Algorithmik
Fachbereich IV – Informatik
Universität Trier

Trier, April 29, 2025

Prof. Dr. Stefan Näher

Exercise sheet 2

Visualization of Graphs

Exercise 1 – Characterizations of trees

Prove that the four characterizations of a tree G from the lecture are all equivalent:

(1) there is exactly one v-w-path between any $v, w \in V$

(2) G is cycle-free and connected

(3) G is cycle-free and $m = n - 1$

(4) G is connected and $m = n - 1$

Hint: You may use transitivity to show all equivalencies, e.g., it is sufficient to show that (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (1). **8 Points**

Exercise 2 – Binary trees with inorder coordinates

Let $G = (V, E)$ be a binary tree with root r. For each $v \in V$, let $x(v) := \mathrm{preorder}(v)$ and $y(v) := \mathrm{depth}(v)$. Prove that this coordinate assignment yields a planar drawing of G.

Hint: Use induction on n. **4 Points**

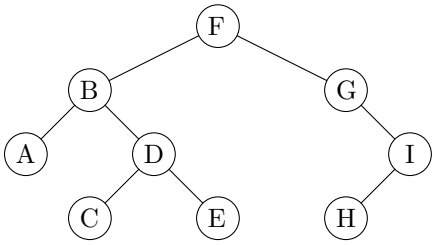
Exercise 3 – Binary trees with pre- and postorder coordinates

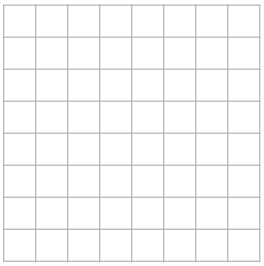
Let $G = (V, E)$ be a binary tree with root r. For each $v \in V$, let $x(v) := \mathrm{preorder}(v)$ and $y(v) := \mathrm{postorder}(v)$.

You may use the graph and grid below to try an example.

a) Prove that this coordinate assignment yields a planar drawing of G. **4 Points**

b) What is the area requirement of the generated drawing? Give tight bounds. **2 Points**





c) Prove that if you direct all edges of G such that they "point away" from r – that is, all vertices are reachable from r – then all arcs in the drawing point downwards and rightwards. **2 Points**

This assignment is due at the beginning of the next lecture, that is, on April 20 at 08:30. Please hand in your solutions online via Moodle. The exercises on this assignment will be discussed in the first tutorial session after April 20.