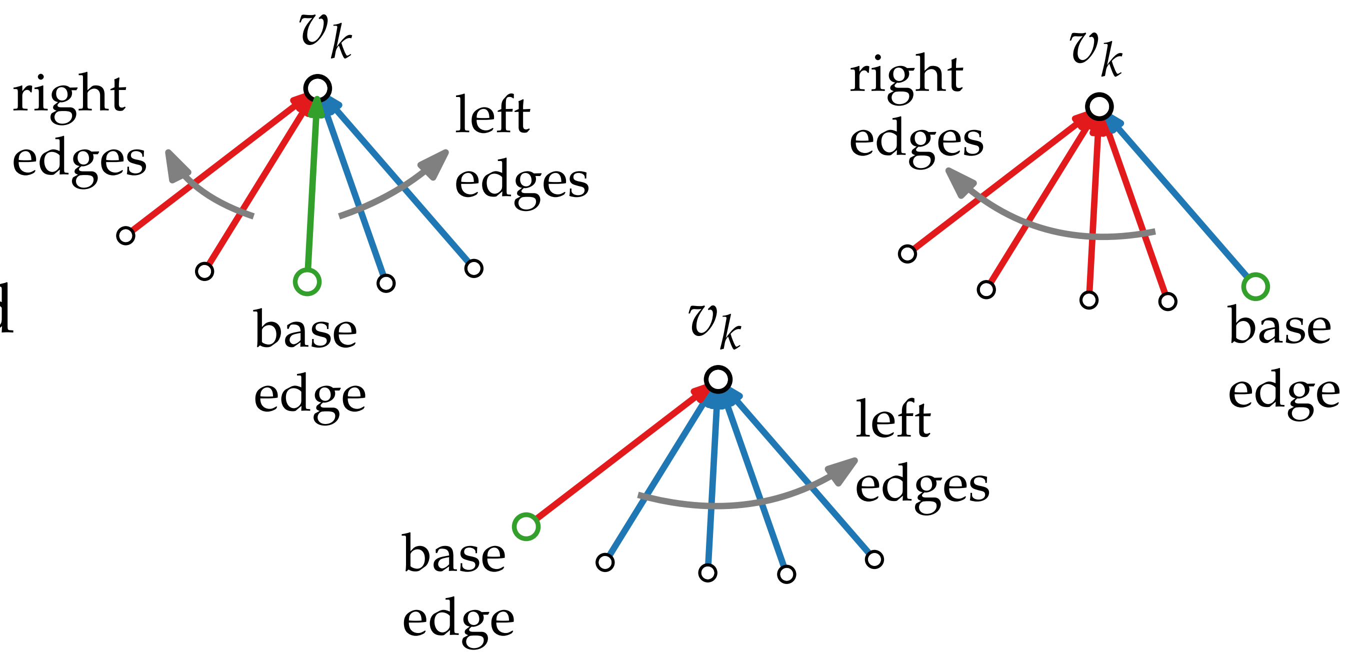
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.



Refined Canonical Order → REL

Coloring.

- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

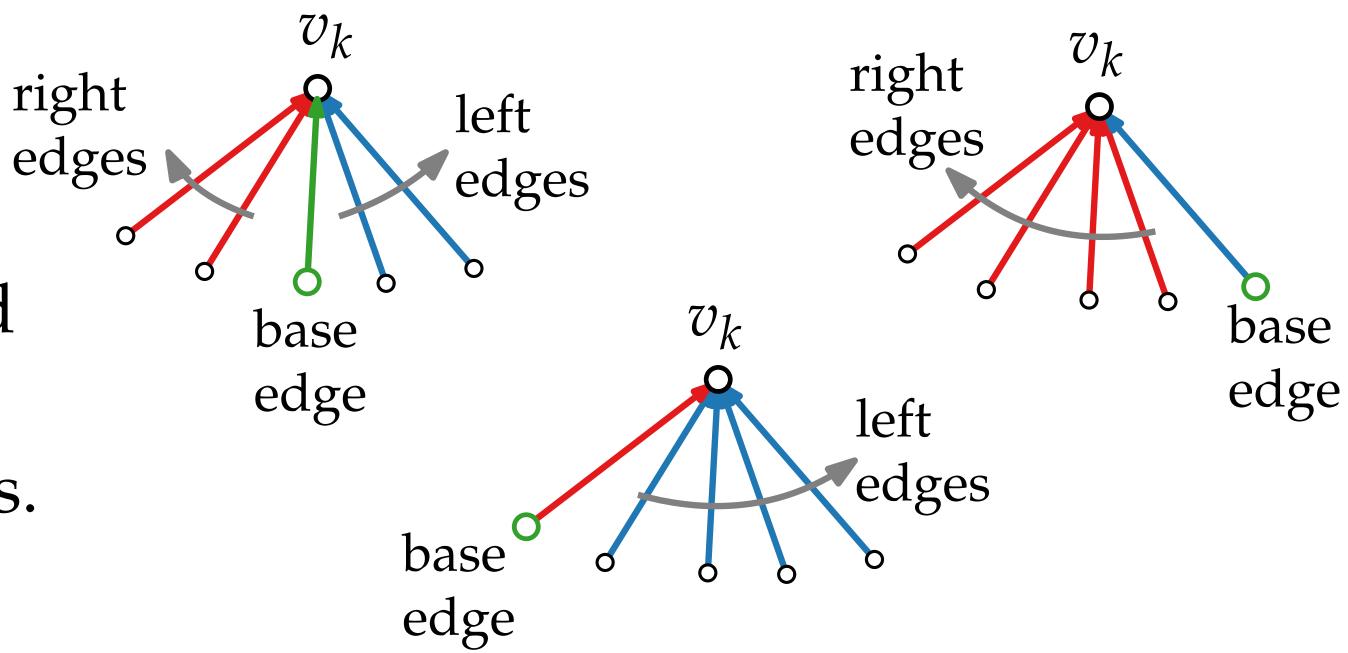


Refined Canonical Order → REL

Coloring.

- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.



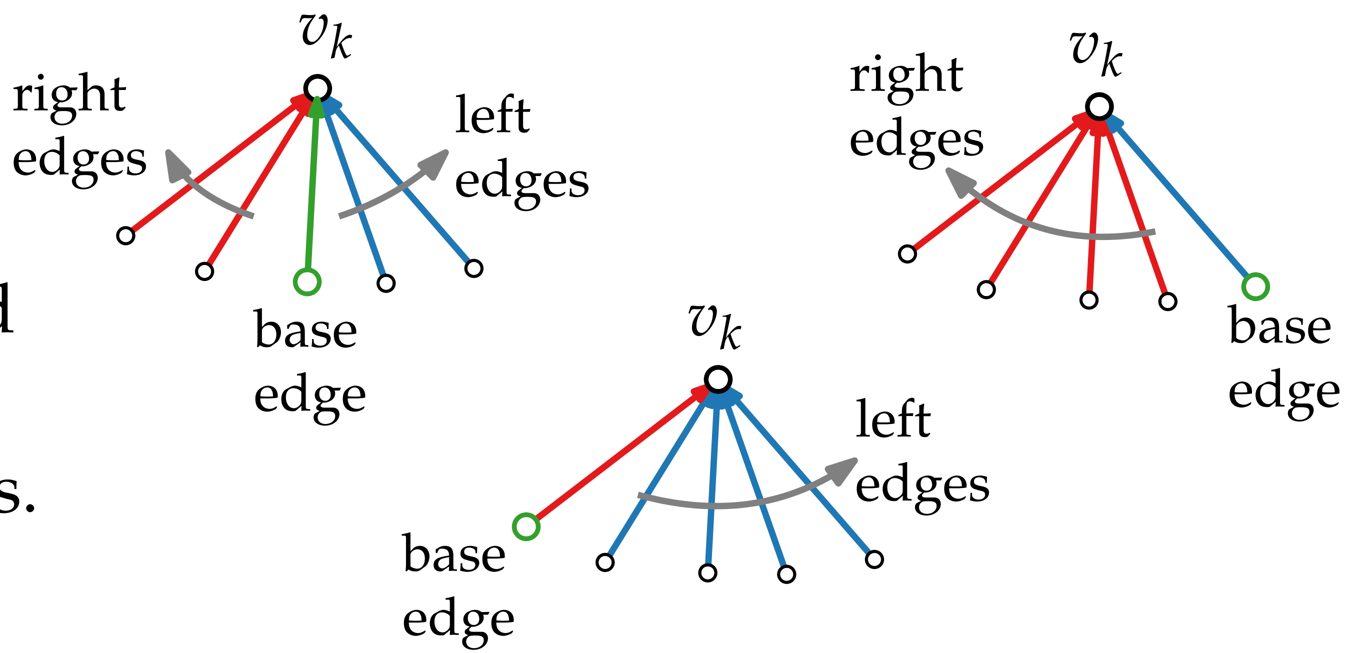
Refined Canonical Order → REL

Coloring.

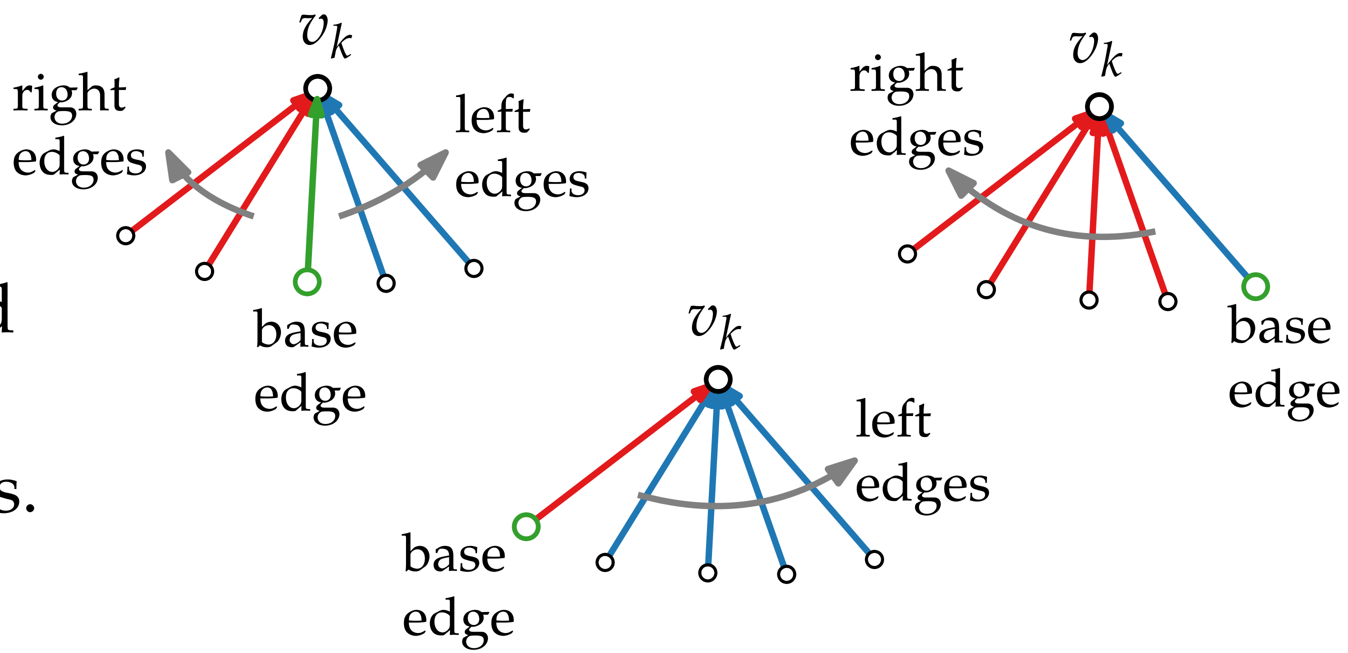
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.

Lemma 3.
$\{T_r, T_b\}$ is a regular edge labeling.



Refined Canonical Order → REL



Coloring.

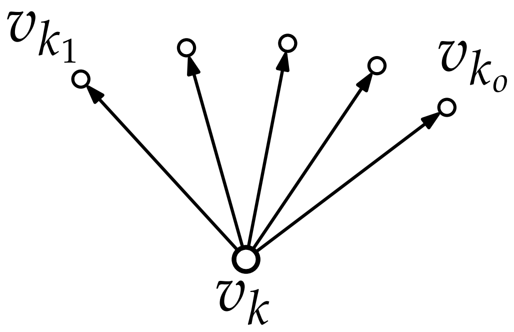
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.

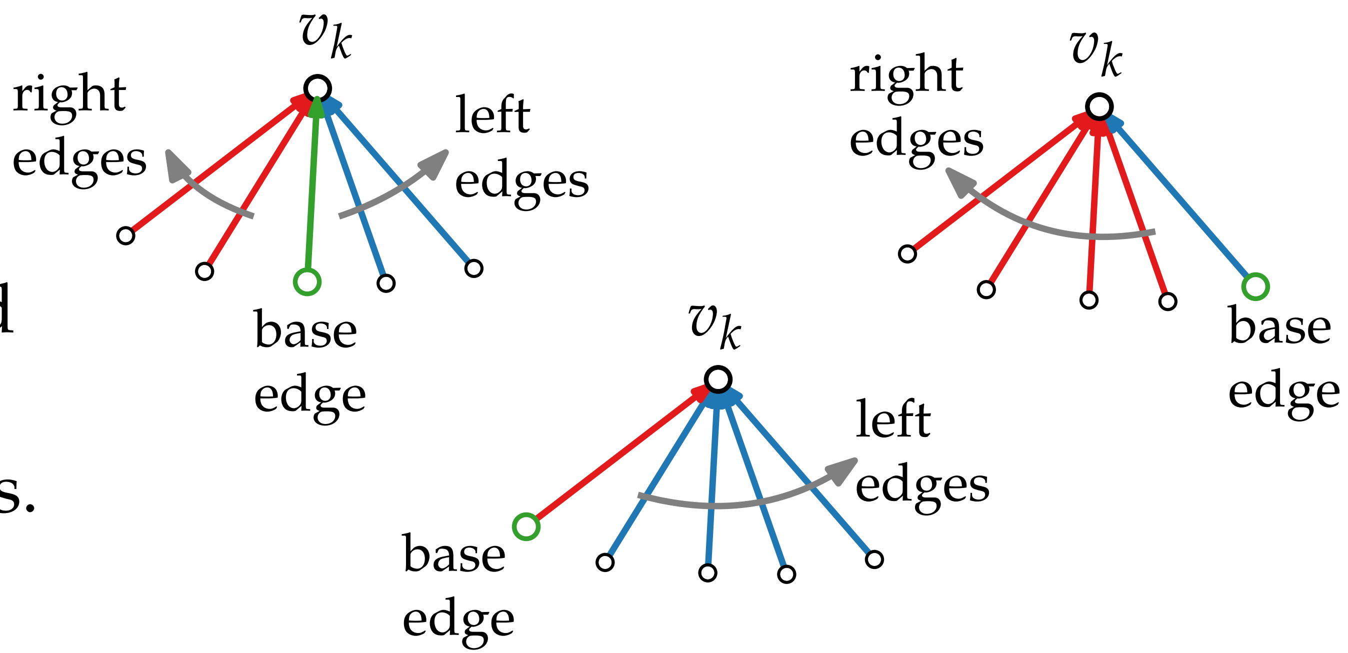
Lemma 3.
$\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$k_o \geq 2$



Refined Canonical Order → REL



Coloring.

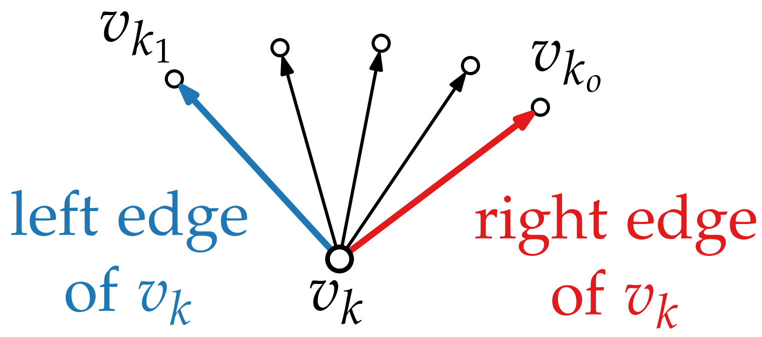
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.

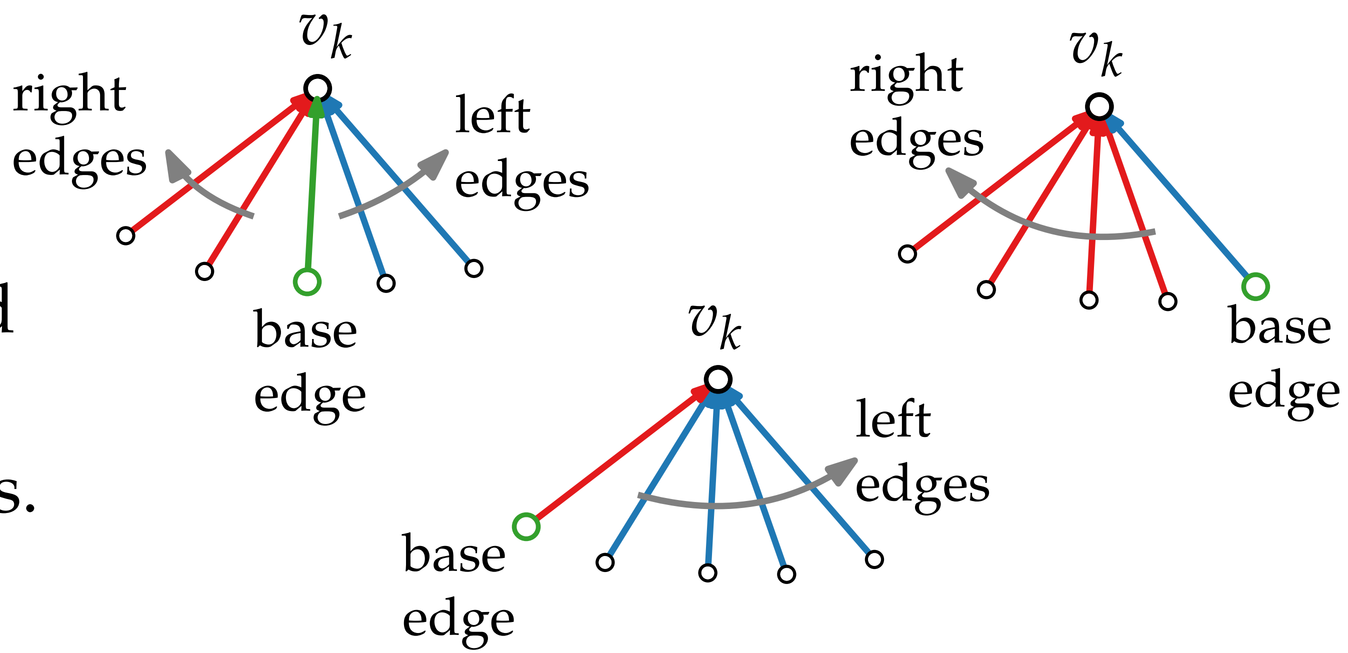
> **Lemma 3.**
> $\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$k_o \geq 2$



Refined Canonical Order → REL



Coloring.

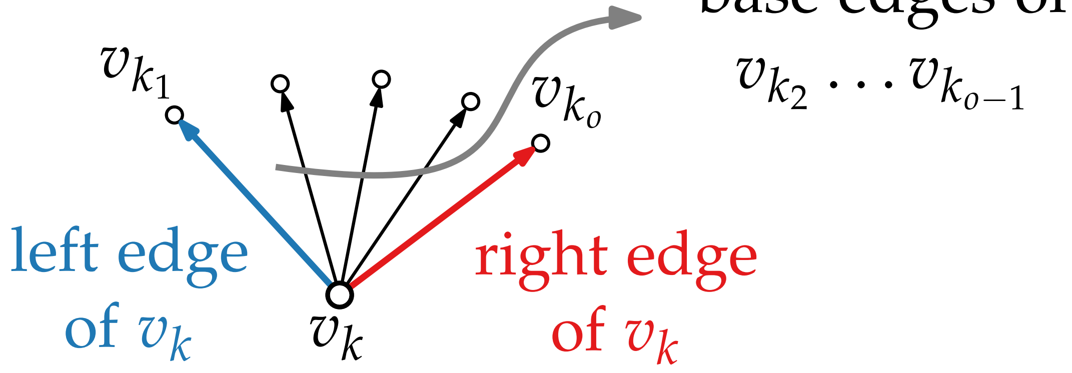
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.

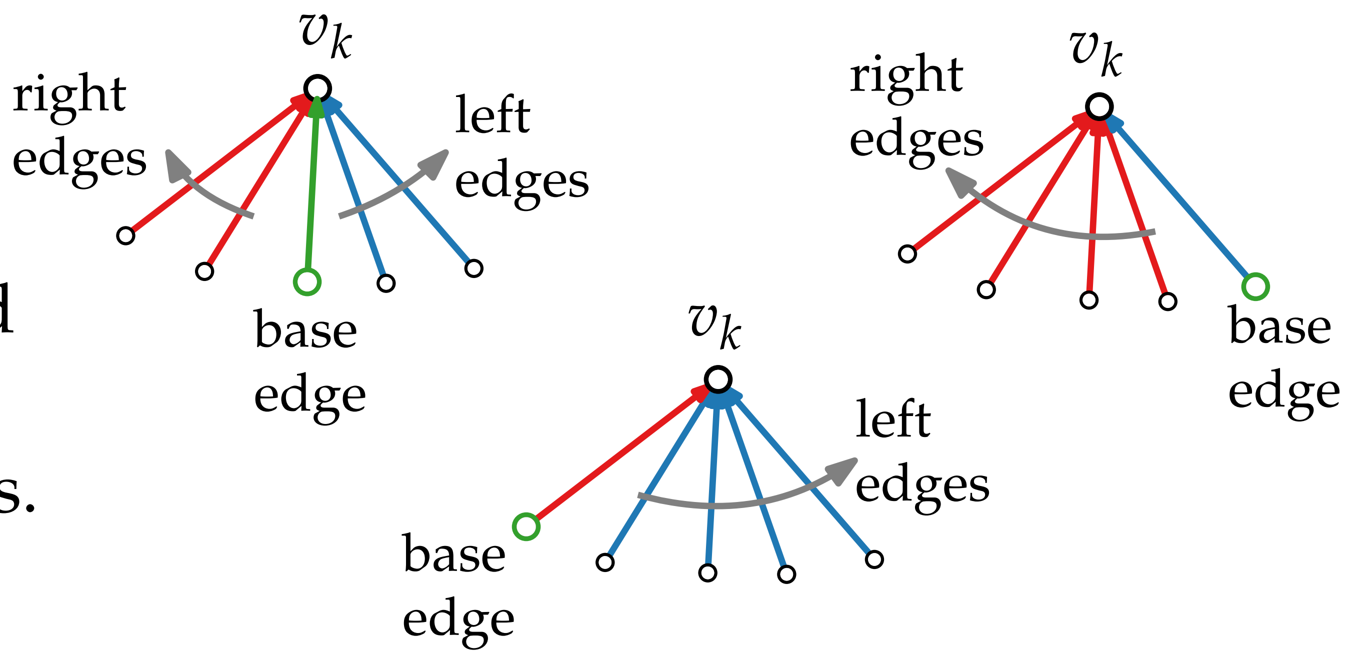
> **Lemma 3.**
> $\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$k_o \geq 2$



Refined Canonical Order → REL



Coloring.

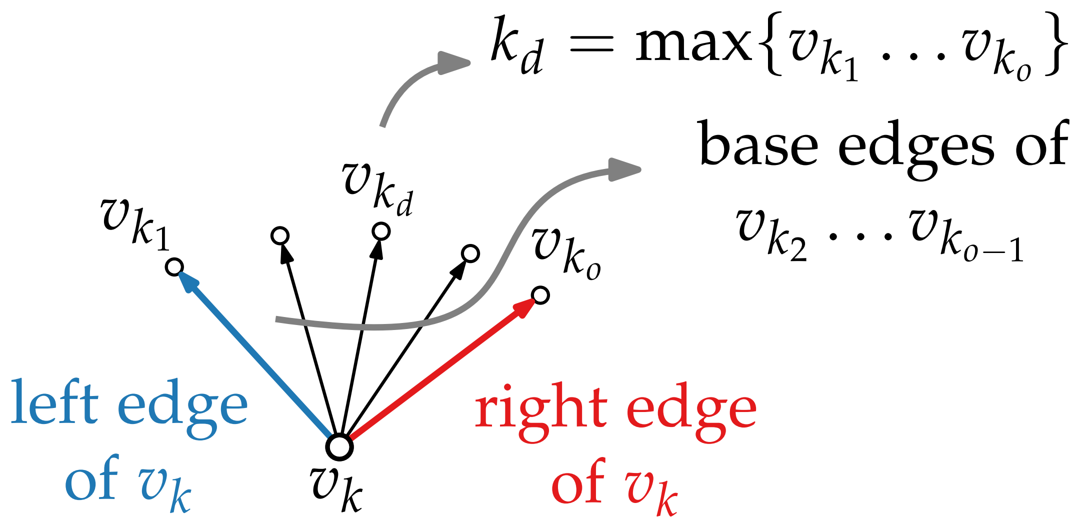
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.

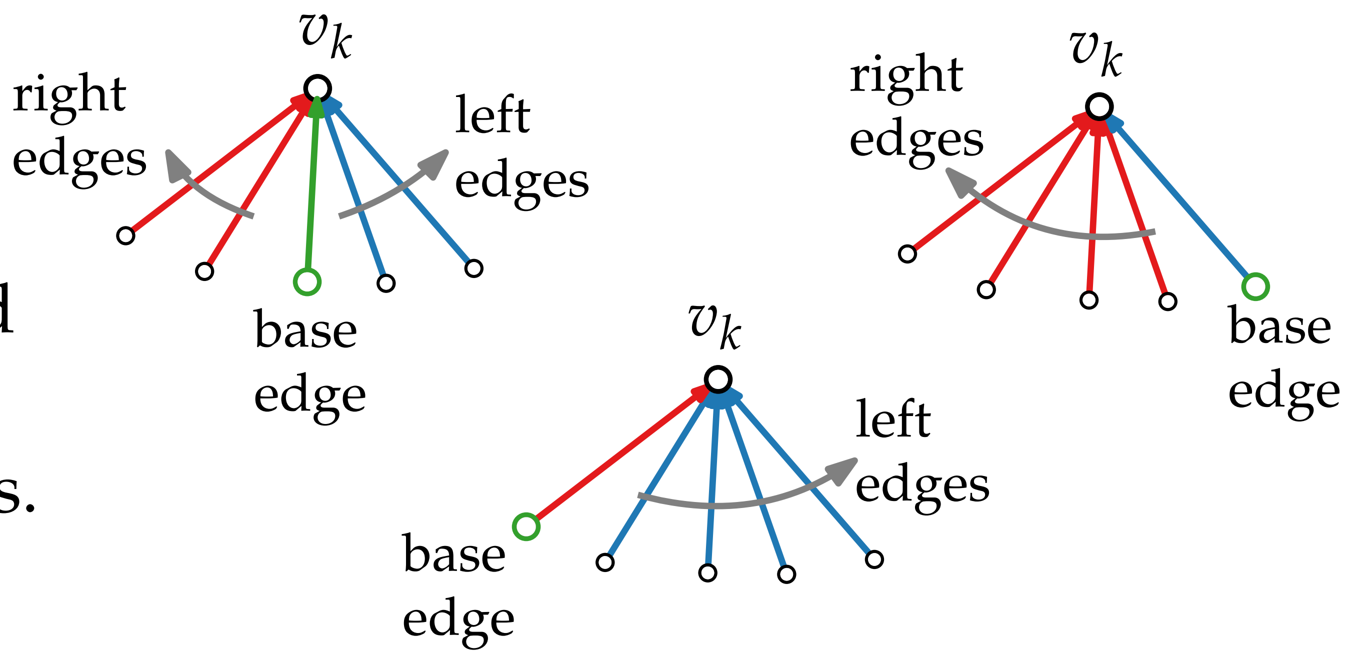
Lemma 3.
$\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$k_o \geq 2$



Refined Canonical Order → REL



Coloring.

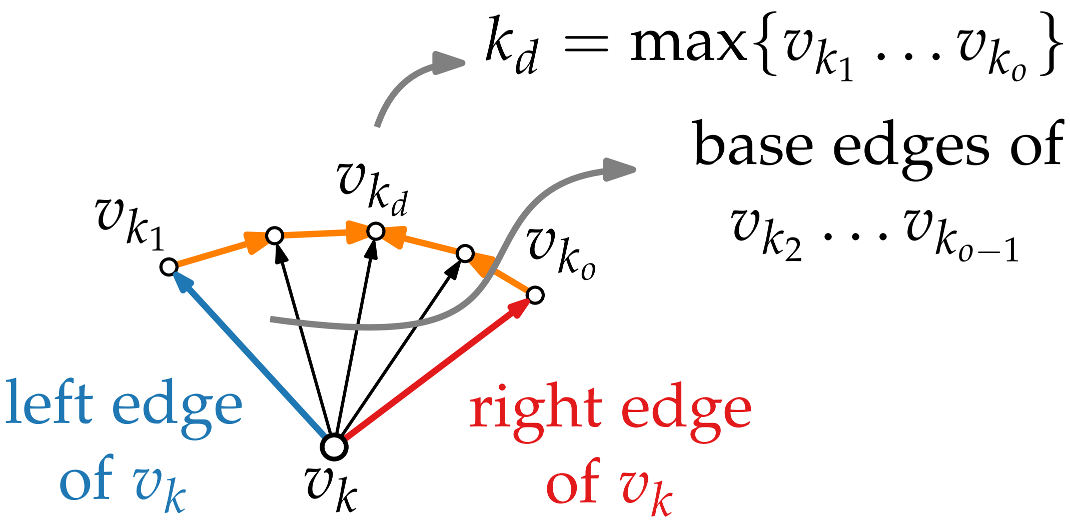
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.

> **Lemma 3.**
> $\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$k_o \geq 2$

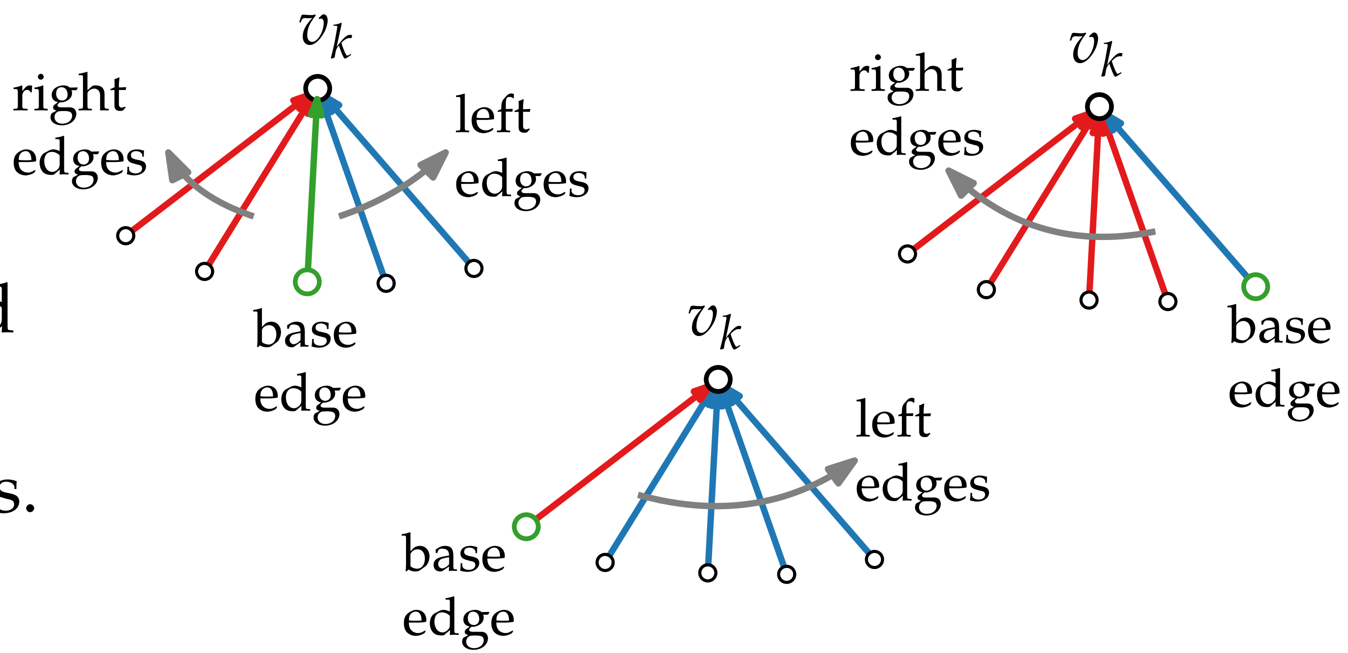


- $k_1 < k_2 < \ldots < k_d$ and $k_d > k_{d+1} > \ldots > k_o$

Refined Canonical Order → REL

Coloring.

- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

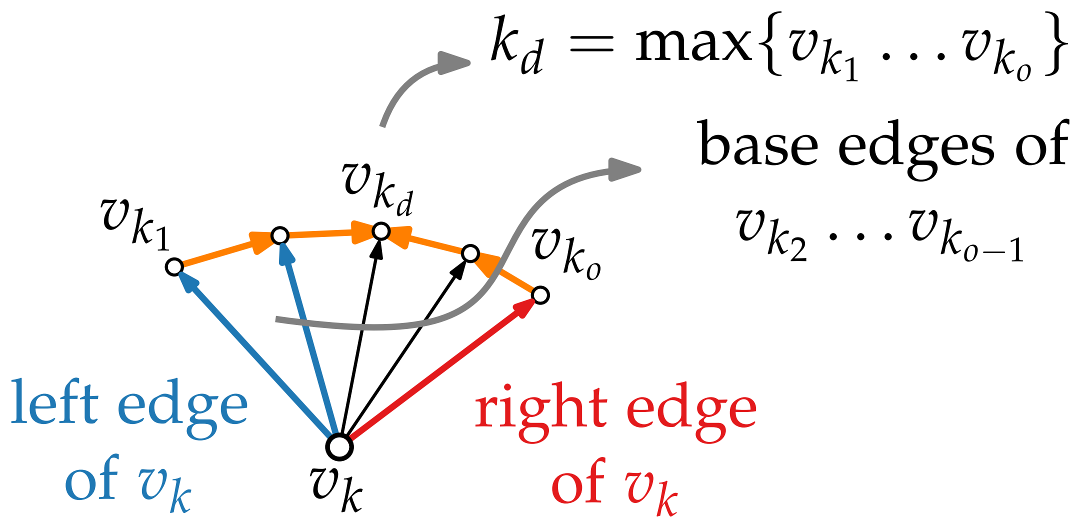


Let T_r be the red edges and T_b the blue edges.

