

Exercise sheet 8

Visualization of Graphs

Exercise 1 – Minimum Feedback (Arc) Set

Let $G = (V, E)$ be a directed graph. For a set of edges $E' \subseteq E$, let $E'_r := \{vu \mid uv \in E'\}$ be the set of reversed edges. A set of edges $E^* \subseteq E$ with minimal cardinality

- such that $G - E^*$ is acyclic is called a **MINIMUM FEEDBACK ARC SET**, and
- such that $G - E^* + E_r^*$ is acyclic is called a **MINIMUM FEEDBACK SET**.

Show that $E^* \subseteq E$ is a **MINIMUM FEEDBACK SET** if and only if E^* is a **MINIMUM FEEDBACK ARC SET**. **6 Points**

Hint: Assume that there is a cycle in $G - E^* + E_r^*$ that contains an edge $e \in E_r^*$. Then find a cycle in $G - E^*$.

Exercise 2 – Optimal One-Sided Crossing Minimization

We consider the problem of one-sided crossing minimization, i.e., we are given a bipartite graph $G = (L_1 \cup L_2, E)$ with a permutation π_1 of L_1 and we search for a permutation π_2 of L_2 that minimizes the number of crossings.

Suppose there exists a permutation π_2^* of L_2 such that no edges cross.

- Show that in this case the *barycenter heuristic* also yields a permutation π_2' that results in no crossings. **3 Points**
- Show that in this case the *median heuristic* also yields a permutation π_2'' that results in no crossings. **3 Points**

Exercise 3 – Upward Planar Drawings

By iteratively applying the heuristics from the lecture, does one always find a crossing-free drawing of a graph (with more than two layers, where necessary) if the graph is upward planar? Justify your answer. **2 Points**