Algorithmik
Fachbereich IV – Informatik
Universität Trier

Trier, Juli, 2025

Prof. Dr. Stefan Näher

Exercise sheet 9

Visualization of Graphs

Exercise 1 – Fast Construction of Schnyder Realizer via Contraction

In the lecture, we proved that every triangulated plane graph $G = (V, E)$ has a Schnyder realizer or a respective Schnyder labeling. The proof yields a recursive algorithm: contract an edge ax, find recursively a Schnyder realizer in the resulting graph, and then add x back consistently. A naive implementation of this algorithm yields a running time of $O(n^2)$ because we need to find the contracted edge.

Explain how the algorithm can be improved to get linear runtime. **7 Points**

Hint: Maintain a list of candidate edges for the contraction. How can the list be updated quickly during the algorithm?

Exercise 2 – Fast Calculation of Barycentric Coordinates

Let $G = (V, E)$ be a triangulated plane graph with a Schnyder realizer T_1, T_2, T_3. As in the lecture, for each inner vertex v, let $|R_i(v)|$ be the number of faces in the region R_i with respect to v. Let $v_i = |R_i(v)|$.

Show that the values v_i can be calculated for all inner vertices v at once with a total runtime of $O(n)$.

Hint: For the weak barycentric coordinates, we computed $|V(R_i(v))|$ and $|P_i(v)|$ in $O(n)$ time. How can we use this information to compute $R_i(v)$? **6 Points**