Algorithmik
Fachbereich IV – Informatik
Universität Trier

Trier, June 3, 2025

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Exercise sheet 6

Visualization of Graphs

Exercise 1 – Higher degree vertices in orthogonal layouts

Let $G = (V, E)$ be an arbitrary graph with faces F and outer face f_0. Our goal is to draw G orthogonally, while preserving the embedding, such that all vertices of degree greater than 4 are represented by rectangles instead of points. To achieve this, we replace every vertex v having $\deg(v) > 4$ by a ring of vertices $v_1, \ldots, v_{\deg(v)}$, such that the edges incident to v are distributed among the vertices $v_1, \ldots, v_{\deg v}$ (see figure below). The embedding $\mathcal{E}$ is modified accordingly during this step. Let G' with faces F' and outer face f'_0 be the result of this replacement step.

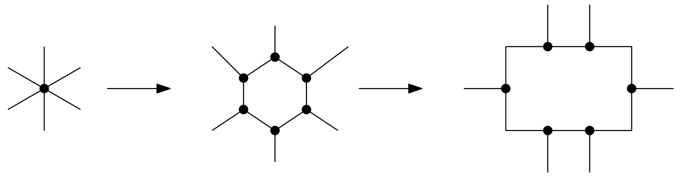

Figure 1: Replacement of a vertex of higher degree by a ring, which shall be represented by a rectangle in the orthogonal drawing.

Modify the flow network by Tamassia such that it provides a bend-minimal orthogonal description of (G', F', f'_0) in which every ring representing a vertex of higher degree is a rectangle such that no vertices are placed in any of its four corners and such that every side of the rectangle contains at least one vertex. **7 Points**

Hint: Consider the set V' of the new vertices and the set E' of the new edges of a ring that are added to the graph after the modification. Think of the additional constraints to the flow model to enforce the structure of the ring and its vertices.

Exercise 2 – From flow to orthogonal representation

Let $G = (V, E)$ be a plane graph with faces F and outer face f_0. Let X be a flow of cost k in the corresponding flow network $N(G)$. We consider the orthogonal description $H(G)$ belonging to X (as in the lecture).

Show that $H(G)$ fulfills property (H3) on the angle sum of the faces for orthogonal descriptions, that is, argue that

$$\sum_{r \in H(f)} C(r) = \begin{cases} -4 & \text{if } f = f_0 \\ +4 & \text{otherwise.} \end{cases}$$

holds for every face f.

4 Points

Exercise 3 – Edge bending left and right in orthogonal representation

Let $G = (V, E)$ be a plane graph with faces F and outer face f_0 and let $H(G)$ be a bend-minimal orthogonal representation of (G, F, f_0). Is it possible that there exists an edge such that, in $H(G)$, it bends to the right as well as to the left?

Prove this claim (by giving an example) or disprove it (by showing that such an edge cannot exist). **3 Points**