Lemma 3.
$\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$k_o \geq 2$



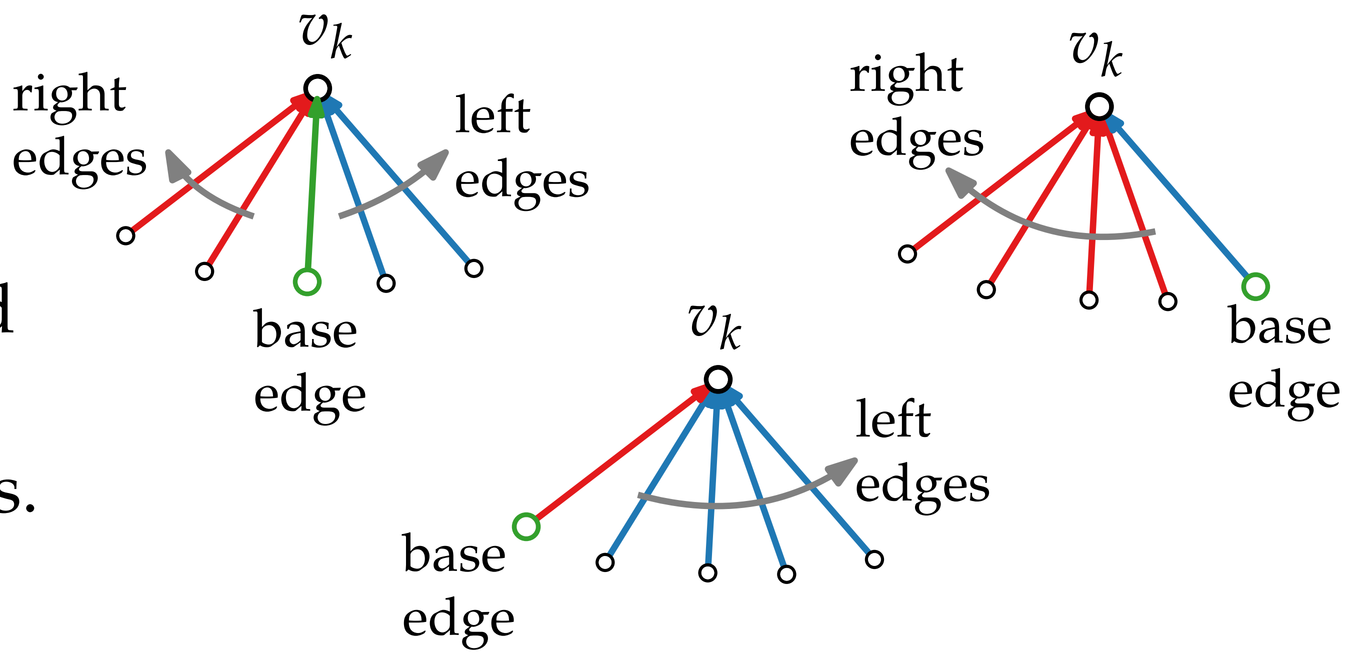
- $k_1 < k_2 < \ldots < k_d$ and $k_d > k_{d+1} > \ldots > k_o$
- (v_k, v_{k_i}), $2 \leq i \leq d-1$ are blue

Refined Canonical Order → REL

Coloring.

- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

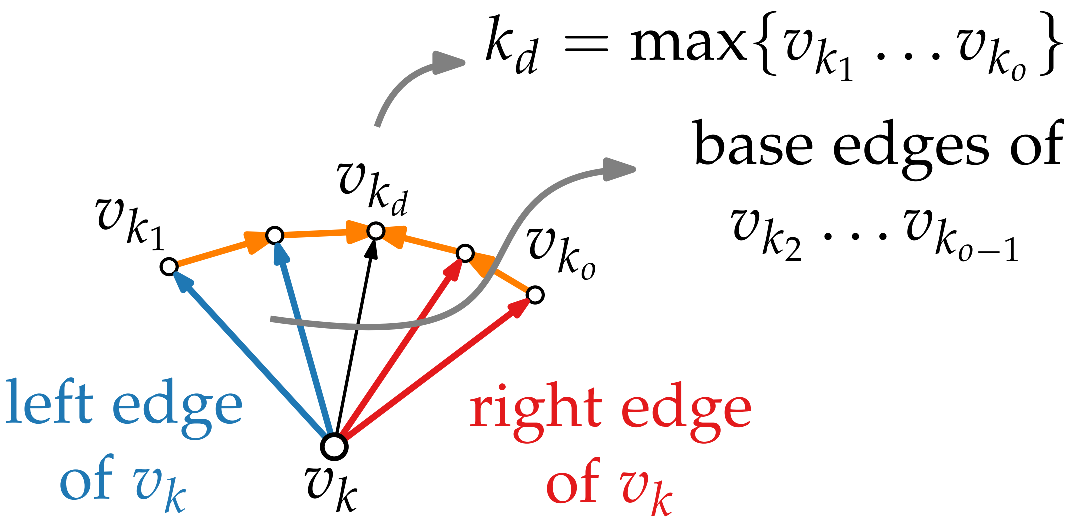
Let T_r be the red edges and T_b the blue edges.



Lemma 3.
$\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$k_o \geq 2$

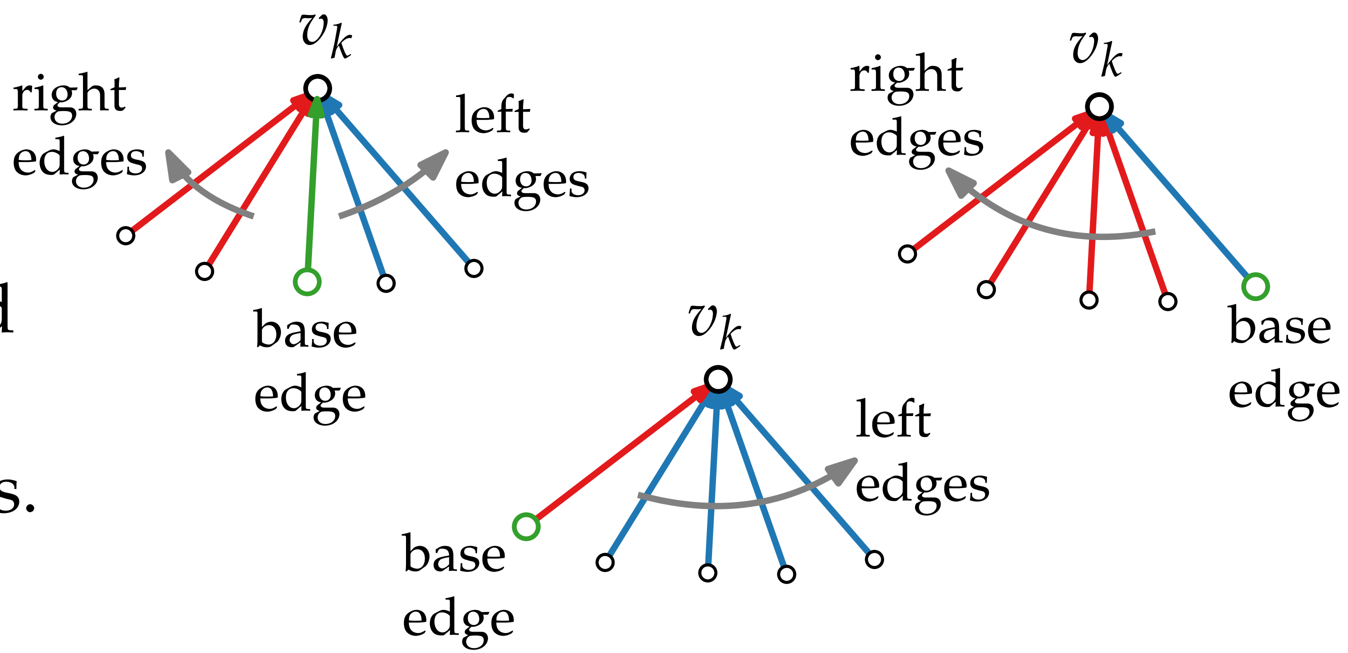


- $k_1 < k_2 < \dots < k_d$ and $k_d > k_{d+1} > \dots > k_o$
- (v_k, v_{k_i}), $2 \leq i \leq d-1$ are blue
- (v_k, v_{k_i}), $d+1 \leq i \leq o-1$ are red

Refined Canonical Order → REL

Coloring.

- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

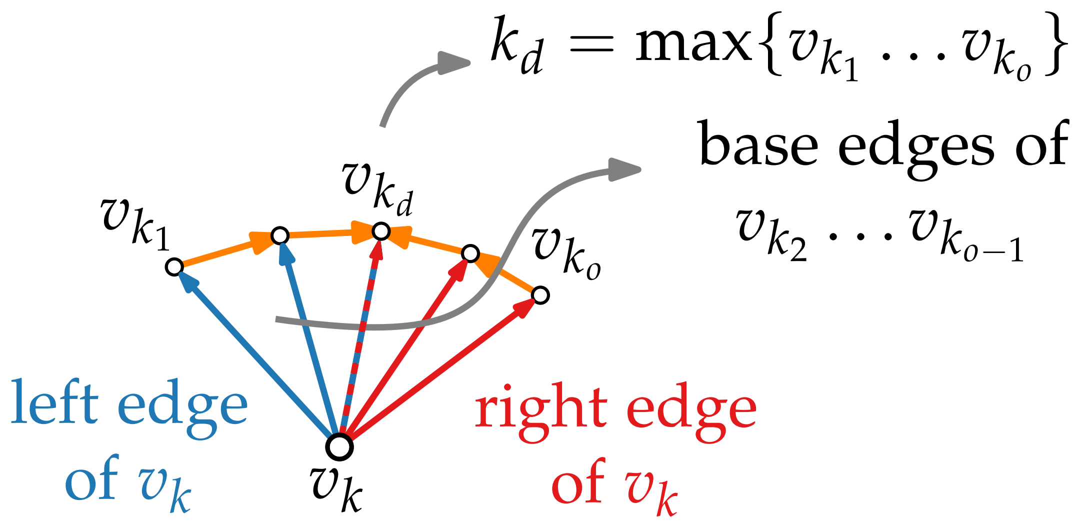


Let T_r be the red edges and T_b the blue edges.

Lemma 3.
$\{T_r, T_b\}$ is a regular edge labeling.

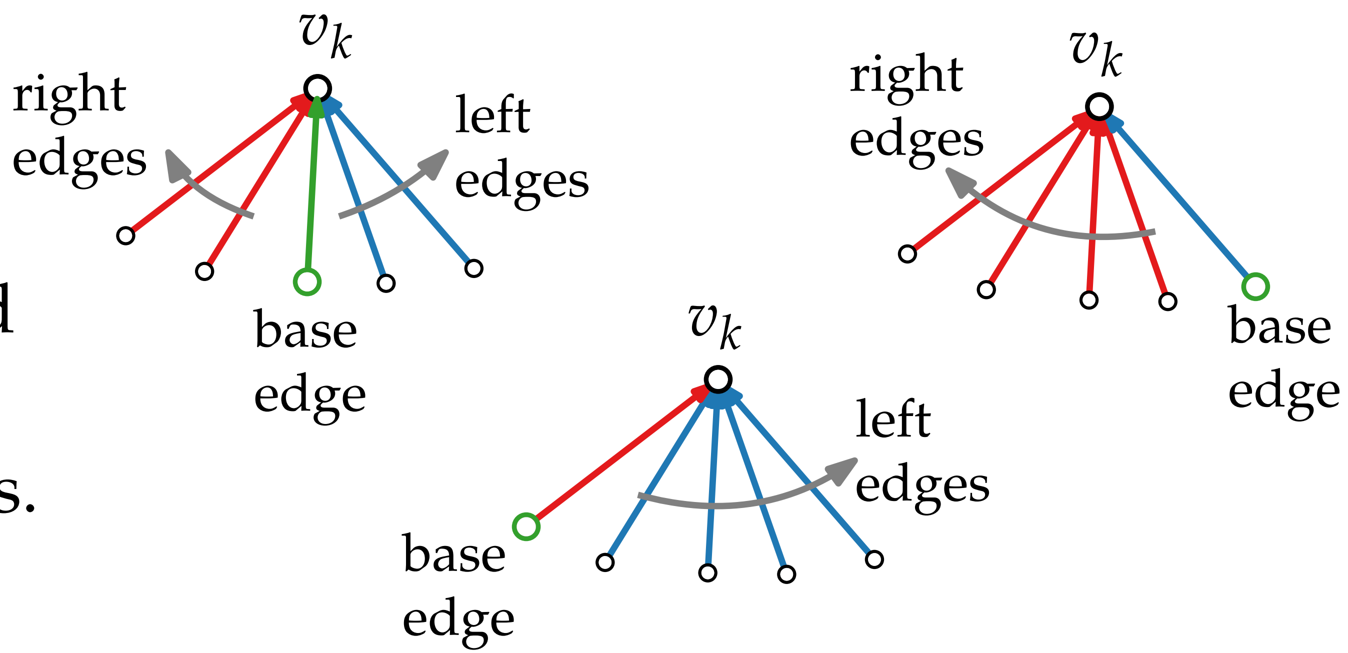
Proof.

$k_o \geq 2$



- $k_1 < k_2 < \dots < k_d$ and $k_d > k_{d+1} > \dots > k_o$
- (v_k, v_{k_i}), $2 \leq i \leq d-1$ are blue
- (v_k, v_{k_i}), $d+1 \leq i \leq o-1$ are red
- (v_k, v_{k_d}) is either red or blue

Refined Canonical Order → REL



Coloring.

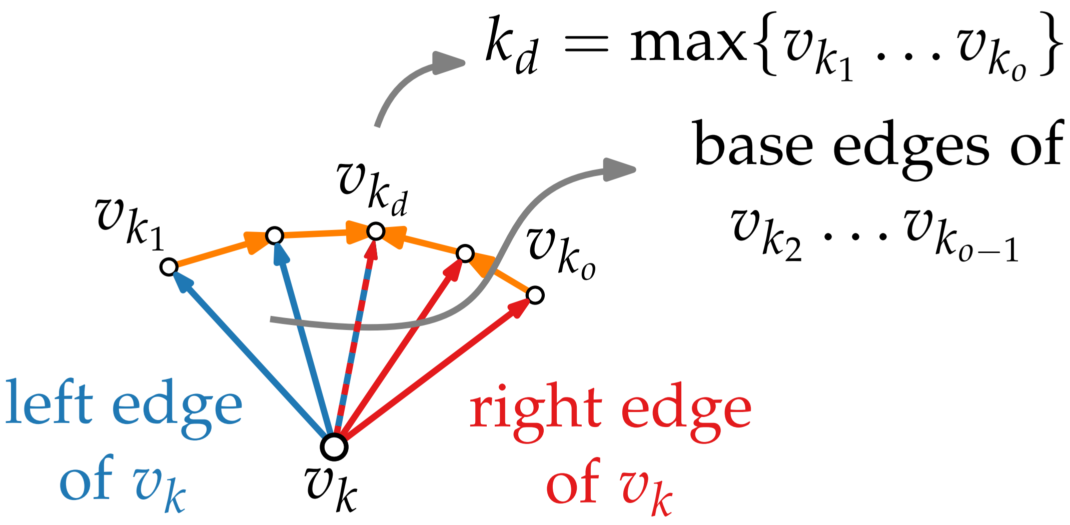
- Color right (left) edges in red (blue).
- Color a base edge (v_{t_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.

> **Lemma 3.**
> $\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$k_o \geq 2$

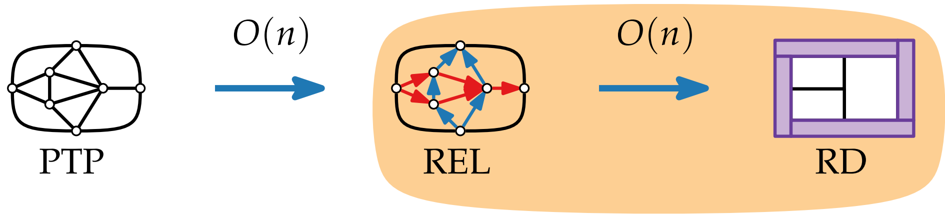


- $k_1 < k_2 < \ldots < k_d$ and $k_d > k_{d+1} > \ldots > k_o$
- (v_k, v_{k_i}), $2 \leq i \leq d-1$ are blue
- (v_k, v_{k_i}), $d+1 \leq i \leq o-1$ are red
- (v_k, v_{k_d}) is either red or blue

$\Rightarrow$ circular order of outgoing edges at v_k correct

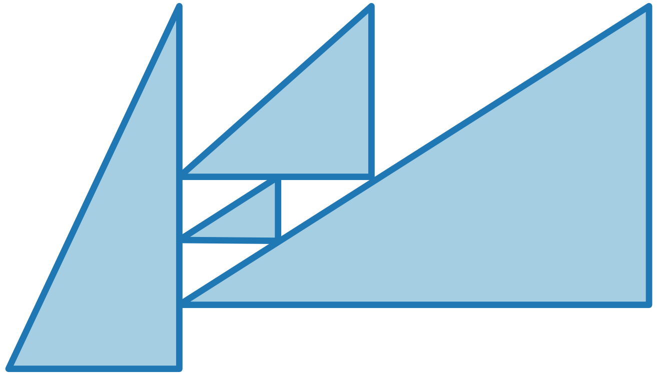
Universität Trier

[Kant, He '94]:

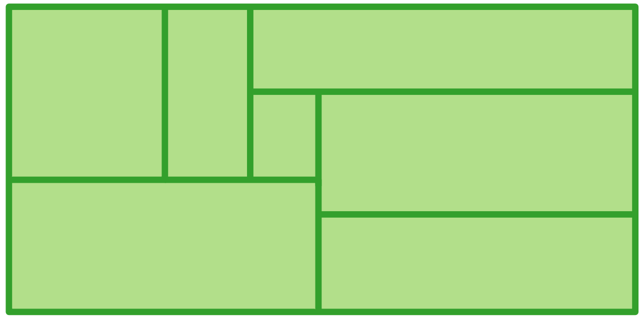


Visualization of Graphs

Lecture 9:
Contact Representations of Planar Graphs: Triangle Contacts and Rectangular Duals

