Algorithmik
Fachbereich IV – Informatik
Universität Trier

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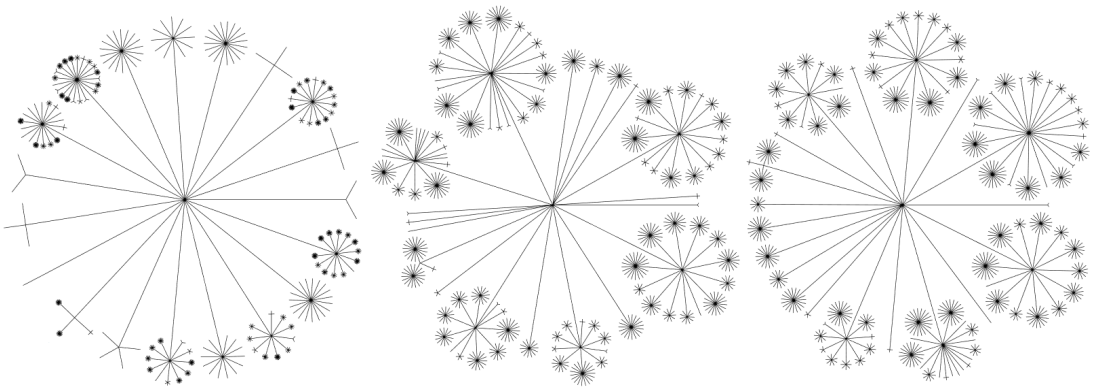
Exercise sheet 3

Visualization of Graphs

Exercise 1 – Drawing conventions & aesthetics of balloon layouts

The three drawings of the same tree below are drawn with a so-called *balloon layout*. Try to find at least two common drawing conventions and two possible drawing aesthetics for this layout style. **2 Points**



Exercise 2 – Space-saving layered drawings of trees

Consider the layered drawing style without the drawing conventions that the parent should be centered above its children and that isomorphic subtrees should have identical drawings.

a) Describe an algorithm that creates such a drawing of a tree with *minimum* area. **4 Points**

b) Are there binary trees that still require $\Omega(n^2)$ area in this drawing style? **2 Points**

Exercise 3 – Space-saving HV-drawings of complete binary trees

Let T be a *complete binary tree* of height h, that is, a binary tree where all vertices of depth $1, \ldots, h-1$ have exactly 2 children and all vertices of depth h are leaves. Consider the following HV-drawing algorithm.

Algorithm 1: BalancedHVDraw(node v, depth d)

if $v ==$ nil **then return** $\emptyset$
$v_l, v_r \leftarrow$ left / right child of v
$\Gamma_1 \leftarrow$ BalancedHVDraw(v_l, $d+1$)
$\Gamma_2 \leftarrow$ BalancedHVDraw(v_r, $d+1$)
if d odd **then return** horizontal combination of Γ_1 and Γ_2
if d even **then return** vertical combination of Γ_1 and Γ_2

a) How many vertices has T? **2 Points**

b) Prove that the right-heavy HV-layout algorithm from the lecture produces a drawing of T with area $\Omega(n \log n)$. **2 Points**

c) Prove that BalancedHVDraw produces a drawing of T with area $O(n)$. **6 Points**

d) What is the aspect ratio (i.e., the ratio between the width and the height) of the generated drawing in the worst case? **2 Points**

$$\Rightarrow W_1 \leq 2^k W_h + 2^k - 1 = 2^k + 2^k - 1 = 2^{k+1} - 1, H_1 = 2^k H_h + 2^k - 1 = 2^k + 2^k - 1 = 2^{k+1} - 1$$
$$\Rightarrow W_1, H_1 \in O(2^k) = O(2^h) = O(2^{\log n}) = O(n)$$