Algorithmik
Fachbereich IV – Informatik
Universität Trier

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Exercise sheet 4

Visualization of Graphs

Exercise 1 – Unit edge lengths

In a drawing of a graph G with *unit edge lengths* each edge is drawn as a line segment of length 1.

a) Prove or disprove that all trees admit a *crossing-free* drawing with unit edge lengths. **2 Points**

We now go one step further and consider drawings with unit edge lengths of G where the Euclidean distance between two vertices u and v is equal to the distance of u and v in G (i.e. equal to the number of edges of a shortest path from u to v).

b) Characterize the set of connected graphs that can be drawn in this way. **2 Points**

Exercise 2 – Adapting forces for positioning

In the force-directed approach, we may add additional forces to all or some vertices. Describe functions for forces that are suitable to

a) keep a vertex v close to a specified position,

b) position a vertex u close to the x-axis,

c) align an edge $\{a, b\}$ parallel to the y-axis (approximately),

d) draw directed edges upward.

6 Points

Exercise 3 – Adapting forces for vertices with area > 0

The force-directed methods introduced in the lecture assume that all vertices are represented as points, i.e., disks with radius 0. Which modifications are necessary to represent vertices as disks? **3 Points**

Hint: Consider a physical analogy again first.

Exercise 4 – Tutte Drawings

Prove the following properties for Tutte drawings.

a) If G is connected, then a Tutte drawing can have vertex overlaps. **1 Point**

b) If G is 2-connected, then a Tutte drawing can have vertex overlaps.

   Hint: Try to find small examples. You don't need many vertices! What is the smallest example you can find? **2 Points**

c) In the literature, the Tutte forces are often described without dividing by the degree of the vertex:

$$f_{\text{attr}}(u, v) = \begin{cases} 0 & u \text{ fixed} \\ ||p_u - p_v|| & \text{else} \end{cases}$$

   Find an example where iteratively applying these forces does not find the equilibrium. Does that mean that no equilibrium exists?

   Hint: Find a situation where a vertex "shoots" too far over the optimum position.

   4 Points