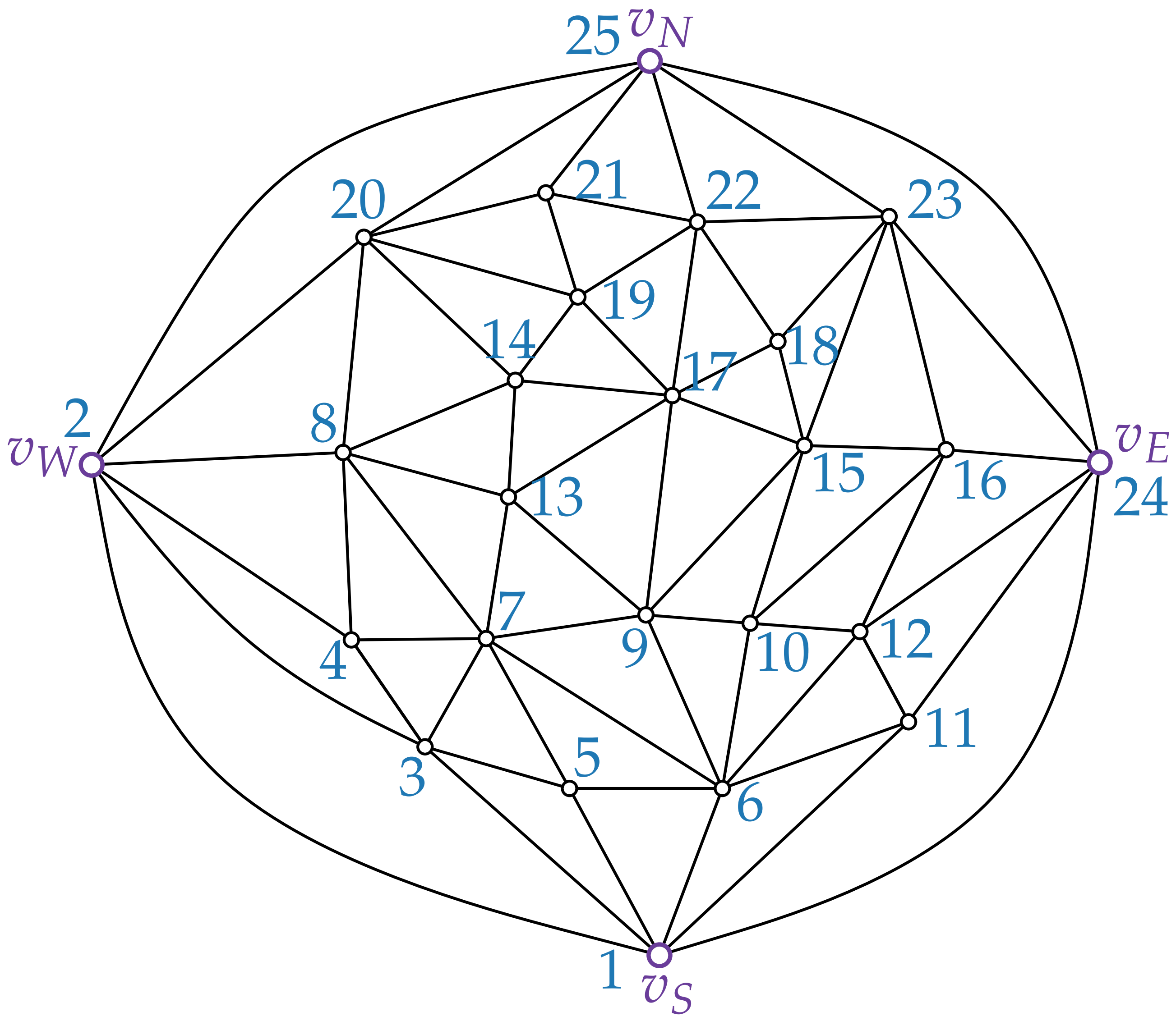


Part V:
Computing the Coordinates

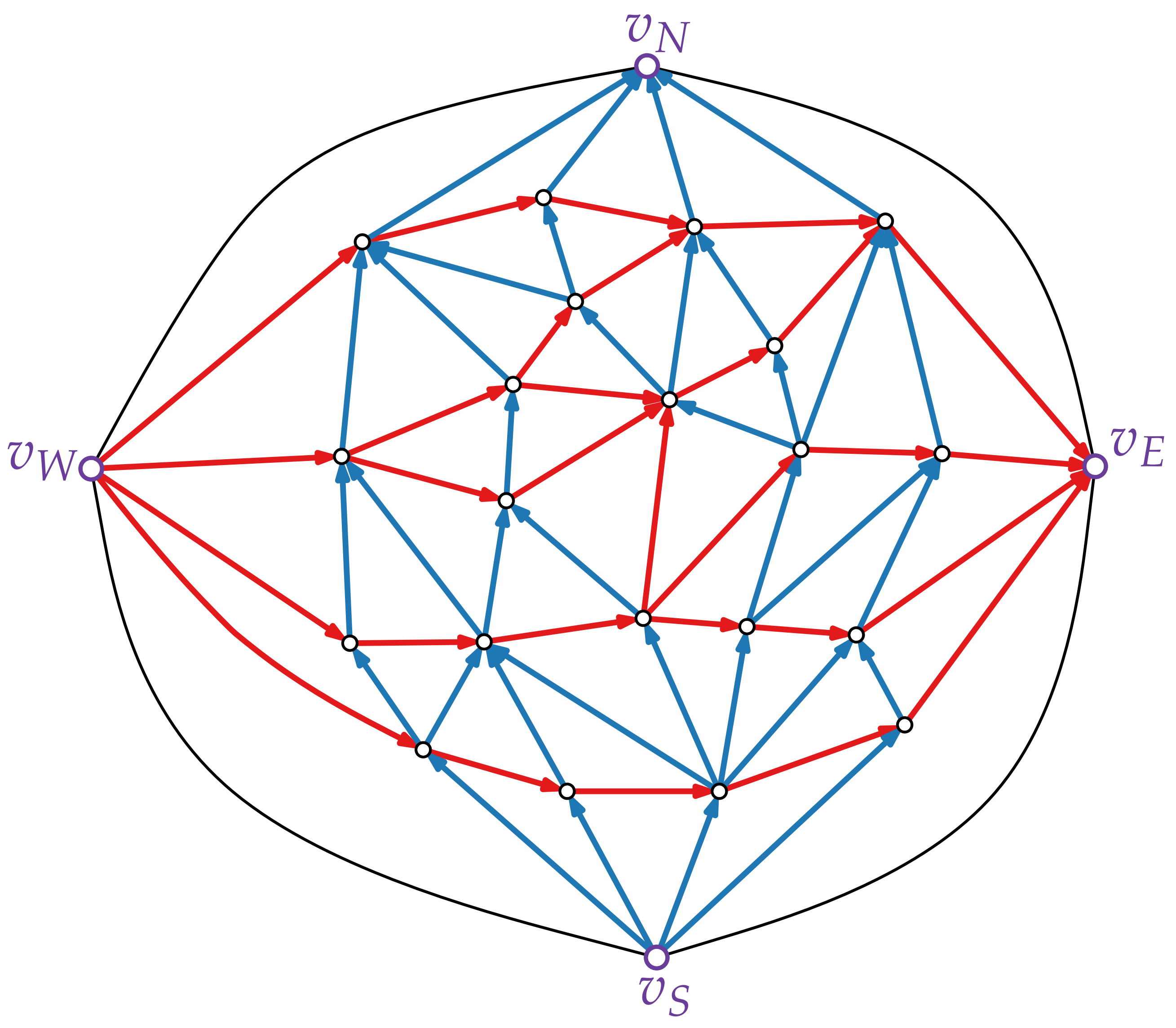


Philipp Kindermann

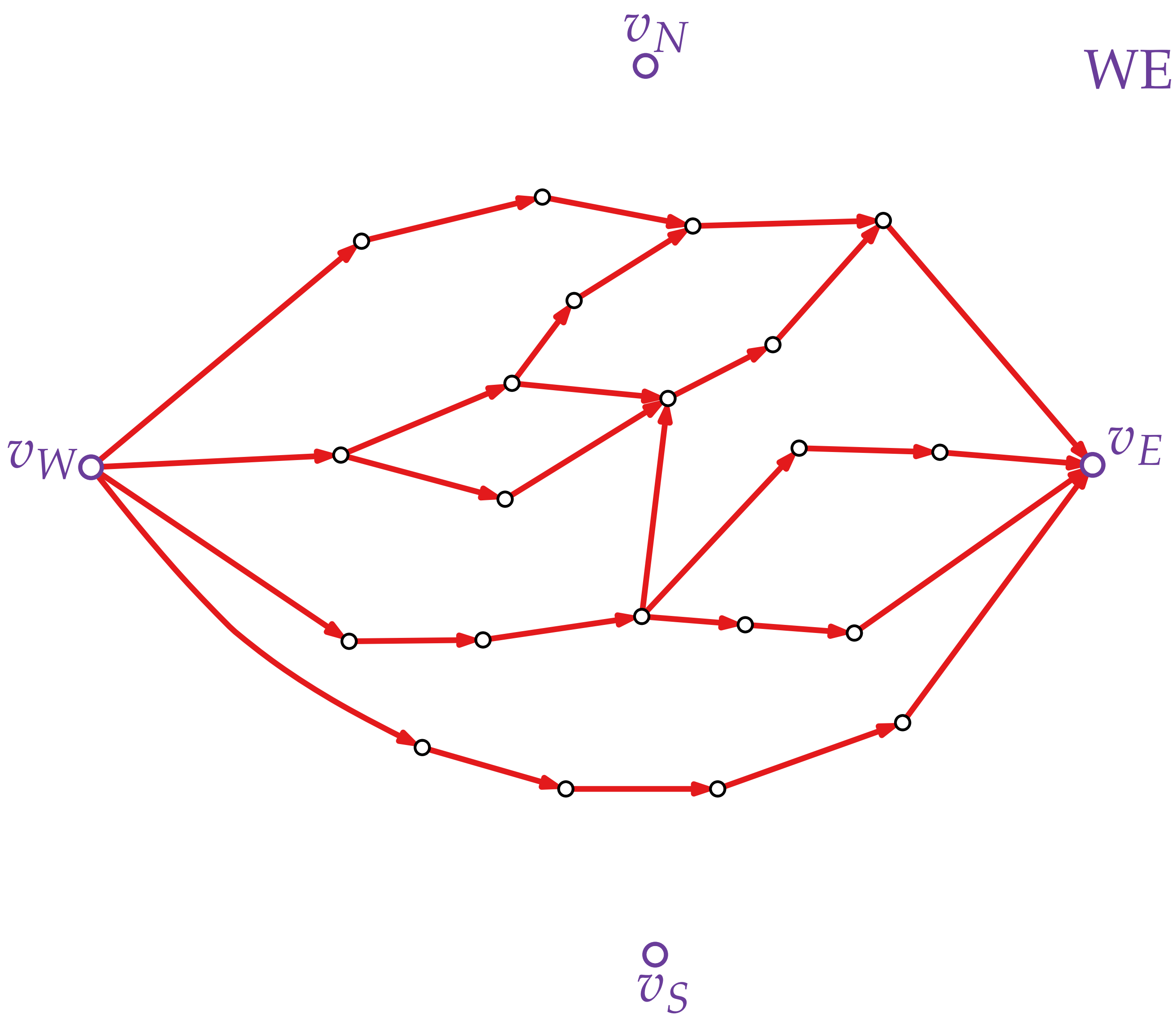
From REL to st-digraphs to Coordinates



From REL to st-digraphs to Coordinates

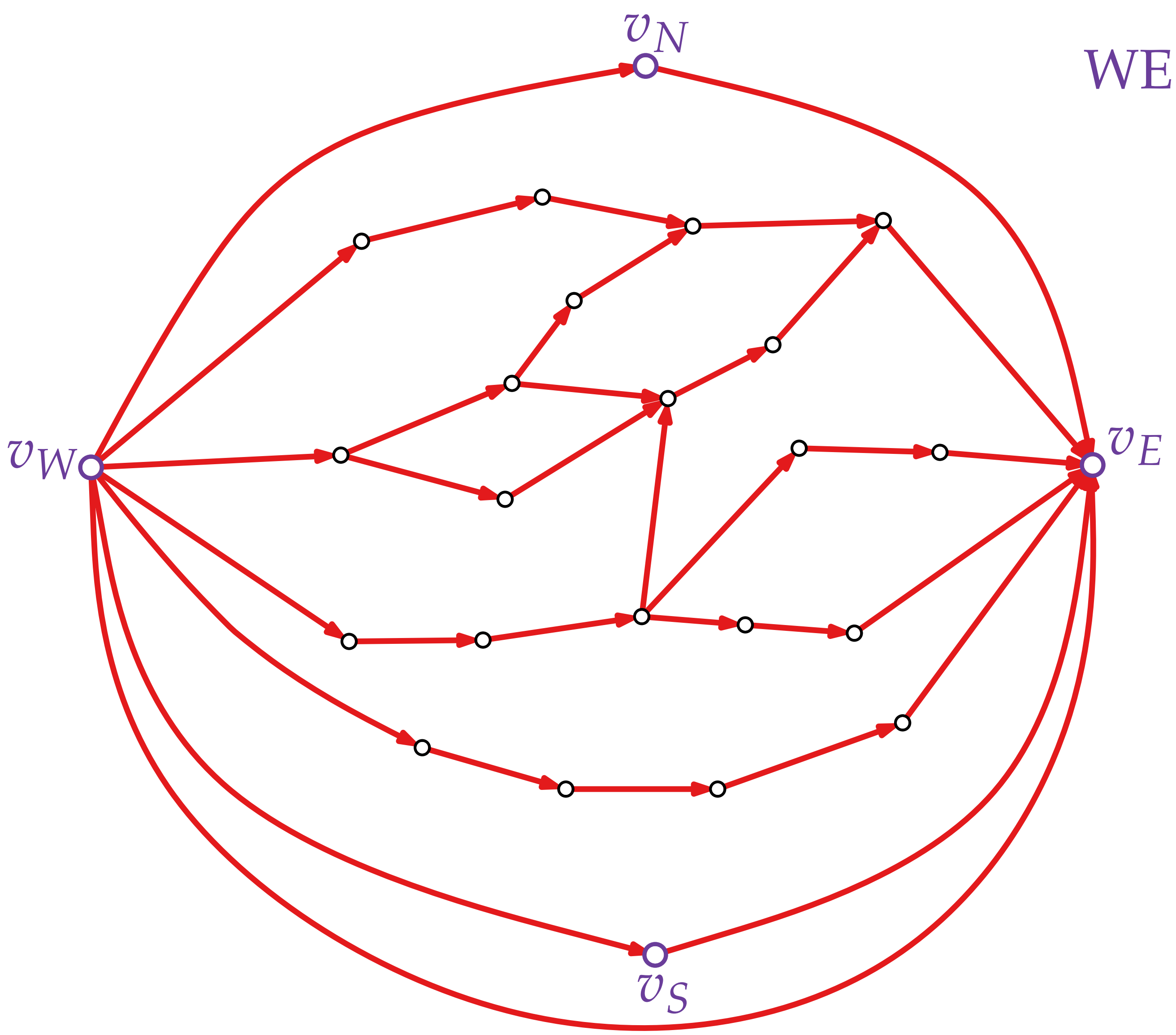


From REL to st-digraphs to Coordinates



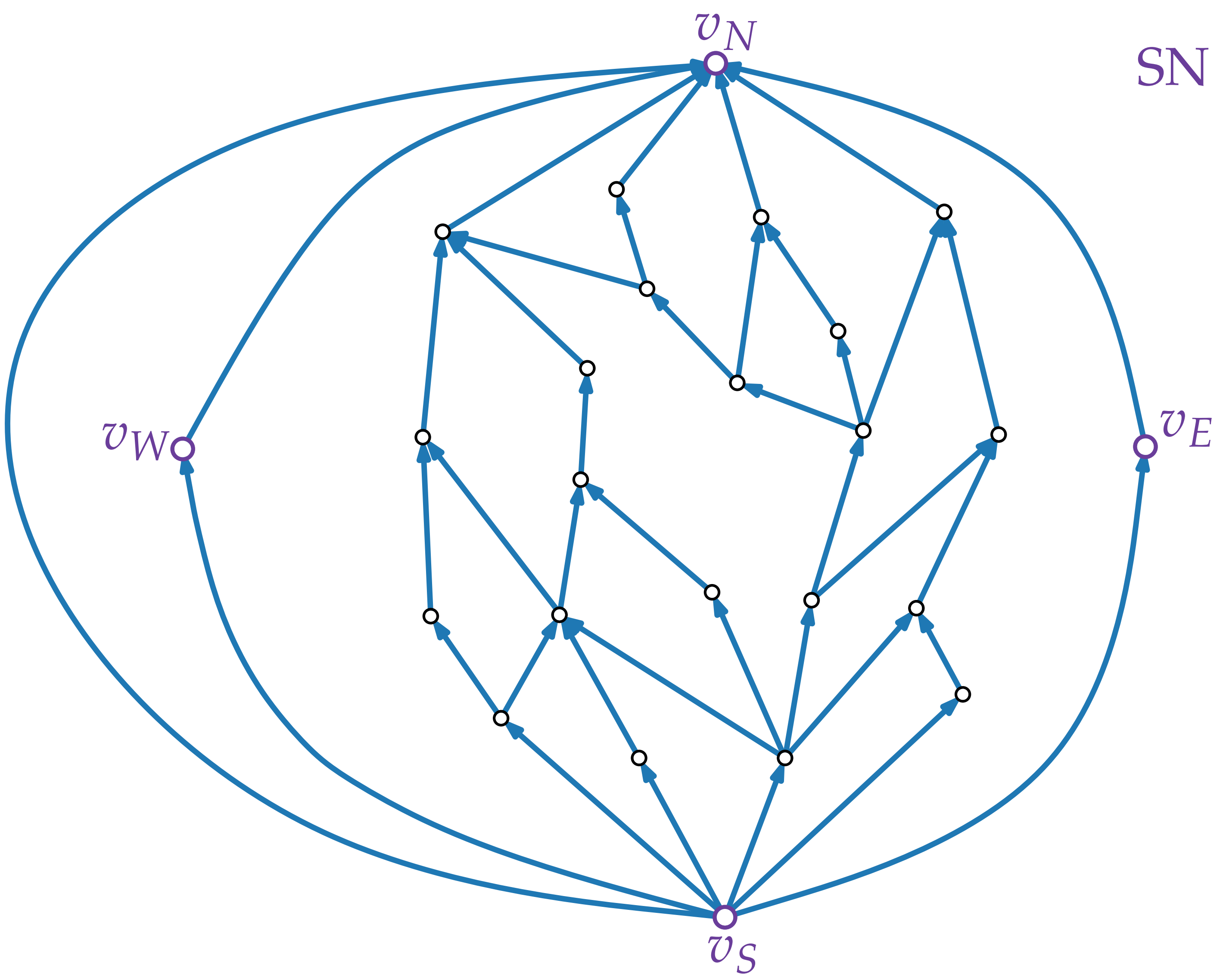
WE network G_{hor}

From REL to st-digraphs to Coordinates



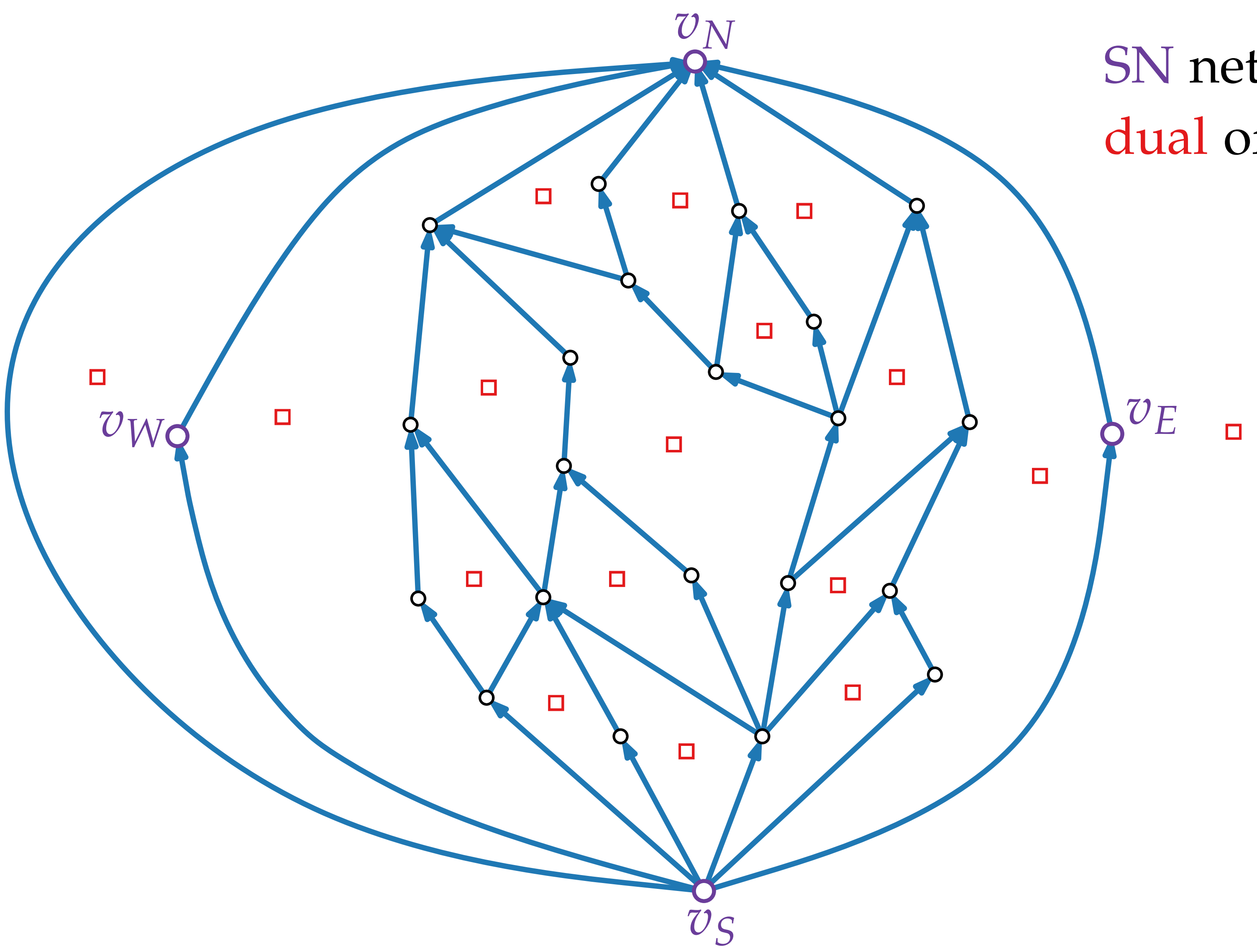
WE network G_{hor}

From REL to st-digraphs to Coordinates



SN network G_{ver}

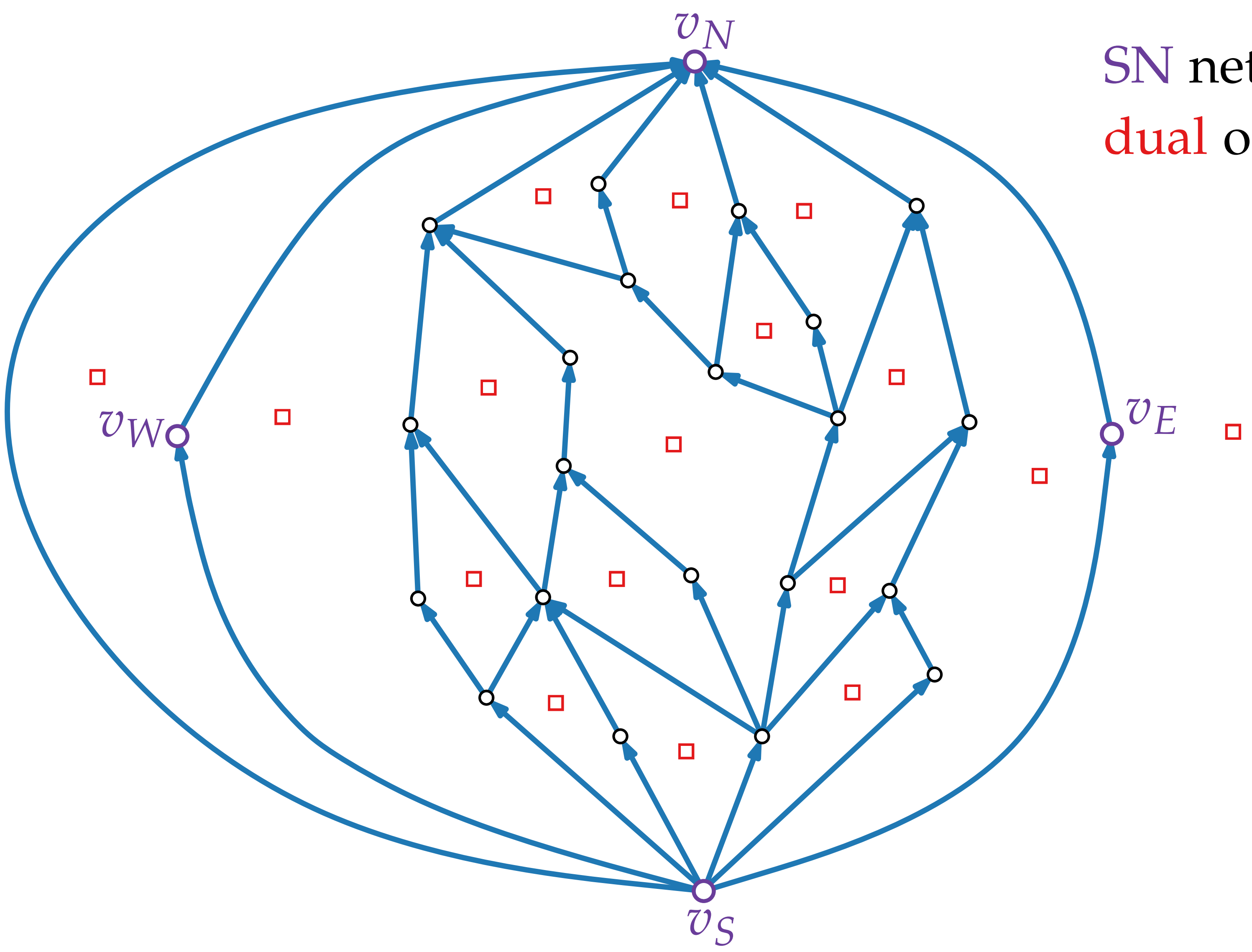
From REL to st-digraphs to Coordinates



SN network G_{ver}

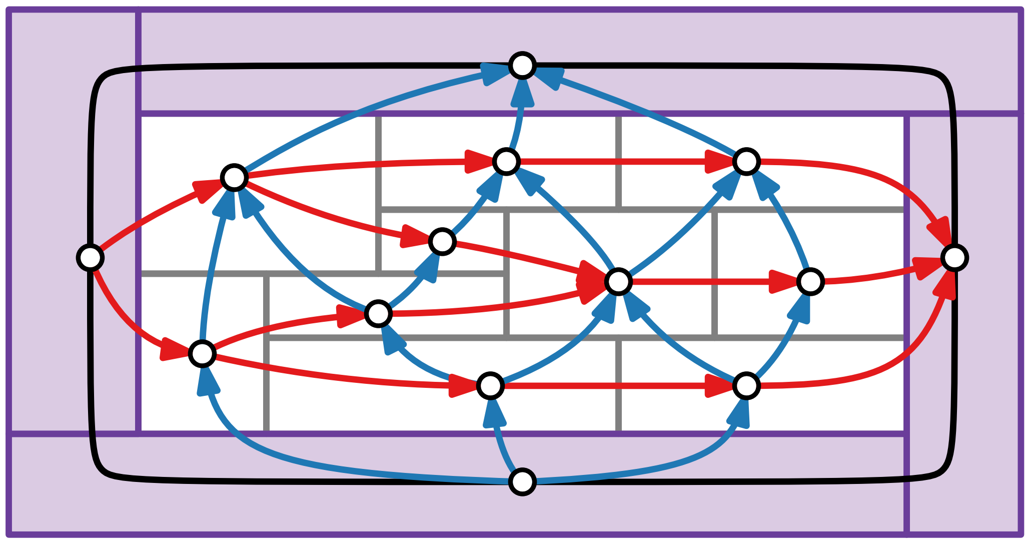
dual of G_{ver}

From REL to st-digraphs to Coordinates

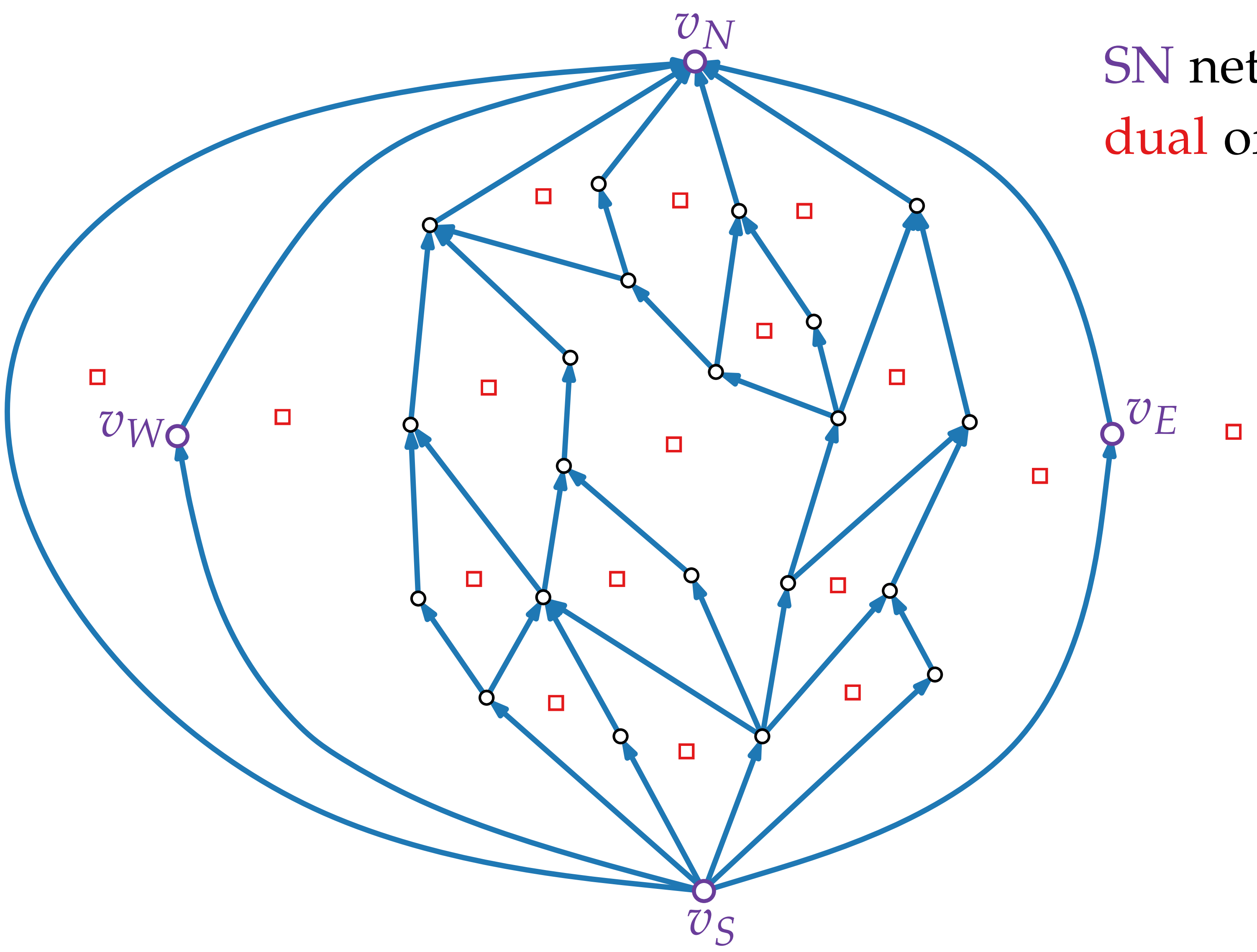


SN network G_{ver}

dual of G_{ver}

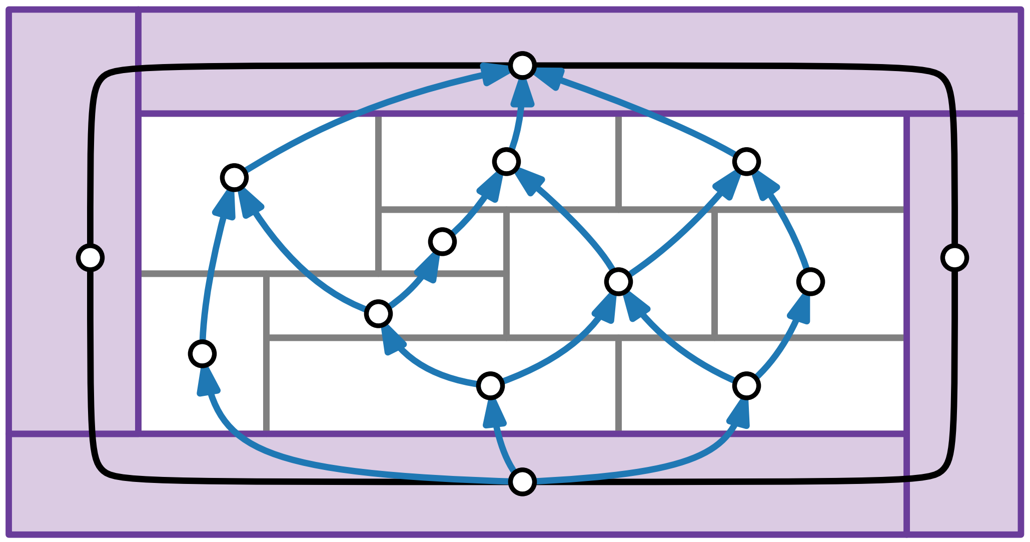


From REL to st-digraphs to Coordinates

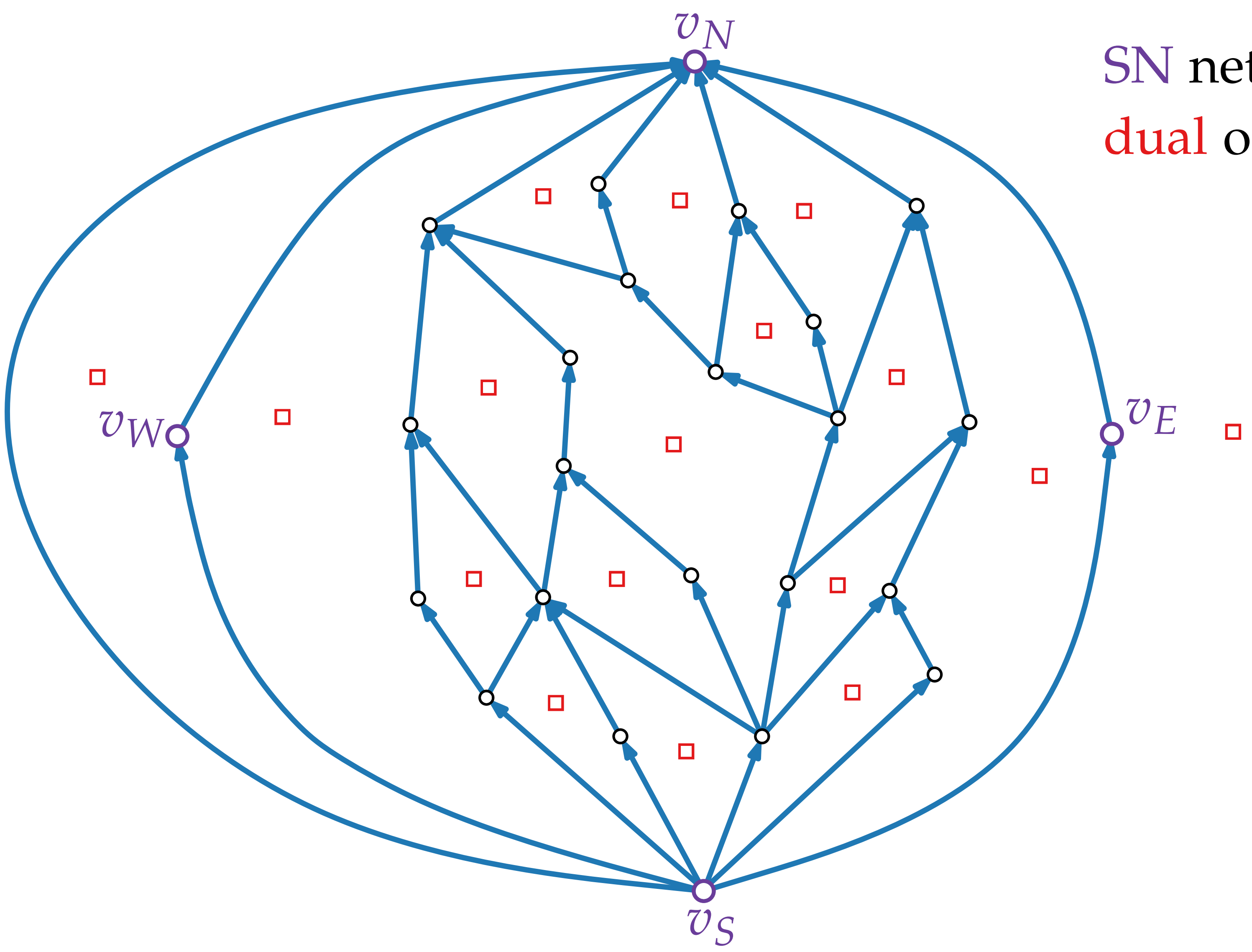


SN network G_{ver}

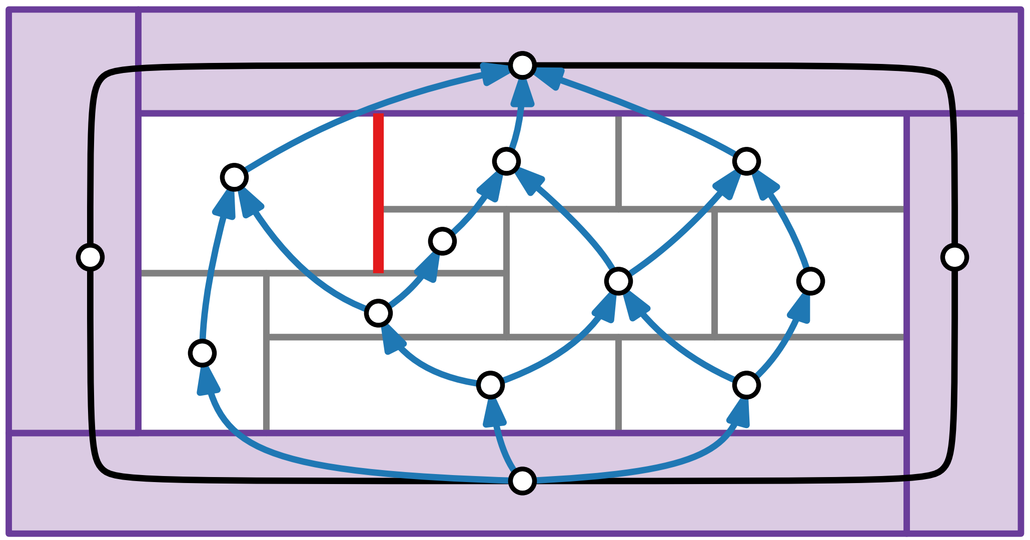
dual of G_{ver}



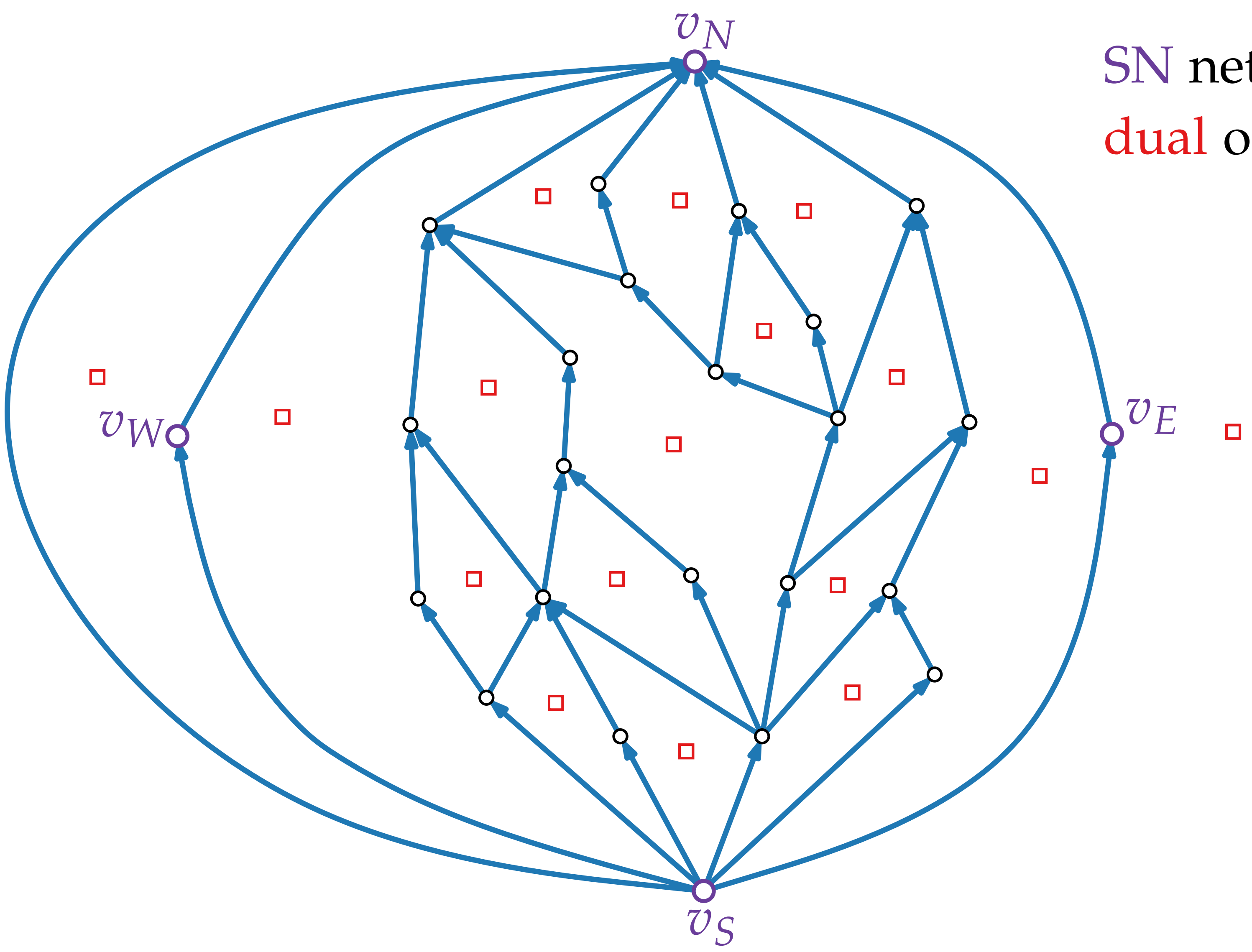
From REL to st-digraphs to Coordinates



SN network G_{ver}
dual of G_{ver}

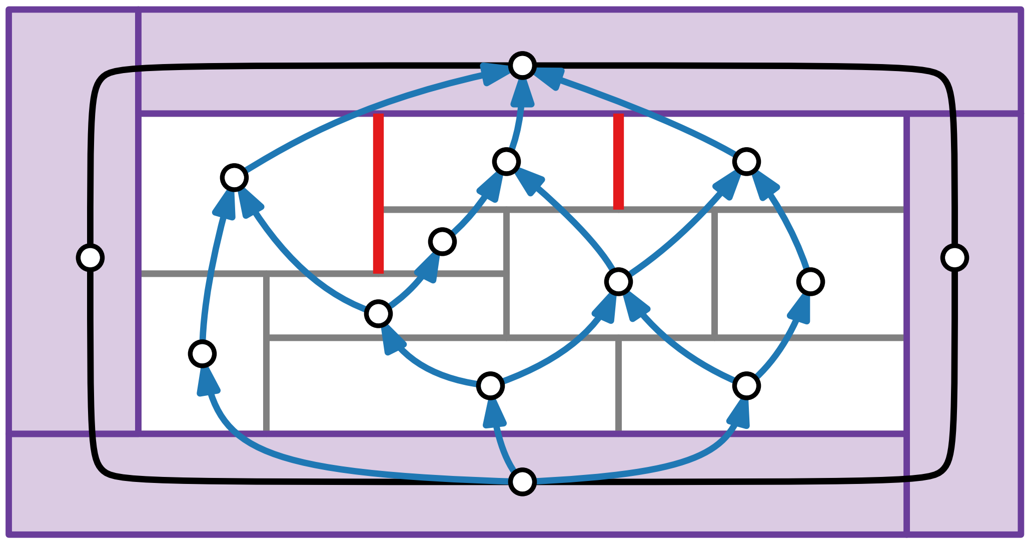


From REL to st-digraphs to Coordinates

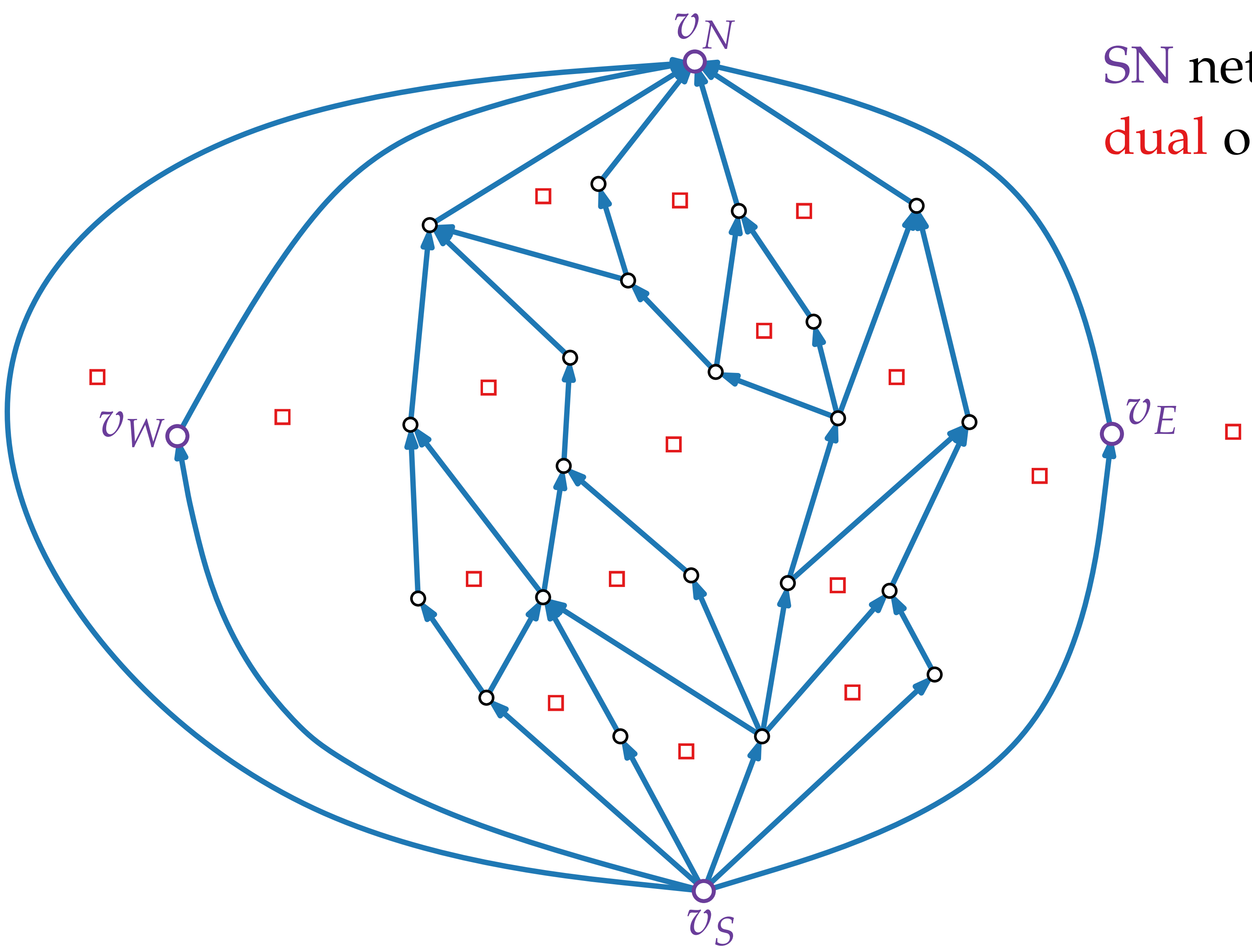


SN network G_{ver}

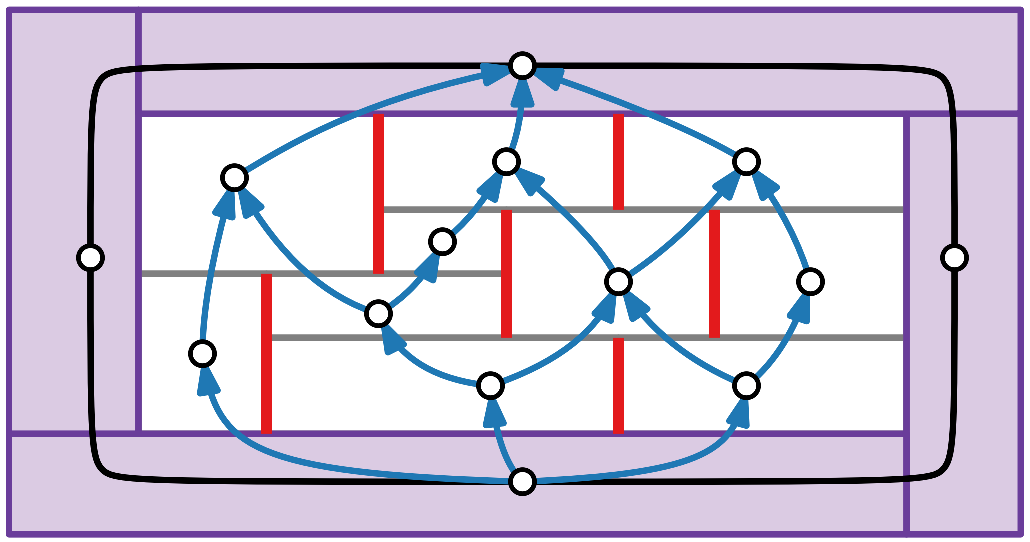
dual of G_{ver}



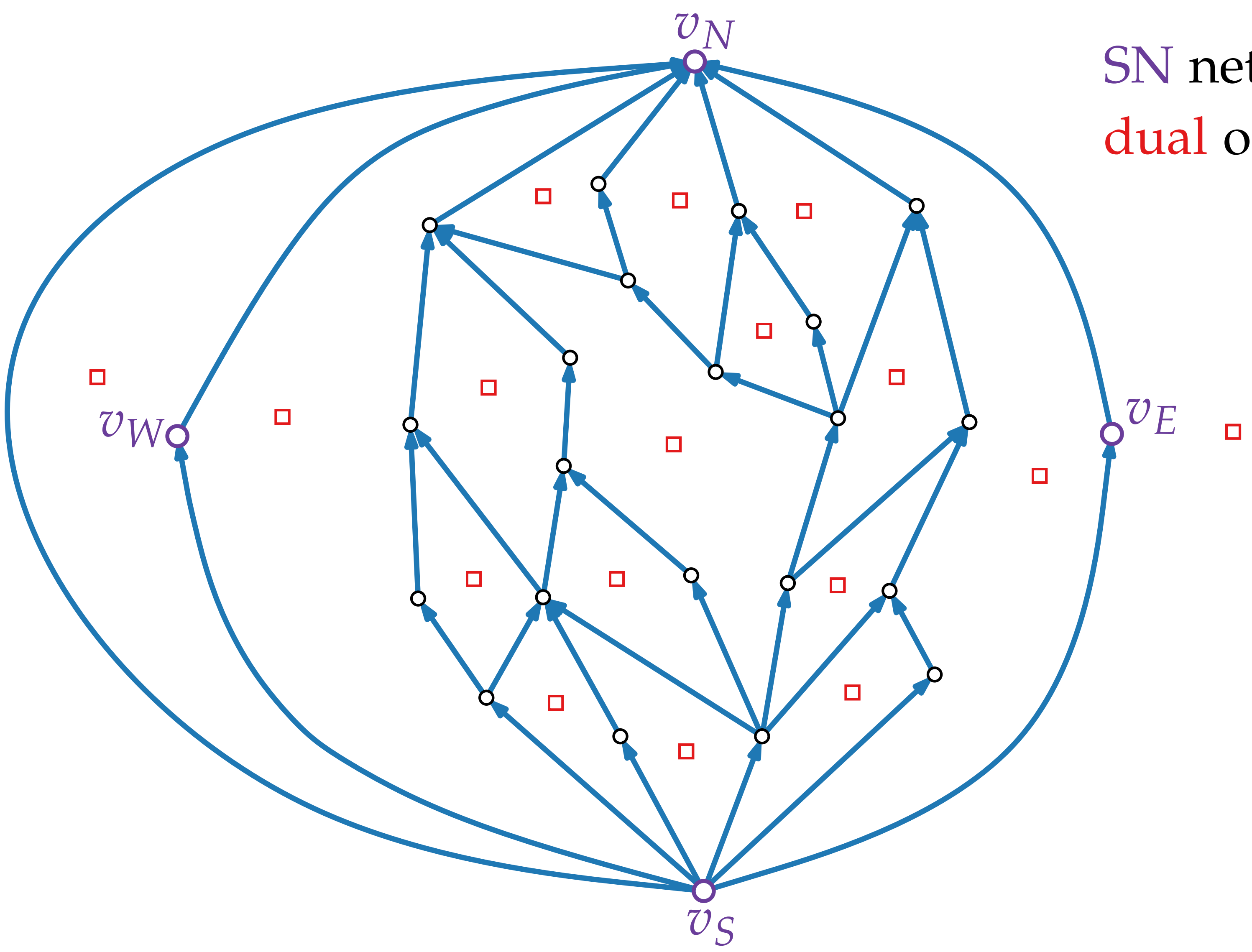
From REL to st-digraphs to Coordinates



SN network G_{ver}
dual of G_{ver}

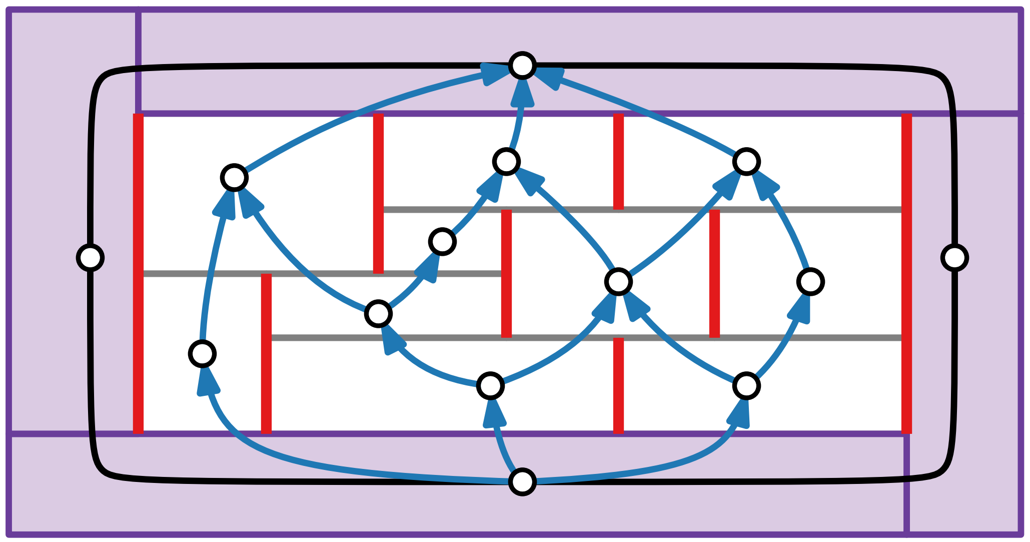


From REL to st-digraphs to Coordinates

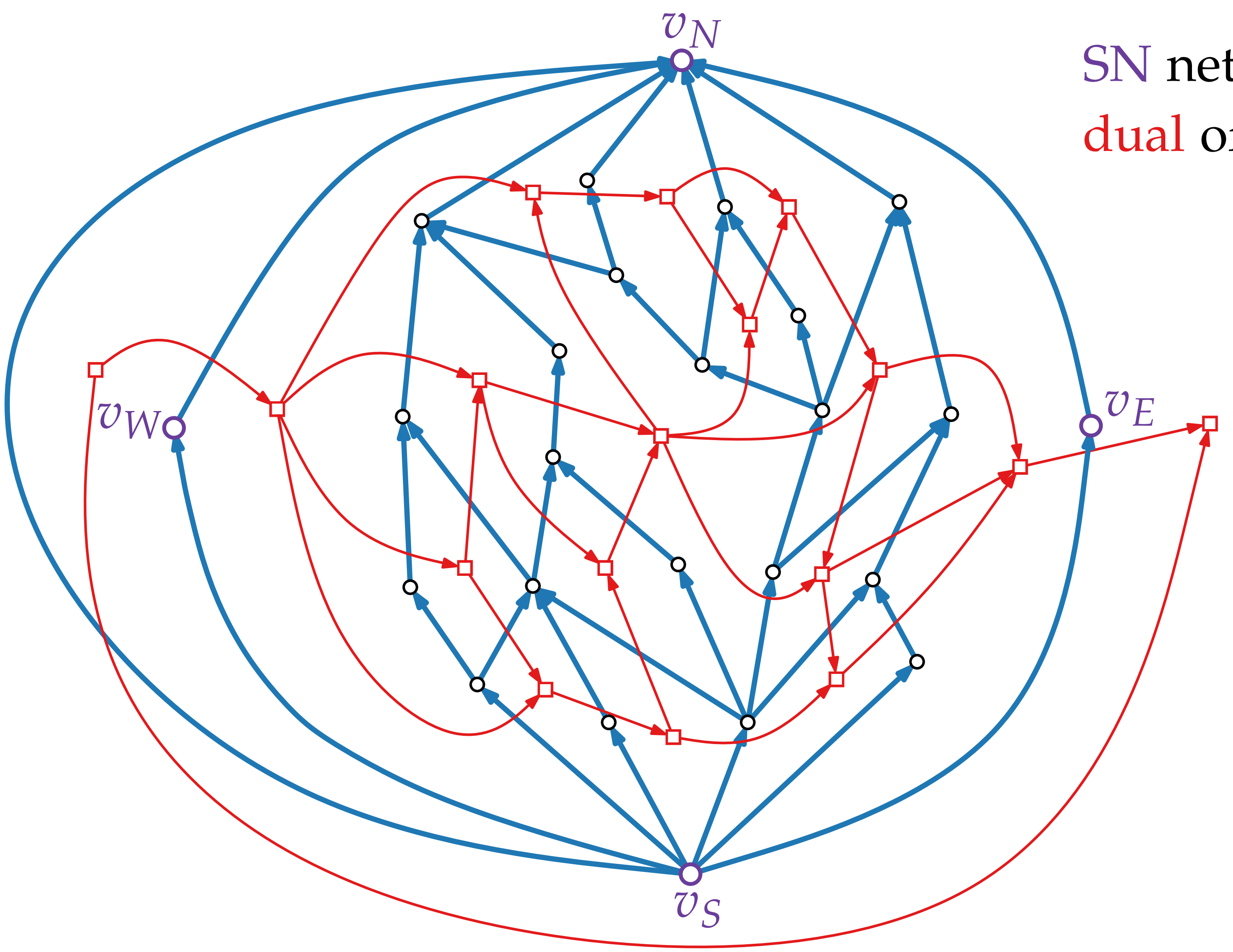


SN network G_{ver}

dual of G_{ver}



From REL to st-digraphs to Coordinates

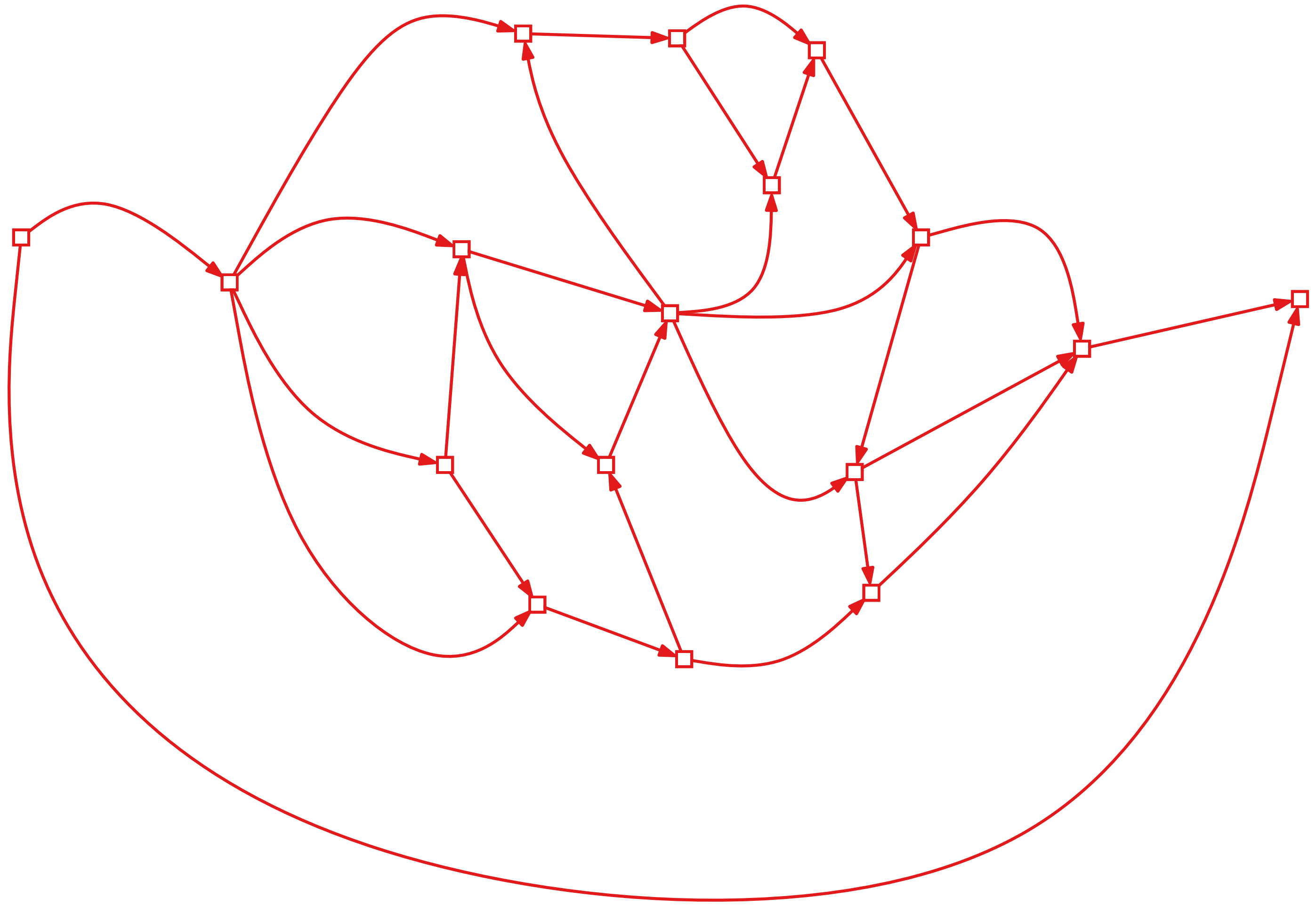


SN network G_{ver}

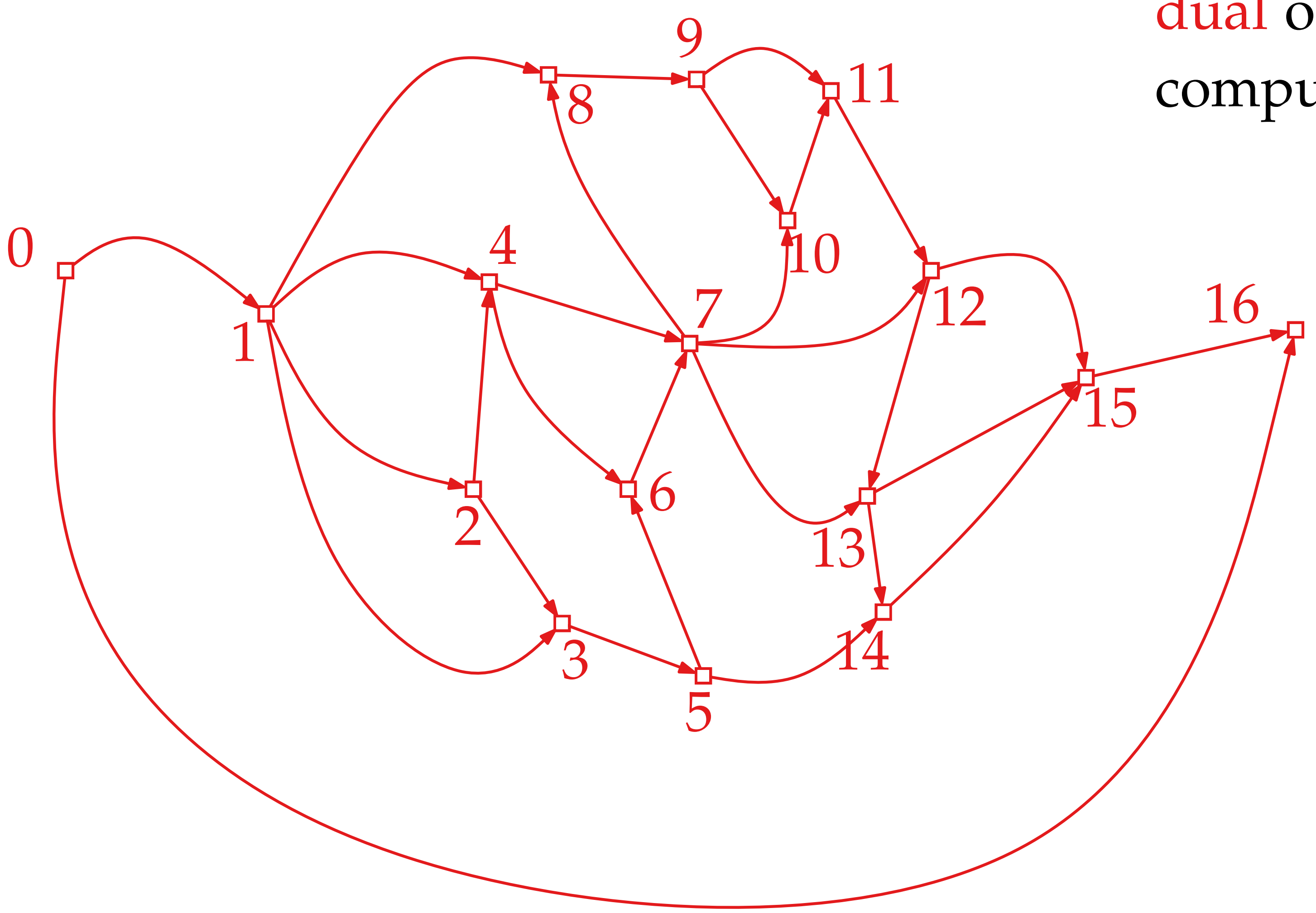
dual of G_{ver}

From REL to st-digraphs to Coordinates

dual of G_{ver}



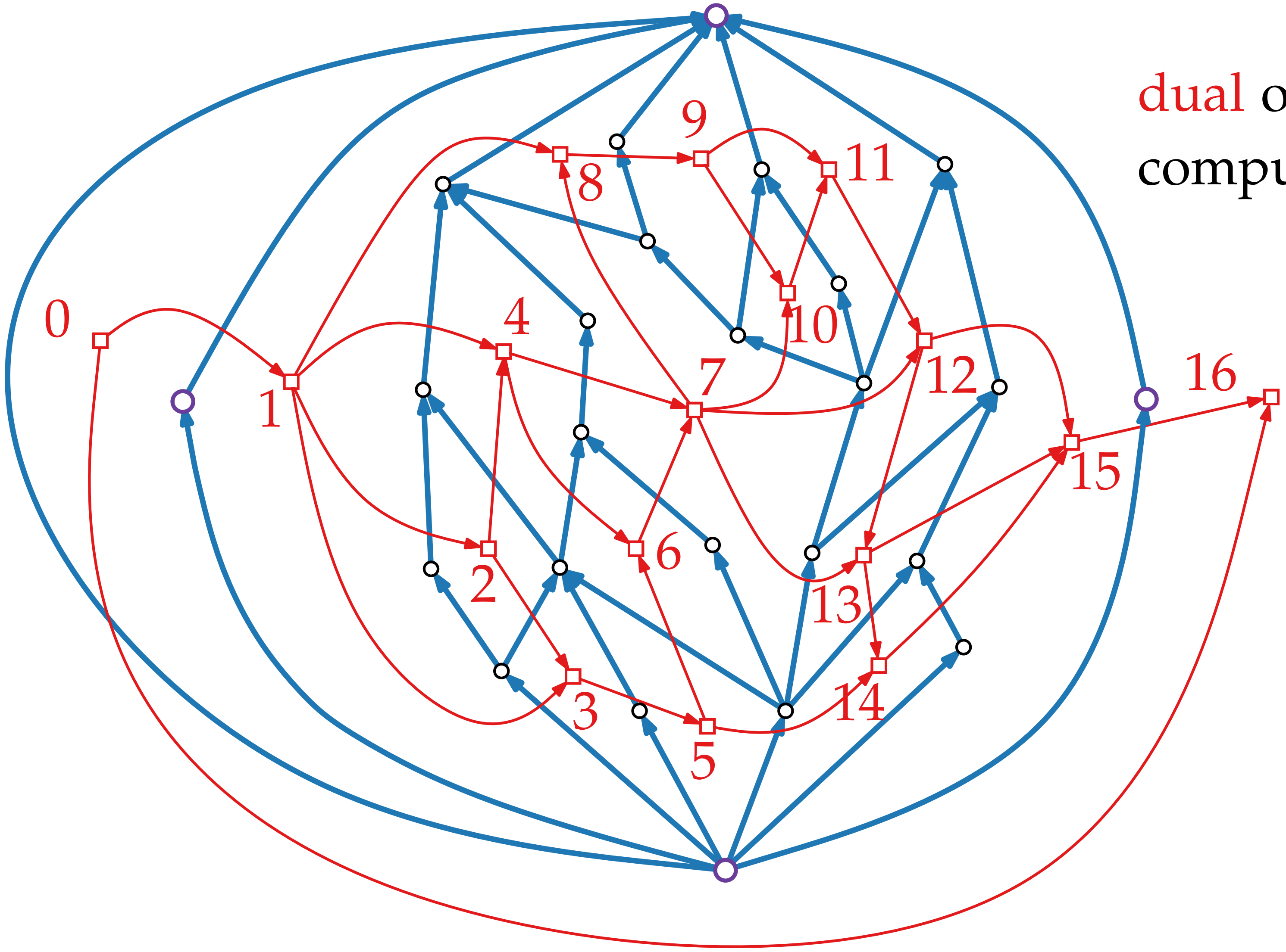
From REL to st-digraphs to Coordinates



dual of G_{ver}

compute topological order

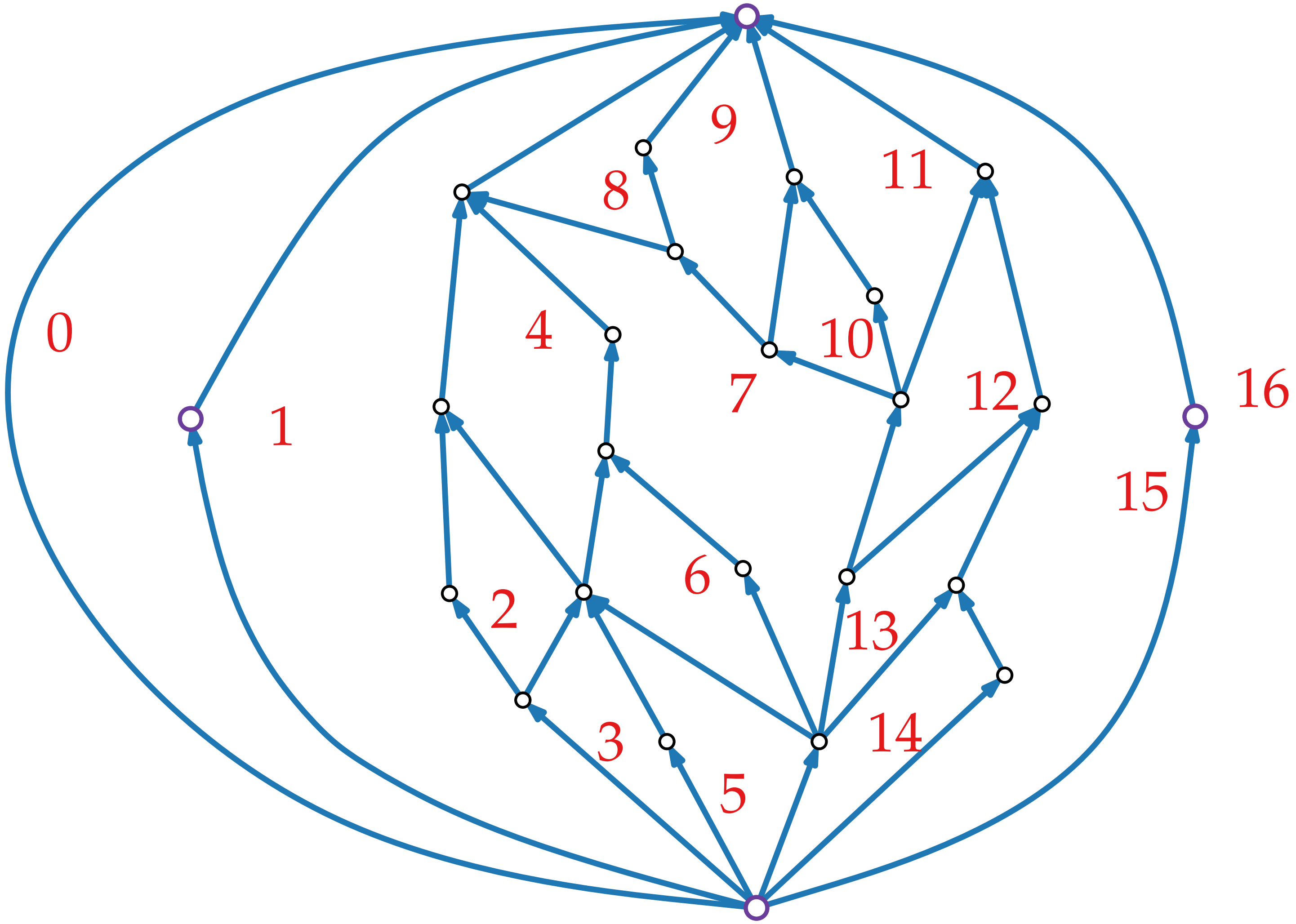
From REL to st-digraphs to Coordinates



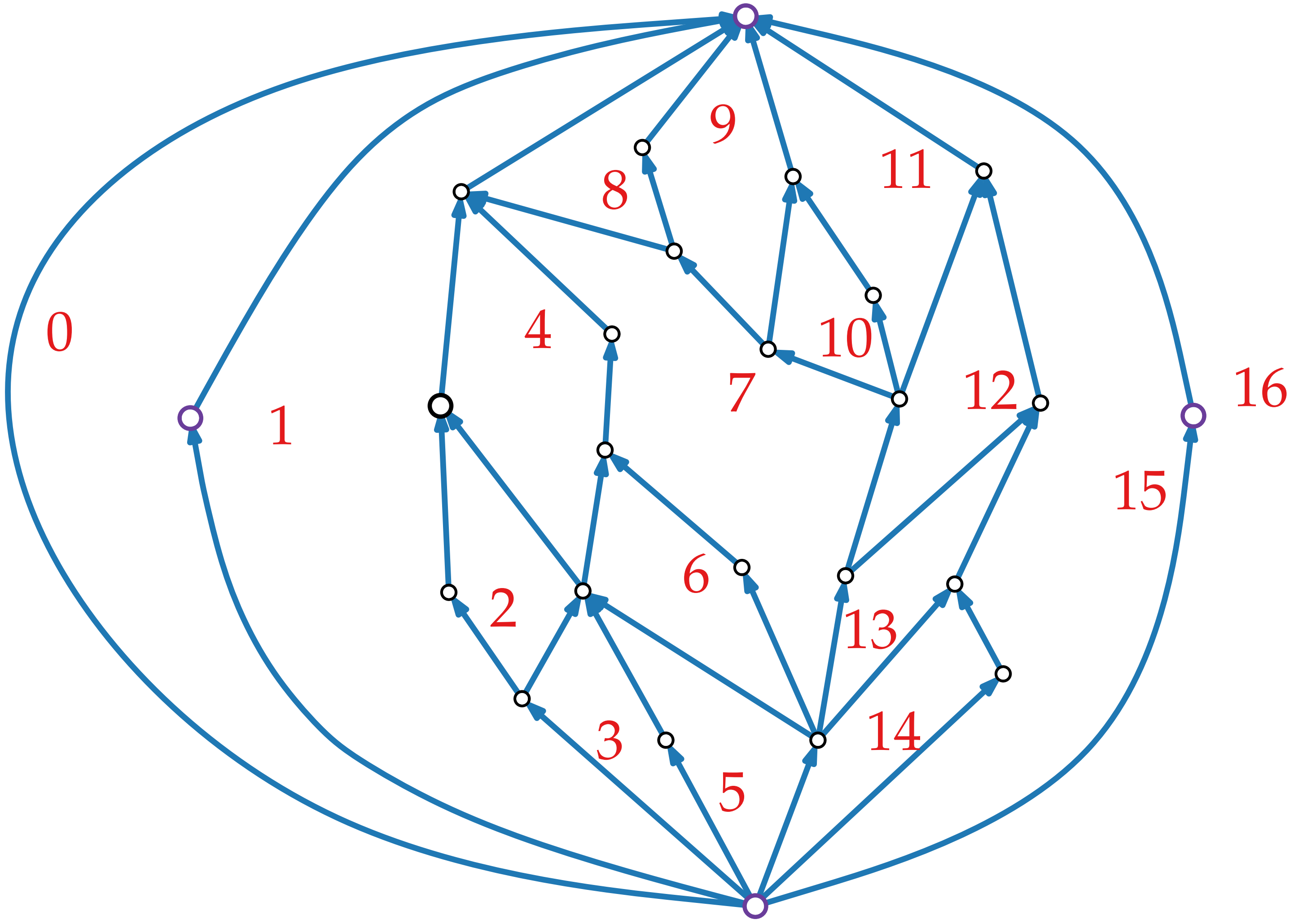
dual of G_{ver}

compute topological order

