

## Exercise sheet 8

### Visualization of Graphs

#### Exercise 1 – Minimum Feedback (Arc) Set

Let  $G = (V, E)$  be a directed graph. For a set of edges  $E' \subseteq E$ , let  $E'_r := \{vu \mid uv \in E'\}$  be the set of reversed edges. A set of edges  $E^* \subseteq E$  with minimal cardinality

- such that  $G - E^*$  is acyclic is called a **MINIMUM FEEDBACK ARC SET**, and
- such that  $G - E^* + E_r^*$  is acyclic is called a **MINIMUM FEEDBACK SET**.

Show that  $E^* \subseteq E$  is a **MINIMUM FEEDBACK SET** if and only if  $E^*$  is a **MINIMUM FEEDBACK ARC SET**. **6 Points**

*Hint:* Assume that there is a cycle in  $G - E^* + E_r^*$  that contains an edge  $e \in E_r^*$ . Then find a cycle in  $G - E^*$ .

#### Exercise 2 – Optimal One-Sided Crossing Minimization

We consider the problem of one-sided crossing minimization, i.e., we are given a bipartite graph  $G = (L_1 \cup L_2, E)$  with a permutation  $\pi_1$  of  $L_1$  and we search for a permutation  $\pi_2$  of  $L_2$  that minimizes the number of crossings.

Suppose there exists a permutation  $\pi_2^*$  of  $L_2$  such that no edges cross.

- Show that in this case the *barycenter heuristic* also yields a permutation  $\pi_2'$  that results in no crossings. **3 Points**
- Show that in this case the *median heuristic* also yields a permutation  $\pi_2''$  that results in no crossings. **3 Points**

#### Exercise 3 – Upward Planar Drawings

By iteratively applying the heuristics from the lecture, does one always find a crossing-free drawing of a graph (with more than two layers, where necessary) if the graph is upward planar? Justify your answer. **2 Points**