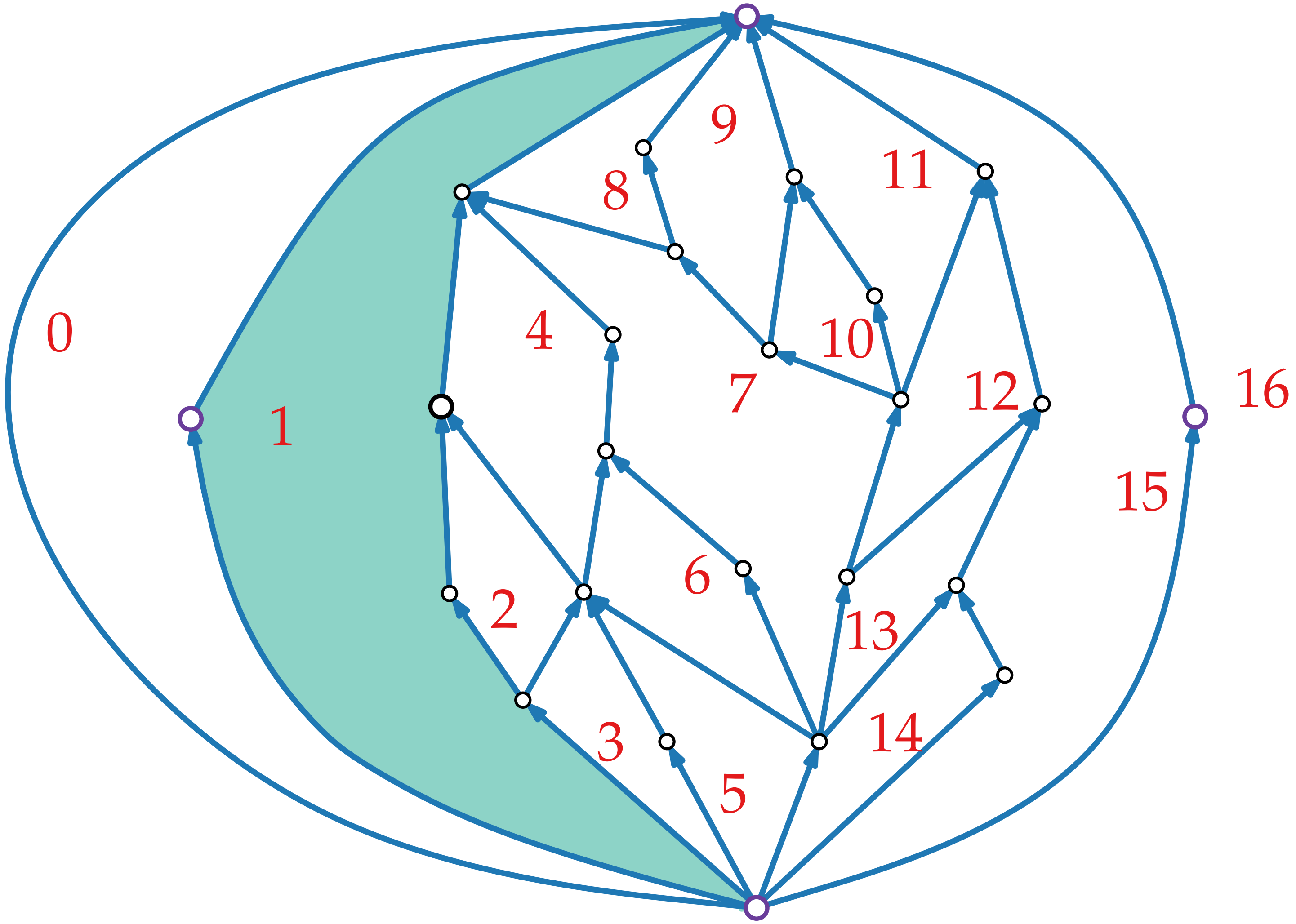
From REL to st-digraphs to Coordinates



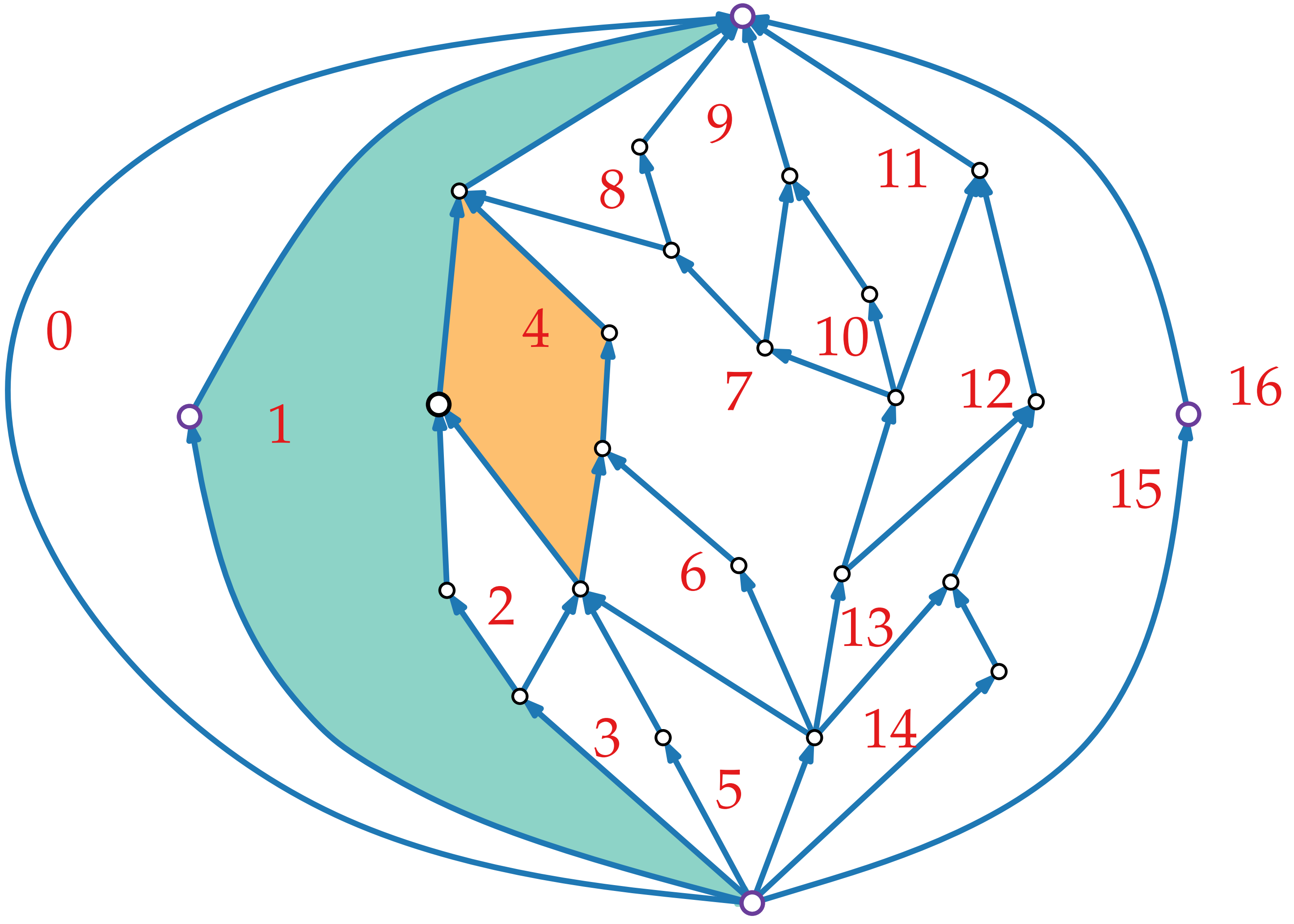
From REL to st-digraphs to Coordinates



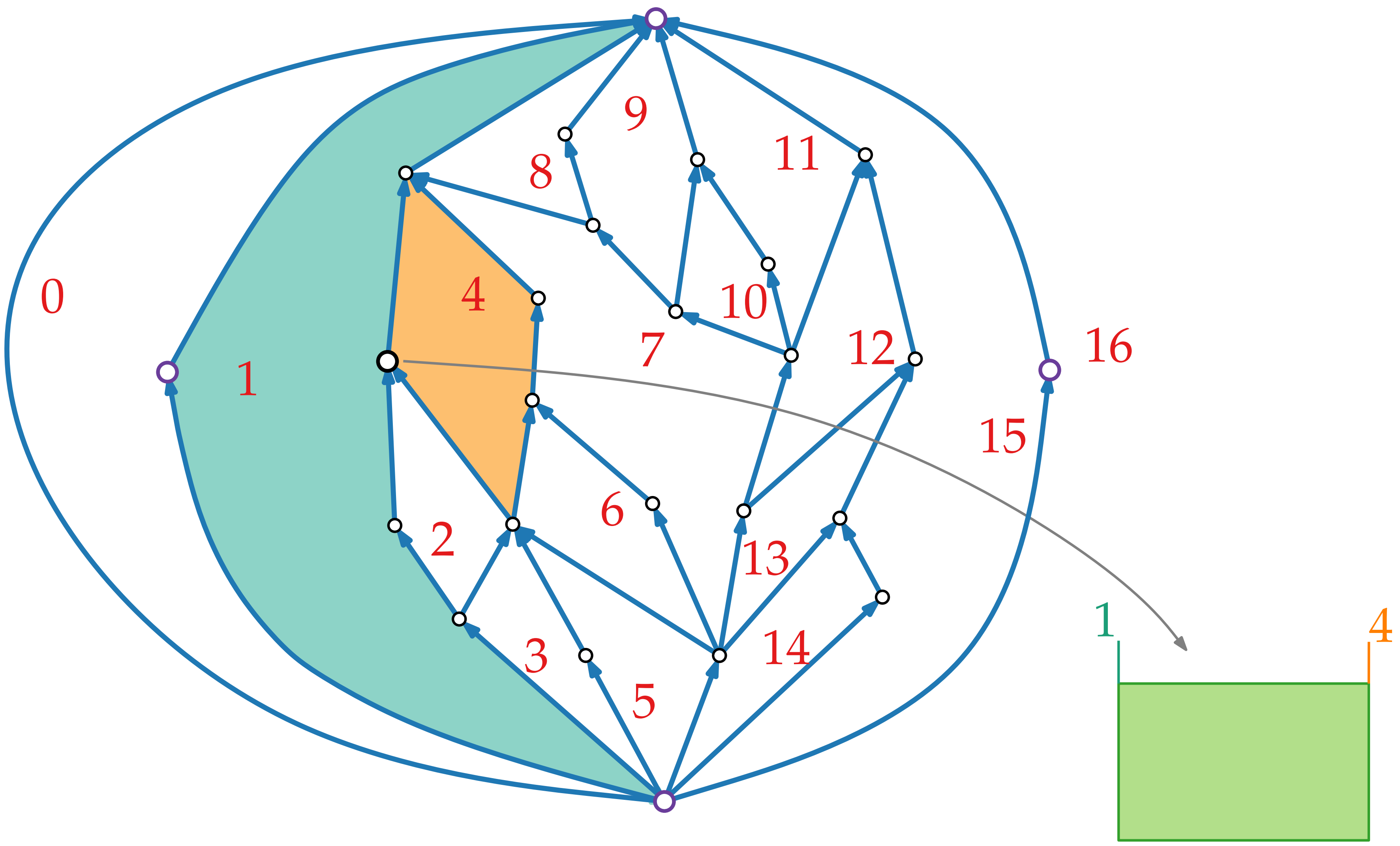
From REL to st-digraphs to Coordinates



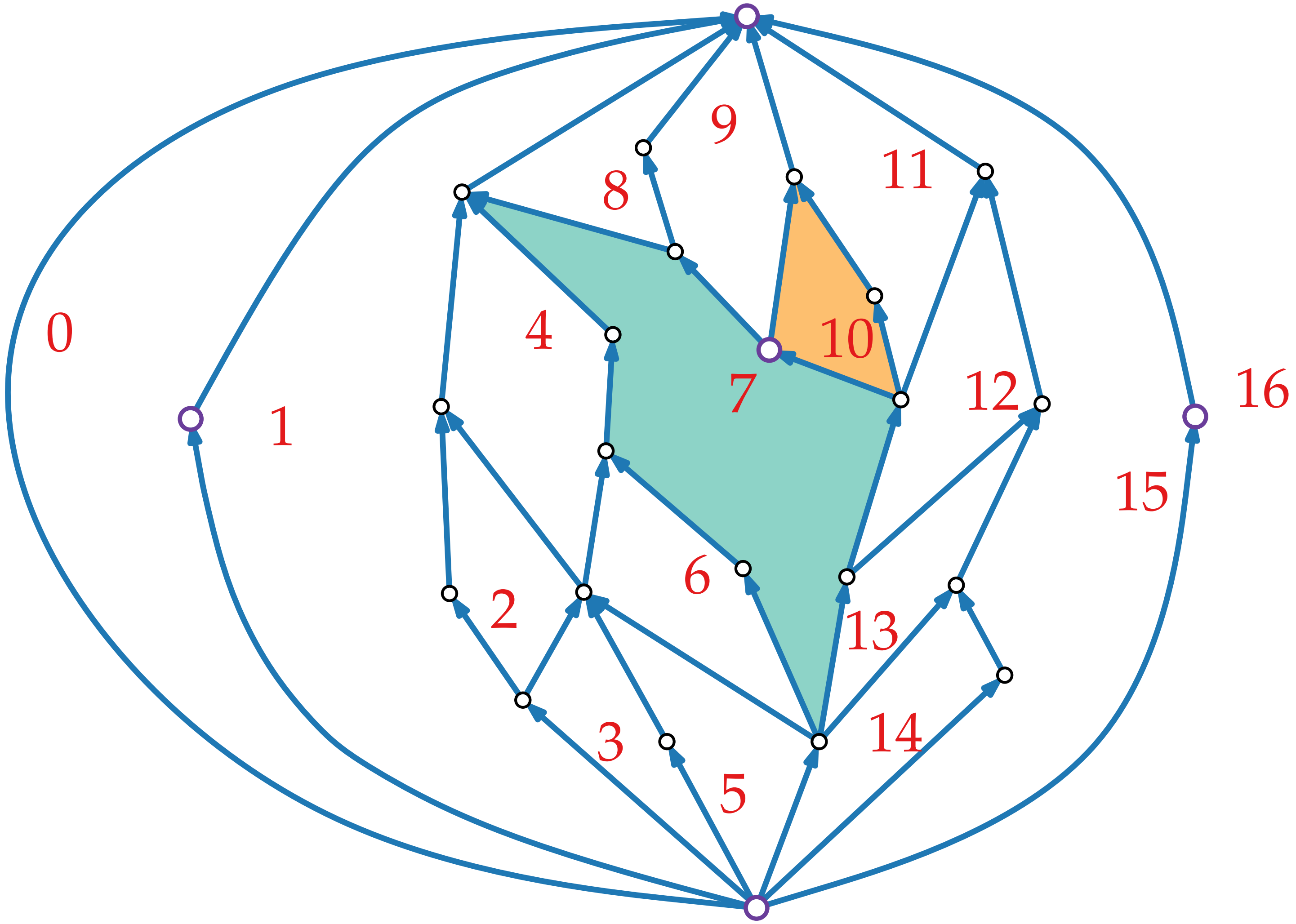
From REL to st-digraphs to Coordinates



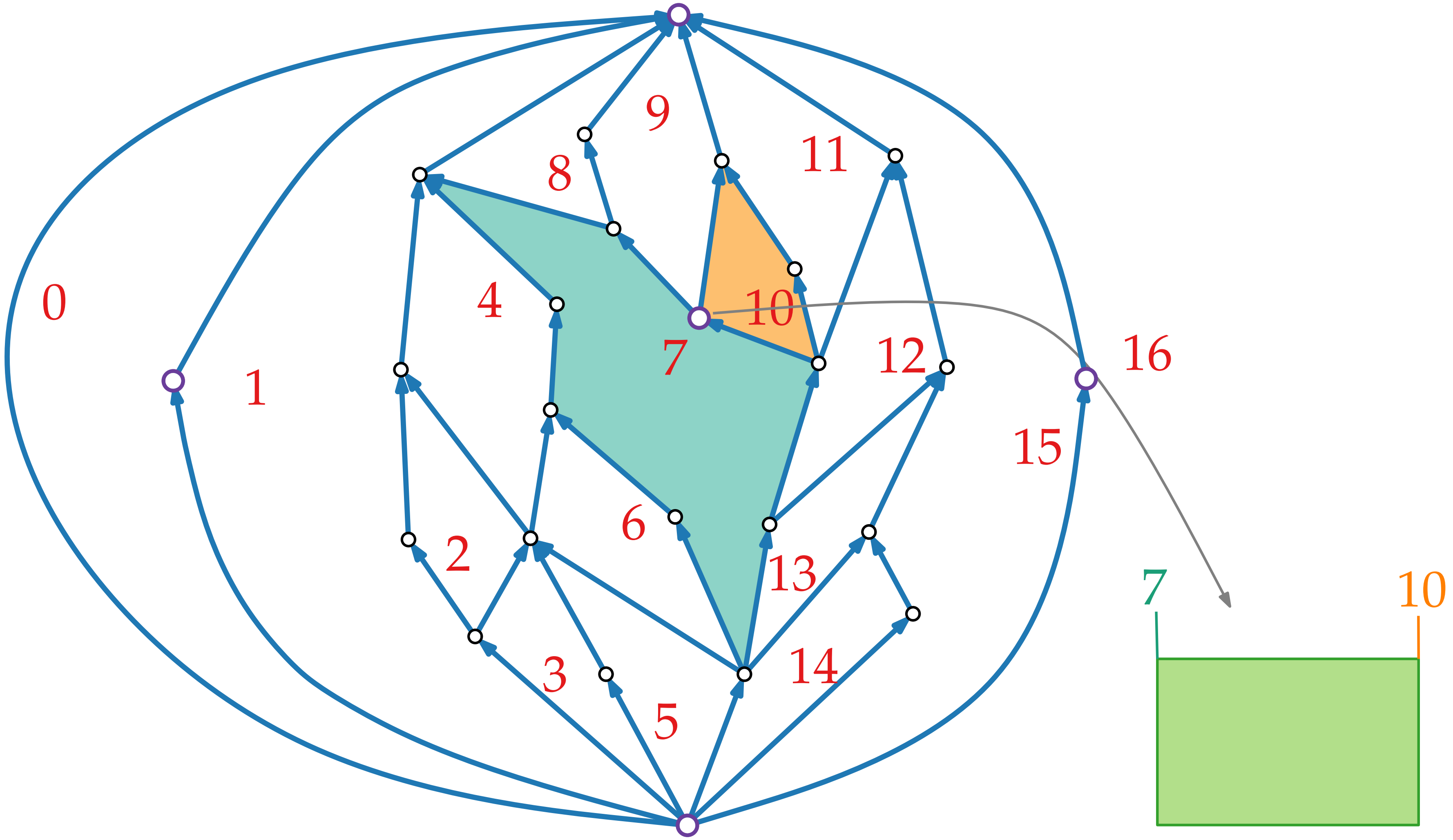
From REL to st-digraphs to Coordinates



From REL to st-digraphs to Coordinates



From REL to st-digraphs to Coordinates



Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

- Find a REL $\{T_r, T_b\}$ of G;

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

- Find a REL $\{T_r, T_b\}$ of G;
- Construct a SN network G_{ver} of G (consists of T_b plus outer edges)

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

- Find a REL $\{T_r, T_b\}$ of G;
- Construct a SN network G_{ver} of G (consists of T_b plus outer edges)
- Construct the dual $G^\star_{\text{ver}}$ of G_{ver} and compute a topological ordering f_{ver} of $G^\star_{\text{ver}}$

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

- Find a REL $\{T_r, T_b\}$ of G;
- Construct a SN network G_{ver} of G (consists of T_b plus outer edges)
- Construct the dual $G^\star_{\text{ver}}$ of G_{ver} and compute a topological ordering f_{ver} of $G^\star_{\text{ver}}$
- For each vertex $v \in V$, let g and h be the face on the left and face on the right of v.

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

- Find a REL $\{T_r, T_b\}$ of G;
- Construct a SN network G_{ver} of G (consists of T_b plus outer edges)
- Construct the dual $G^\star_{\text{ver}}$ of G_{ver} and compute a topological ordering f_{ver} of $G^\star_{\text{ver}}$
- For each vertex $v \in V$, let g and h be the face on the left and face on the right of v. Set $x_1(v) = f_{\text{ver}}(g)$ and $x_2(v) = f_{\text{ver}}(h)$.

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

- Find a REL $\{T_r, T_b\}$ of G;
- Construct a SN network G_{ver} of G (consists of T_b plus outer edges)
- Construct the dual $G^{\star}_{\text{ver}}$ of G_{ver} and compute a topological ordering f_{ver} of $G^{\star}_{\text{ver}}$
- For each vertex $v \in V$, let g and h be the face on the left and face on the right of v. Set $x_1(v) = f_{\text{ver}}(g)$ and $x_2(v) = f_{\text{ver}}(h)$.
- Define $x_1(v_N) = 1, x_1(v_S) = 2$ and $x_2(v_N) = \max f_{\text{ver}} - 1, x_2(v_S) = \max f_{\text{ver}}$

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

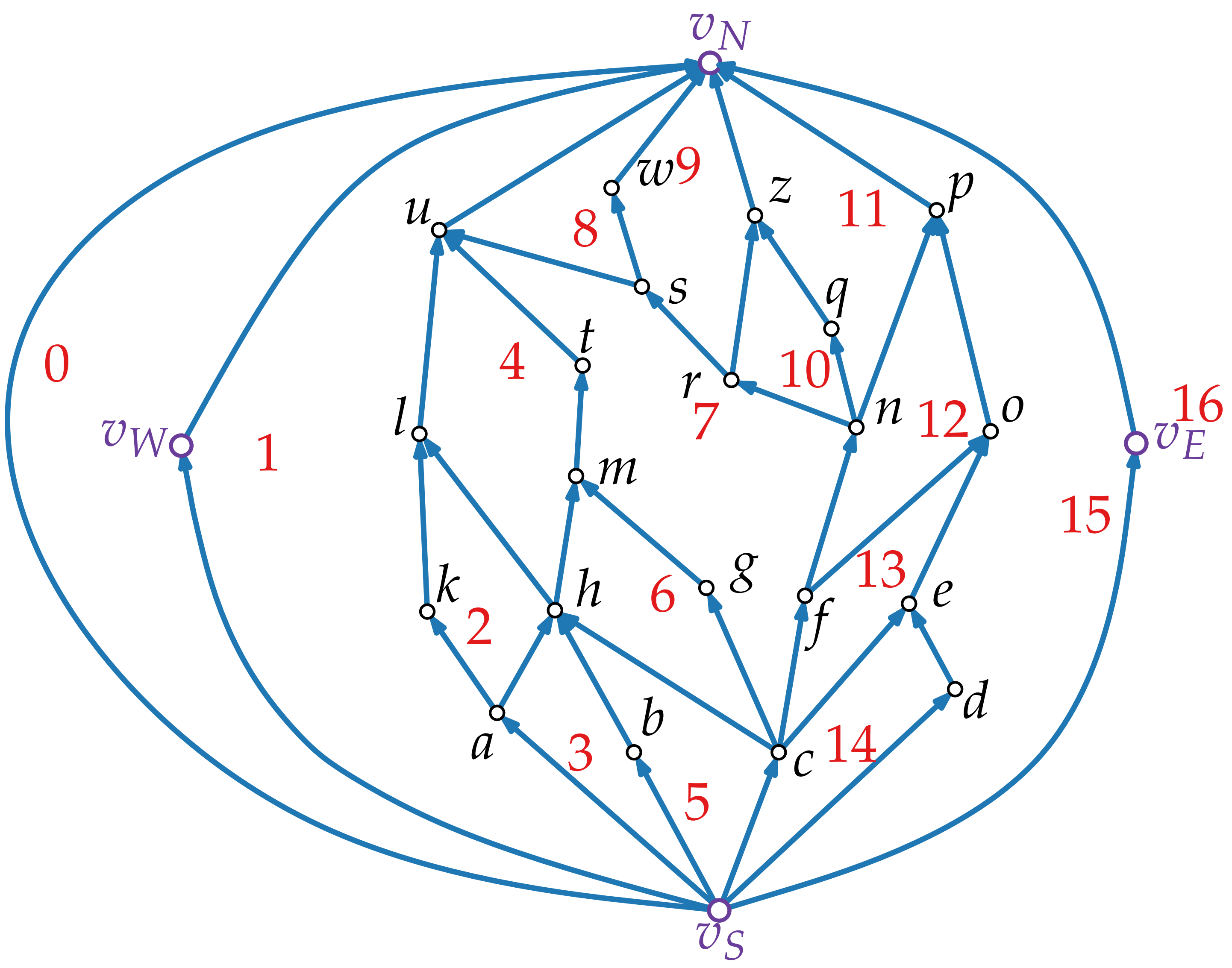
- Find a REL $\{T_r, T_b\}$ of G;
- Construct a SN network G_{ver} of G (consists of T_b plus outer edges)
- Construct the dual $G^\star_{\text{ver}}$ of G_{ver} and compute a topological ordering f_{ver} of $G^\star_{\text{ver}}$
- For each vertex $v \in V$, let g and h be the face on the left and face on the right of v. Set $x_1(v) = f_{\text{ver}}(g)$ and $x_2(v) = f_{\text{ver}}(h)$.
- Define $x_1(v_N) = 1, x_1(v_S) = 2$ and $x_2(v_N) = \max f_{\text{ver}} - 1, x_2(v_S) = \max f_{\text{ver}}$
- Analogously compute y_1 and y_2 with G_{hor}.

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

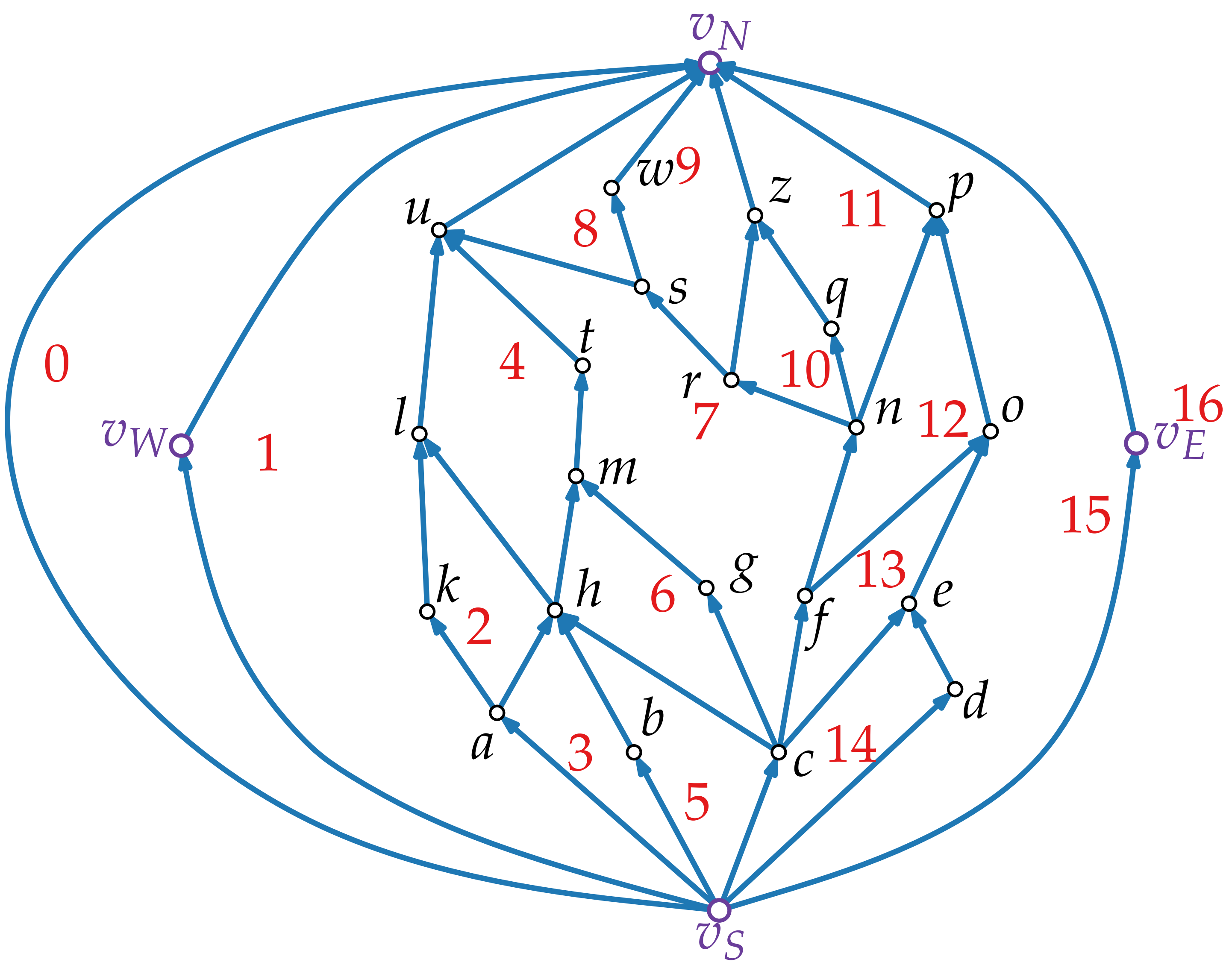
- Find a REL $\{T_r, T_b\}$ of G;
- Construct a SN network G_{ver} of G (consists of T_b plus outer edges)
- Construct the dual $G^\star_{\text{ver}}$ of G_{ver} and compute a topological ordering f_{ver} of $G^\star_{\text{ver}}$
- For each vertex $v \in V$, let g and h be the face on the left and face on the right of v. Set $x_1(v) = f_{\text{ver}}(g)$ and $x_2(v) = f_{\text{ver}}(h)$.
- Define $x_1(v_N) = 1, x_1(v_S) = 2$ and $x_2(v_N) = \max f_{\text{ver}} - 1, x_2(v_S) = \max f_{\text{ver}}$
- Analogously compute y_1 and y_2 with G_{hor}.
- For each $v \in V$, assign a rectangle $R(v)$ bounded by x-coordinates $x_1(v),\ x_2(v)$ and y-coordinates $y_1(v),\ y_2(v)$.

Reading off Coordinates to get Rectangular Dual

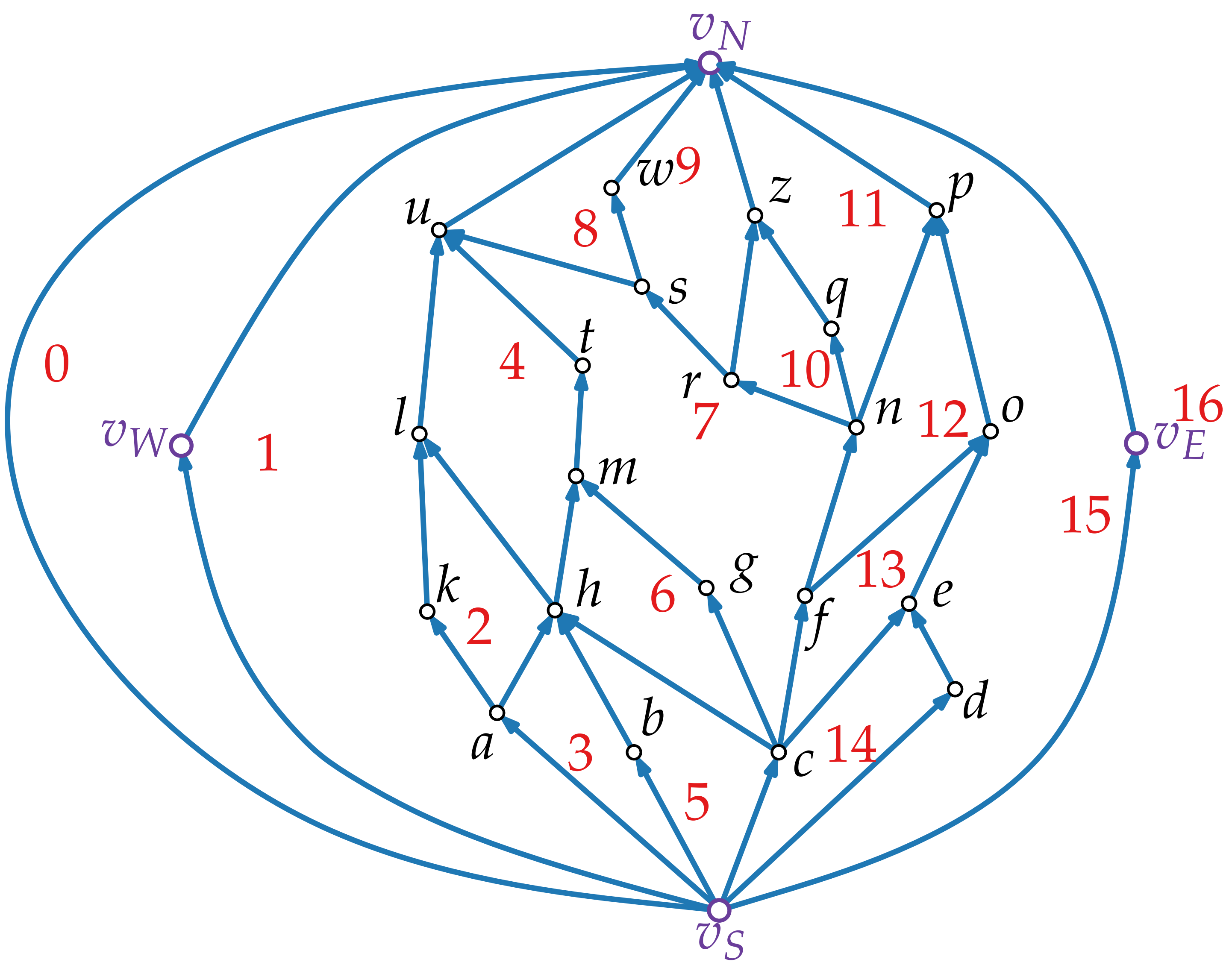


Reading off Coordinates to get Rectangular Dual

$x_1(v_N) = 1,\ x_2(v_N) = 15$



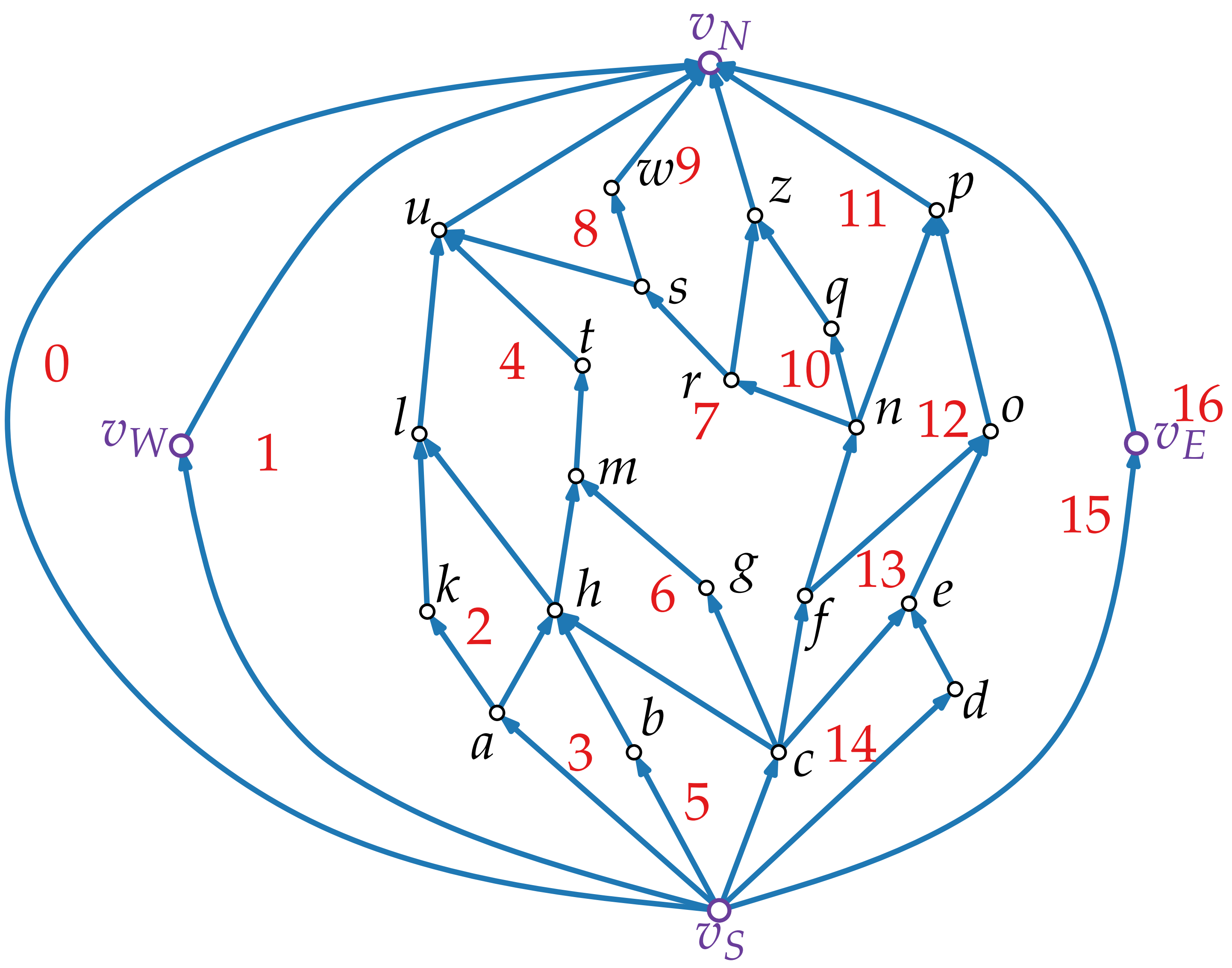
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$

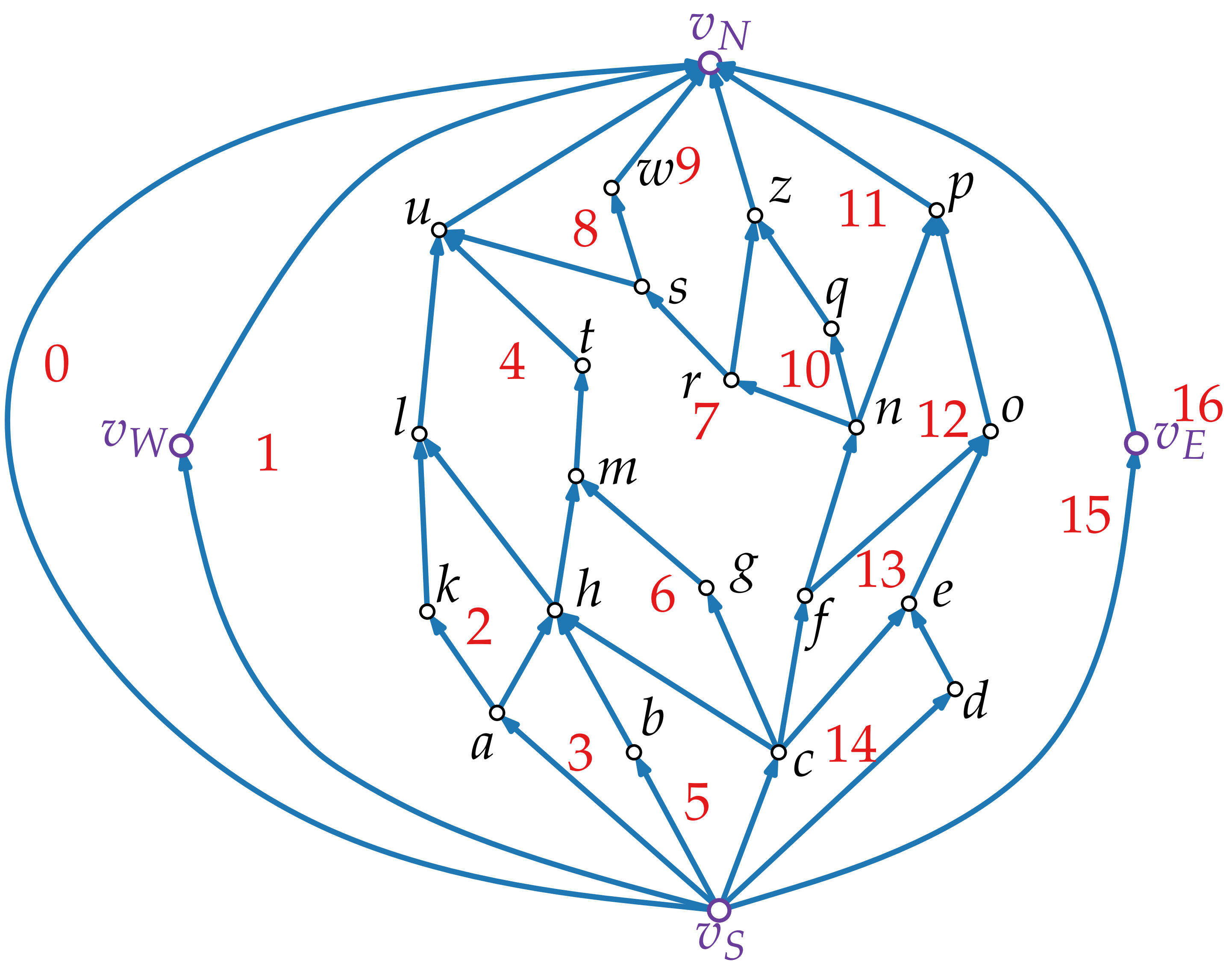
$x_1(v_S) = 2,\ x_2(v_S) = 16$

Reading off Coordinates to get Rectangular Dual



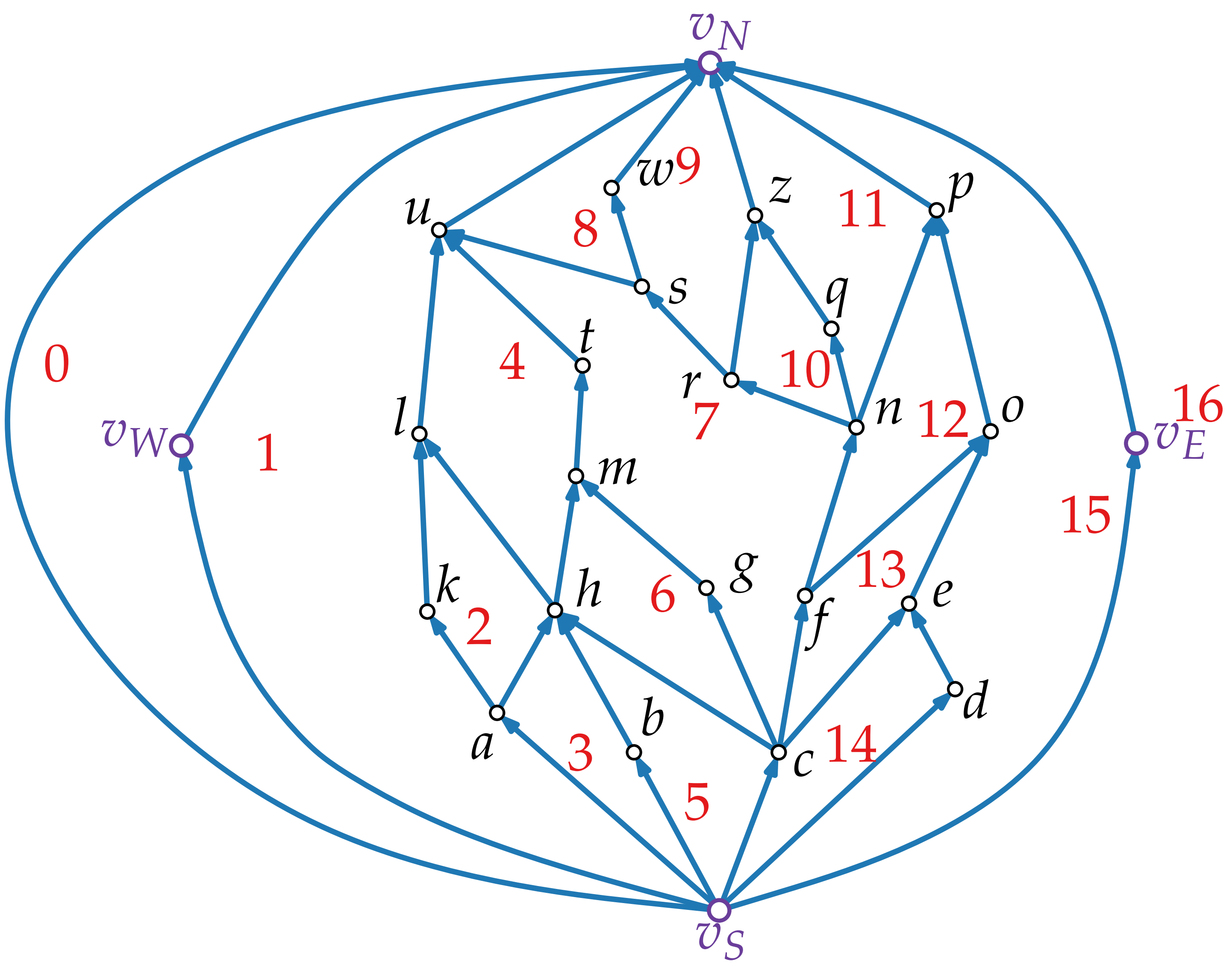
$$x_1(v_N) = 1,\ x_2(v_N) = 15$$
$$x_1(v_S) = 2,\ x_2(v_S) = 16$$
$$x_1(v_W) = 0, x_2(v_W) = 1$$

Reading off Coordinates to get Rectangular Dual



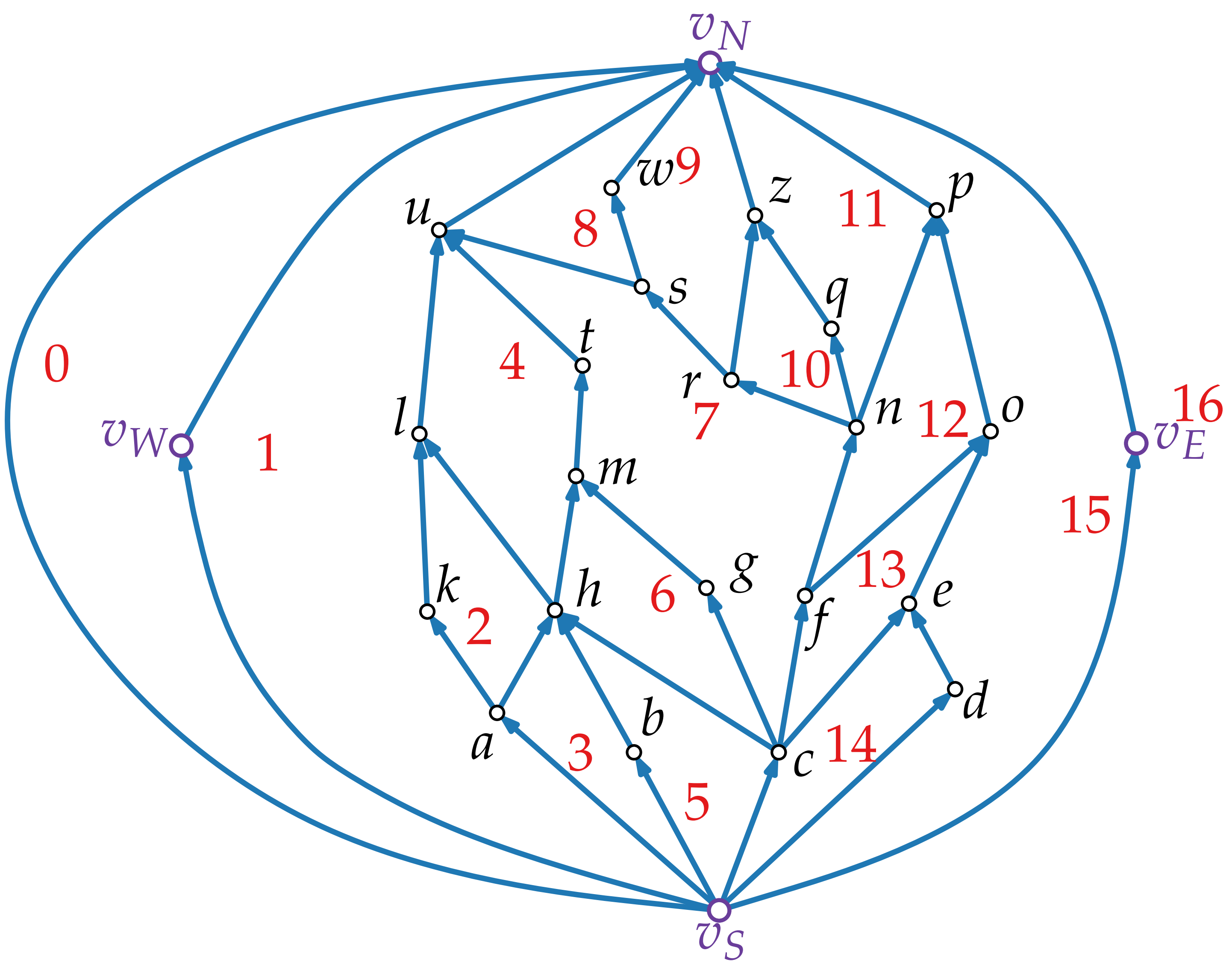
$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$

Reading off Coordinates to get Rectangular Dual



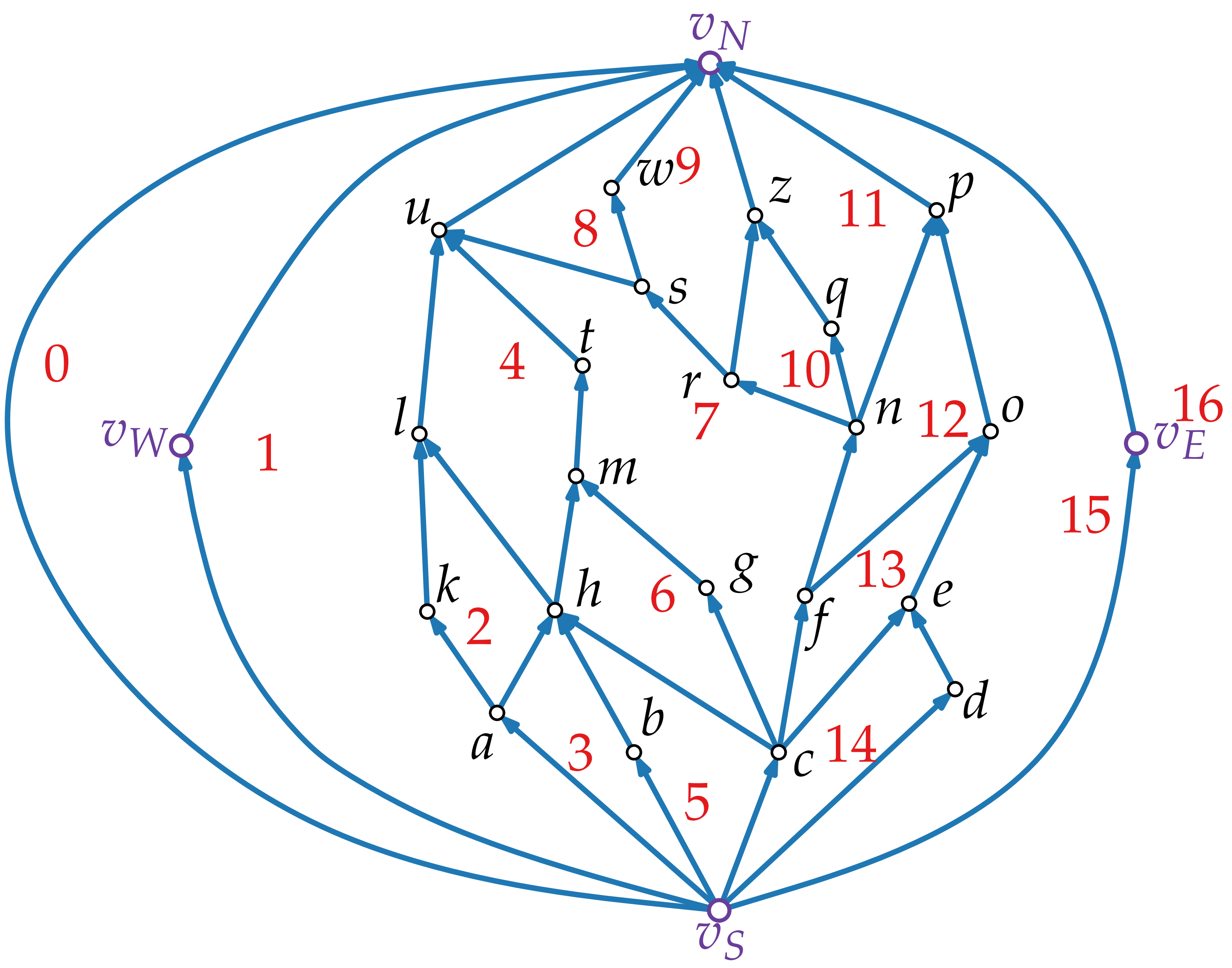
$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$

$x_1(v_S) = 2,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

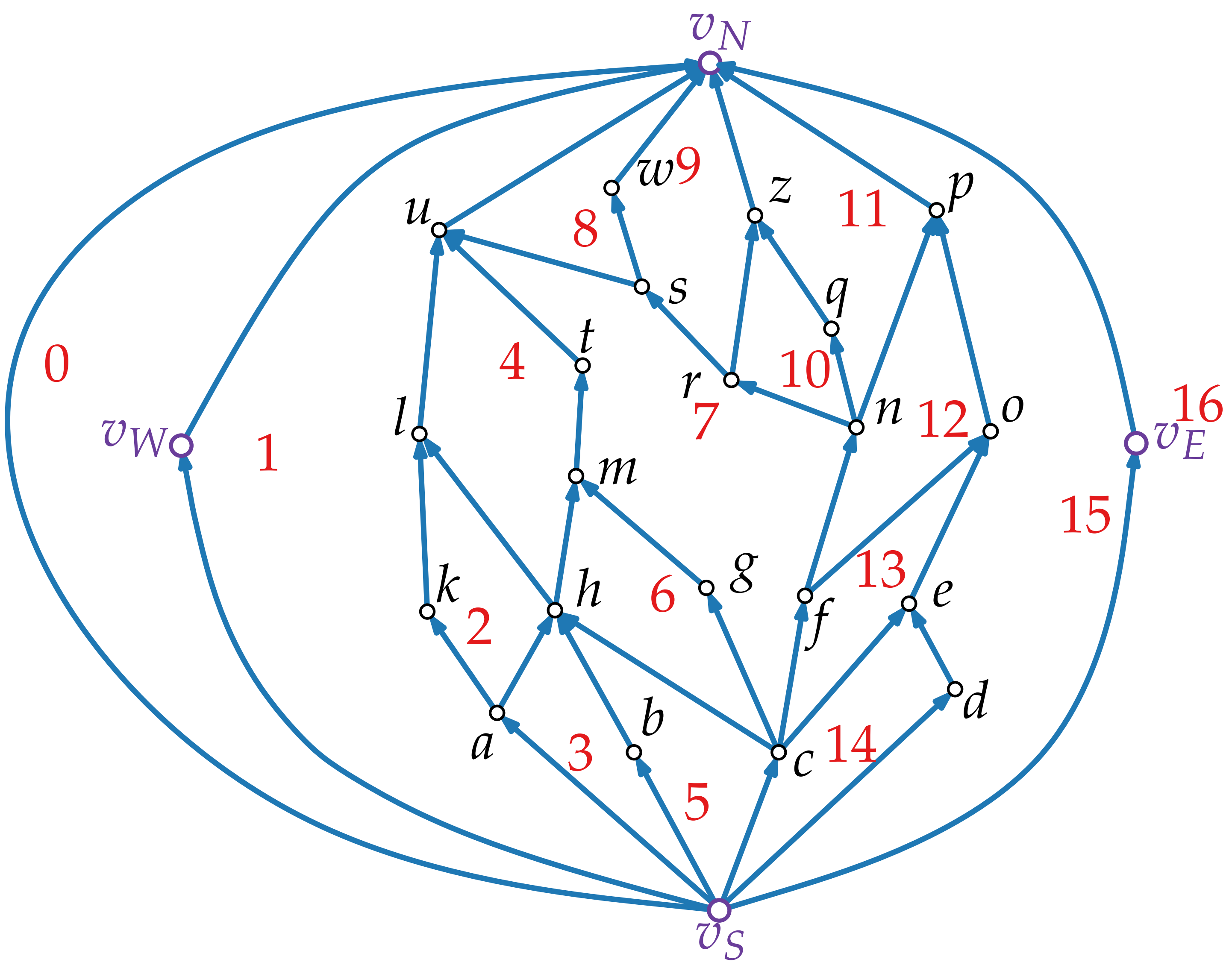
$x_1(v_E) = 15,\ x_2(v_E) = 16$

$x_1(a) = 1,\ x_2(a) = 3$

$x_1(b) = 3,\ x_2(b) = 5$

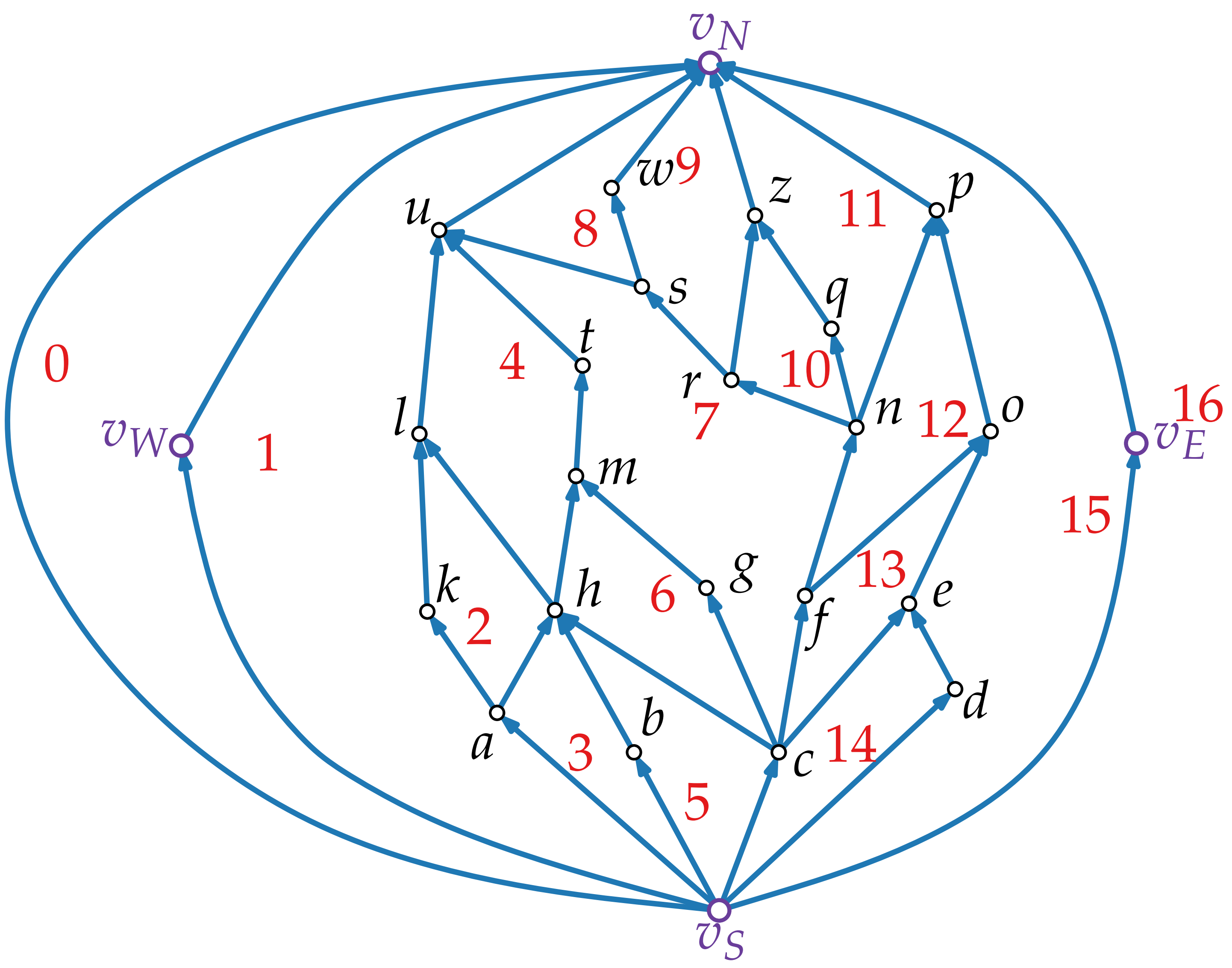
$x_1(c) = 5,\ x_2(c) = 14$

Reading off Coordinates to get Rectangular Dual



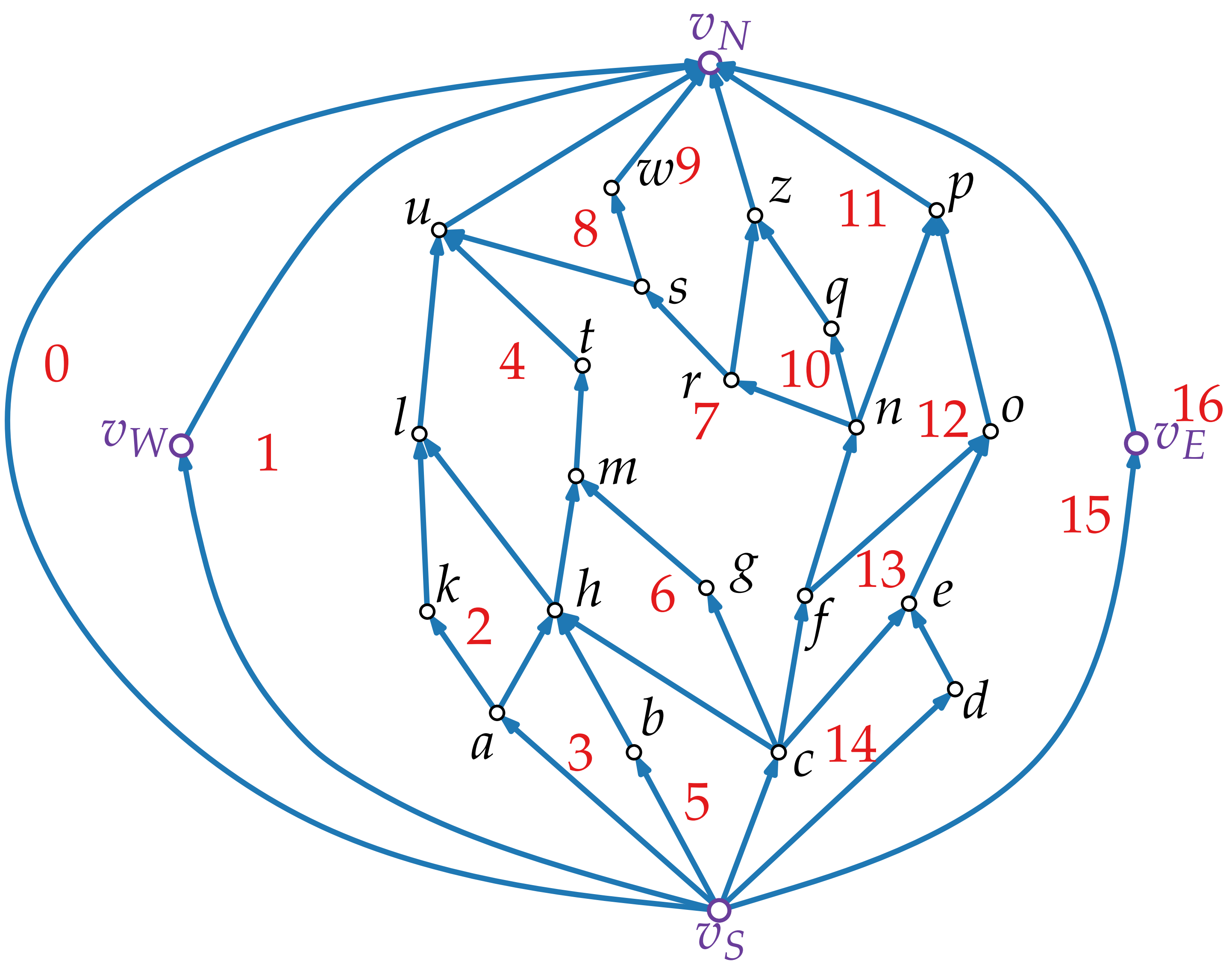
$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$

$x_1(v_S) = 2,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

$x_1(v_E) = 15,\ x_2(v_E) = 16$

$x_1(a) = 1,\ x_2(a) = 3$

$x_1(b) = 3,\ x_2(b) = 5$

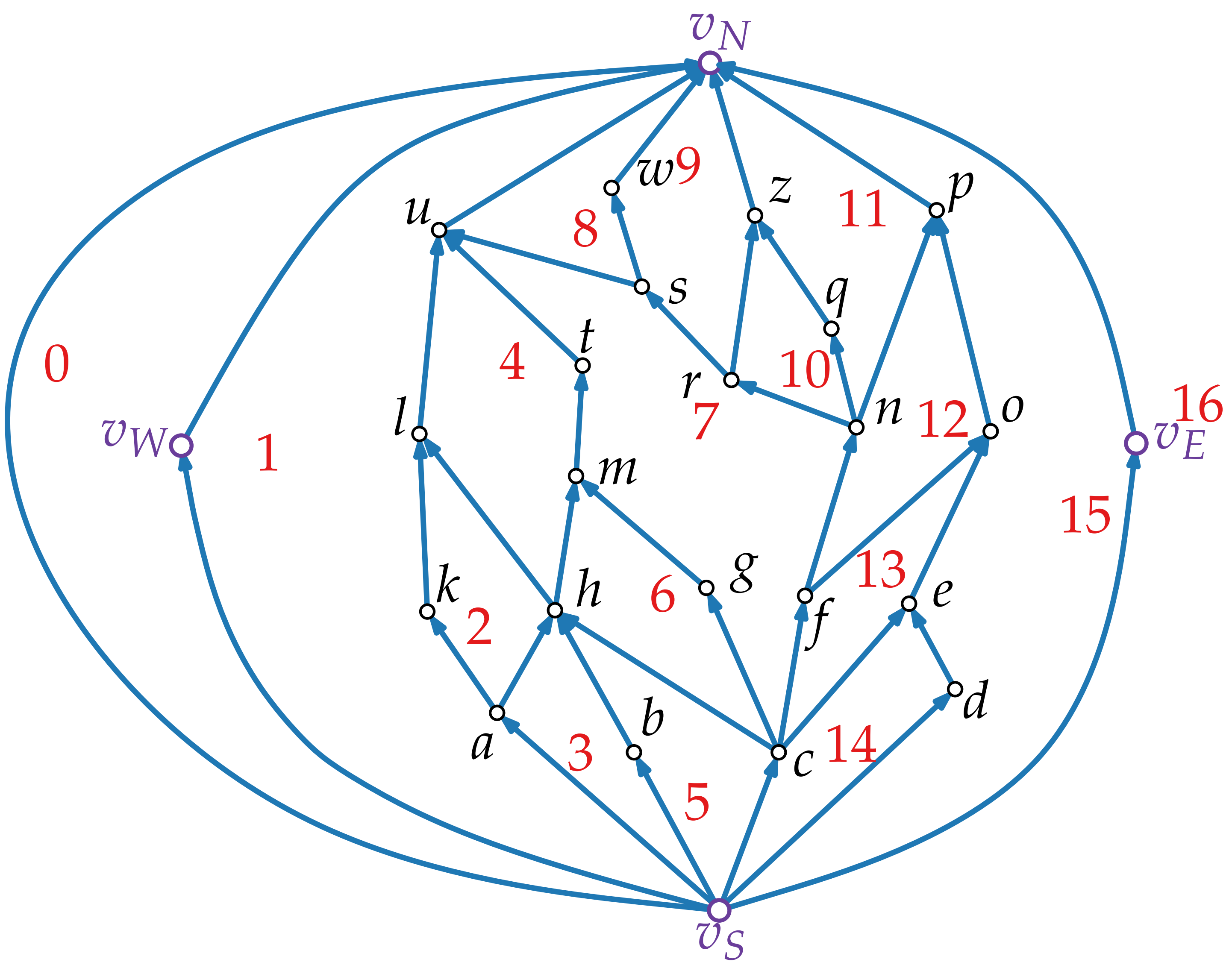
$x_1(c) = 5,\ x_2(c) = 14$

$x_1(d) = 14,\ x_2(d) = 15$

$x_1(e) = 13,\ x_2(e) = 15$

$\ldots$

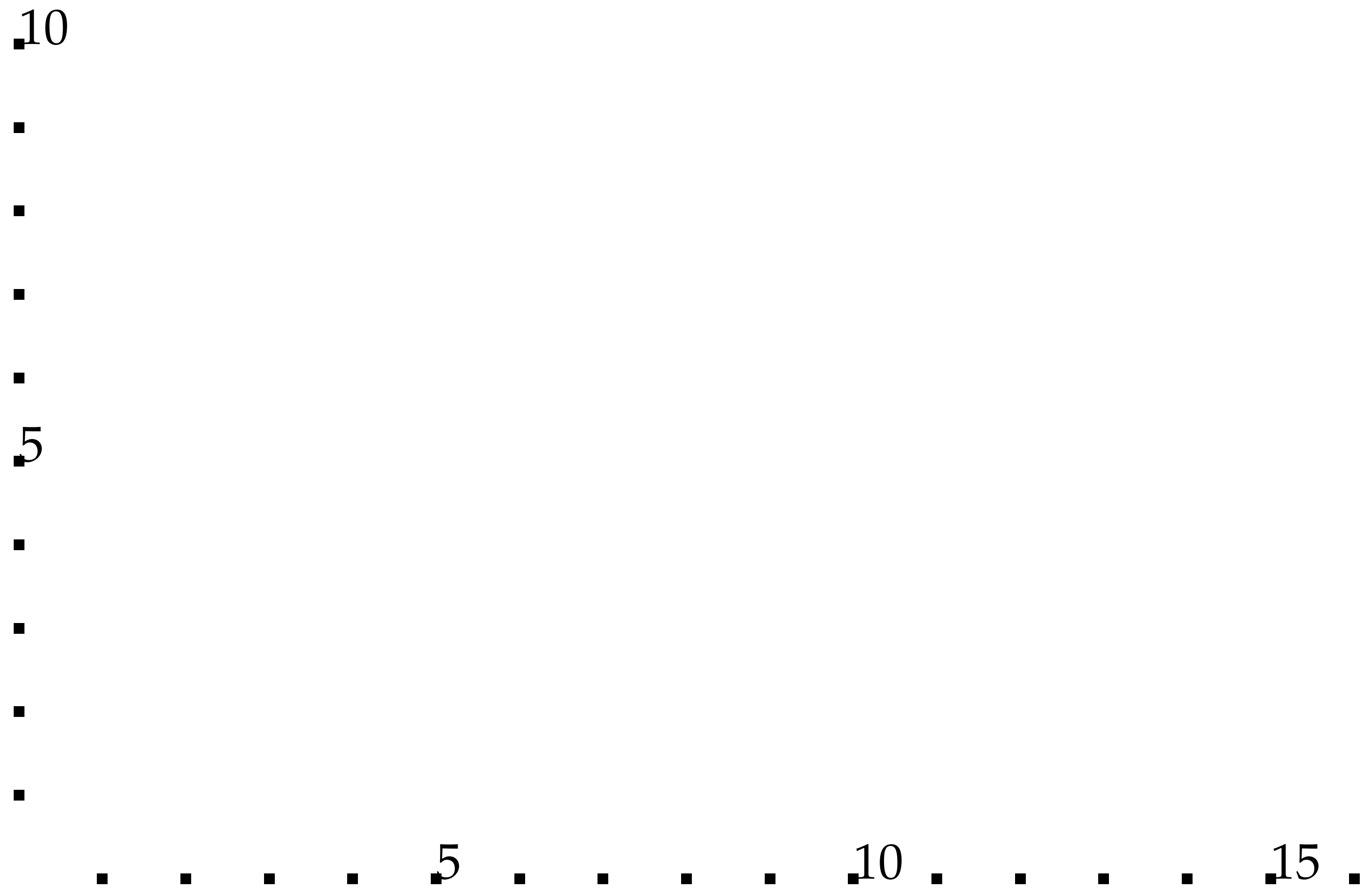
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

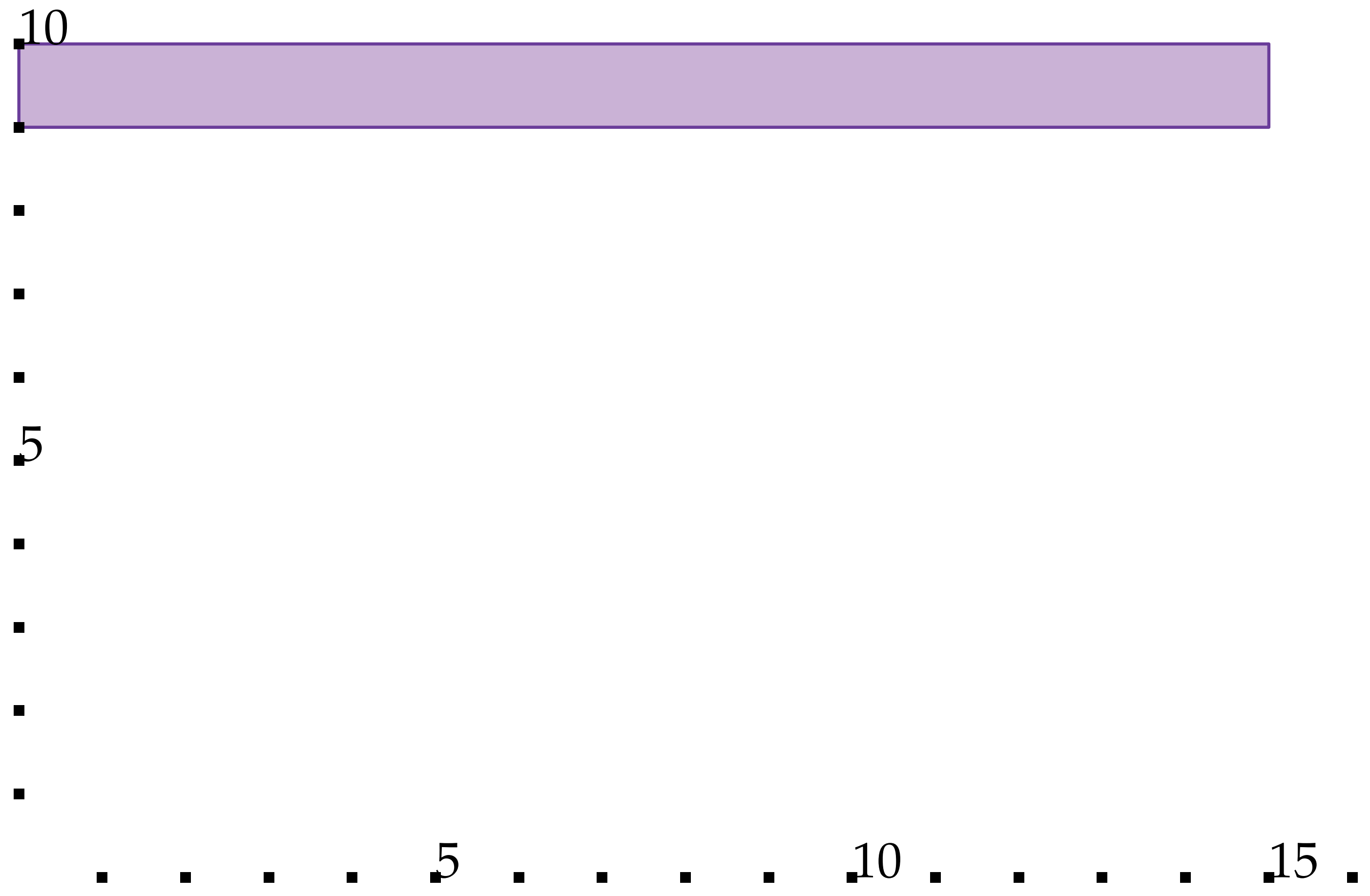
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

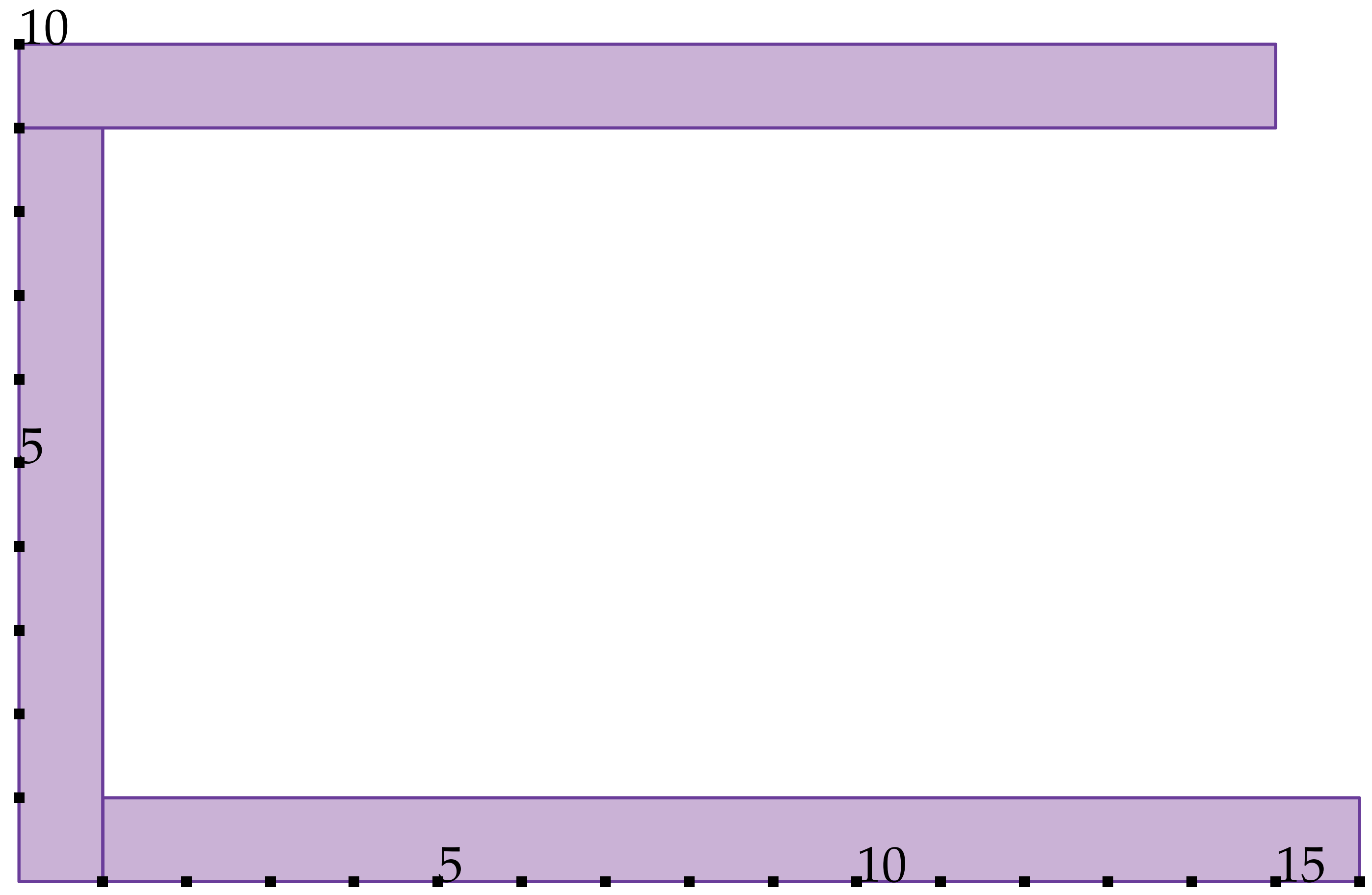
$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

Reading off Coordinates to get Rectangular Dual

$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

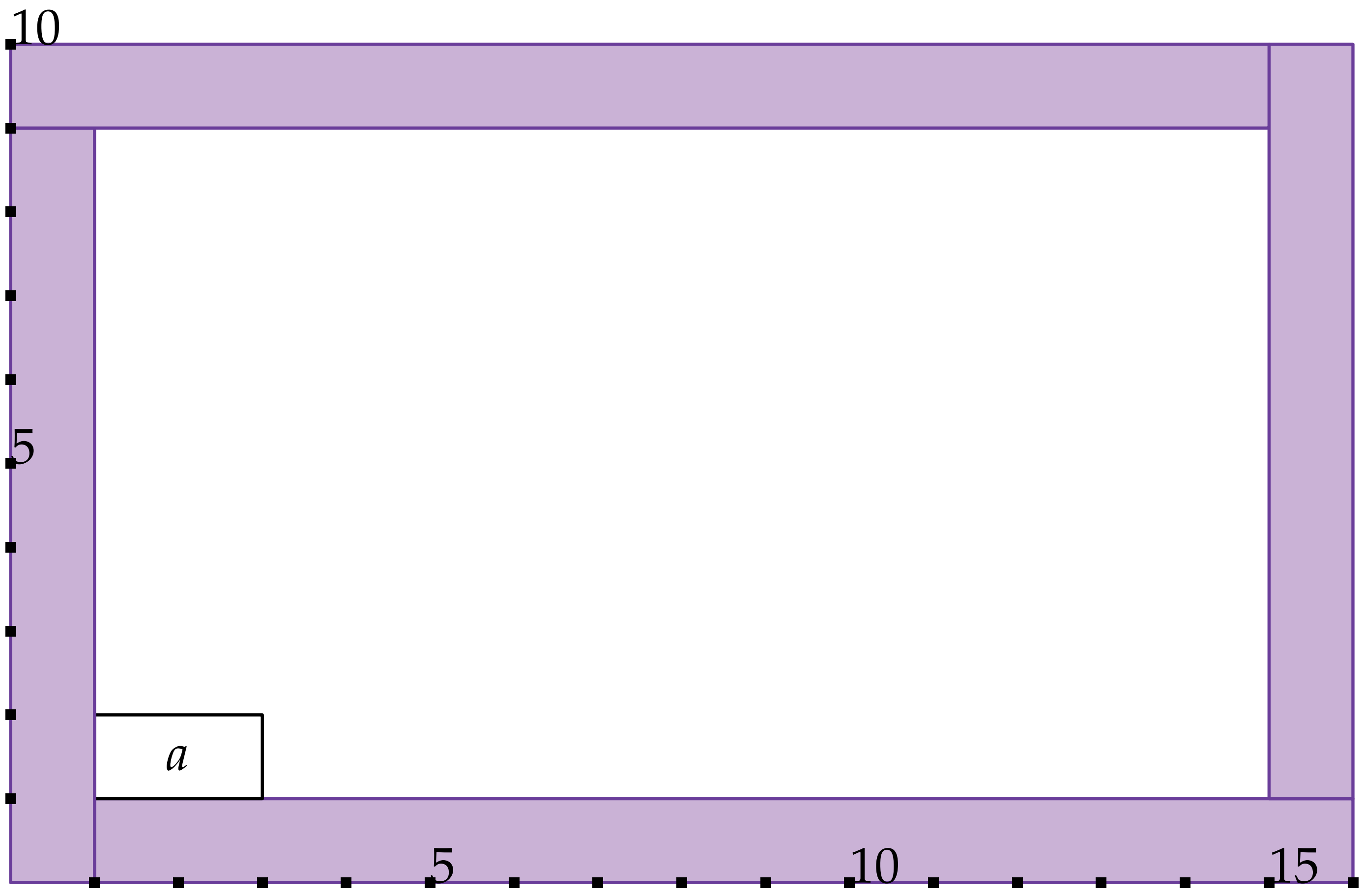
$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

Reading off Coordinates to get Rectangular Dual

$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

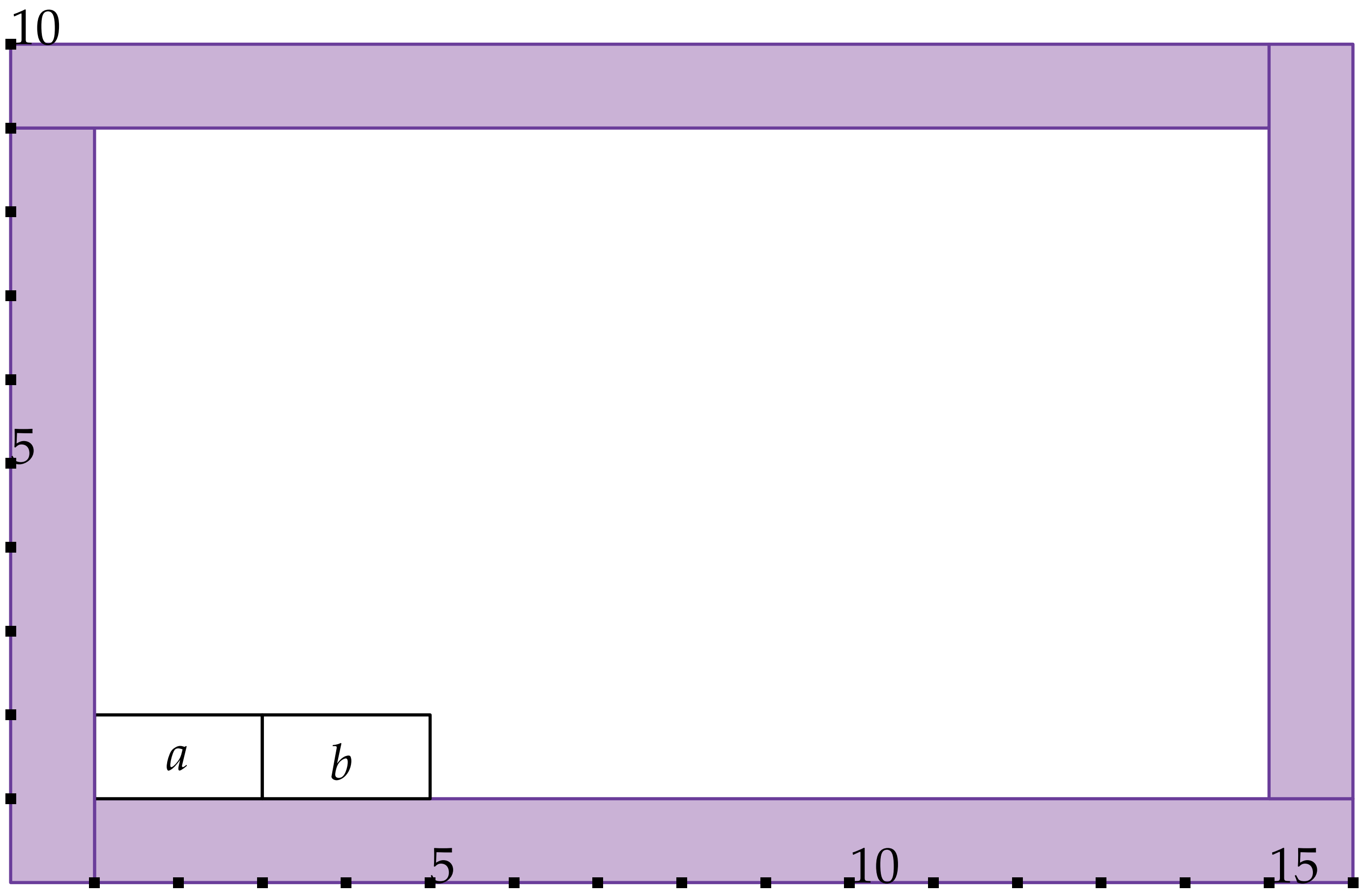
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

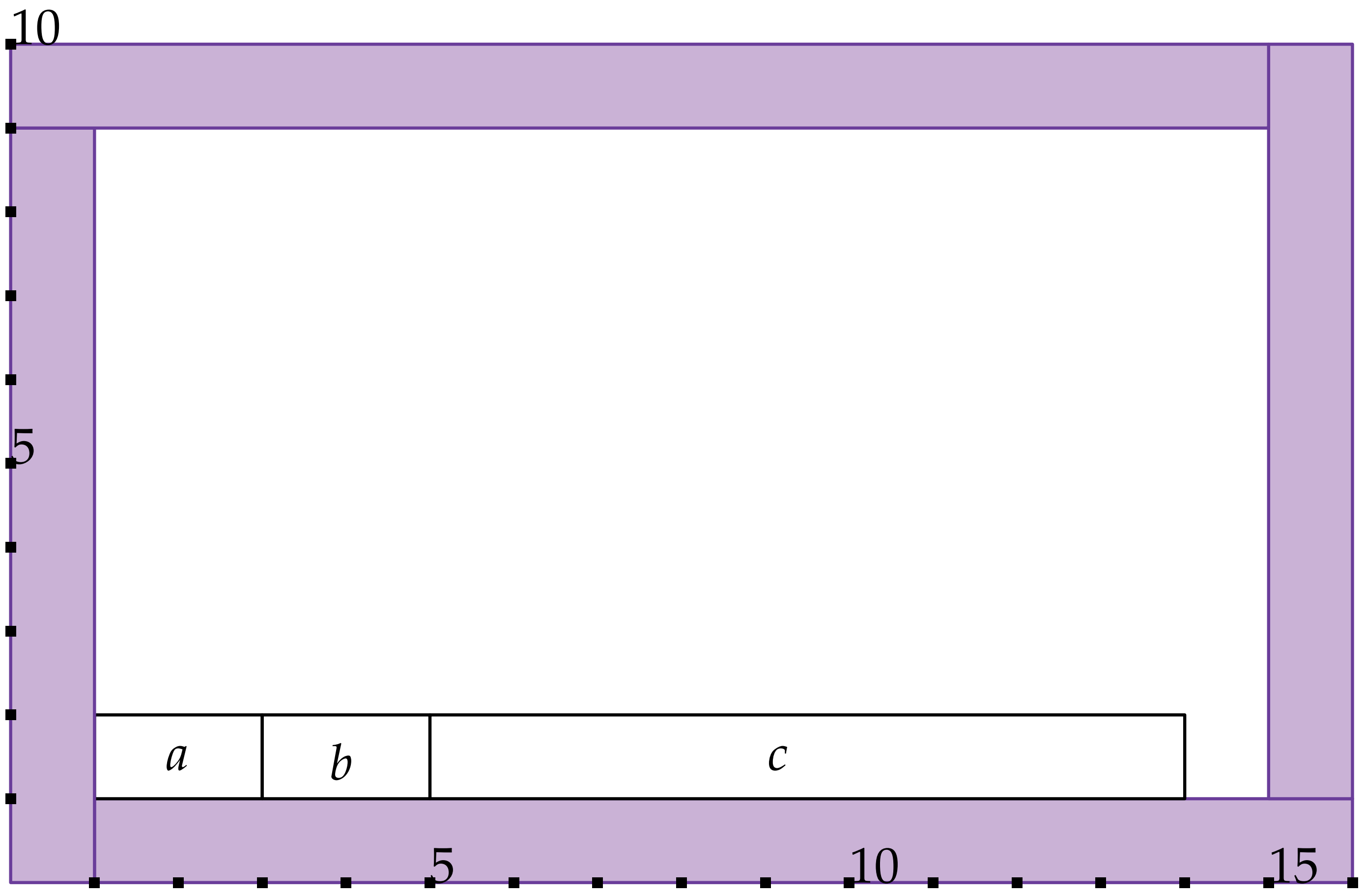
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

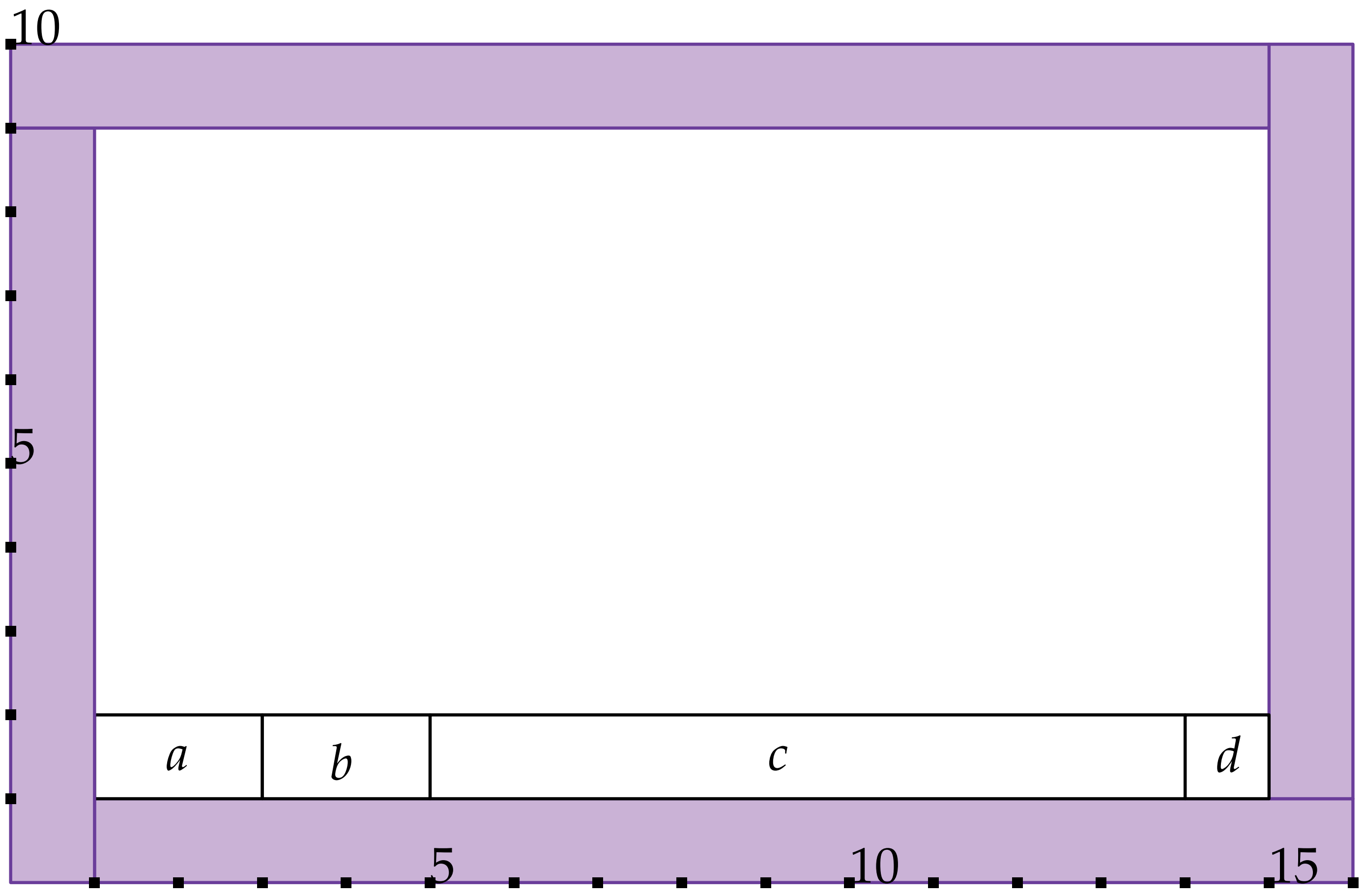
Reading off Coordinates to get Rectangular Dual



$$x_1(v_N) = 1,\ x_2(v_N) = 15$$
$$x_1(v_S) = 2,\ x_2(v_S) = 16$$
$$x_1(v_W) = 0, x_2(v_W) = 1$$
$$x_1(v_E) = 15,\ x_2(v_E) = 16$$
$$x_1(a) = 1,\ x_2(a) = 3$$
$$x_1(b) = 3,\ x_2(b) = 5$$
$$x_1(c) = 5,\ x_2(c) = 14$$
$$x_1(d) = 14,\ x_2(d) = 15$$
$$x_1(e) = 13,\ x_2(e) = 15$$
$$\ldots$$

$$y_1(v_W) = 0, y_2(v_W) = 9$$
$$y_1(v_E) = 1,\ y_2(v_E) = 10$$
$$y_1(v_N) = 9,\ y_2(v_N) = 10$$
$$y_1(v_S) = 0,\ y_2(v_S) = 1$$
$$y_1(a) = 1,\ y_2(a) = 2$$
$$y_1(b) = 1,\ y_2(b) = 2$$
$$\ldots$$

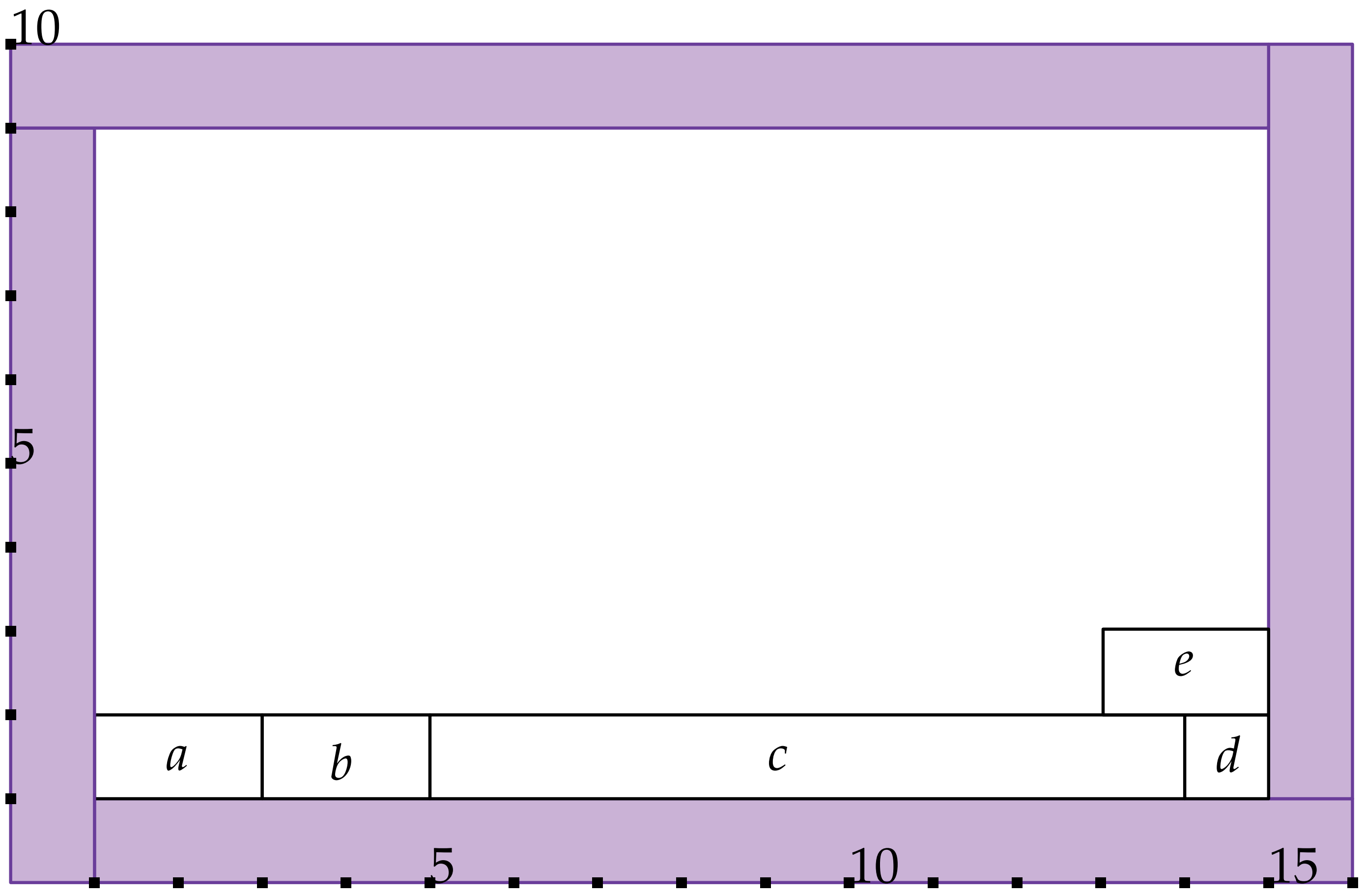
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

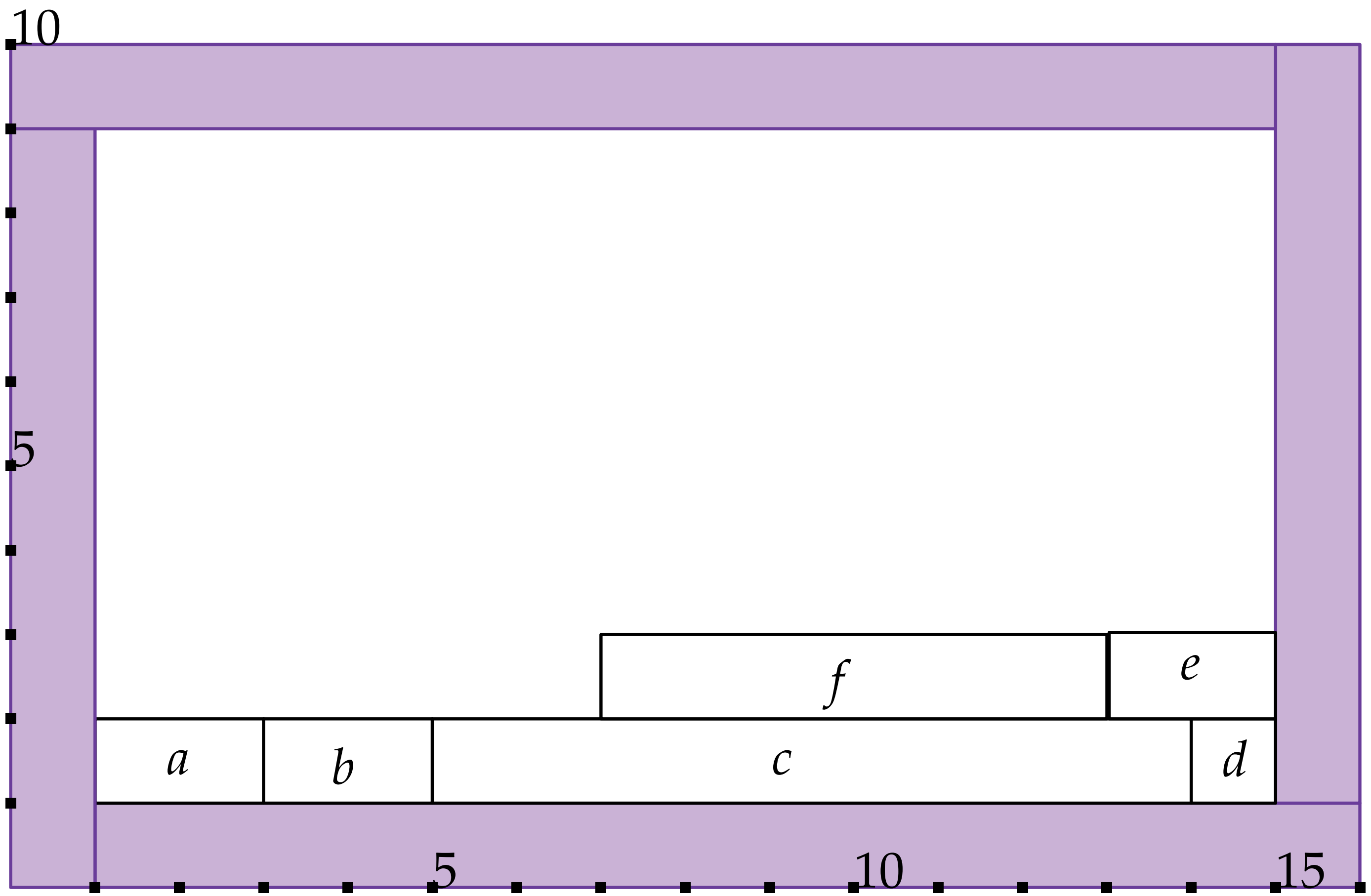
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

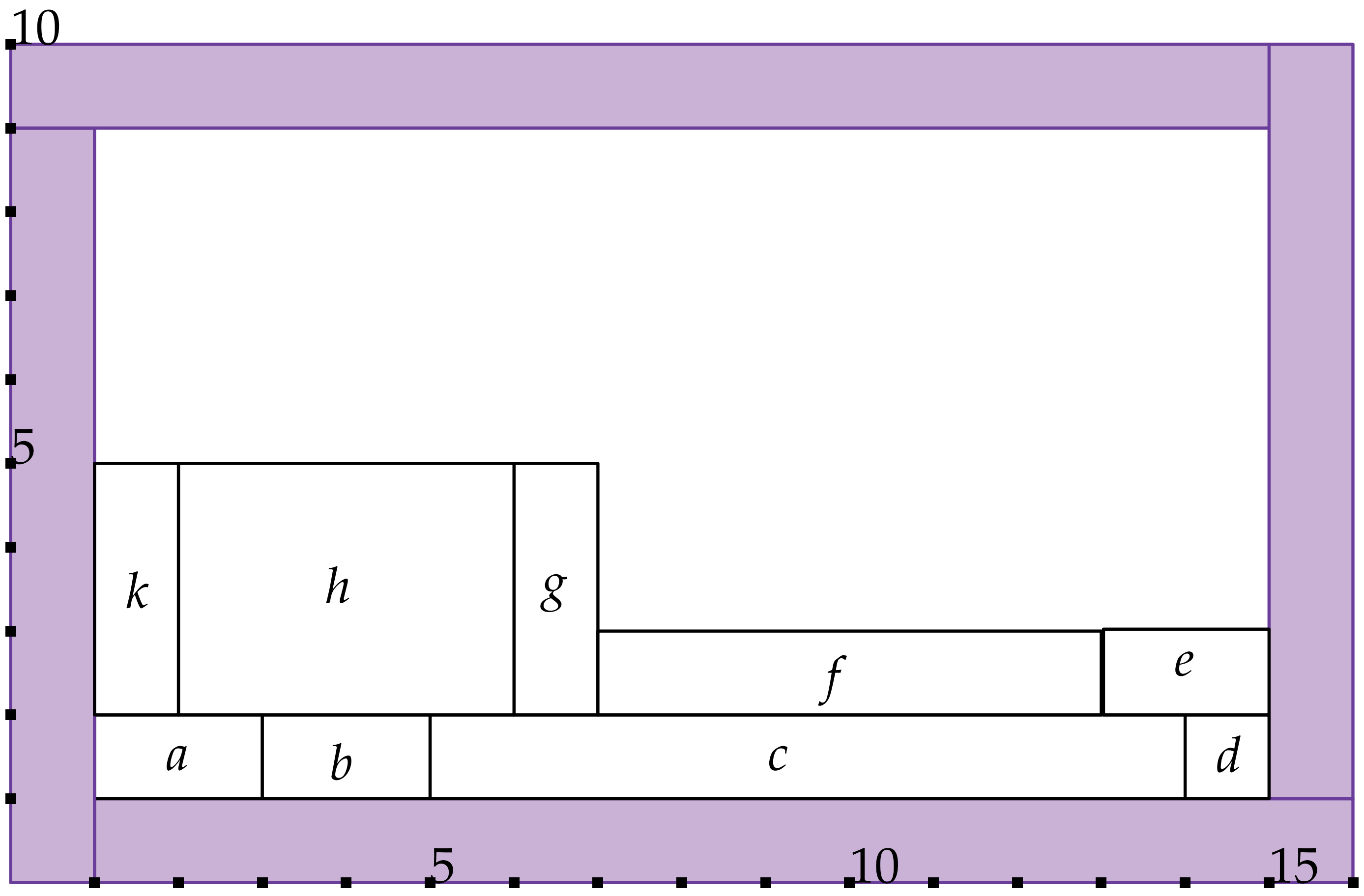
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$

$x_1(v_S) = 2,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

$x_1(v_E) = 15,\ x_2(v_E) = 16$

$x_1(a) = 1,\ x_2(a) = 3$

$x_1(b) = 3,\ x_2(b) = 5$

$x_1(c) = 5,\ x_2(c) = 14$

$x_1(d) = 14,\ x_2(d) = 15$

$x_1(e) = 13,\ x_2(e) = 15$

$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$

$y_1(v_E) = 1,\ y_2(v_E) = 10$

$y_1(v_N) = 9,\ y_2(v_N) = 10$

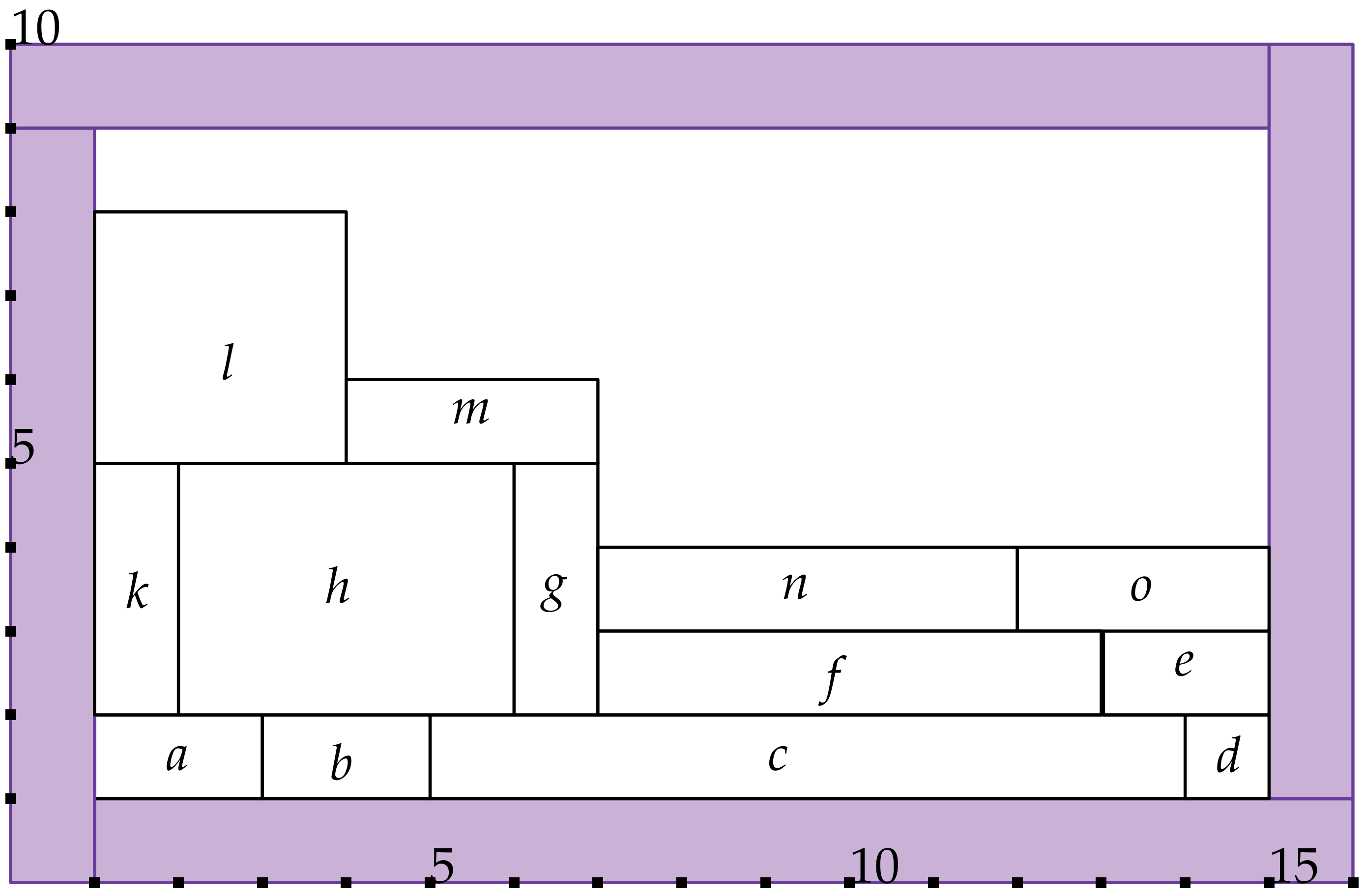
$y_1(v_S) = 0,\ y_2(v_S) = 1$

$y_1(a) = 1,\ y_2(a) = 2$

$y_1(b) = 1,\ y_2(b) = 2$

$\dots$

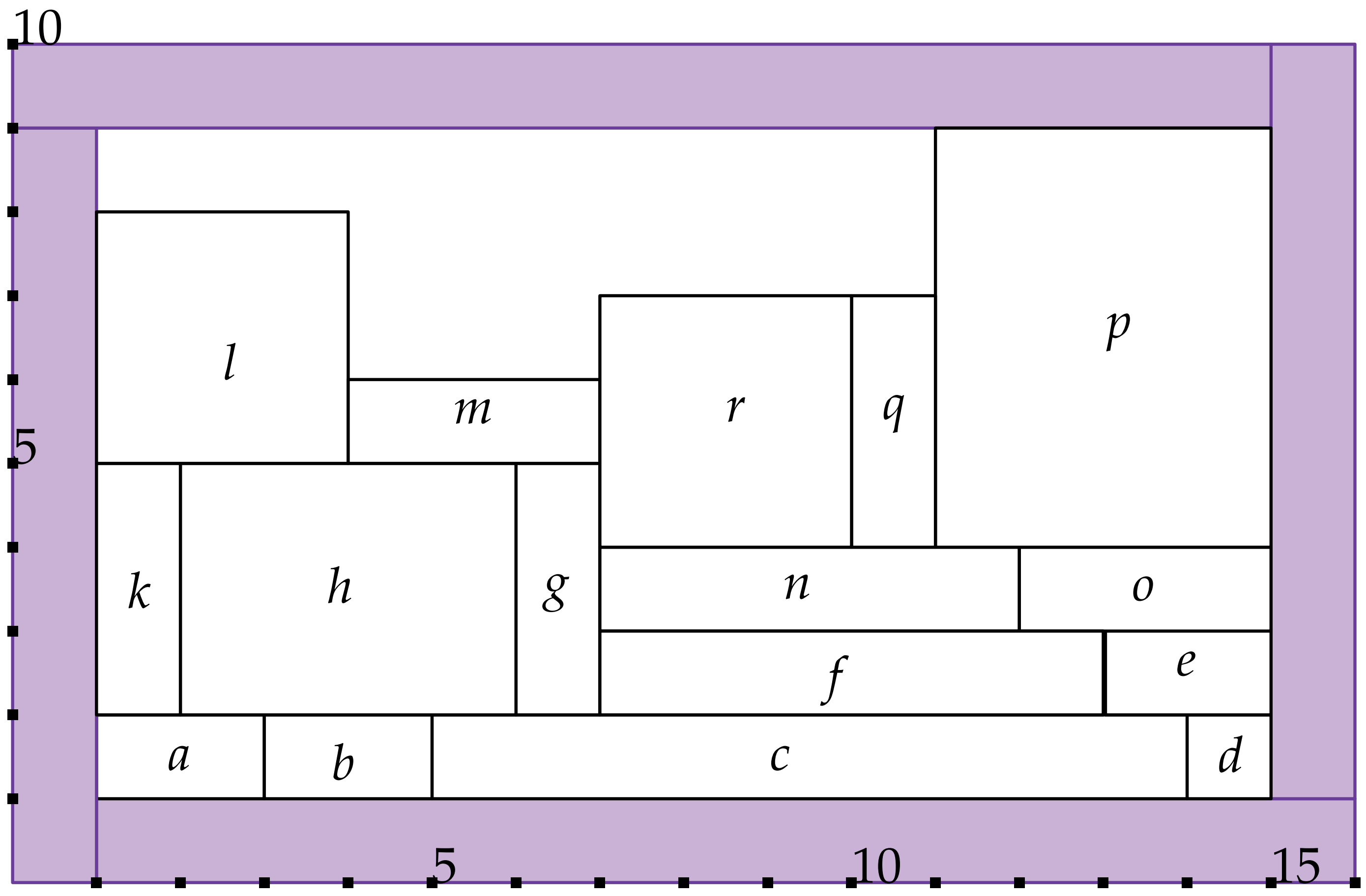
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

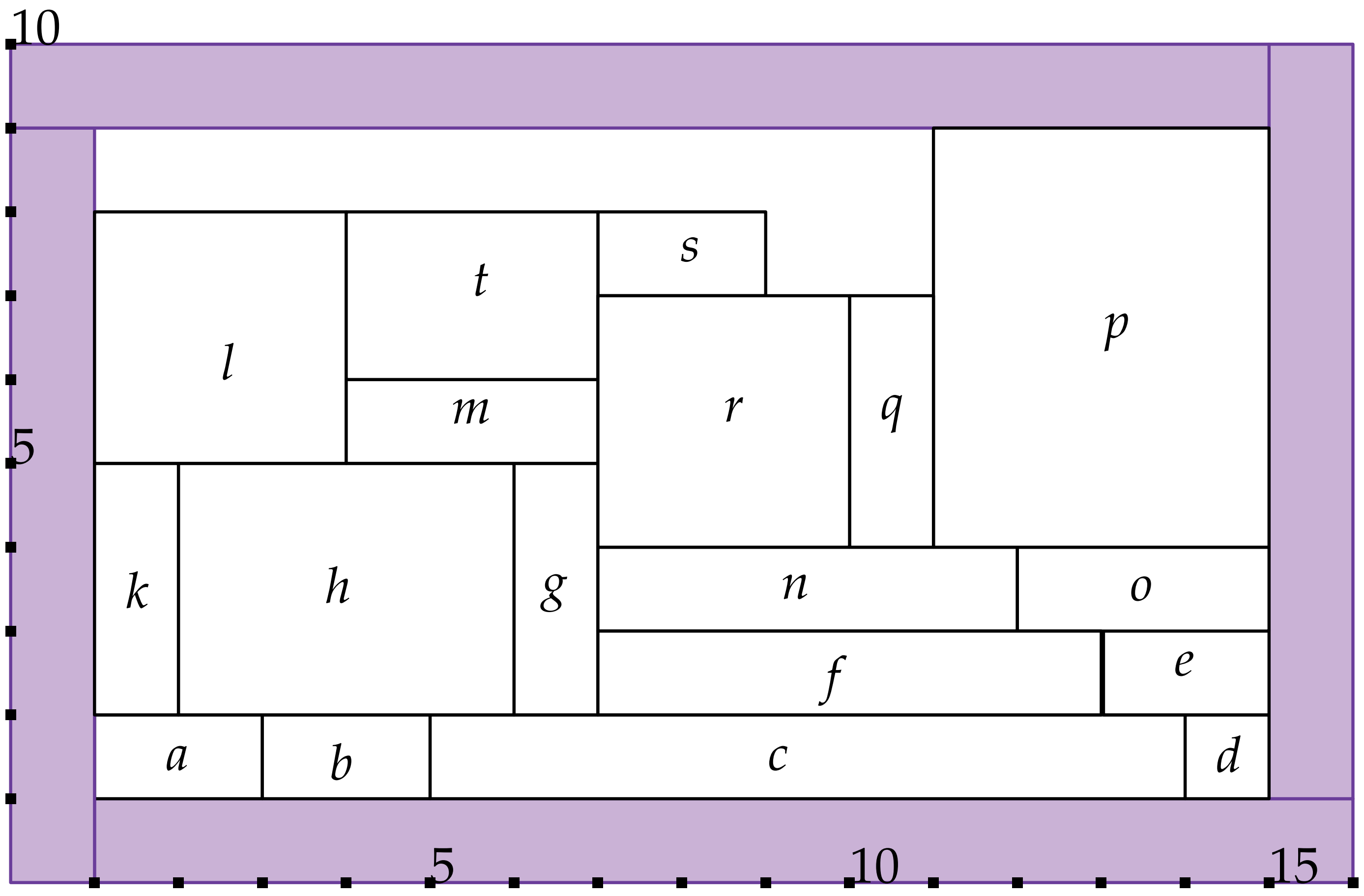
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

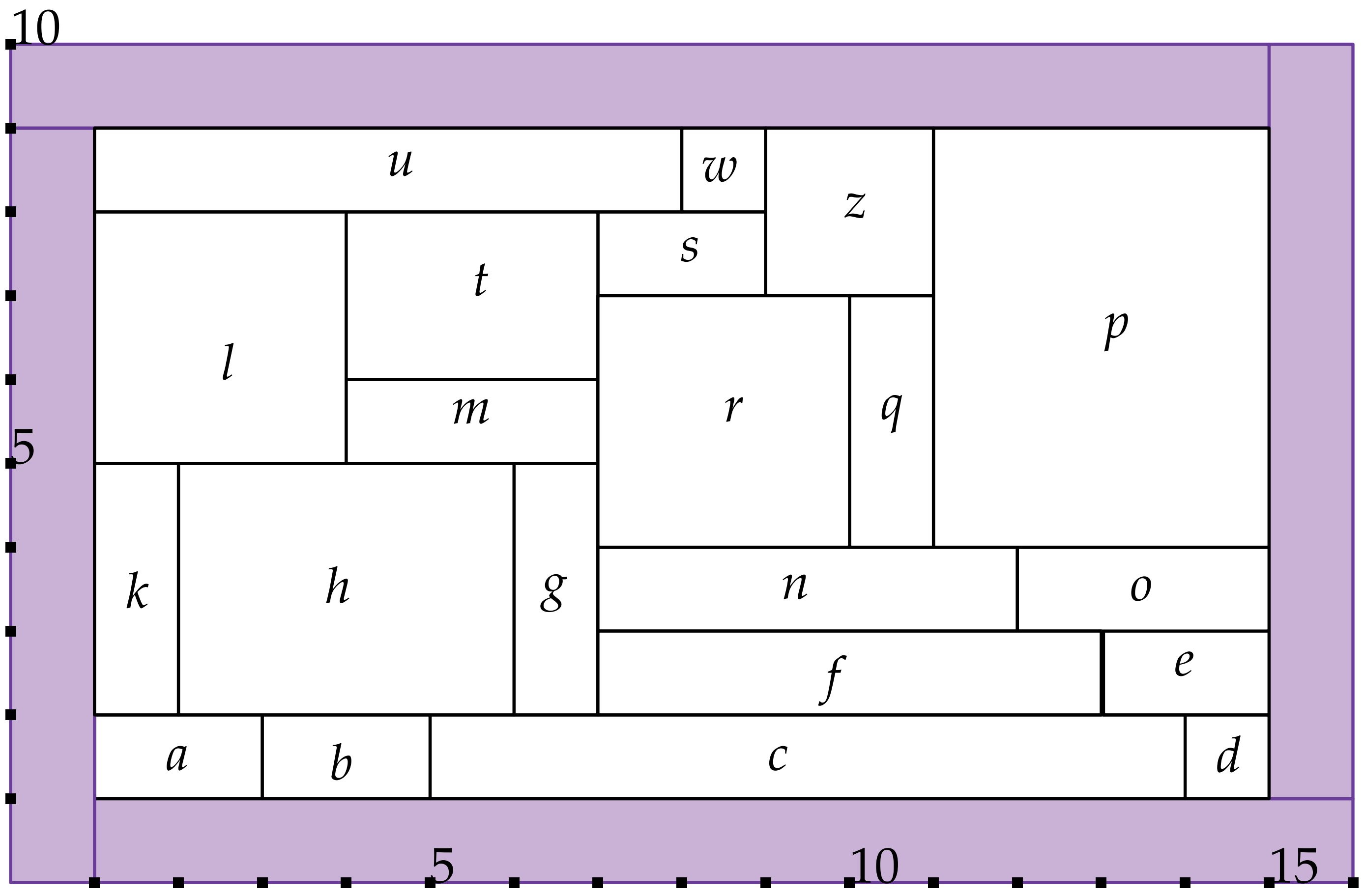
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

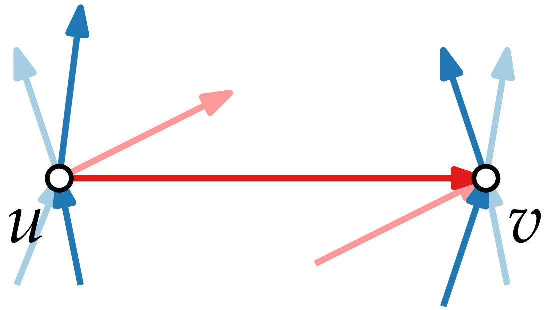
$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$

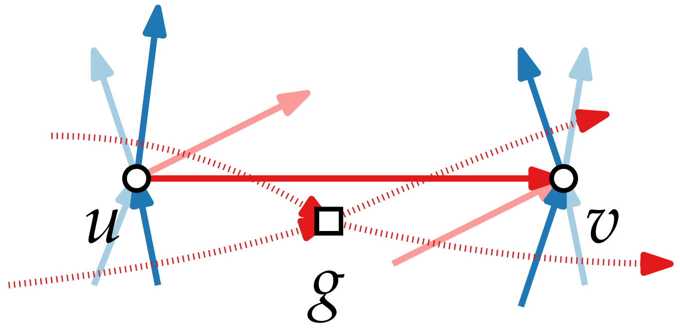
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



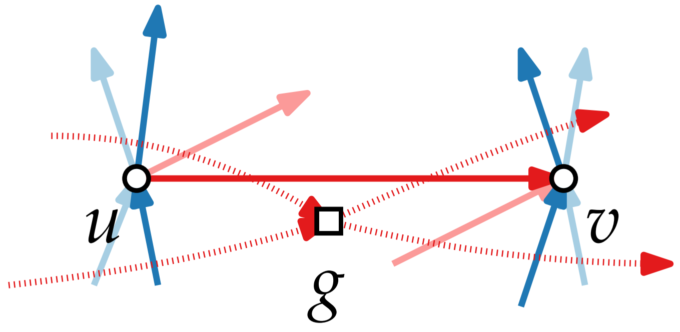
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



Correctness of Algorithm (Sketch)

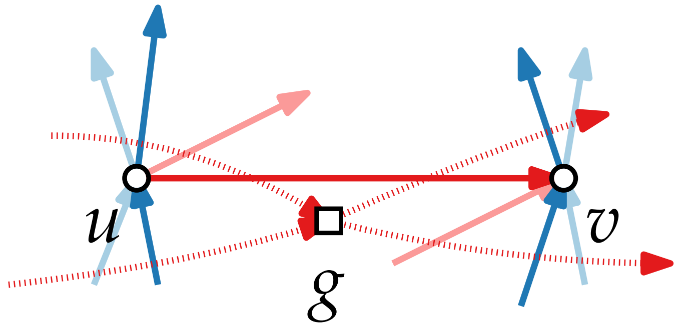
- If edge (u, v) exists, then $x_2(u) = x_1(v)$



$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

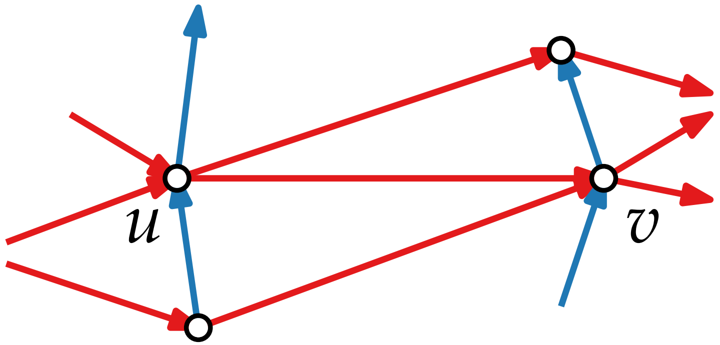
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



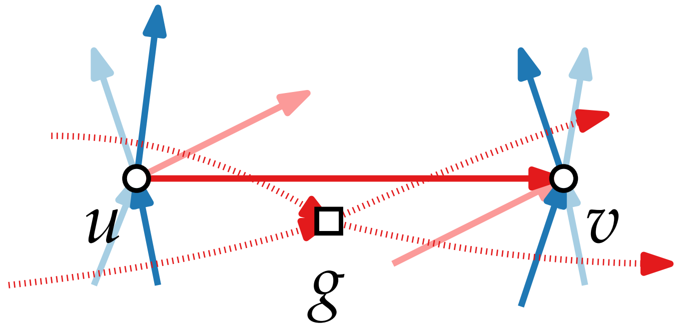
$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

- and the vertical segments of their rectangles overlap



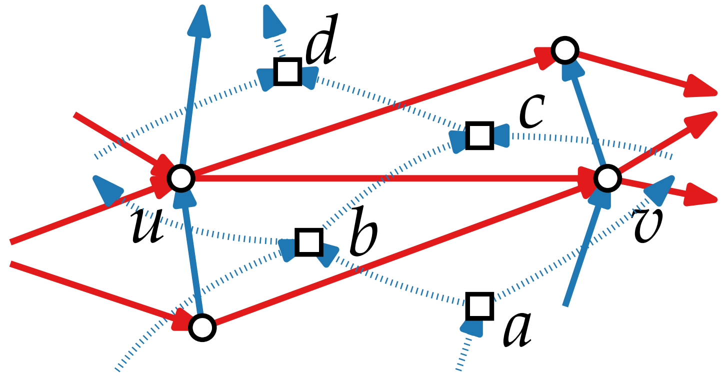
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



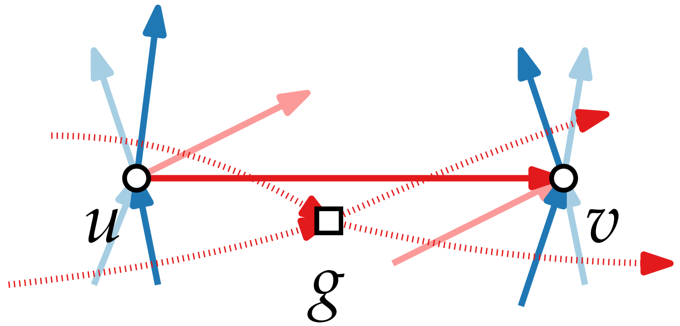
$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

- and the vertical segments of their rectangles overlap



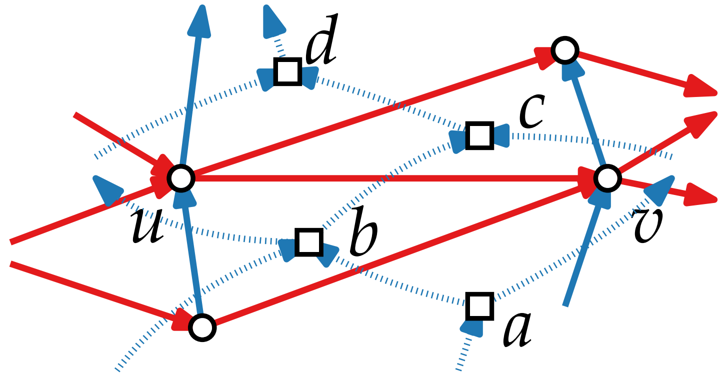
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

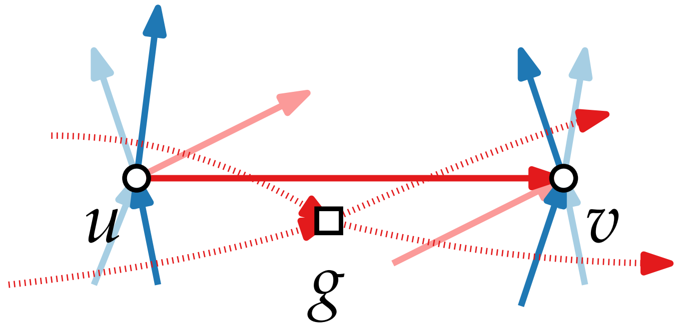
- and the vertical segments of their rectangles overlap



$$y_1(v) = f_{\text{hor}}(a)$$

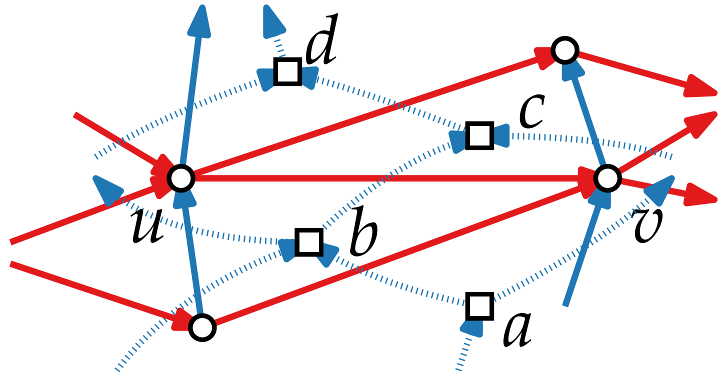
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

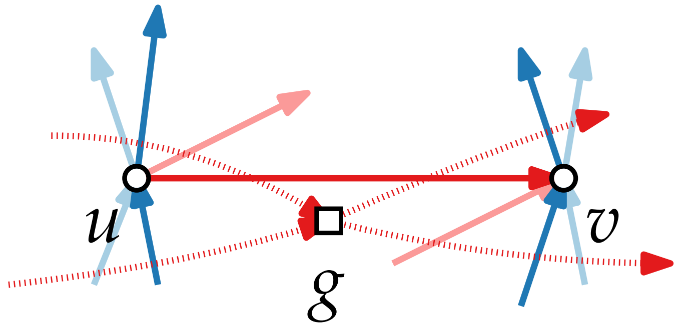
- and the vertical segments of their rectangles overlap



$$y_1(v) = f_{\text{hor}}(a) \leq y_1(u) = f_{\text{hor}}(b)$$

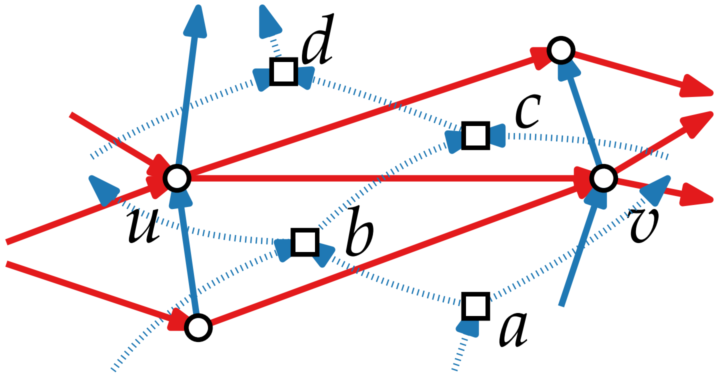
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

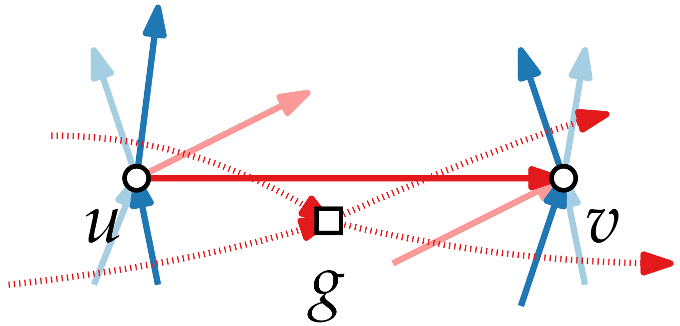
- and the vertical segments of their rectangles overlap



$$y_1(v) = f_{\text{hor}}(a) \leq y_1(u) = f_{\text{hor}}(b)$$
$$< y_2(v) = f_{\text{hor}}(c)$$

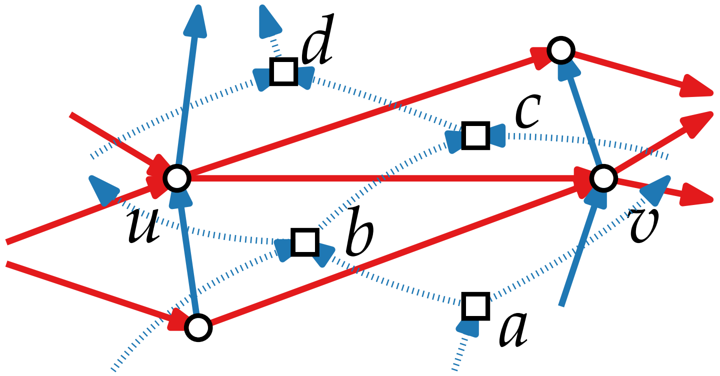
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



$$x_2(u) = f_{\mathrm{ver}}(g) = x_1(v)$$

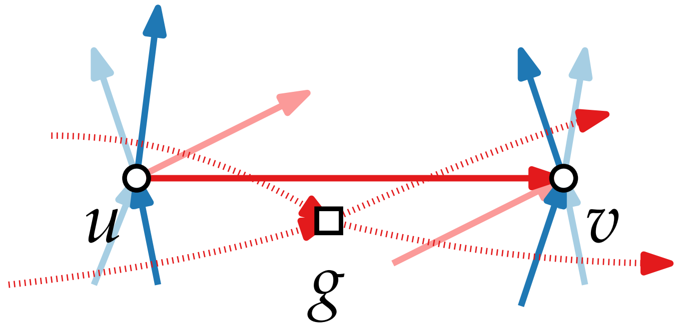
- and the vertical segments of their rectangles overlap



$$y_1(v) = f_{\mathrm{hor}}(a) \leq y_1(u) = f_{\mathrm{hor}}(b)$$
$$< y_2(v) = f_{\mathrm{hor}}(c) \leq y_2(u) = f_{\mathrm{hor}}(d)$$

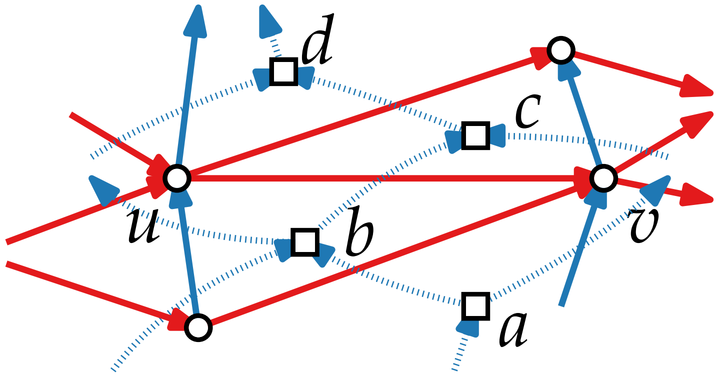
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



$$x_2(u) = f_{\mathrm{ver}}(g) = x_1(v)$$

- and the vertical segments of their rectangles overlap

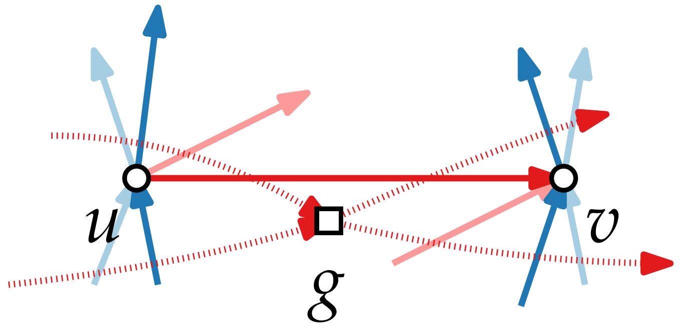


$$y_1(v) = f_{\mathrm{hor}}(a) \leq y_1(u) = f_{\mathrm{hor}}(b)$$
$$< y_2(v) = f_{\mathrm{hor}}(c) \leq y_2(u) = f_{\mathrm{hor}}(d)$$

- If path from u to v in red at least two edges long, then $x_2(u) < x_1(v)$.

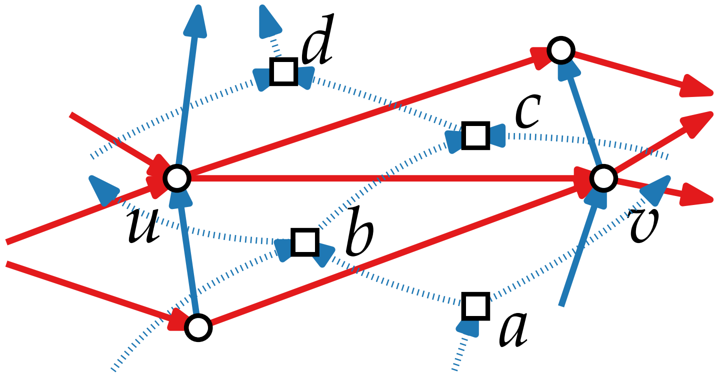
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

- and the vertical segments of their rectangles overlap

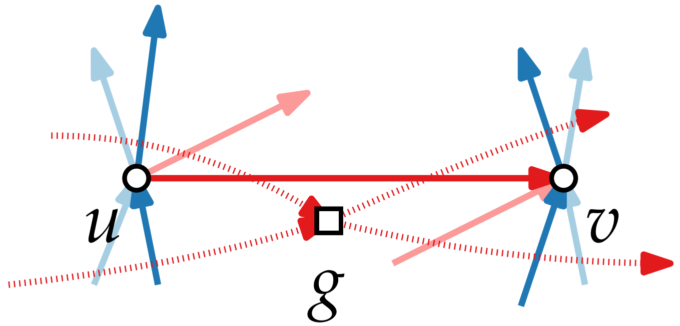


$$y_1(v) = f_{\text{hor}}(a) \leq y_1(u) = f_{\text{hor}}(b)$$
$$< y_2(v) = f_{\text{hor}}(c) \leq y_2(u) = f_{\text{hor}}(d)$$

- If path from u to v in red at least two edges long, then $x_2(u) < x_1(v)$.
- No two boxes overlap.

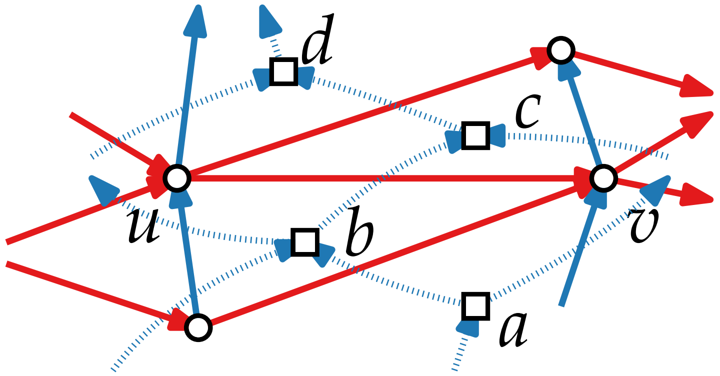
Correctness of Algorithm (Sketch)

- If edge (u, v) exists, then $x_2(u) = x_1(v)$



$$x_2(u) = f_{\mathrm{ver}}(g) = x_1(v)$$

- and the vertical segments of their rectangles overlap



$$y_1(v) = f_{\mathrm{hor}}(a) \leq y_1(u) = f_{\mathrm{hor}}(b)$$
$$< y_2(v) = f_{\mathrm{hor}}(c) \leq y_2(u) = f_{\mathrm{hor}}(d)$$

- If path from u to v in red at least two edges long, then $x_2(u) < x_1(v)$.
- No two boxes overlap.
- for details see He's paper [He '93]

Rectangular Dual Result

Theorem.
Every PTP graph G has a rectangular dual, which can be computed in linear time.

Rectangular Dual Result

Theorem.
Every PTP graph G has a rectangular dual, which can be computed in linear time.

Proof.

- Compute a planar embedding of G.

Rectangular Dual Result

Theorem.
Every PTP graph G has a rectangular dual, which can be computed in linear time.

Proof.

- Compute a planar embedding of G.
- Compute a refined canonical ordering of G.

Rectangular Dual Result

Theorem.
Every PTP graph G has a rectangular dual, which can be computed in linear time.

Proof.

- Compute a planar embedding of G.
- Compute a refined canonical ordering of G.
- Traverse the graph and color the edges.

Rectangular Dual Result

Theorem.
Every PTP graph G has a rectangular dual, which can be computed in linear time.

Proof.

- Compute a planar embedding of G.
- Compute a refined canonical ordering of G.
- Traverse the graph and color the edges.
- Construct G_{ver} and G_{hor}.

Rectangular Dual Result

Theorem.
Every PTP graph G has a rectangular dual, which can be computed in linear time.

Proof.

- Compute a planar embedding of G.
- Compute a refined canonical ordering of G.
- Traverse the graph and color the edges.
- Construct G_{ver} and G_{hor}.
- Construct their duals $G^{\star}_{\text{ver}}$ and $G^{\star}_{\text{hor}}$.

Rectangular Dual Result

Theorem.
Every PTP graph G has a rectangular dual, which can be computed in linear time.

Proof.

- Compute a planar embedding of G.
- Compute a refined canonical ordering of G.
- Traverse the graph and color the edges.
- Construct G_{ver} and G_{hor}.
- Construct their duals $G^{\star}_{\text{ver}}$ and $G^{\star}_{\text{hor}}$.
- Compute a topological ordering for vertices of $G^{\star}_{\text{ver}}$ and $G^{\star}_{\text{hor}}$.

Rectangular Dual Result

Theorem.
Every PTP graph G has a rectangular dual, which can be computed in linear time.

Proof.

- Compute a planar embedding of G.
- Compute a refined canonical ordering of G.
- Traverse the graph and color the edges.
- Construct G_{ver} and G_{hor}.
- Construct their duals $G^{\star}_{\text{ver}}$ and $G^{\star}_{\text{hor}}$.
- Compute a topological ordering for vertices of $G^{\star}_{\text{ver}}$ and $G^{\star}_{\text{hor}}$.
- Assing coordinates to the rectangles representing vertices.

Discussion

- A layout is **area-universal** if any assignment of areas to rectangles can be realized by a combinatorially equivalent rectangular layout.

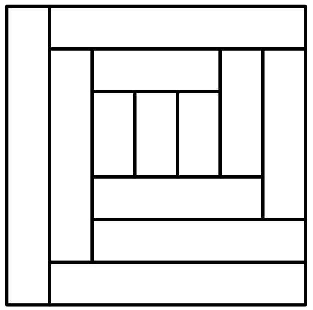
Discussion

- A layout is **area-universal** if any assignment of areas to rectangles can be realized by a combinatorially equivalent rectangular layout.
- A rectangular layout is **area-universal** if and only if it is **one-sided**. [Eppstein et al. SIAM J. Comp. 2012]

Discussion

- A layout is **area-universal** if any assignment of areas to rectangles can be realized by a combinatorially equivalent rectangular layout.
- A rectangular layout is **area-universal** if and only if it is **one-sided**.
  [Eppstein et al. SIAM J. Comp. 2012]

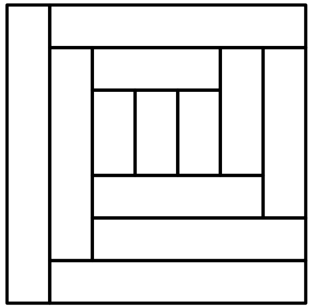
one-sided

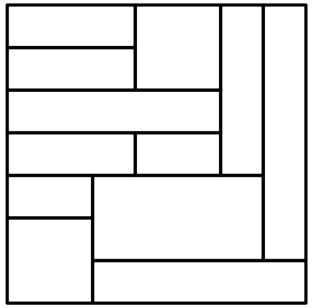


Discussion

- A layout is **area-universal** if any assignment of areas to rectangles can be realized by a combinatorially equivalent rectangular layout.
- A rectangular layout is **area-universal** if and only if it is **one-sided**.
  [Eppstein et al. SIAM J. Comp. 2012]

one-sided



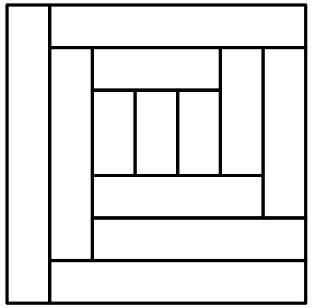


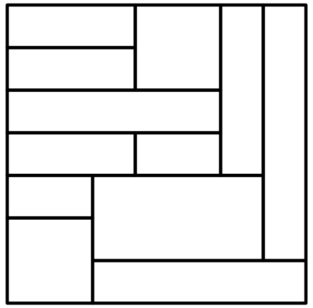
not one-sided

Discussion

- A layout is **area-universal** if any assignment of areas to rectangles can be realized by a combinatorially equivalent rectangular layout.
- A rectangular layout is **area-universal** if and only if it is **one-sided**. [Eppstein et al. SIAM J. Comp. 2012]

one-sided





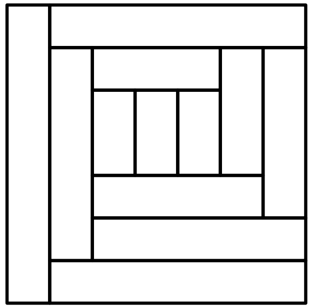
not one-sided

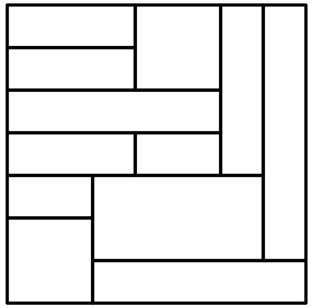
- Area-universal **rectlinear** representation: possible for all planar graphs
- [Alam et al. 2013]: 8 sides (matches the lower bound)

Discussion

- A layout is **area-universal** if any assignment of areas to rectangles can be realized by a combinatorially equivalent rectangular layout.
- A rectangular layout is **area-universal** if and only if it is **one-sided**. [Eppstein et al. SIAM J. Comp. 2012]

one-sided





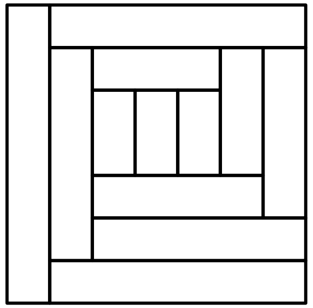
not one-sided

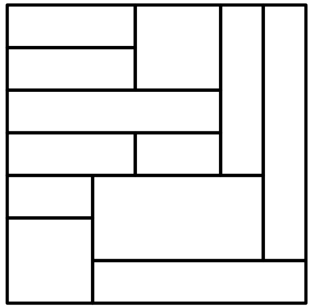
- Area-universal **rectlinear** representation: possible for all planar graphs
- [Alam et al. 2013]: 8 sides (matches the lower bound)

Discussion

- A layout is **area-universal** if any assignment of areas to rectangles can be realized by a combinatorially equivalent rectangular layout.
- A rectangular layout is **area-universal** if and only if it is **one-sided**. [Eppstein et al. SIAM J. Comp. 2012]

one-sided





not one-sided

- Area-universal **rectlinear** representation: possible for all planar graphs
- [Alam et al. 2013]: 8 sides (matches the lower bound)